Planar Symmetry

The fascinating and hidden beauty
of planar symmetries.

Pamphlet 21

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In this pamphlet we shall only touch on the particular aspects of **planar symmetry** associated with **rectangular mats** having a **rectangular central hole**. It might well be a subject to which nobody has even bothered giving any attention to. Most likely such **mats** are considered too mundane for any form of serious investigative considerations apart from some rather more or less remotely trivial connections by Kronecker (1823-1891)†. We did spend some time on **regular rectangular mats** in *The Braider*, hence the reader be referred to that publication where we showed how they were very closely associated with **regular cylindrical braids**. We clearly showed in *The Braider* that they were much more complicated than **regular cylindrical braids**. Naturally it should be obvious to look at **regular rectangular mats** with a centrally placed **rectangular hole**. This might well at first sight appear not to offer a great deal of new territory to explore, but it will soon become clear that a whole new world opens up. Soon it will be discovered that the apparent regular flat braid (weave) consists of interwoven single string flat braids and that these single string components show an interesting variety of **symmetries** and furthermore that only restricted consistencies do exist. Soon it will be discovered that a **square regular mat** with a centrally placed **square hole**, the simplest of the various possibilities, requires n strings When the **outside frame** contains n_1 by n_1 bights and the **hole frame** contains n_2 by n_2 bights. The **hole frame** is **empty**, hence has no braid within its inside boundaries. Only the area between the **outside frame** and **hole frame** is totally covered by the braid. It will also soon be discovered that the value of $n = n_1 - n_2$. As we know the **square** mat without the **square hole** requires n_1 strings to fill it, and the **hole frame** would require n_2 strings for a braid to fill it. Sofar it seems all to be very simple, but don't get fooled by trying to predict the results when the **outside frame** has n_{oi}/n_{oo} bights and the **hole frame** has n_{hi}/n_{ho} bights.

There are at least two ways to carry our investigations further. An obvious one is by calculating the sequential braiding steps consisting of 'full' and 'part' half-cycles, the straight line segments between consecutive bights. This method is quite cumbersome to say the least. The alternative method is to use a suitable computer drawing program. Although that method requires a lot of time with its very large number of subsequent print-outs (for record keeping at least), it is nevertheless to be preferred. Very soon it should become clear that single string solutions do exist and also soon it will be discovered that we have entered the world of symmetries. In our earlier Pamphlets we have seen a litle glimps of symmetries as exhibited for example by the RKT.

A typical, but rather general example of the braids we intend to deal with is presented in Fig.1, where $n_{oi}/n_{oo} = 34/33$ and $n_{hi}/n_{ho} = 17/18$. It looks a simple and very regular braid. **How many strings are there required** is the obvious question and **what do the comprising component(s) look like??** It appears that nobody has ever wondered about those obvious questions. Too mundane?? Soon we will show how totally misplaced such an attitude is. The follow up question is: Is there a **consistent** pattern, or in other words a consistent mathematical formula which will give a definite

† String-polygons for flat braids, **without holes**, have been studied as paths of billiard balls on a table with reflecting boundaries, known as König-Szücs polygons, and the more closely related article '*On sequences of compound braids—Some properties and problems*' by A.G. Schaake, presented at the time by John Turner to the Fibonacci Association in mid. 1994.

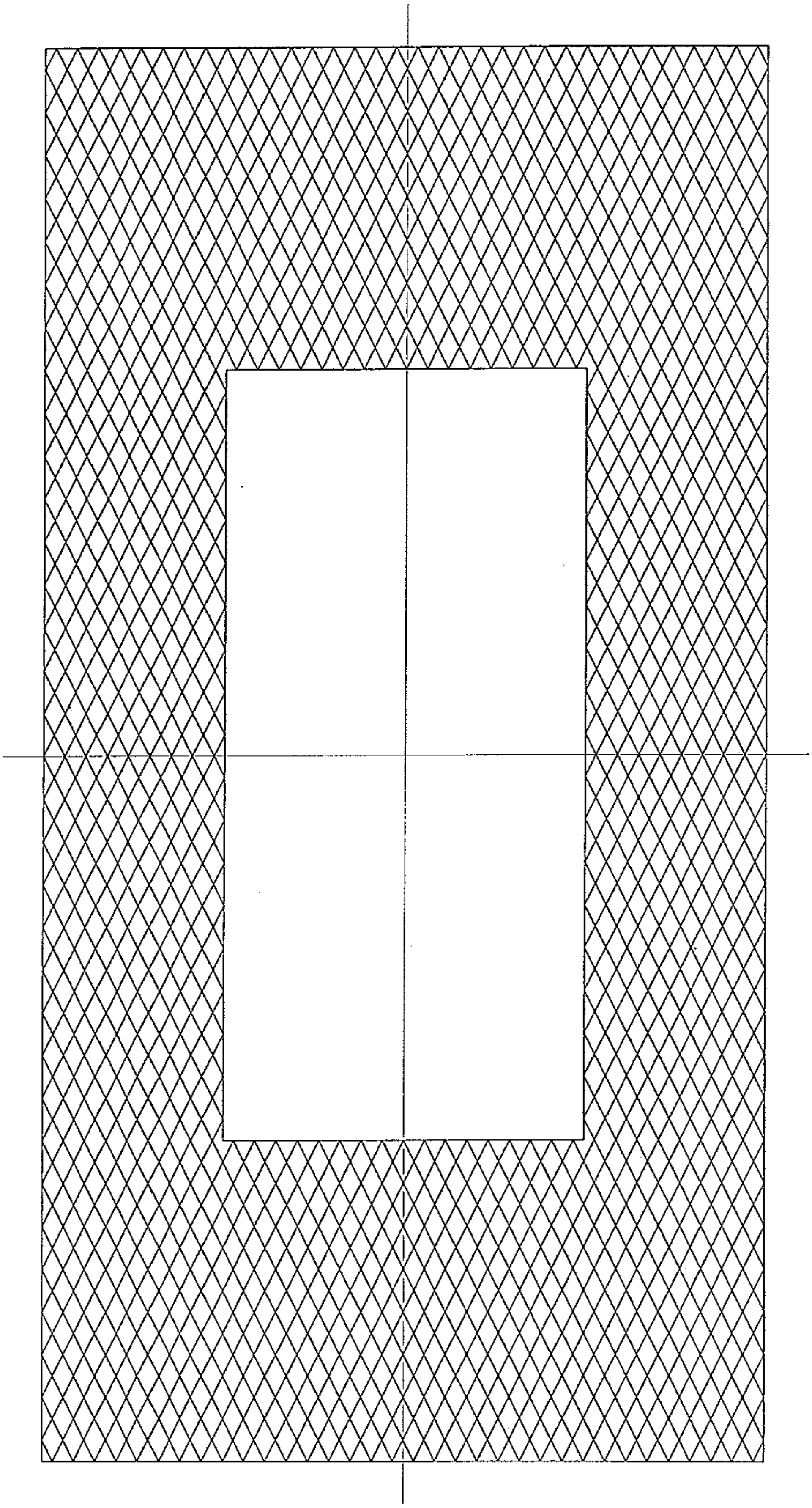


Fig. 1.

answer to the questions posed above??

Obviously the questions posed are simple and so is the braid, at least by the looks of it. It would only be fair to say that **nobody** could possibly be expected to give the correct answer! Of course, we could go a step further after telling an enquirer that this particular braid requires **three** single string components and then ask if any do have or do not have a symmetry. All simple questions looking for an answer. Such questions are surely the beginning of a more comprehensive and may be quite lengthy exploration. For the moment it might pay to let the reader wonder.

After some contemplation it should at least become clear that we understand virtually nothing even about the most simple and well presented phenomena in the Universe. Slowly it should become clear that **symmetry** is a universal and hence a govern, but in generally a well hidden property in the Universe.

The overall braid of a Regular rectangular braid with a central rectangular hole can be designated by $n_{oi}/n_{oo} \times n_{hi}/n_{ho}$. In particular the ones where $n_{oi} = n_{oo} = n_1$ and $n_{hi} = n_{ho} = n_2$ where as mentioned earlier the number of single string components is equal to $n_1 - n_2$, we have a very simple relationship indeed! It tells us, however, absolutely nothing about their apparently inherent internal properties. In fact, initially it might well appear that there is nothing of any interest to be noted other than at best some rather simple symmetry and even that might well be overlooked.

Let $n_{oi} = n_{oo} = n_1 = 17$ and $n_{hi} = n_{ho} = n_2 = 6$. Tis braid can thus be designated by $17/17 \times 6/6$. From the above we already know that this braid requires $17 - 6 = 11$ strings. Let's see what the eleven components look like. They are presented in Figs. 2 – 13. Fig. 2 shows the overall braid wih its eleven components, then follow the eleven diagrams of each component, hence the Figs. 3 – 13. The first component Fig. 3 is totally symmetric. The following four (Figs. 4 – 7) are also very simple ones. Not only has each component a symmetry with itself, but also the component of Fig. 4 has a symmetry with the component of Fig. 5 and the component of Fig. 6 has a symmetry with the component of Fig. 7. Furtheron the component of Fig. 8 has a symmetry with the component of Fig. 9, the component of Fig. 10 with the component of Fig. 11 and the component of Fig. 12 with the component of Fig. 13. The symmetries are slowly getting more complicated, an inherent but apparently a natural process.

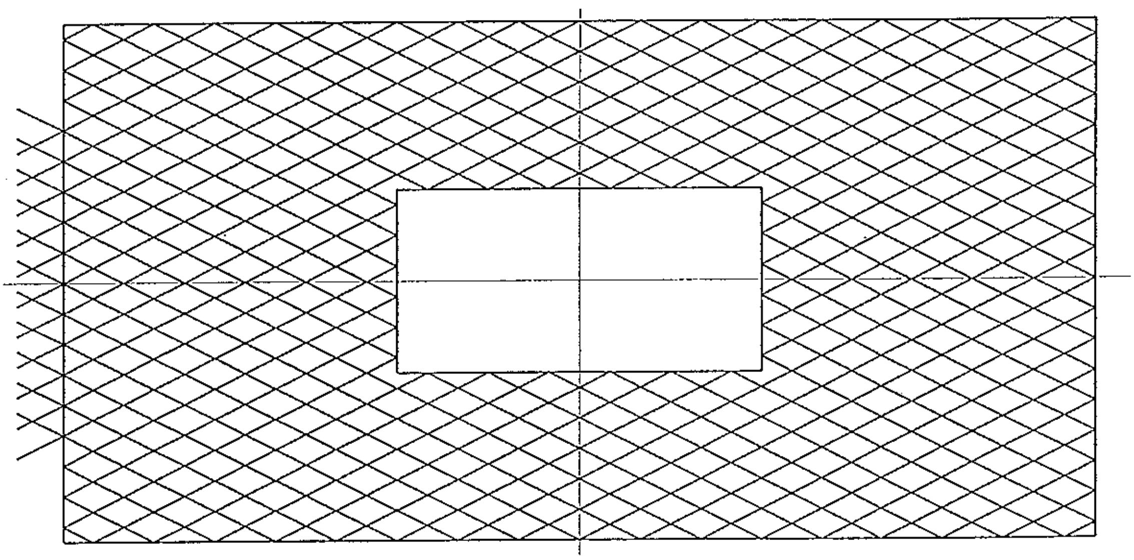
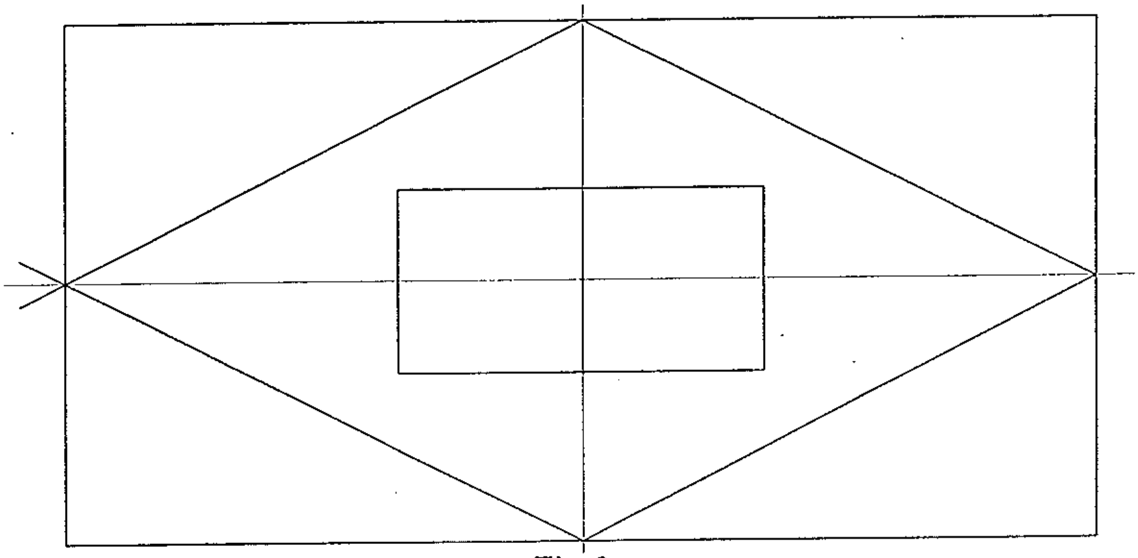


Fig. 2.



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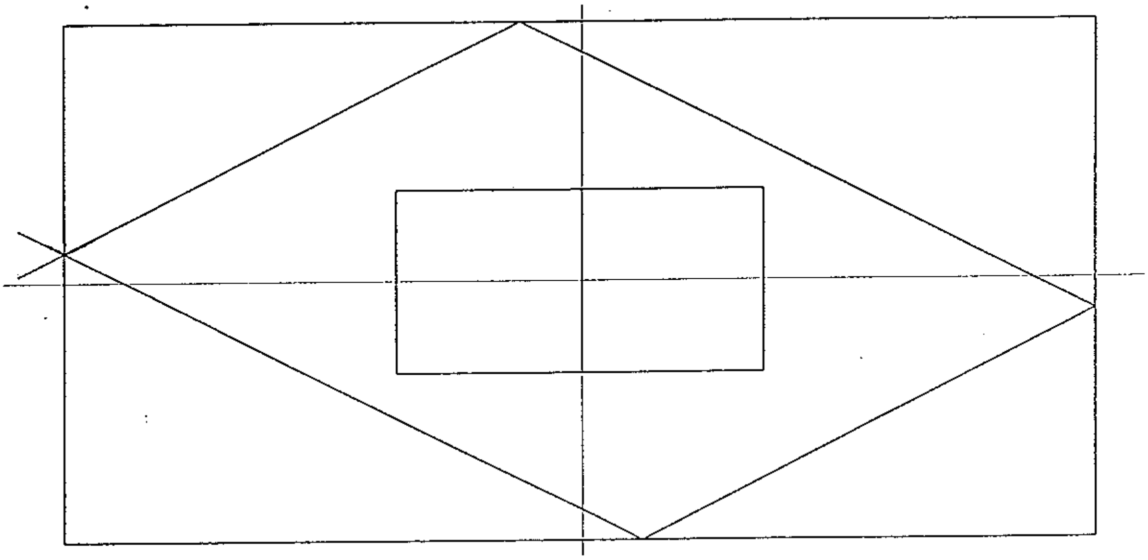

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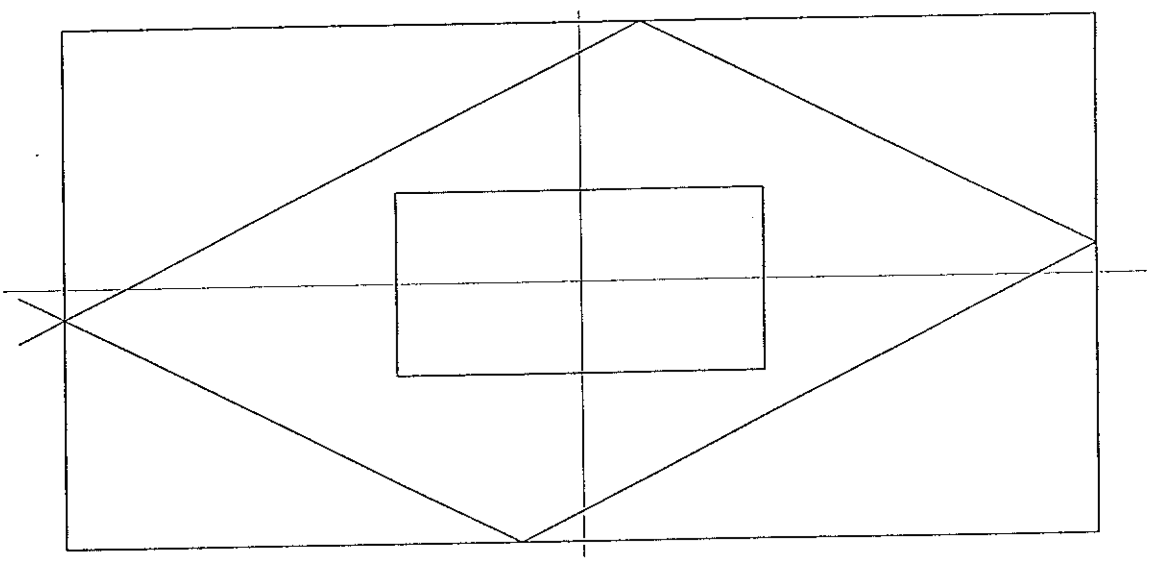

