THE BRAIDER

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THE BRAIDER'S NOTEBOOK

Since braiding is still in the infant stage of development, it is not surprising that one can often encounter knots which have a seemingly good braided pattern. However, a closer inspection will reveal an unbalance in the design. Such unbalanced, but seemingly good patterns are commonly encountered in braid-work produced by pattern-braiders. In general, pattern-braiders don't use grid-diagrams in the design stage, which is an essential preliminary requirement in good braid design. We shall discuss here a few examples of braids which seemingly appear to have a good pattern, but which in fact are unbalanced, and show how with a small modification a good balanced pattern can be created.

A commonly encountered colour-pattern in Regular Knots is shown in the left-hand grid-diagrams of Fig. 1153. The coding-pattern is unbalanced and should in general be avoided.

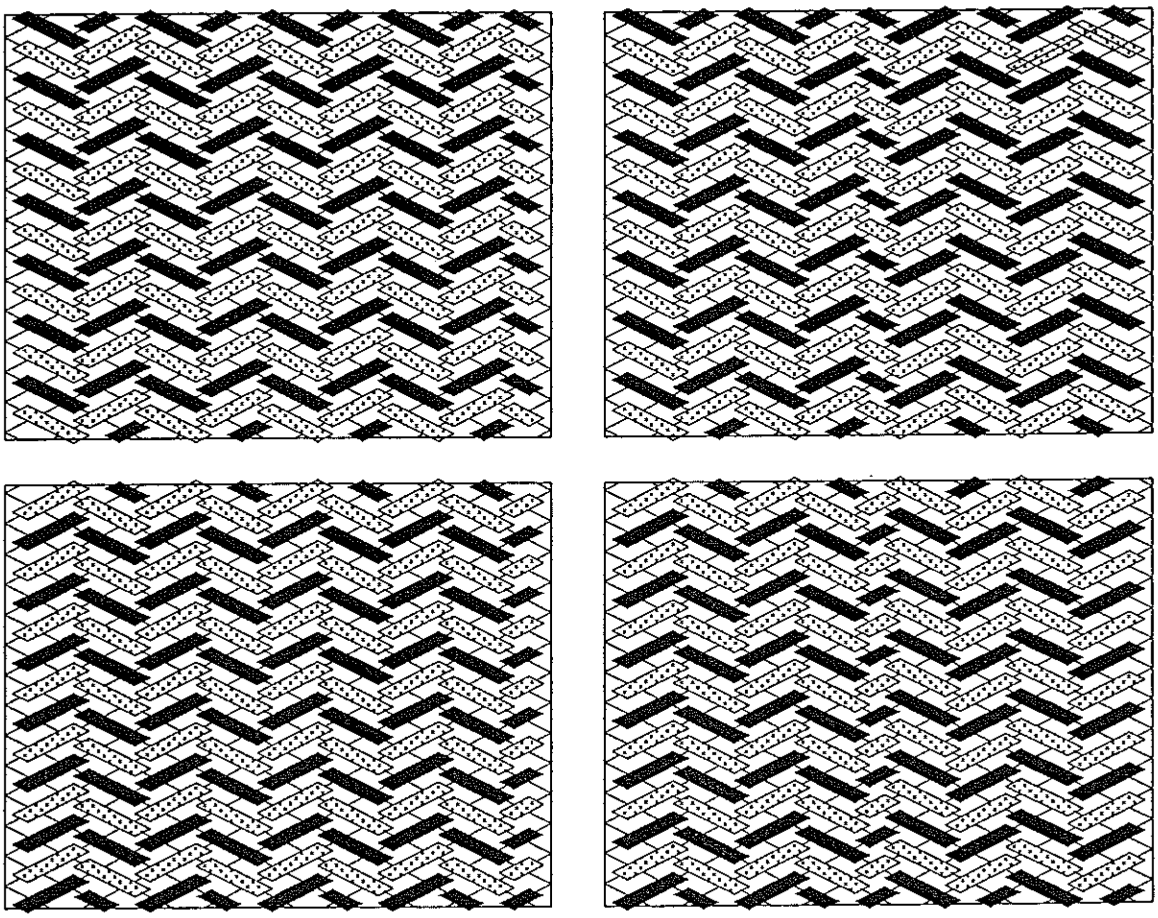


Fig. 1153.

For such a colour-pattern the g.c.d. (p, b) has to be equal to 2, hence a true two-pass coding-pattern cannot give us such a colour-pattern since in that case $p = 2n+1 = odd$. A balanced pattern (as for example shown in the right-hand grid-diagrams of Fig. 1153) can only be obtained when $p = 4n + 2$.

The coding-pattern in the left-hand grid-diagrams of Fig. 1154 is again unbalanced and although in the finished knot the pattern seems to be a good one, it should in general be avoided. A balanced pattern (as for example shown in the right-hand grid-diagrams of Fig. 1154) can only be obtained when $p = 8n + 4$.

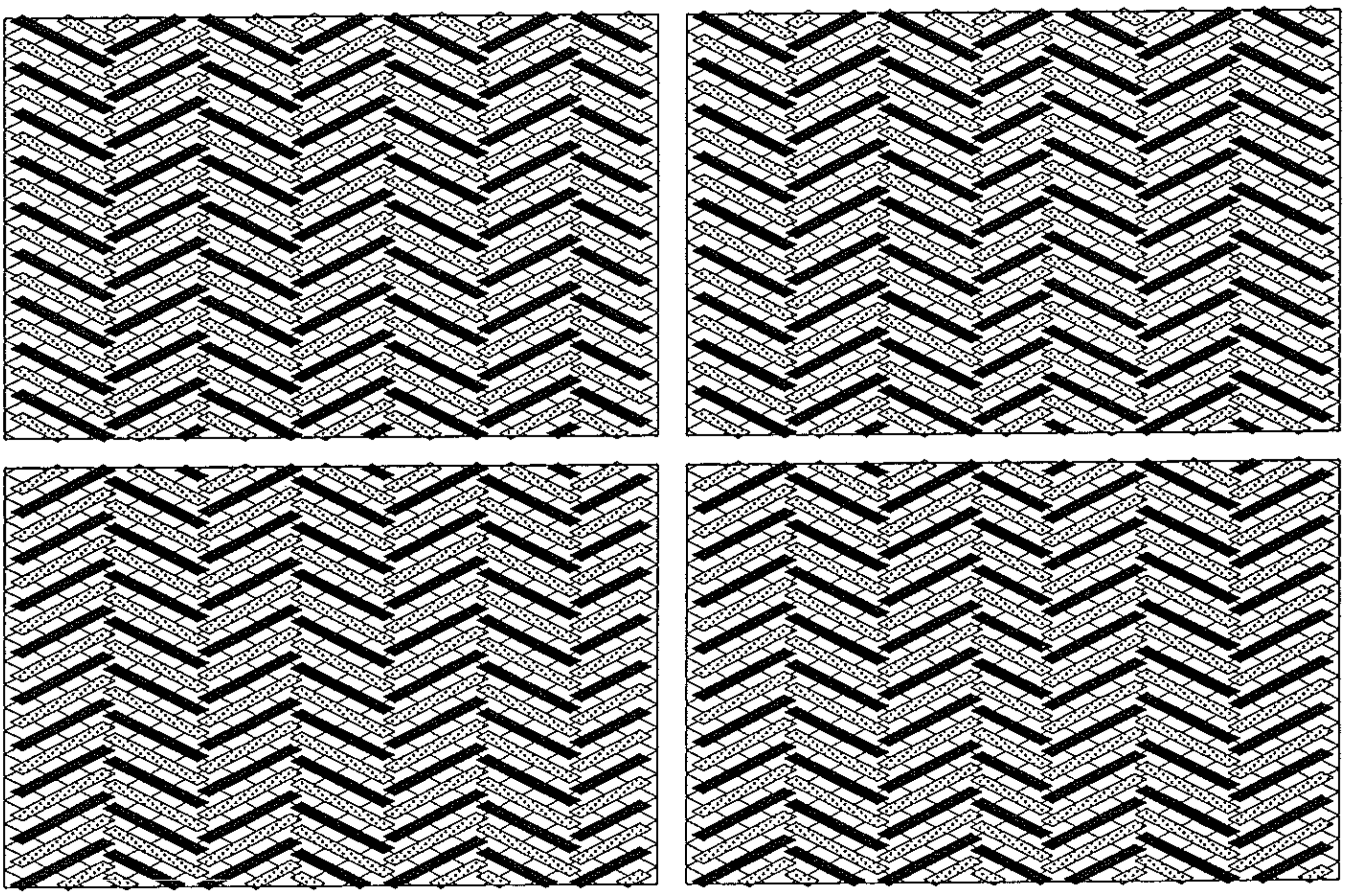


Fig. 1154.