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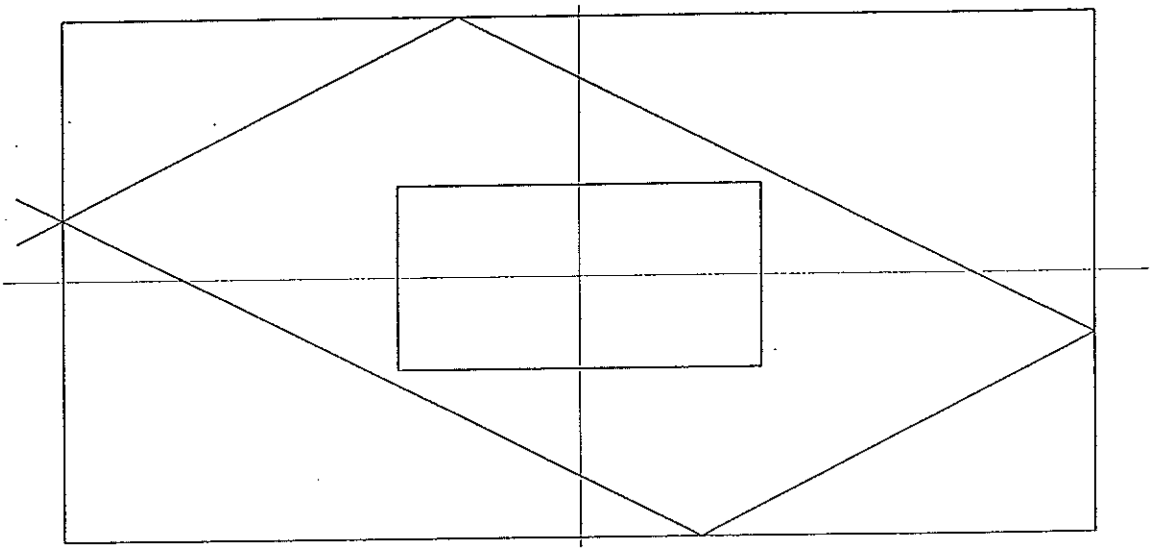

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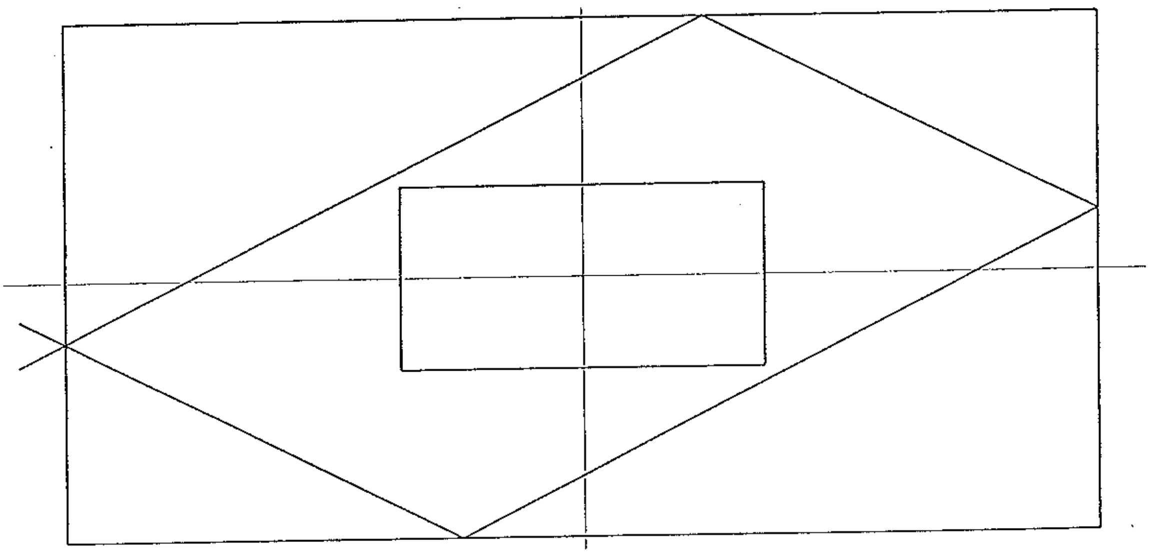

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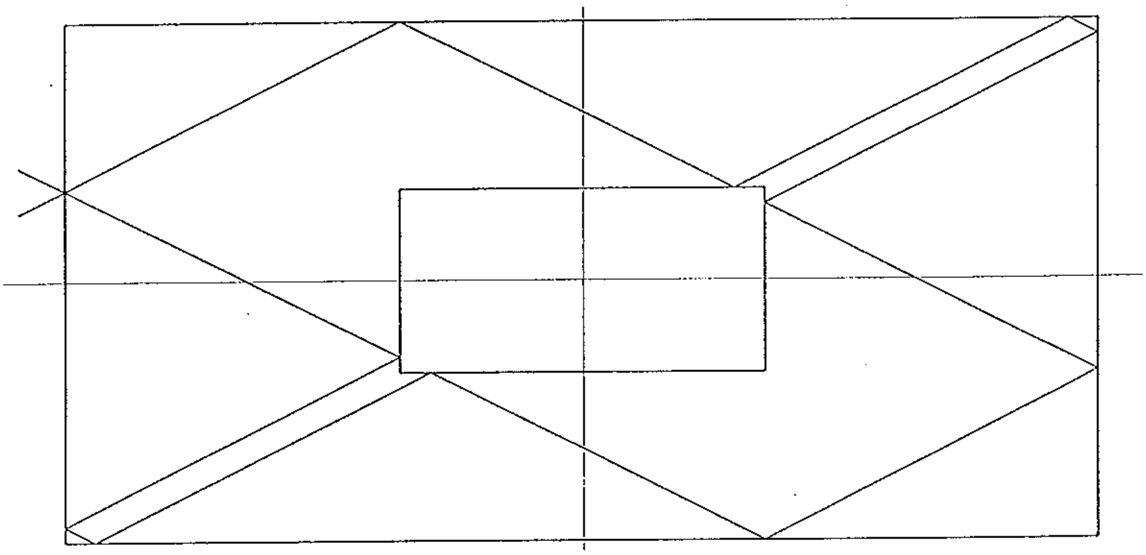

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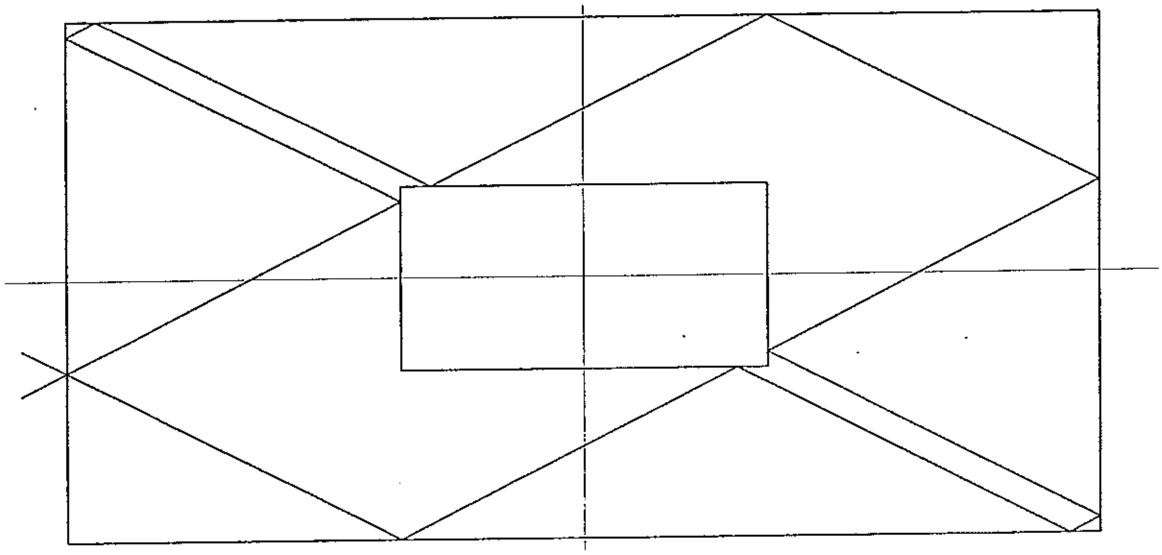

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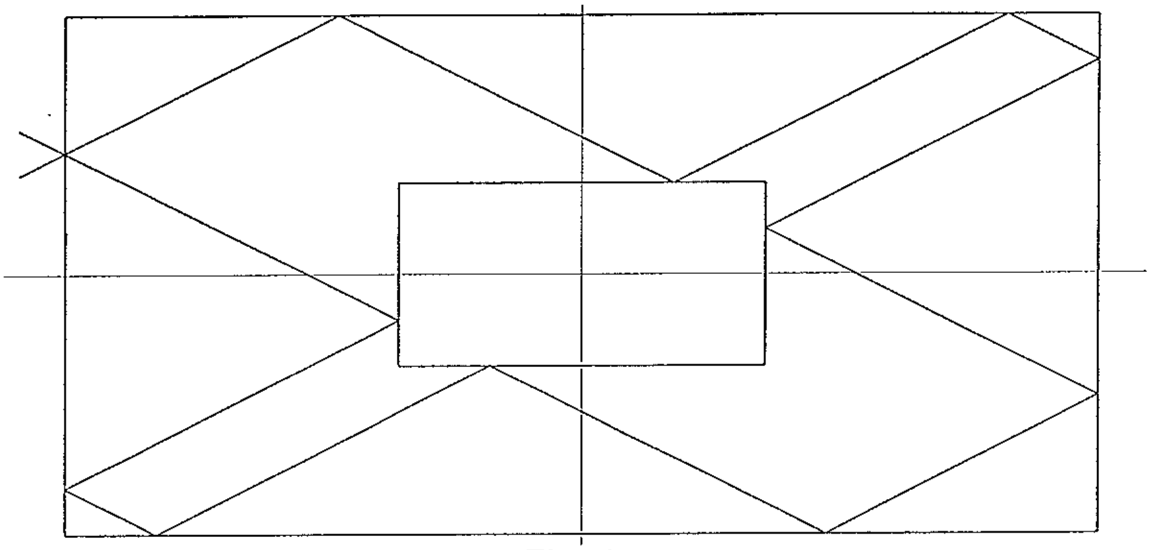

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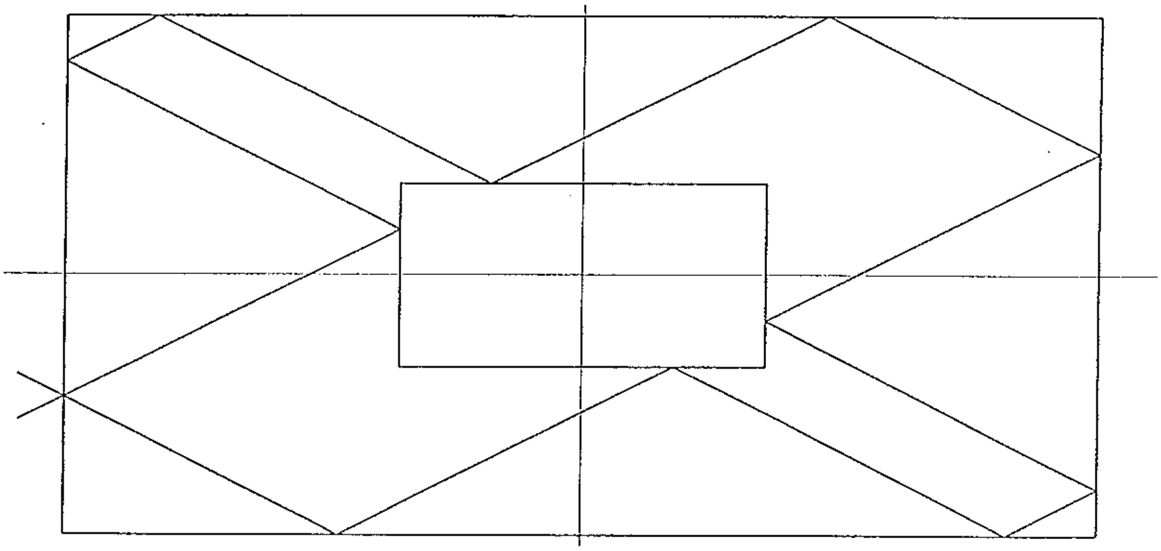

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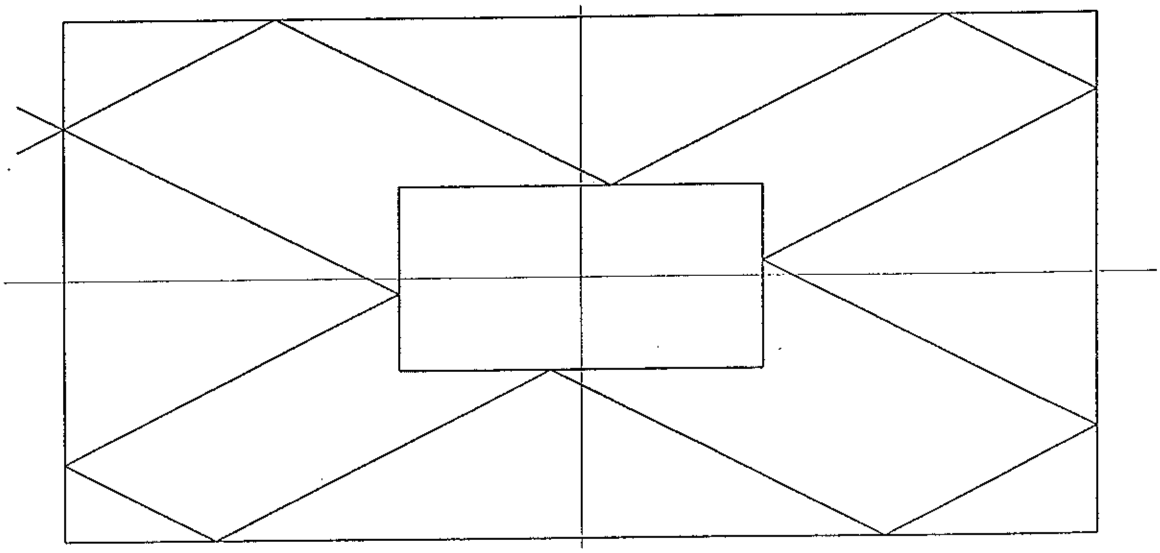

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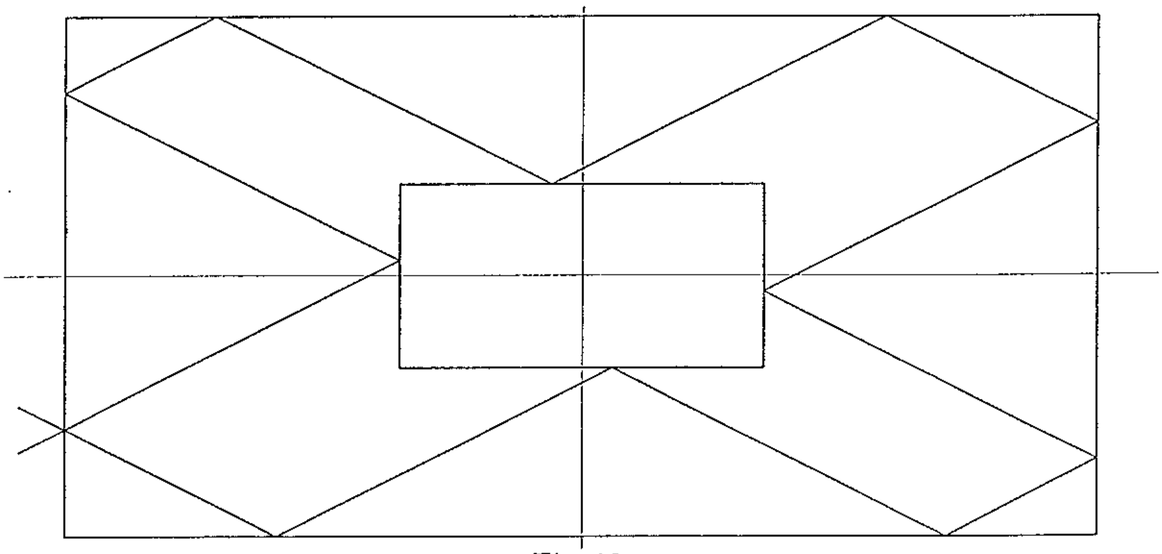
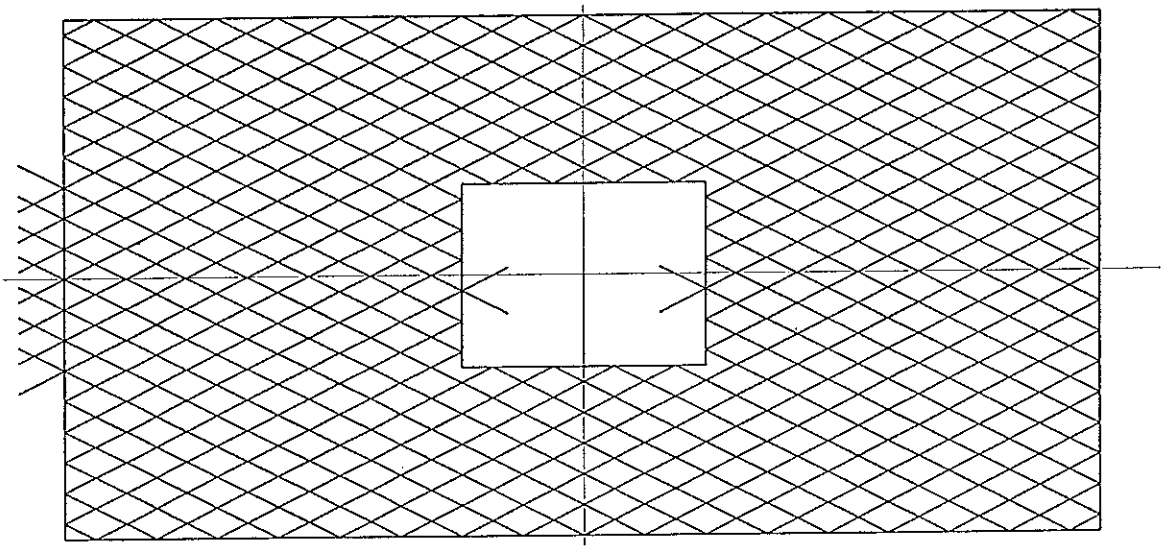


Fig. 13.

When we decrease the hole size from 6/6 to 6/4 while leaving the outside frame at 17/17, the initial seven components are similar to the initial five components of the $17/17 \times 6/6$ braid. The total number of components in the $17/17 \times 6/4$ braid decreases to nine components, hence a decrease of two components from the $17/17 \times 6/6$ braid to the $17/17 \times 6/4$ braid $(11 - 9 = 2)$. The last two components of the $17/17 \times 6/4$ braid have an identical symmetry.