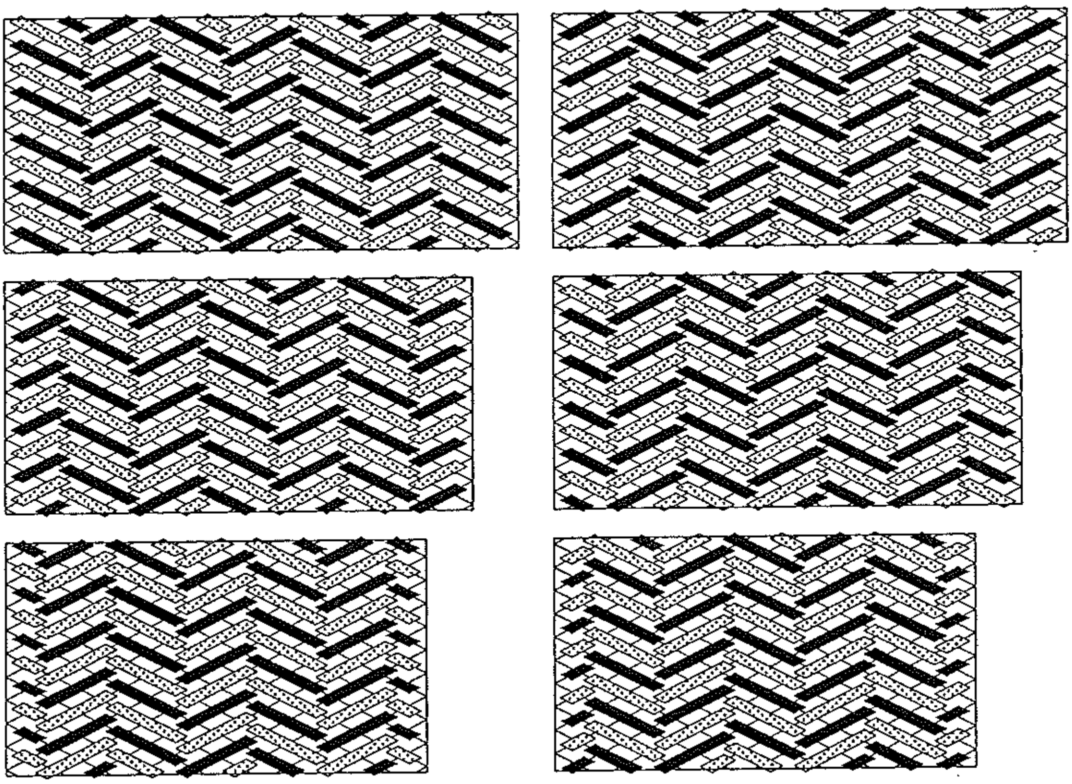


Fig. 1155.

The patterns in the grid-diagrams of Fig. 1155 form helixes and are all balanced. Again the g.c.d.(p, b) has to be equal to 2. For the pattern in the uppermost two grid-diagrams, $p = 3n + 1$ and hence n has to be *odd*. For the pattern in the central two grid-diagrams, $p = 3n + 5$ and hence n has to be *odd*. For the pattern in the lowermost two grid-diagrams, $p = 3n + 3$ and hence n has to be *odd*. Ensure that left and right helix angle of the string-run are the same.

A very common pattern-braider's knot is a column-coded α-pass column-coded Regular Knot (hence $p = \alpha m + 1$) where $b = p - 1 = \alpha m$. Hence p and b are coprime and although such a knot can be braided from a single string, it is not surprising that it may be encountered in a two colour braid-pattern where $\left\lfloor \frac{b}{2} \right\rfloor$ adjacent bights are of one colour and $\left\lceil \frac{b}{2} \right\rceil$ adjacent bights are of another colour.† These two bight numbers are then the same when $\alpha m = even$ and differ by 1 when both α and m are *odd*. When both α and m are *odd*, such two-colour knots may indicate the skill-level of the braider involved. A skilled braider would ensure that the pattern in the knot is balanced and would have been aided by the grid-diagram before starting the braiding process. A braider who does not possess a sufficient skill-level may for example produce the knot depicted by the left-hand grid-diagram in Fig. 1156.