By decreasing the hole further to 6/2, the braid $17/17 \times 6/2$ has eleven components, hence the number of components is rising again. The initial nine components are very simmilar to the initial seven components in the $17/17 \times 6/4$ braid and the initial five components of the $17/17 \times 6/6$ braid.

By decreasing the hole still further to 6/0, the braid $17/17 \times 6/0$ has seventeen components. The initial eleven components are very simmilar to the initial nine components in the $17/17 \times 6/2$ braid, the initial seven components in the $17/17 \times 6/4$ braid and, the initial five components in the $17/17 \times 6/6$ braid. Then there are a further six somewhat similar components which gives us the total of seventeen components.



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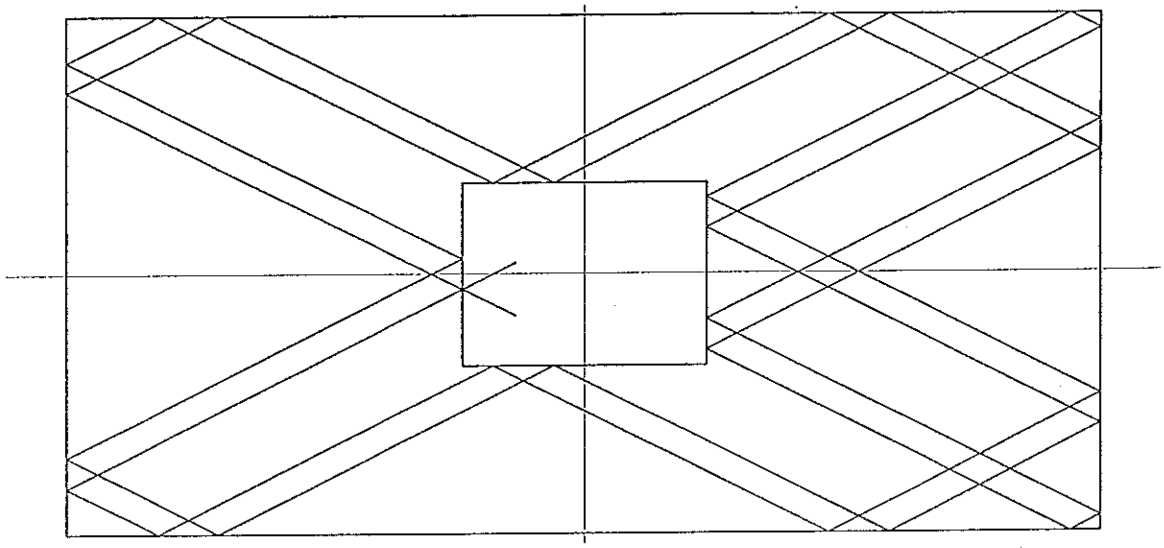

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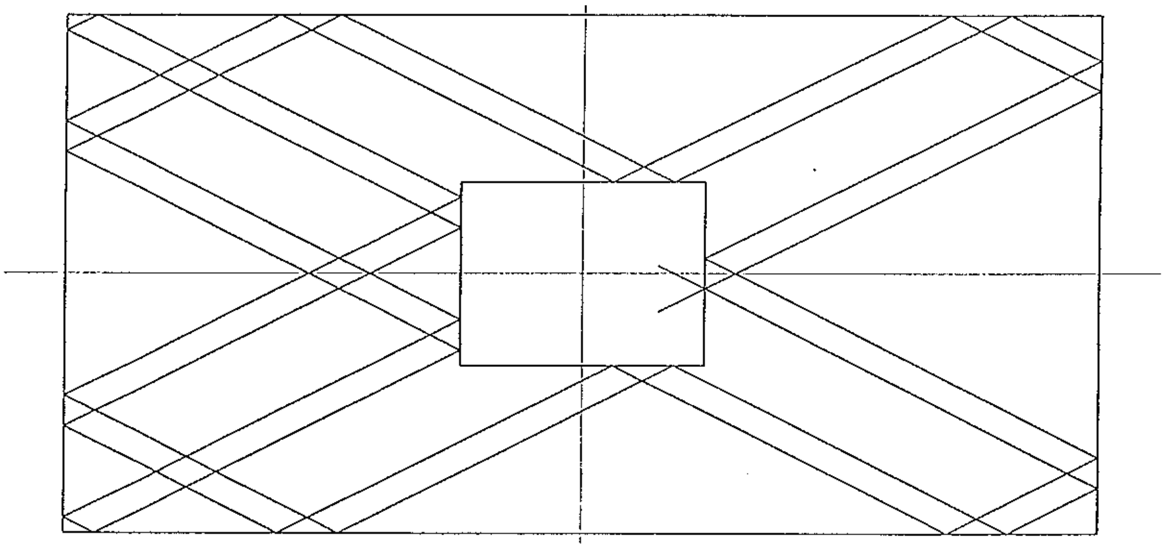

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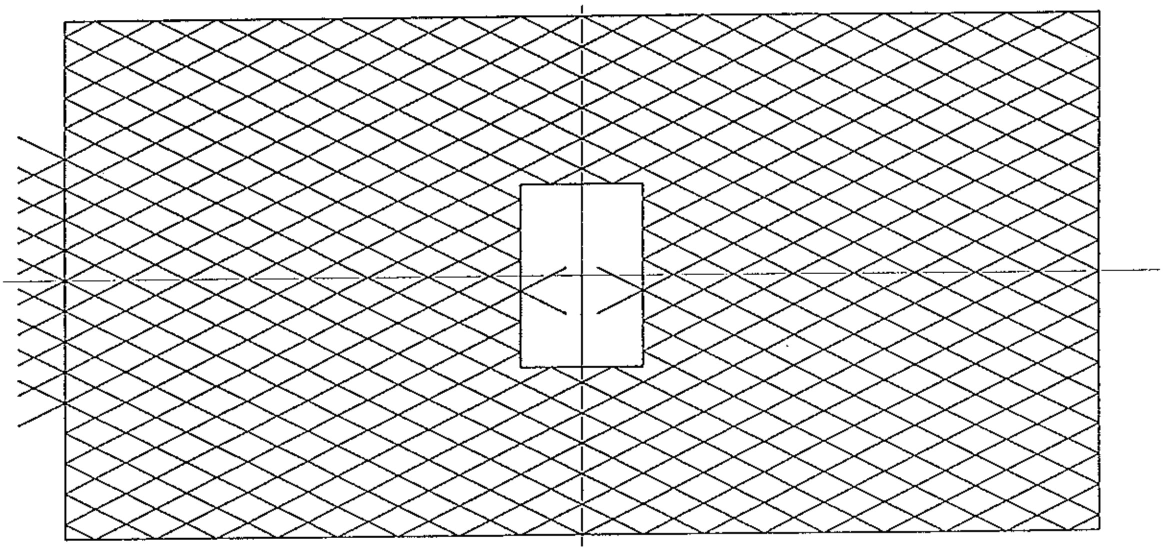

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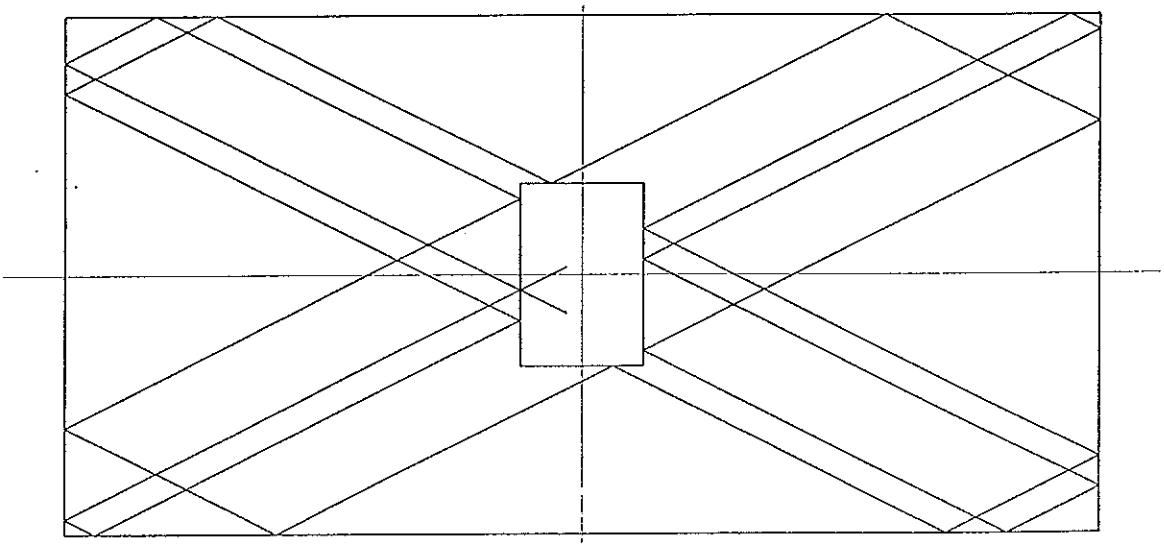

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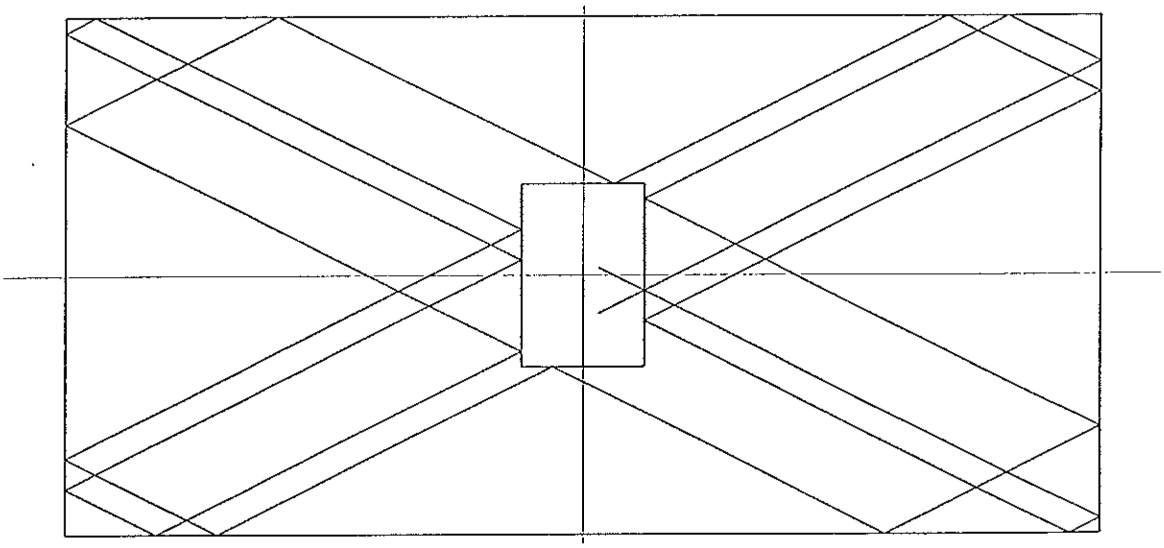

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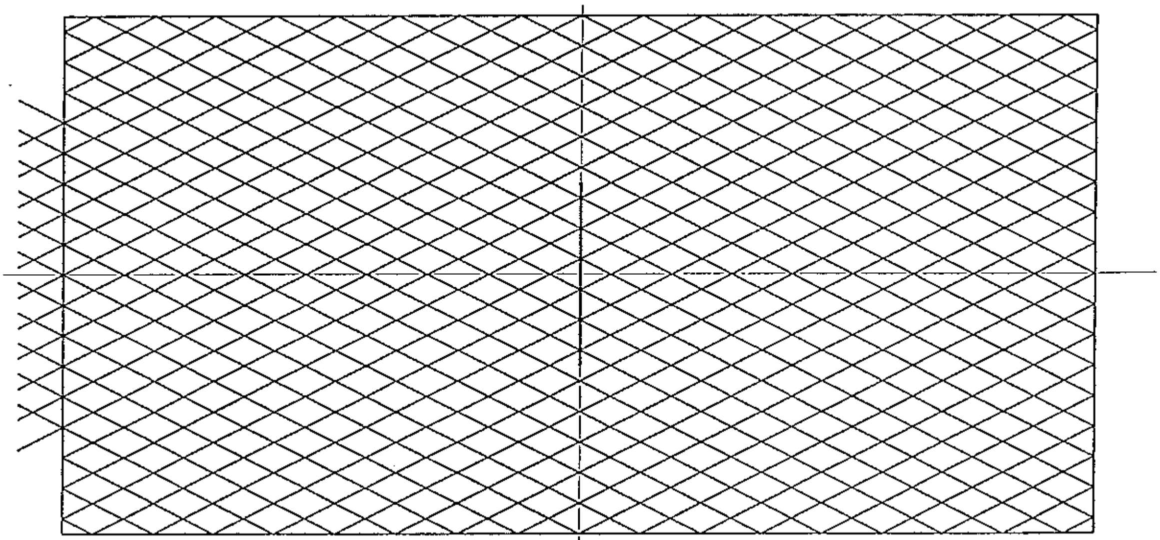

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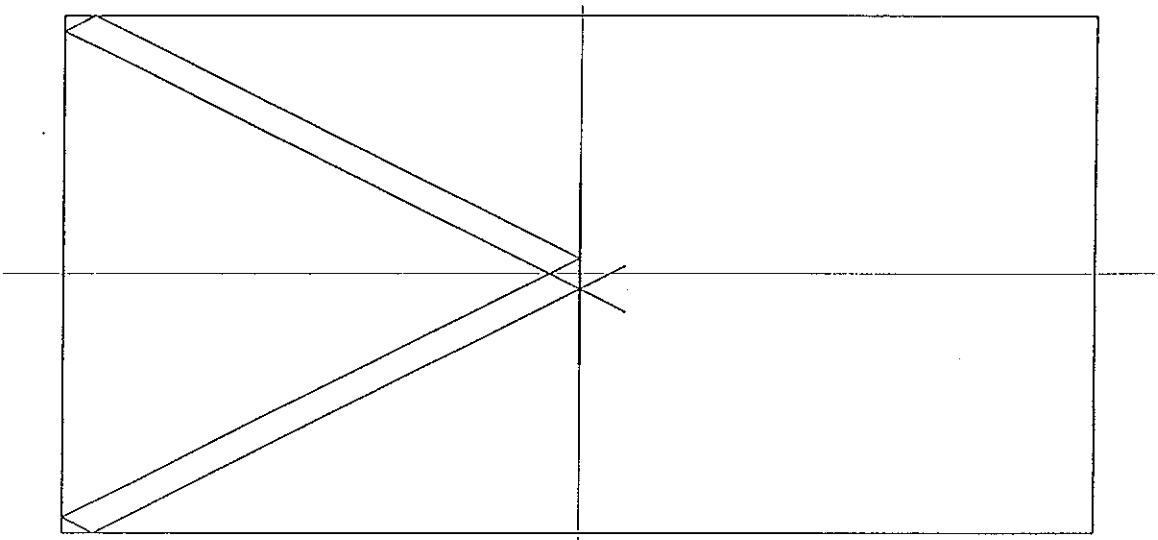

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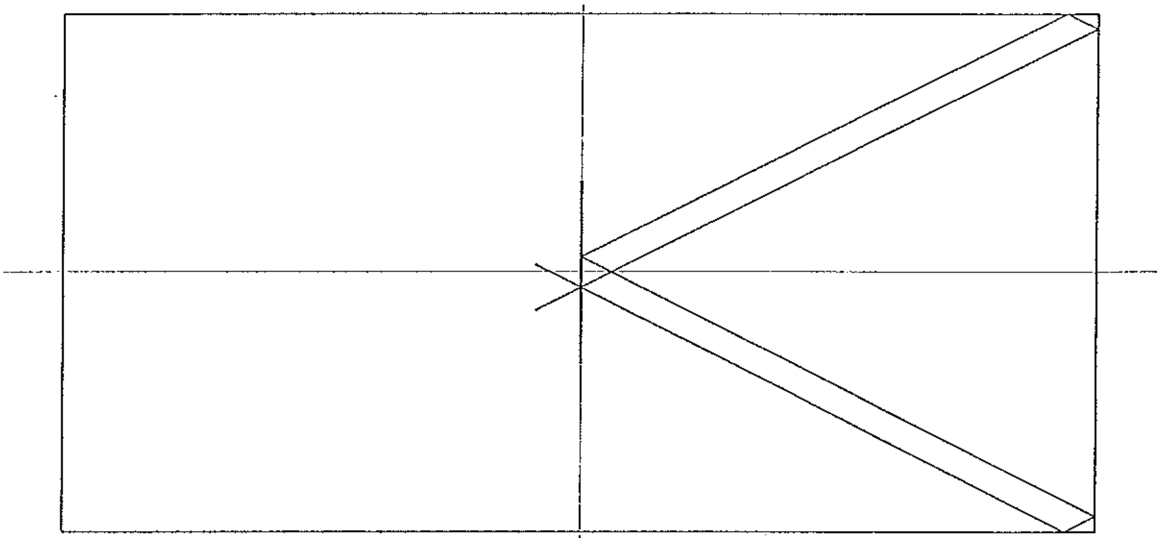