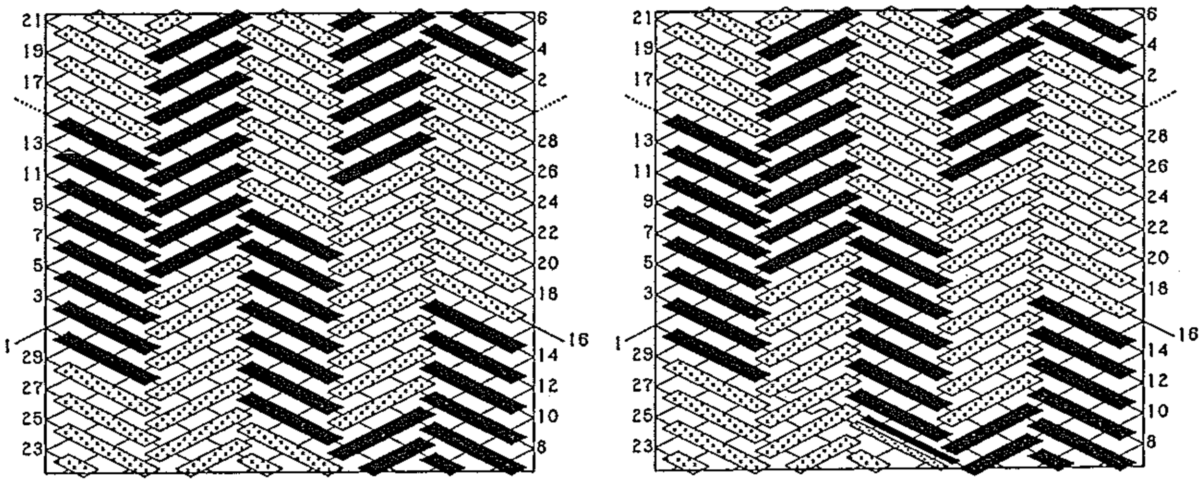


Fig. 1156.

The pattern of this knot is unbalanced (compare the circled areas in Fig. 1157). We have come across this knot in 2 mm. round string where the braider had tried to rectify the unbalance (see the right-hand grid-diagram in Fig. 1156). The pattern in the actual finished knot may then seem to be balanced since the local string-run irregularity created may not be obvious, but will nevertheless be revealed by a closer examination. Instead of braiding the two strings as indicated in the left-hand grid-diagram of Fig. 1156, where one string first braids the half-cycles 1–14 and the other string then braids the half-cycles 16–29 followed by working away the Standing End of half-cycle 1 with the string-end of half-cycle 29 and the string-end of half-cycle 14 with the Standing End of half-cycle 16,

† $\left\lfloor \frac{b}{2} \right\rfloor$ denotes the greatest whole number equal to or smaller than $\frac{b}{2}$.

$\left\lceil \frac{b}{2} \right\rceil$ denotes the smallest whole number equal to or greater than $\frac{b}{2}$.

we can braid the two strings as indicated in the grid-diagram of Fig. 1158, where one string first braids the half-cycles 1–15 and the other string then braids the half-cycles 17–29 followed by working away the Standing End of half-cycle 1 with the string-end of half-cycle 29 and the string-end of half-cycle 15 with the Standing End of half-cycle 17. This creates the balanced pattern shown in the grid-diagram of Fig. 1158.

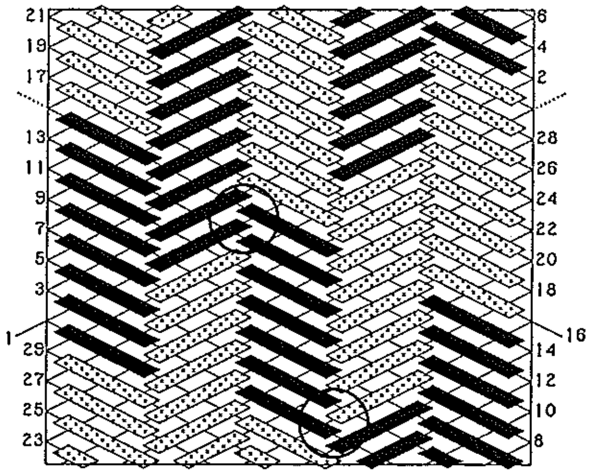


Fig. 1157.