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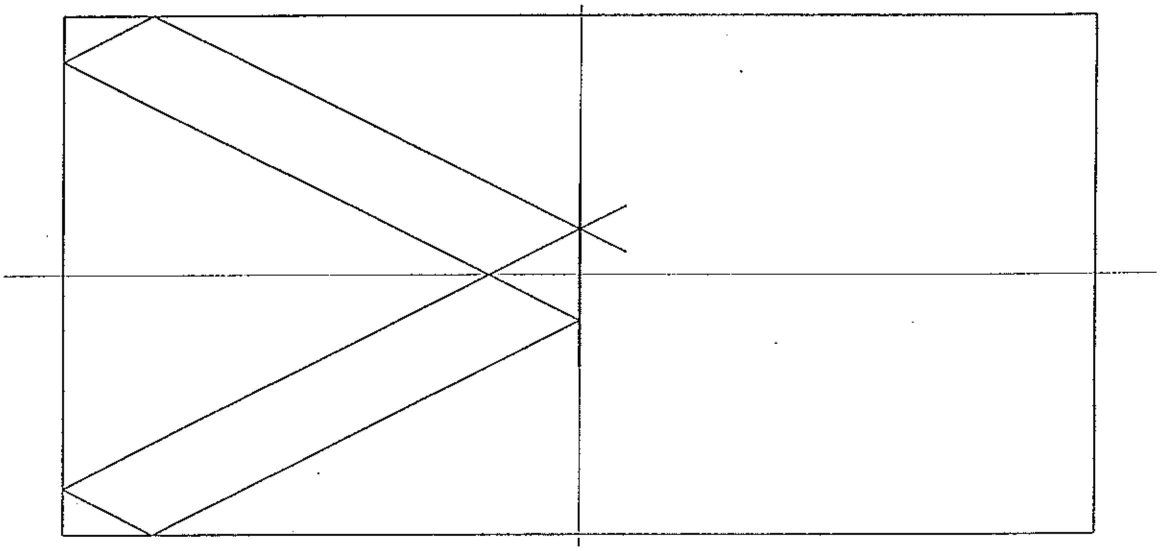

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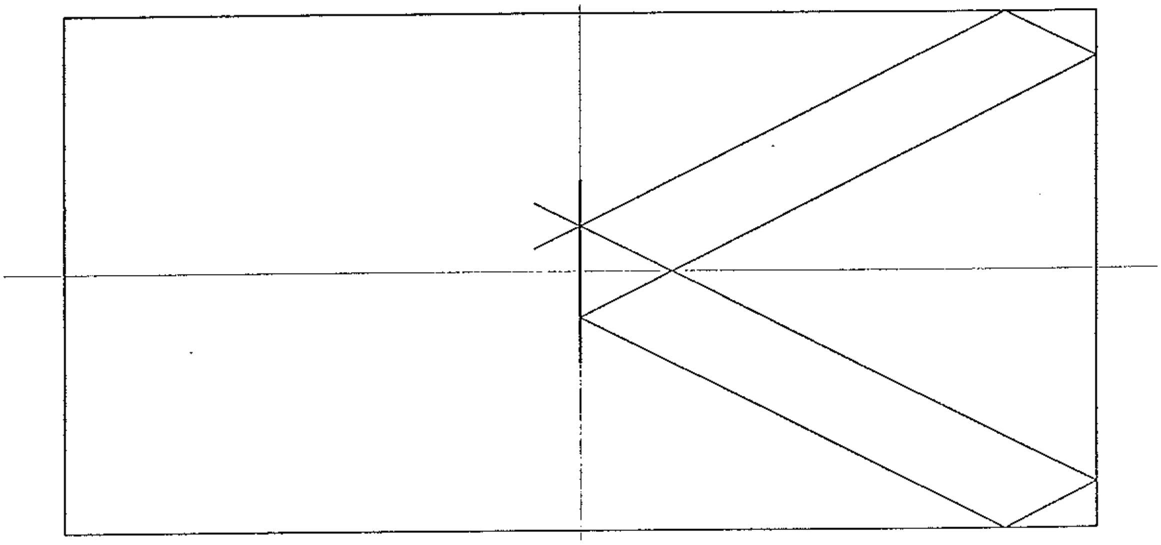

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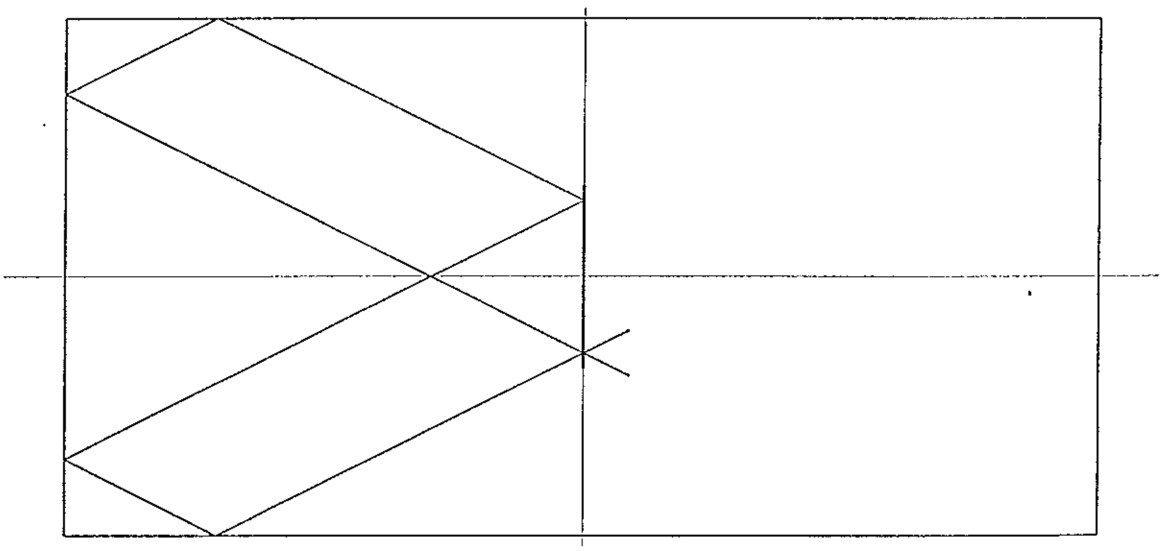

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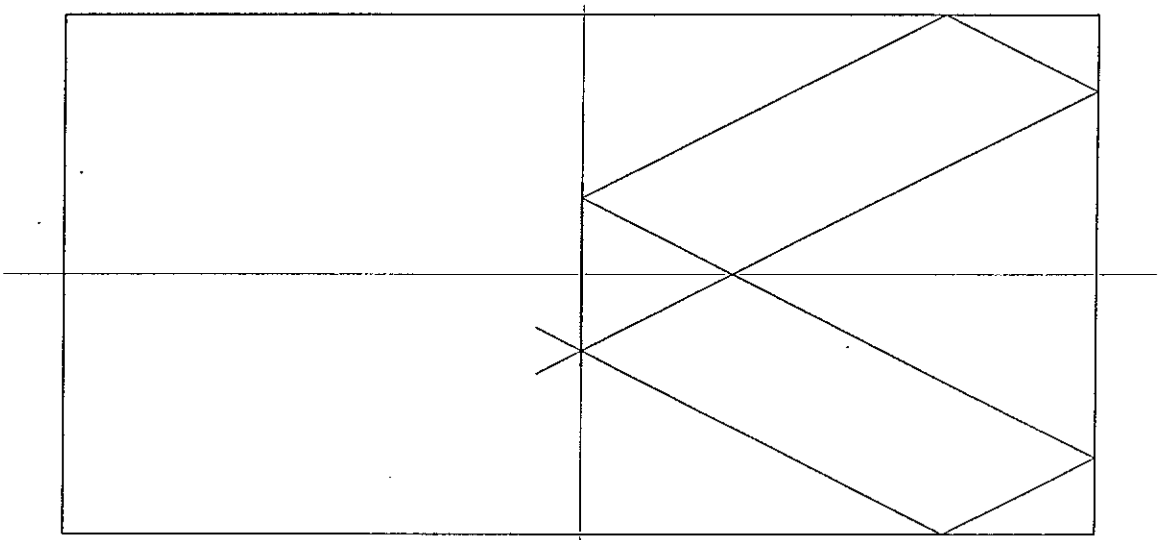

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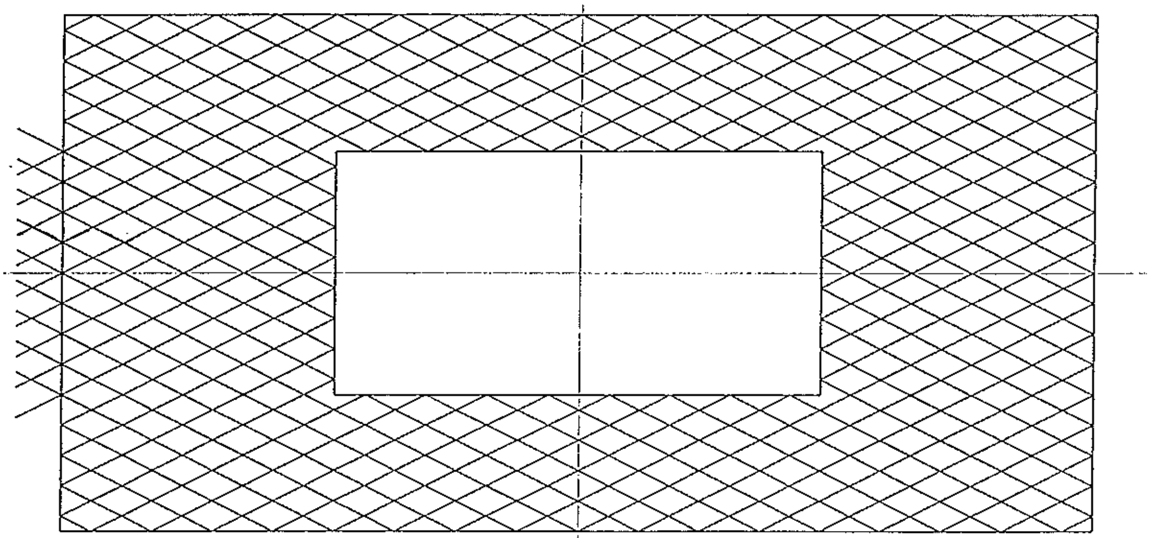

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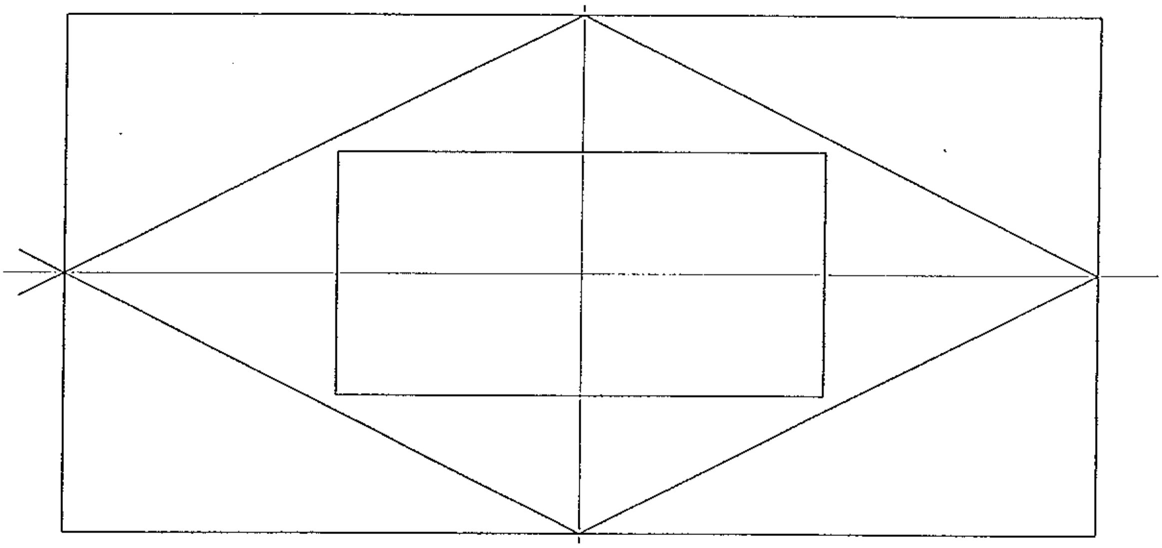
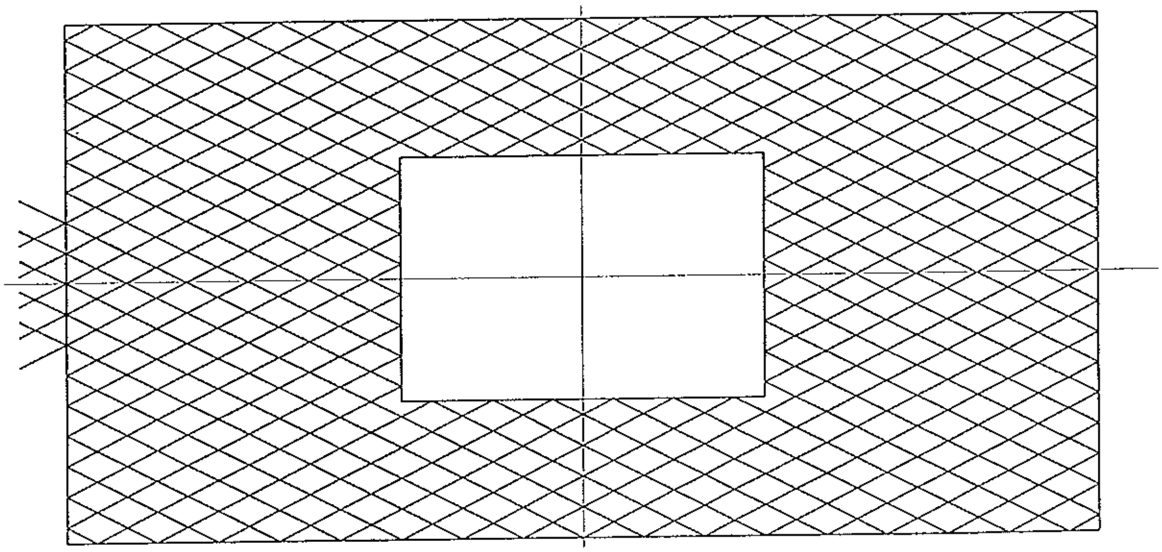


Fig. 28.

Note especially that in the braid $17/17 \times 6/0$ we came back to eleven components and that $17 = 11 + 6$, furthermore recall from pg. 1 that $11 = n_1 - n_2 = 17 - 6$. All simple, easy and apparently natural relationships! We see here a somewhat similar sort of behaviour with the number of components as that of the **RKT** with natural numbers (refer to Pamphlet No. 20 pg. 3) — in our case here we start with the component numbers of the **regular square flat braids** $n_1/n_1 = 17/17$ and $n_2/n_2 = 6/6$ hence with 17 and with 6 which leads to the component numbers of the braid $17/17 \times 6/6$ of $17 - 6 = 11$, flinally to finish up with the number of components of the braid $17/17 \times 6/0$ with $(17 - 6) + 6 = 17$, an interesting and most likely unexpected result.

The first component in Fig. 28 of the $17/17 \times 8/8$ braid in Fig. 27, consisting of $17 - 8 = 9$ components, is again totally symmetric. We do not have components similar to the four in Figs. 4 – 7. The eight components following this totally symmetric component in Fig. 28, are again somewhat very similar.

The braid $17/17 \times 8/6$ in Fig. 29 consists of five components. The first three are again very similar to the ones in Figs. 3 – 7. Then follow the two components in Figs. 30 and 31.



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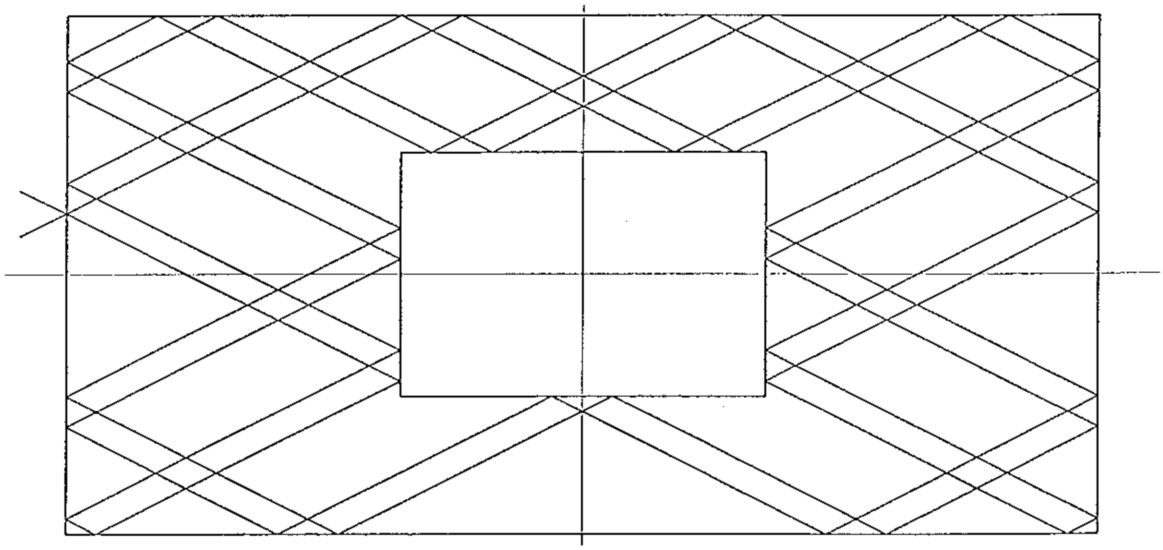

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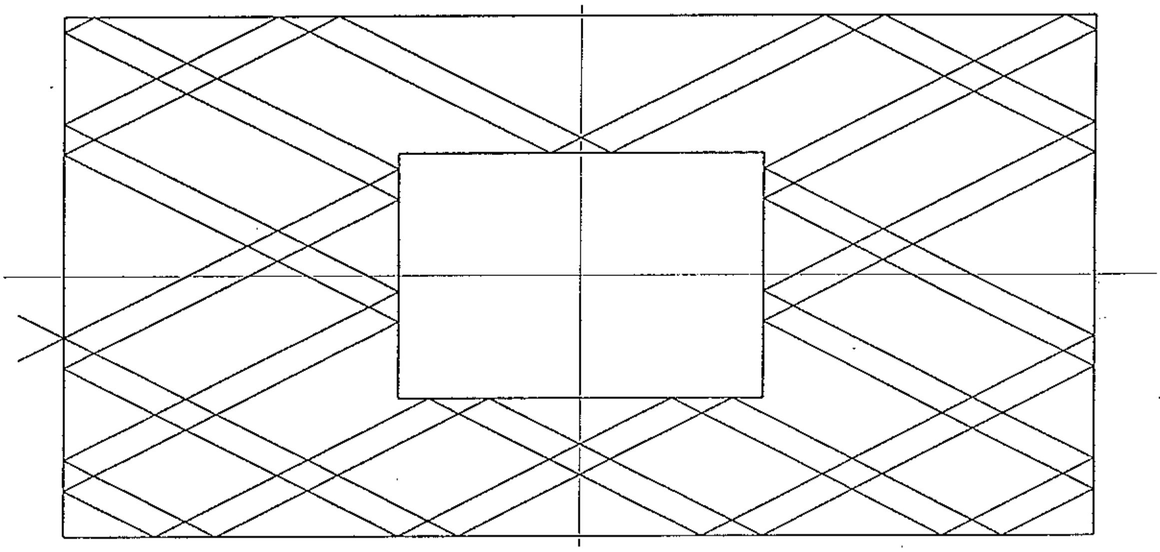

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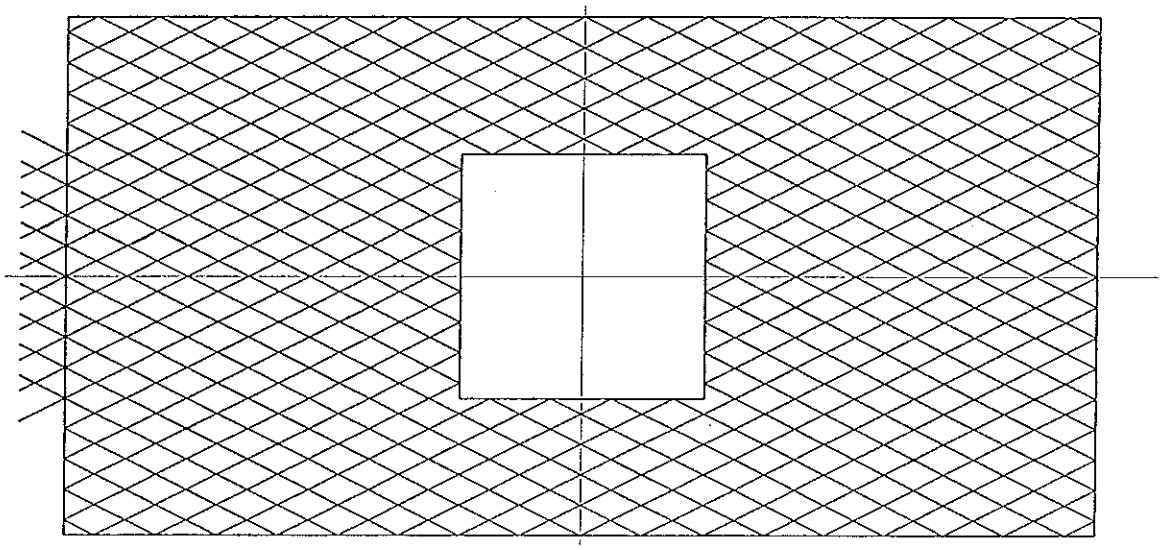

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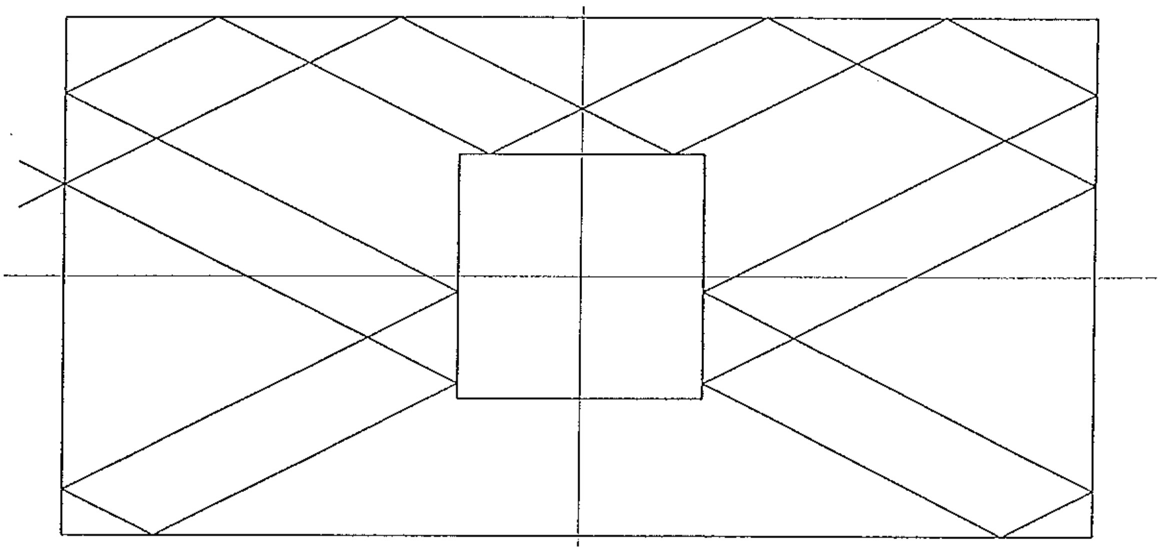

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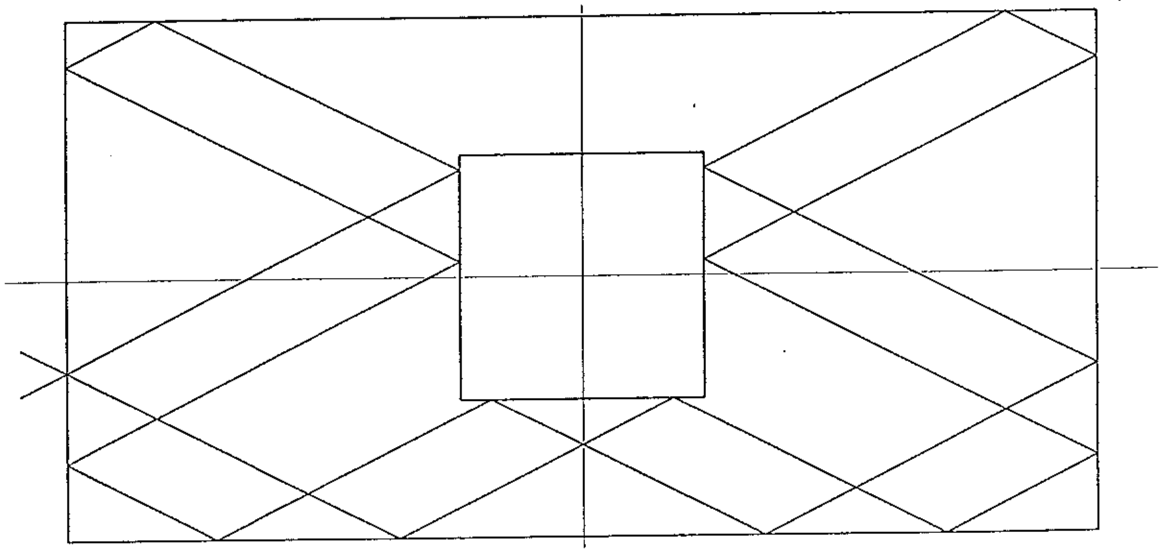

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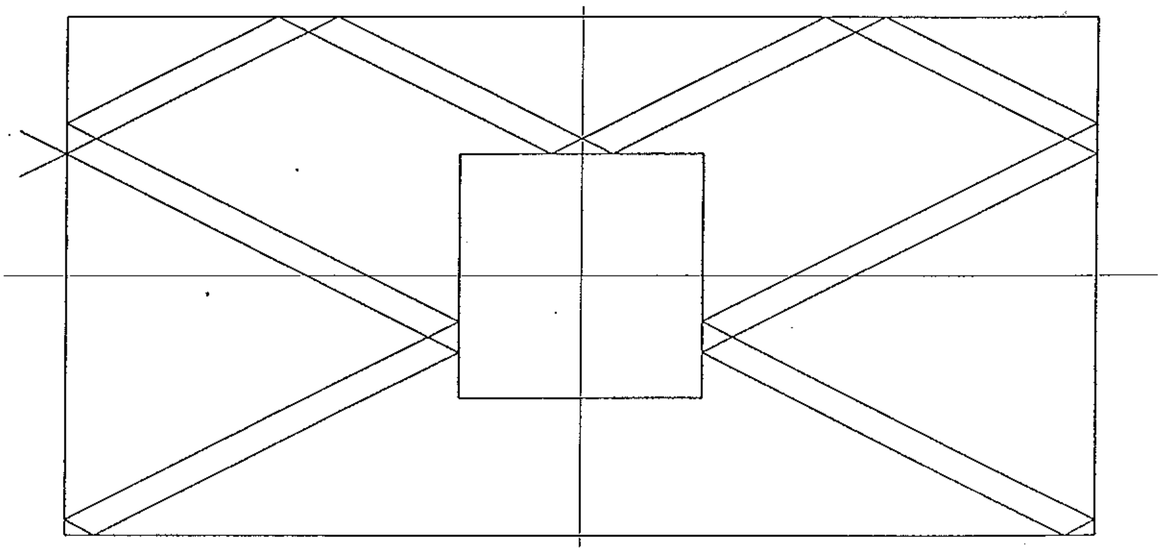

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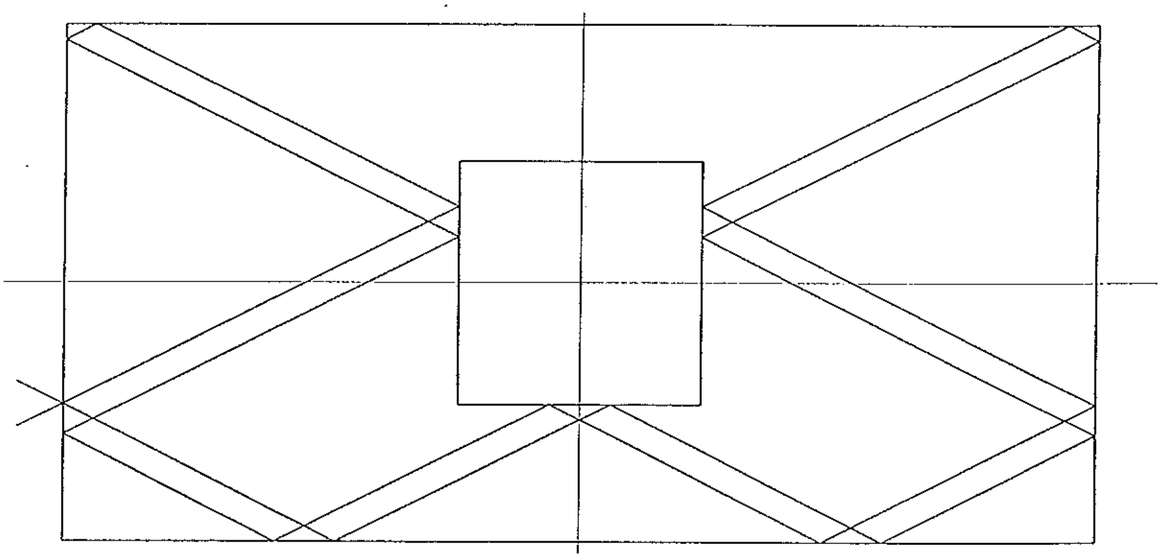

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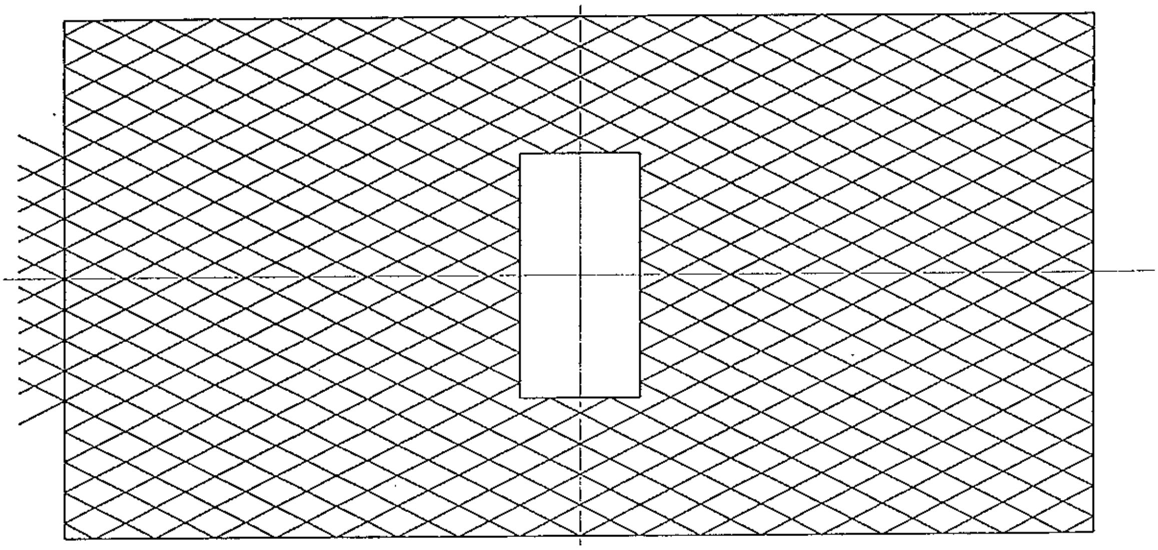
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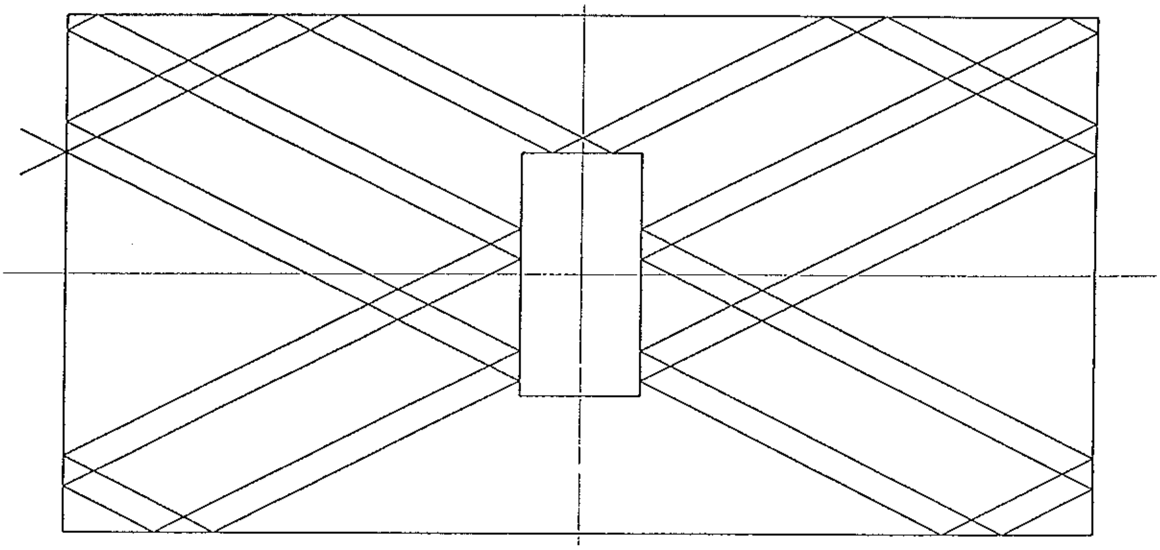
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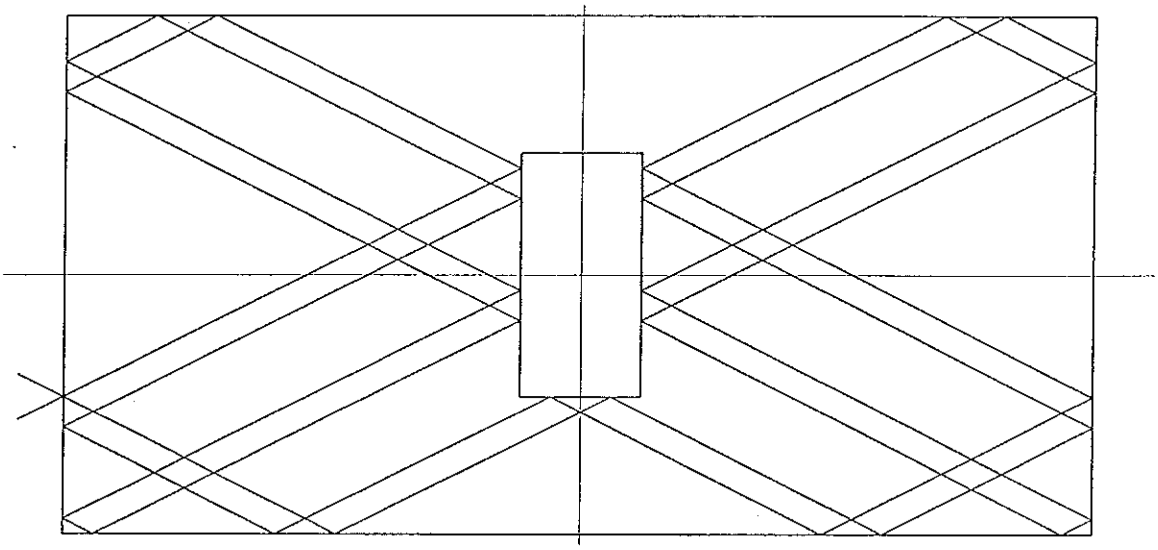


Fig. 39.

The braid $17/17 \times 8/4$ in Fig. 32 consists of nine components. The first five are again very similar to the ones in Figs. 3 – 7. Then follow the four components in Figs. 33 – 36.

The braid $17/17 \times 8/2$ in Fig. 37 consists also of nine components. The first seven are again very similar to the ones in Figs. 3 – 7. Then follow the two components in Figs. 38 and 39.

With the braid $17/17 \times 8/0$ we come back to $(17-8)+8=17$ components.

In the braid $17/17 \times 10/10$ depicted in Fig. 40, its first component is totally symmetric, but as depicted in Fig. 41 quite different to the first component in the braid $17/17 \times 6/6$, hence quite different to the component in Fig. 3. Of its seven components $(17-10=7)$, depicted in Figs. 42 – 47 component two is like component three while component four is like component five and component six is like component seven.

The braid $17/17 \times 10/8$ in Fig. 48 consists of a single component as does the braid $17/17 \times 8/10$.

The braid $17/17 \times 10/6$ in Fig. 49 consists of the three components depicted in the Figs. 50 – 52.

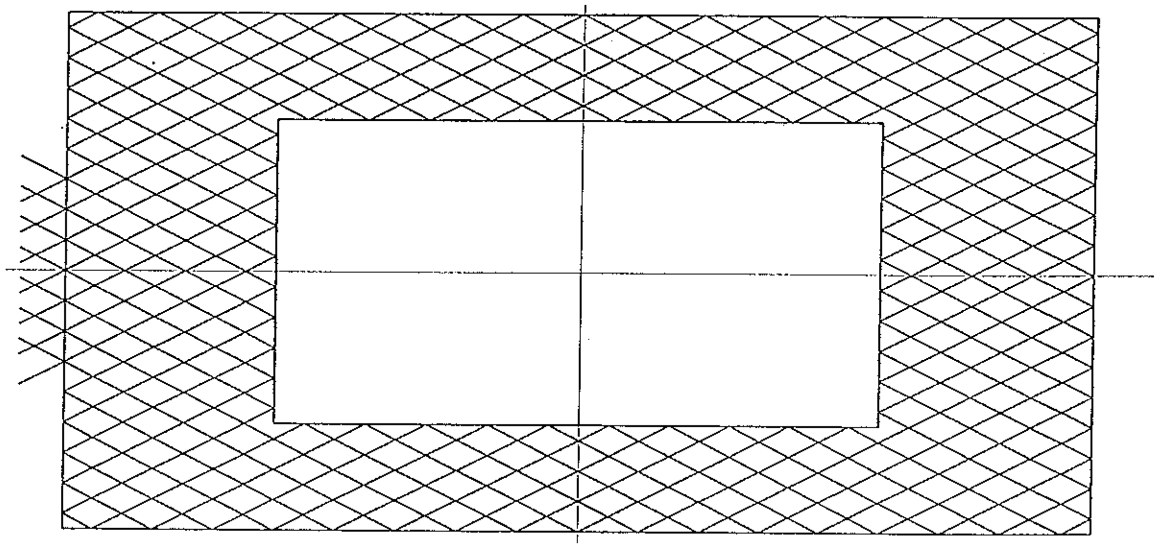
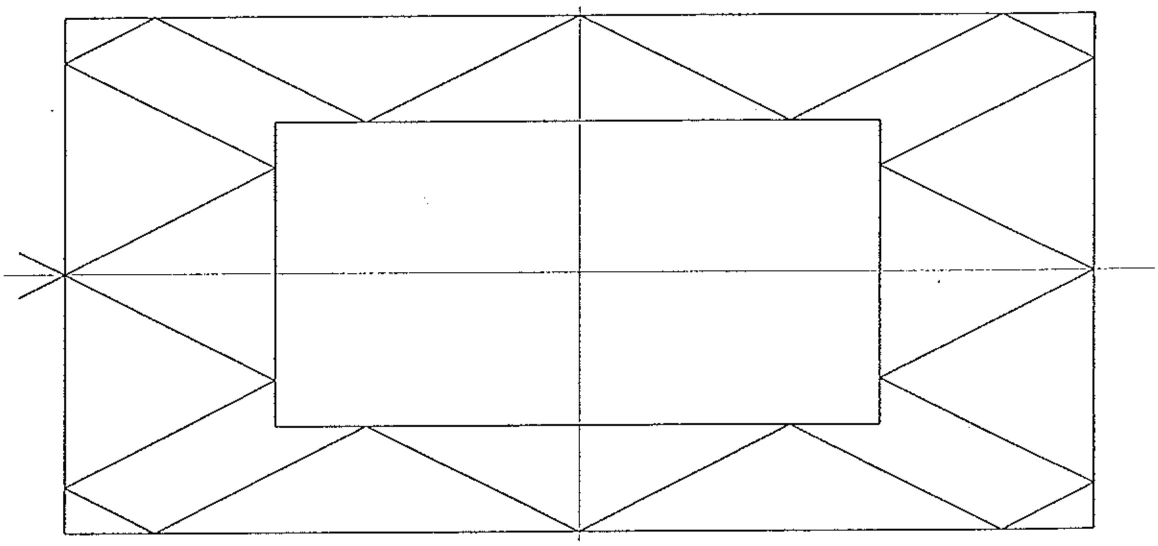


Fig. 40.



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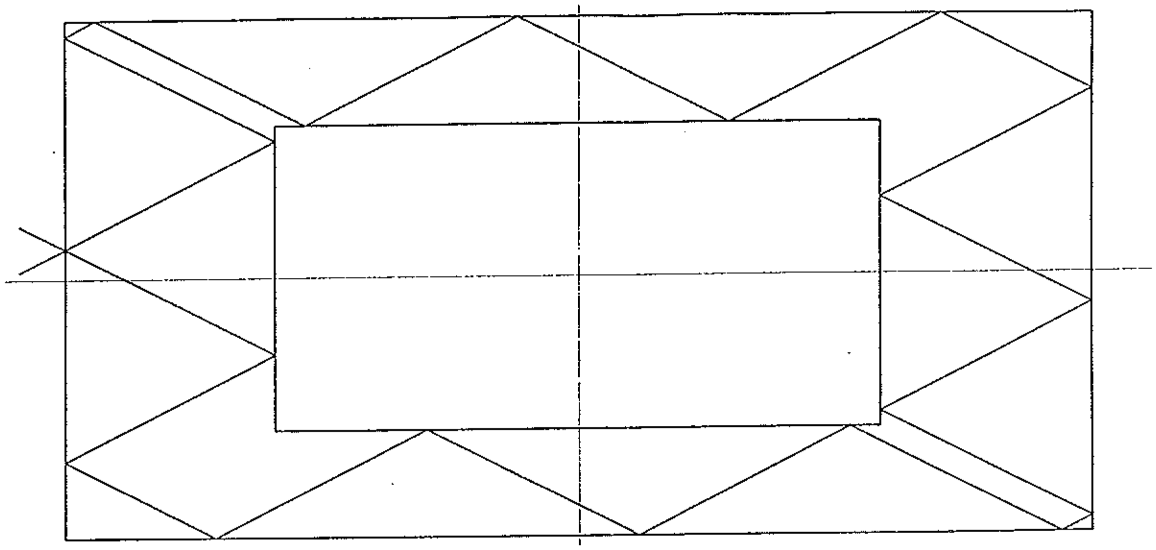

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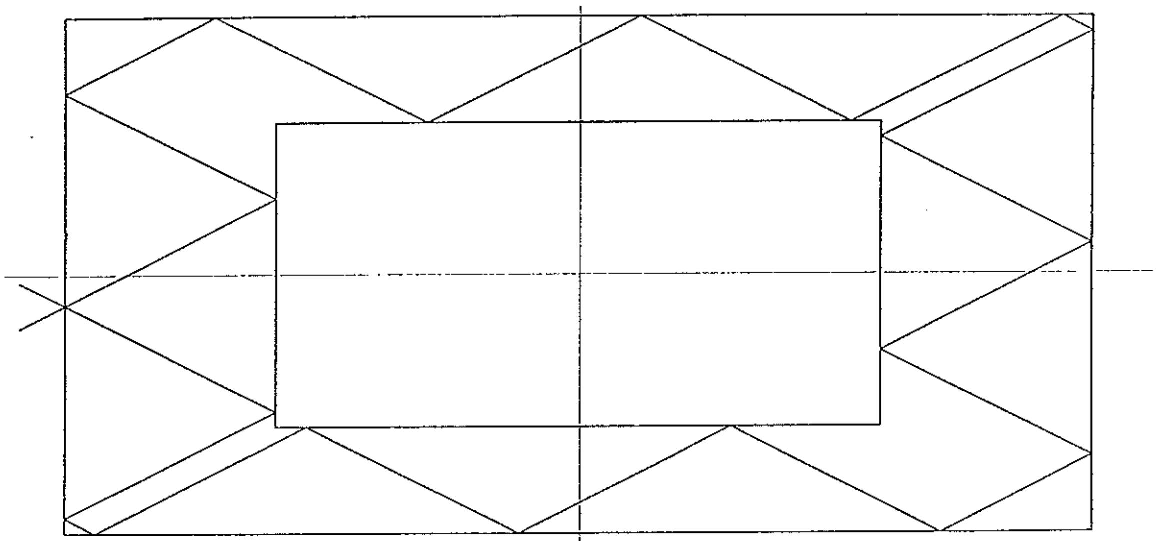

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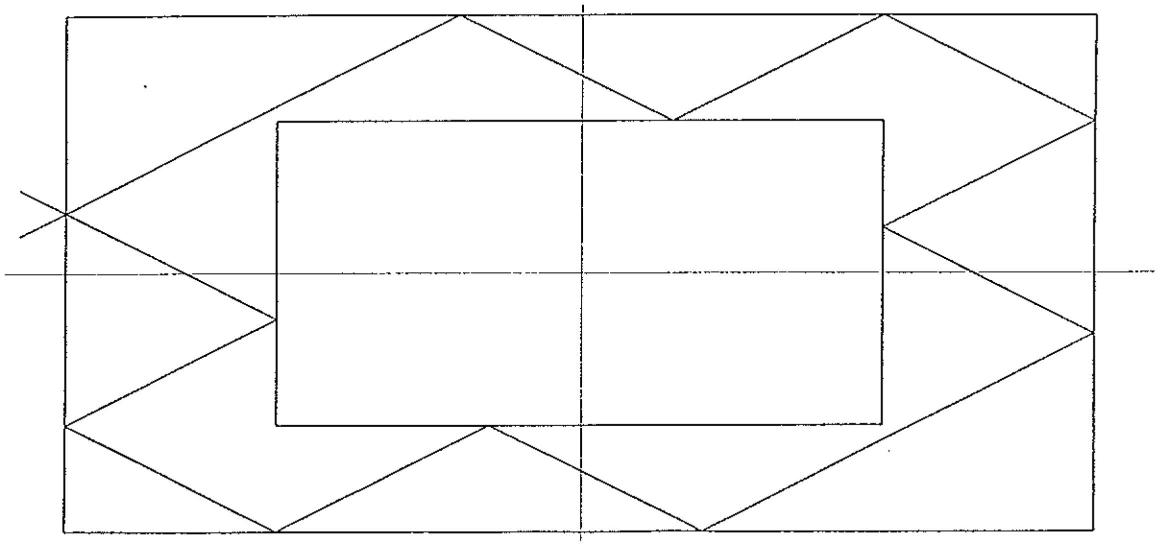

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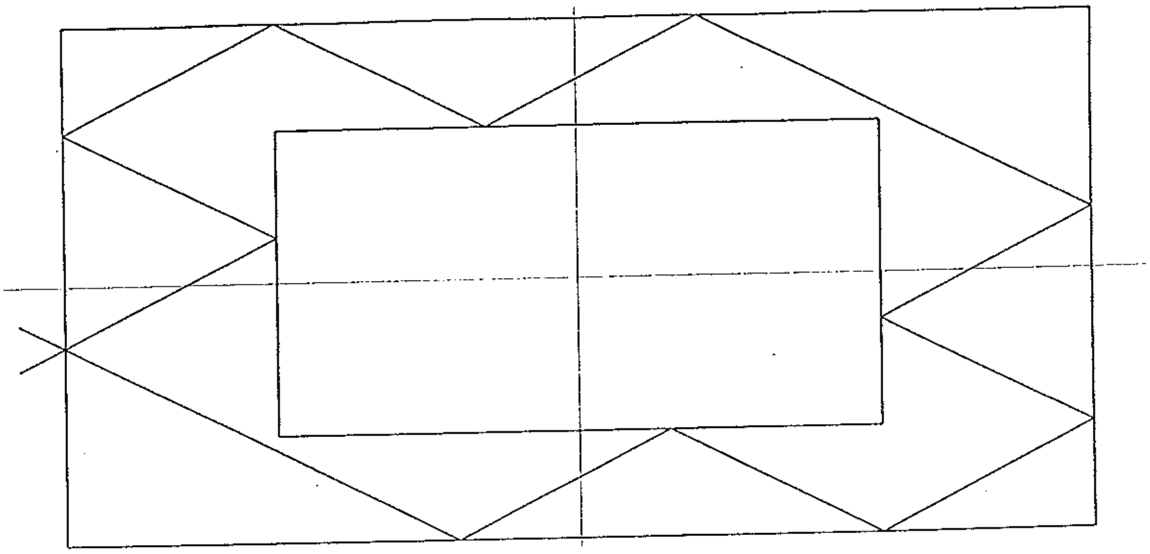

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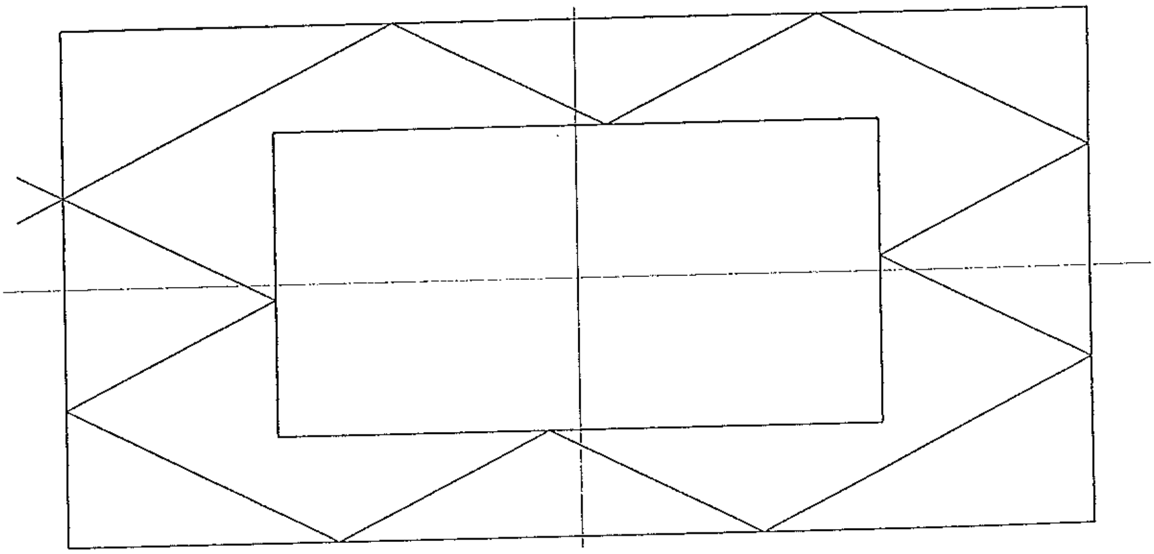

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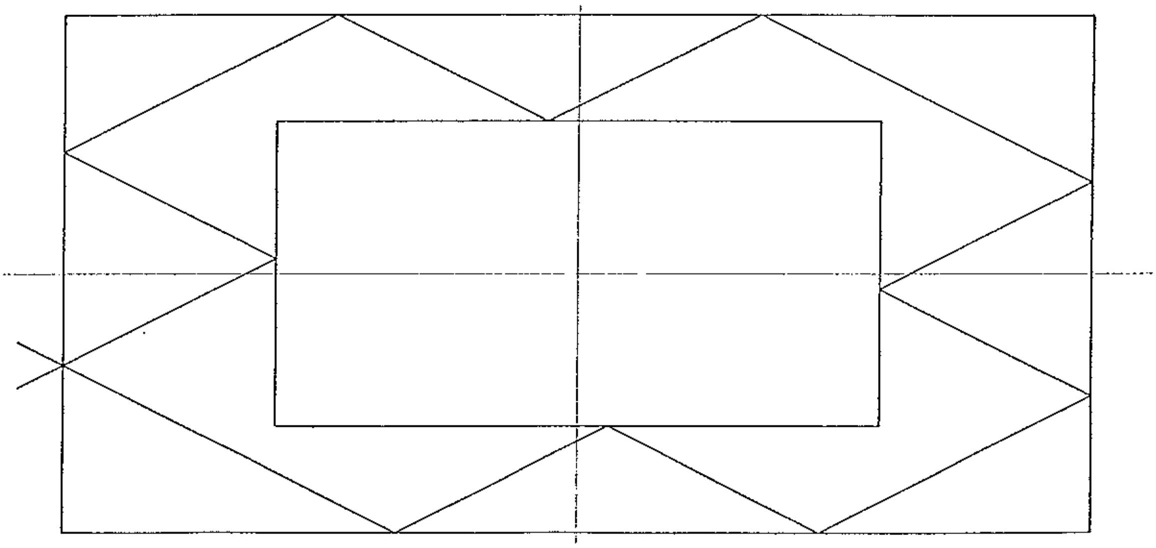

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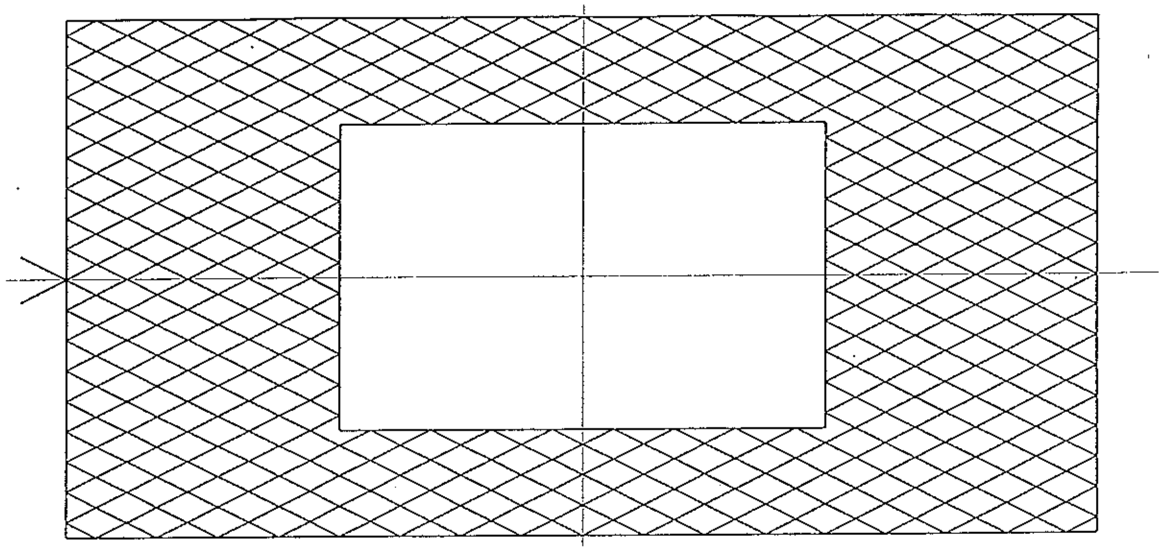

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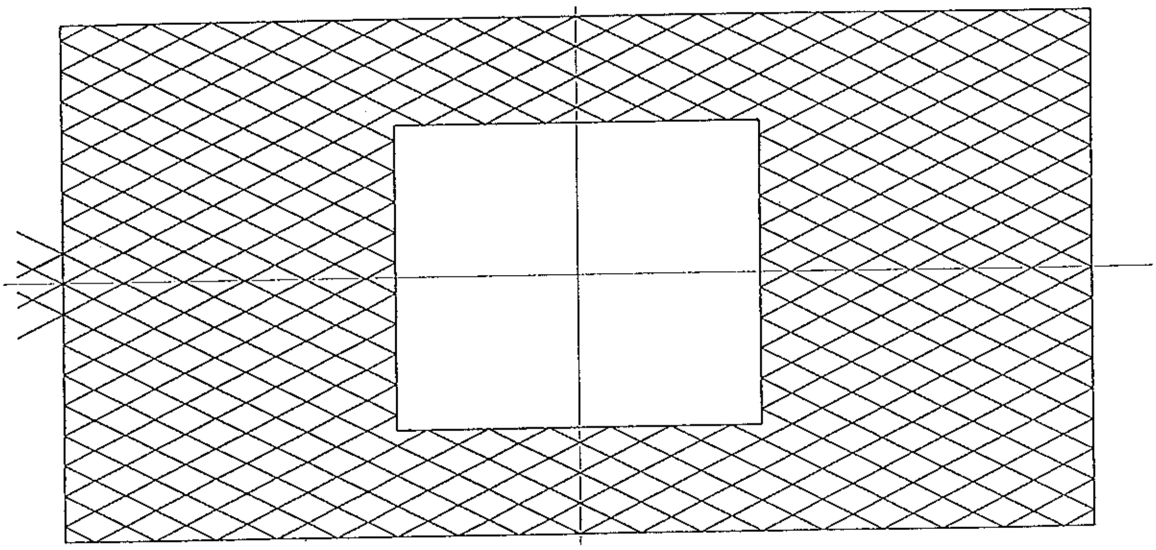

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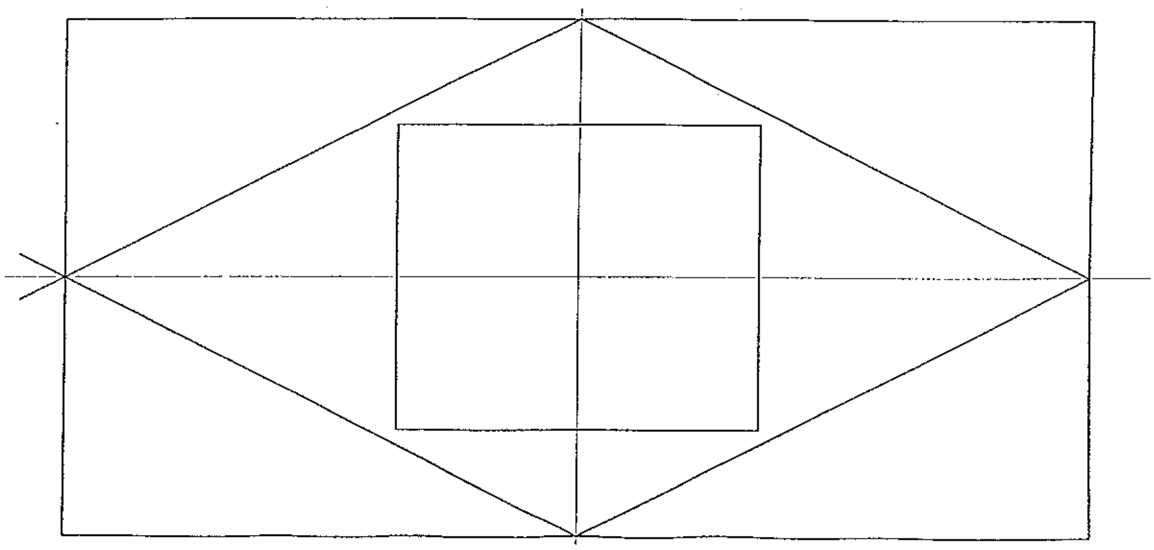


Fig. 50.

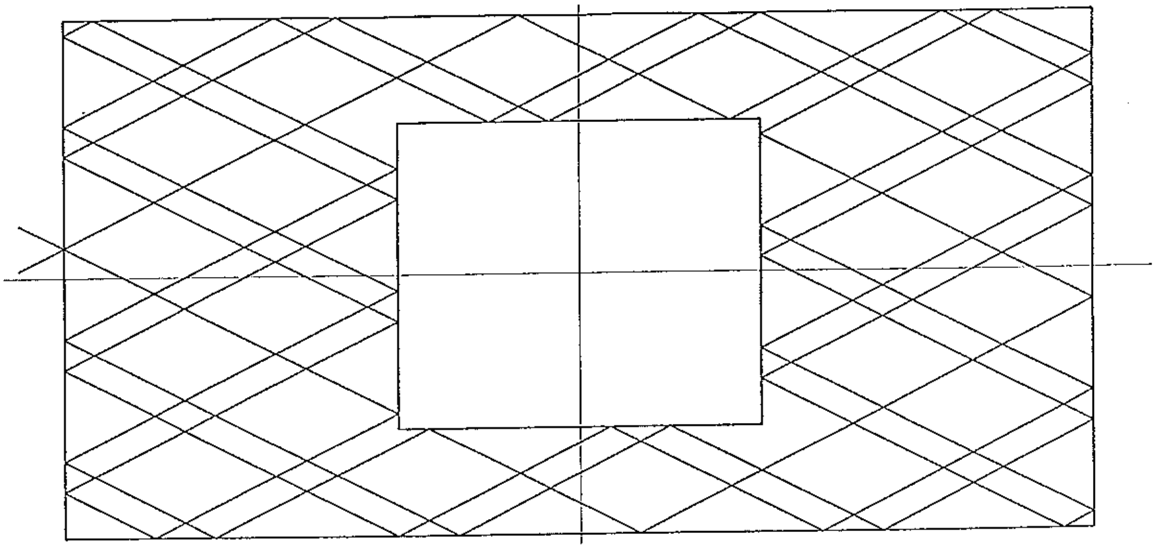


Fig. 51.

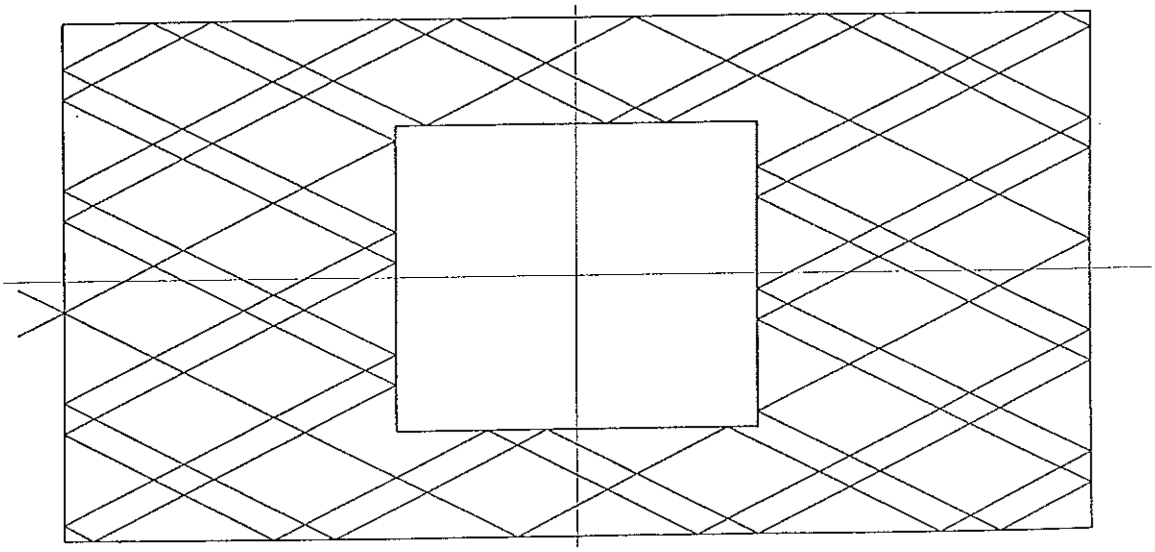
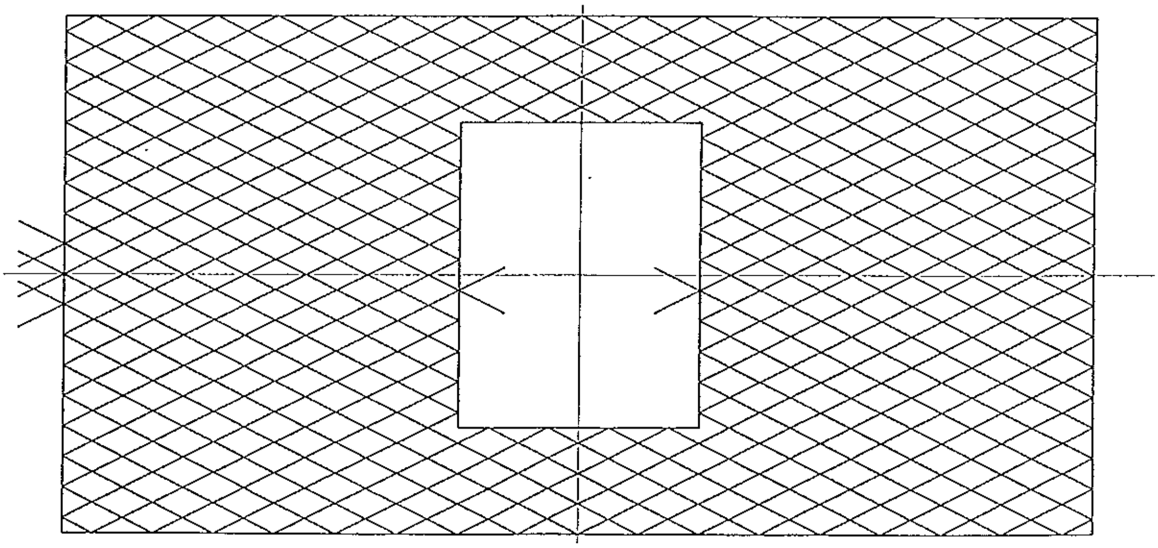


Fig. 52.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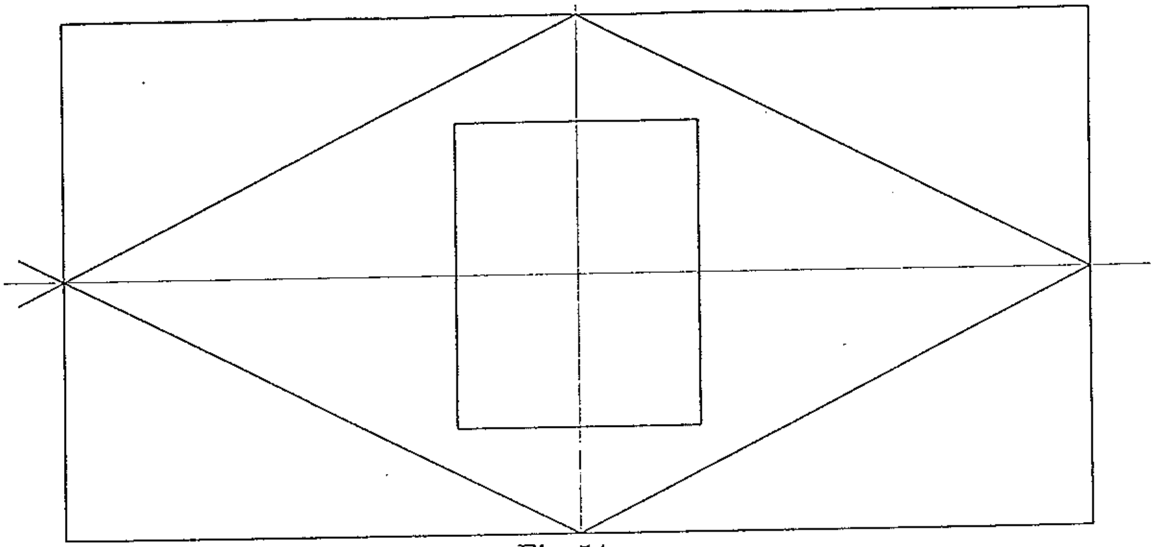
Fig. 54.

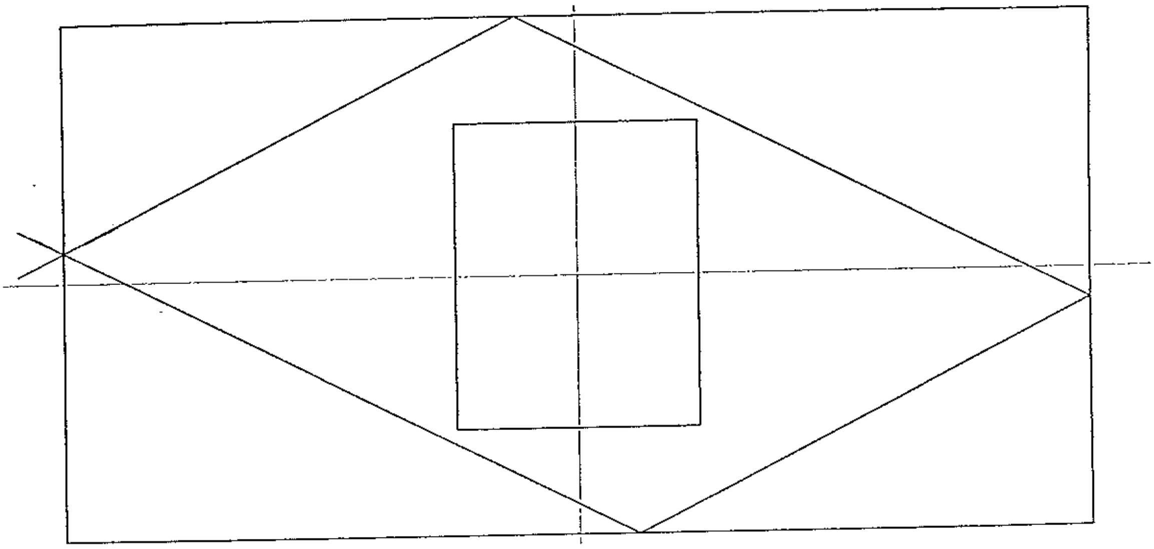
Fig. 55.

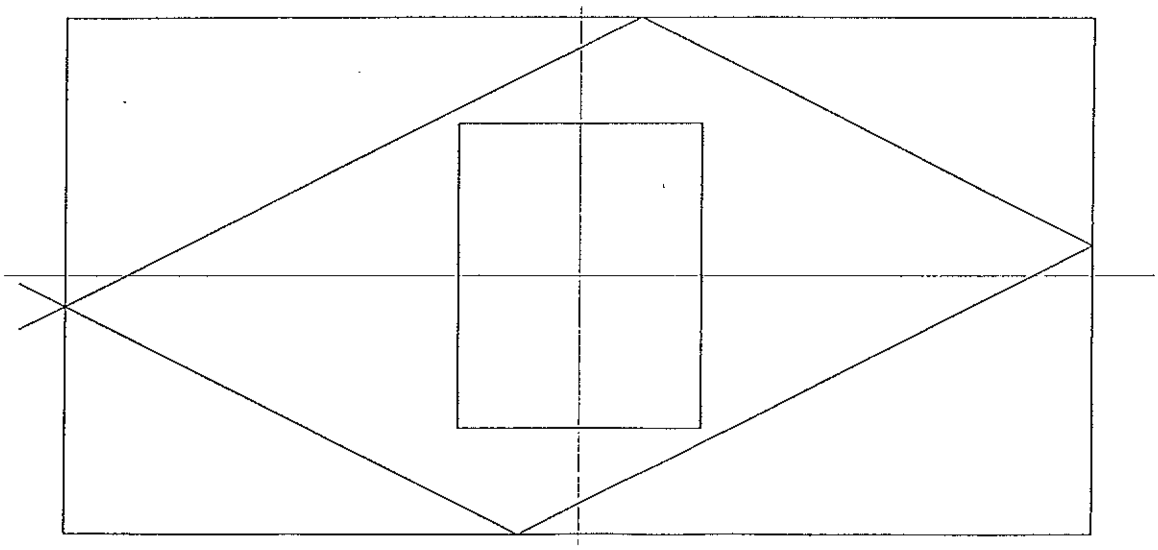
Fig. 56.

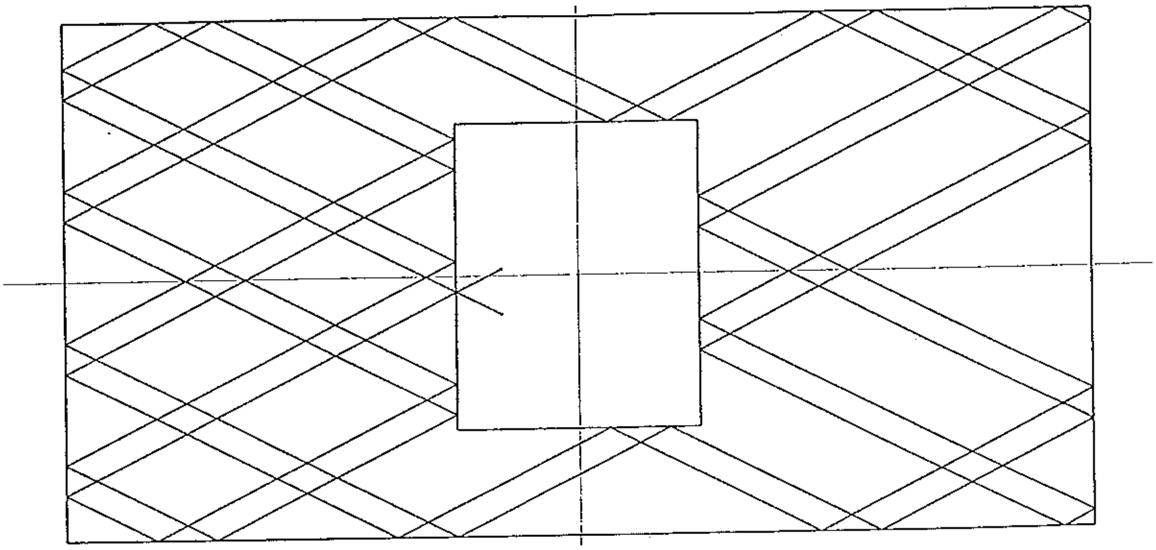

Fig. 57.

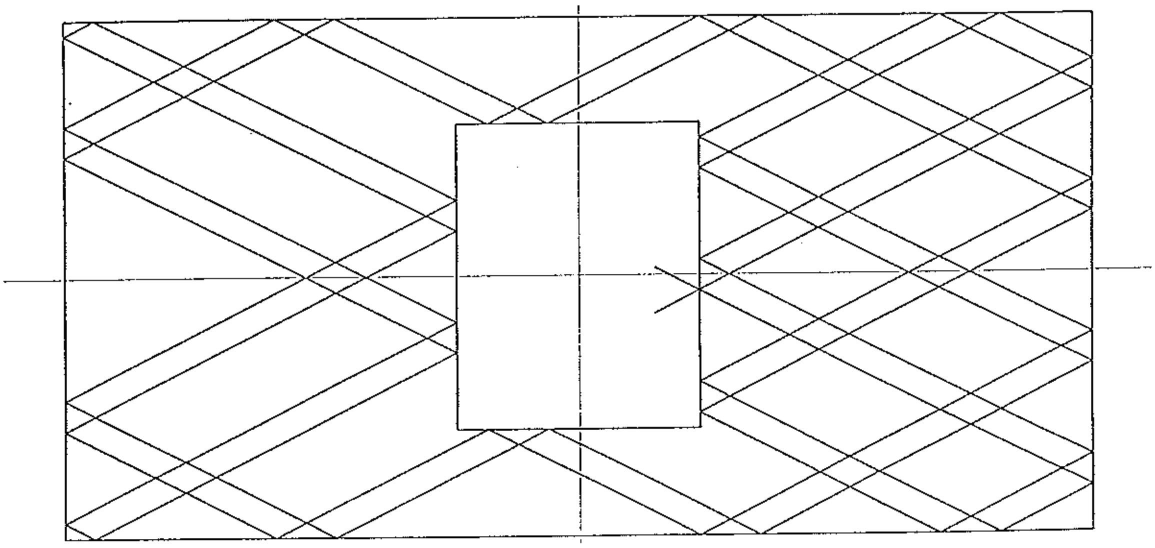

Fig. 58.

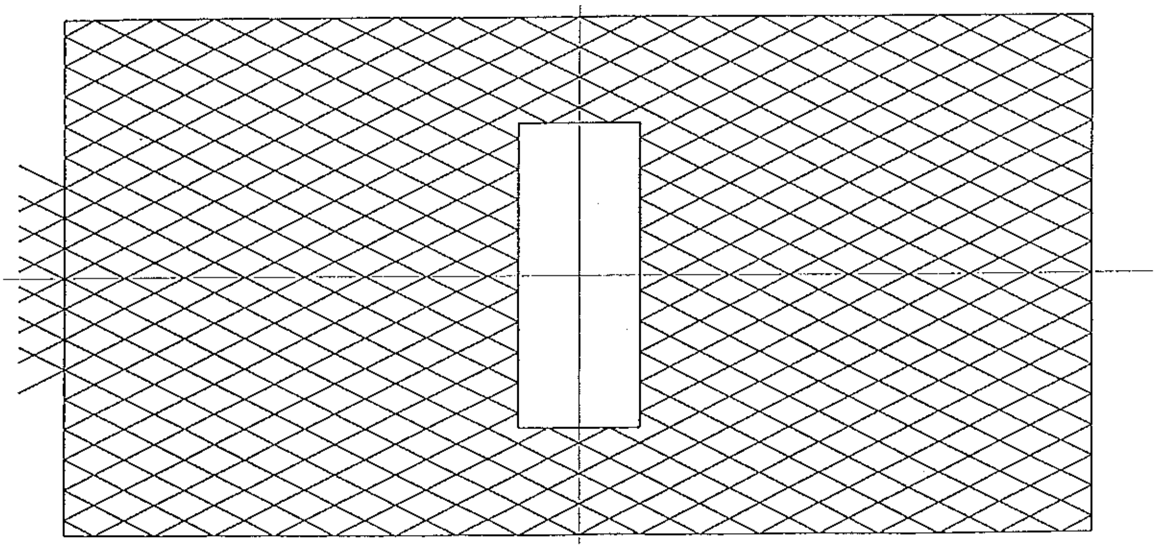

Fig. 59.

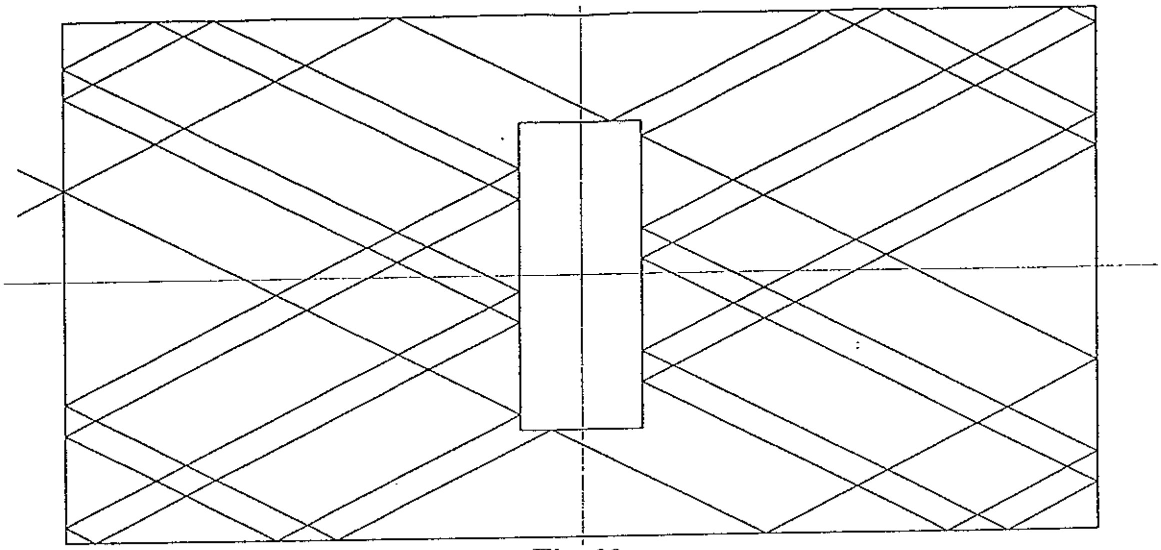

Fig. 60.

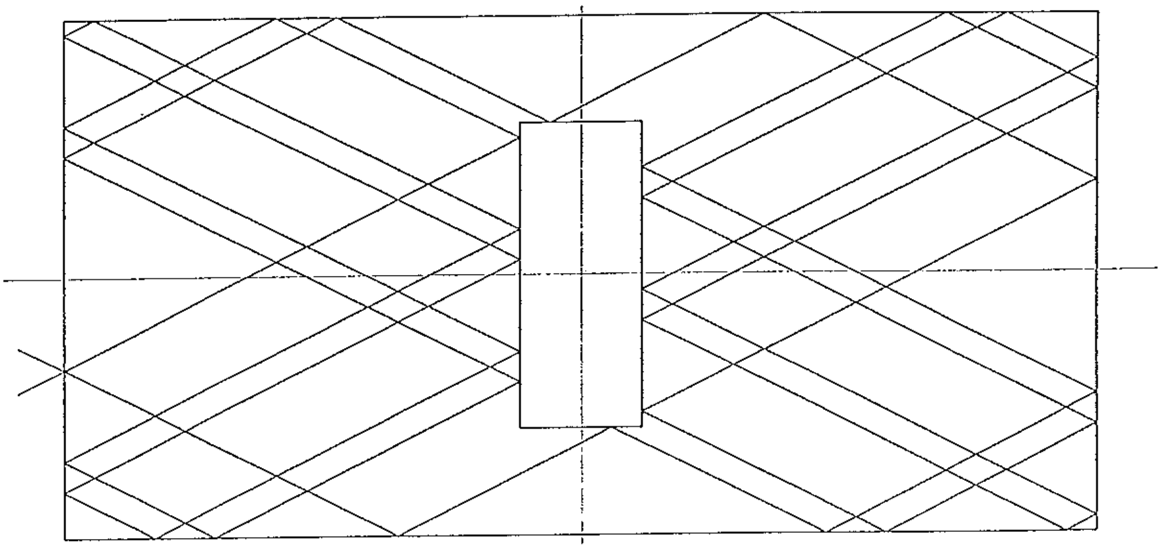

Fig. 61.

In the depicted diagrams we have chosen the start and end of the component, but obviously the start and end of a component can be placed at any point of the component concerned. This must be clearly kept in mind, hence for example the second component in the braid $13/11 \times 3/5$, Fig. 64 of the Figs. 62–64 is the same as the second component, Fig. 67 of the Figs. 65 – 67.

Although only a few braids with their components have been shown, it should be clear that a repeat in pattern is obviously there, but is more complicated. Since the **outside frame** has n_{oi}/n_{oo} bights while the **inside frame** has n_{hi}/n_{ho} bights we have with **odd** and **even** bight numbers $4 \times 2 = 8$ braid-type combinations of $n_{oi}/n_{oo} \times n_{hi}/n_{ho}$ to start with.

As mentioned on pg. 1 there are single-component members of these braids, just as there are two-component members, three-component members, four-component members and n-component members, where n is a natural number, of these braids. Single-component members, for example, are $27/29 \times 18/18$, $13/15 \times 8/8$, $25/27 \times 18/18$, $26/27 \times 17/18$, $28/26 \times 17/17$.

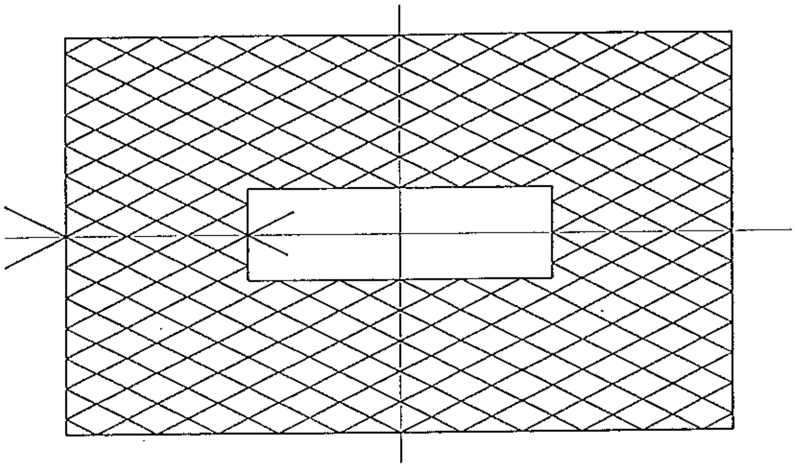

Fig. 62.

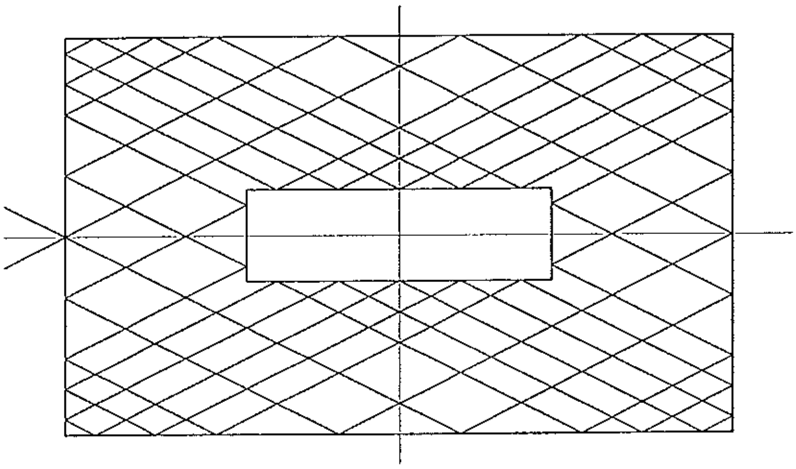

Fig. 63.

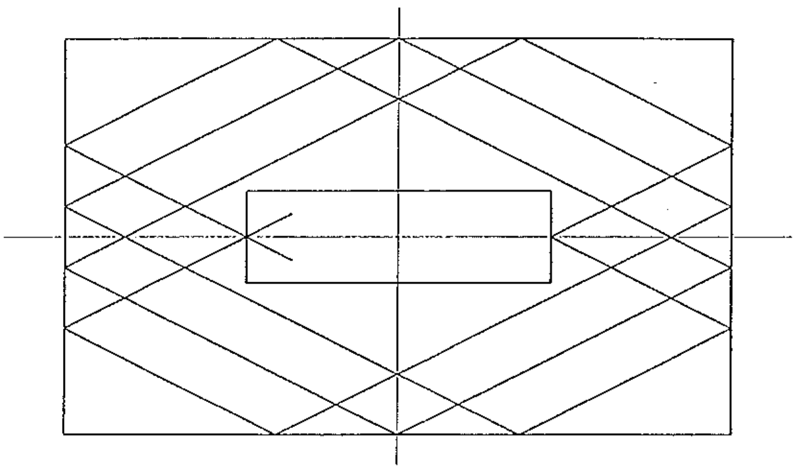

Fig. 64.

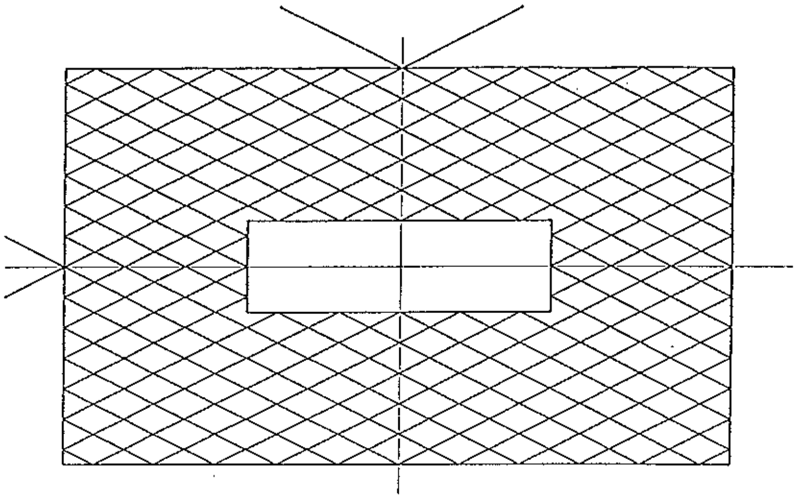

Fig. 65.

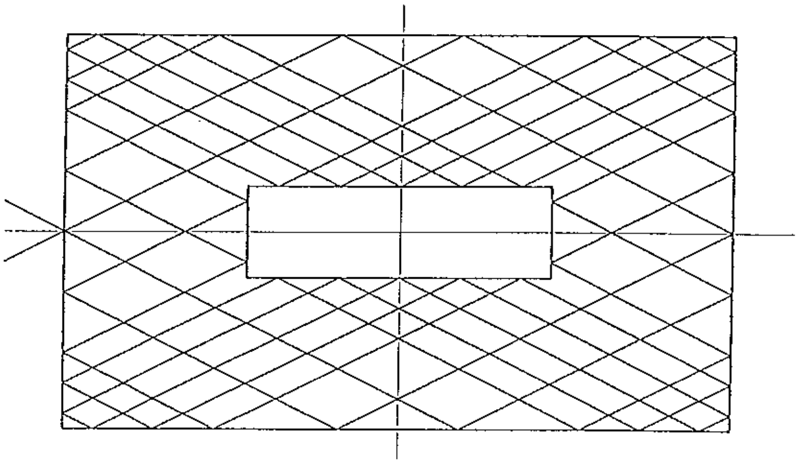

Fig. 66.

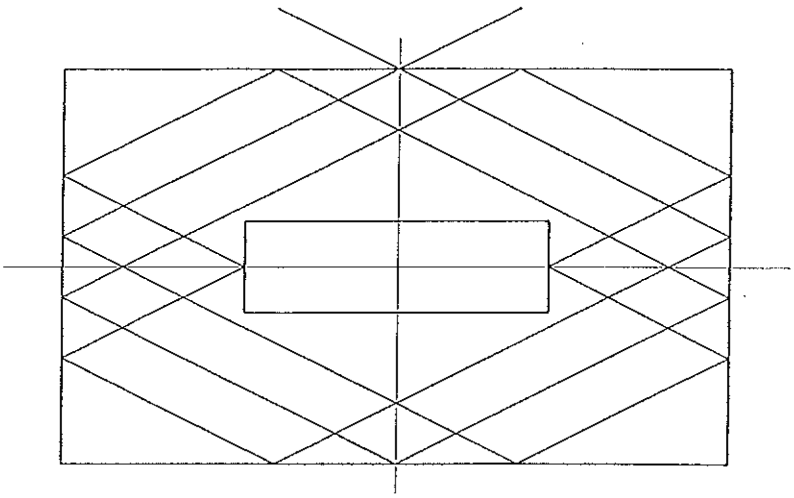

Fig. 67.