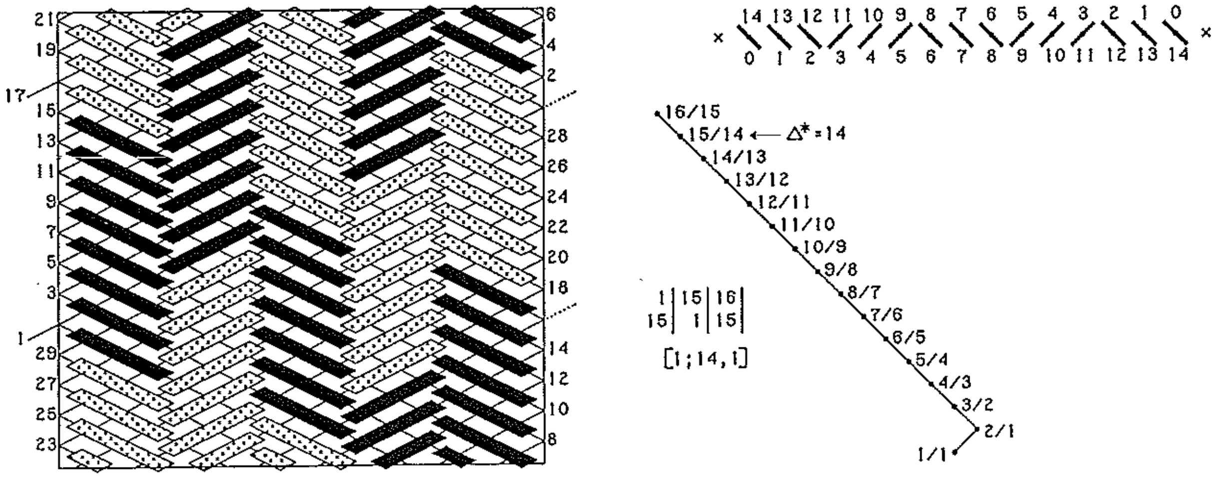


Fig. 1158.

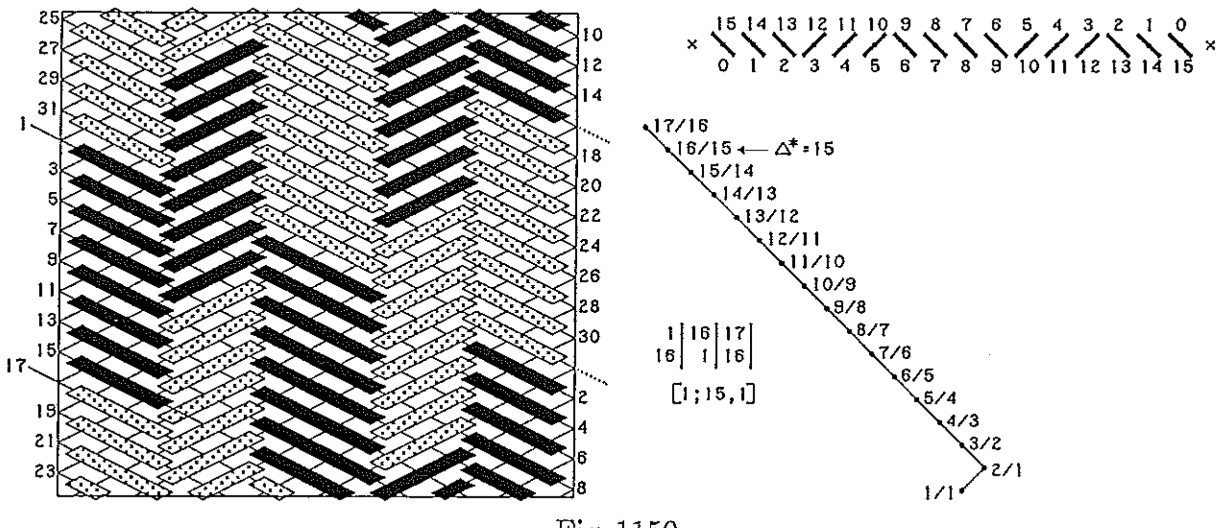


Fig. 1159.

Replacing the central three intersection-columns by four intersection columns gives $p = 17$ and $b = 16$ for $b = p - 1$, and by changing the central three-pass column-coding into a four-pass column-coding, we obtain the grid-diagram in Fig. 1159 with a balanced pattern. First we braid with one of the two strings the half-cycles 1–15, then with the other string we braid the half-cycles 17–31 followed by working away the Standing End of half-cycle 1 with the string-end of half-cycle 31 and the string-end of half-cycle 15 with the Standing End of half-cycle 17.

The Saturn Knots

Saturn Knots are formed by the incorporation of two knot-forms where one forms a collar around the other. The collar forming knot-form is a multi-pointed Star Knot of the type described by P. P. O. Harrison as No. 15 on pp. 6 and 7 in the Harrison Book of Knots. Let's call such a Star Knot the Standard Star Knot. There he describes with the aid of Fig. 1160 the Standard Star Knot with five points as follows:

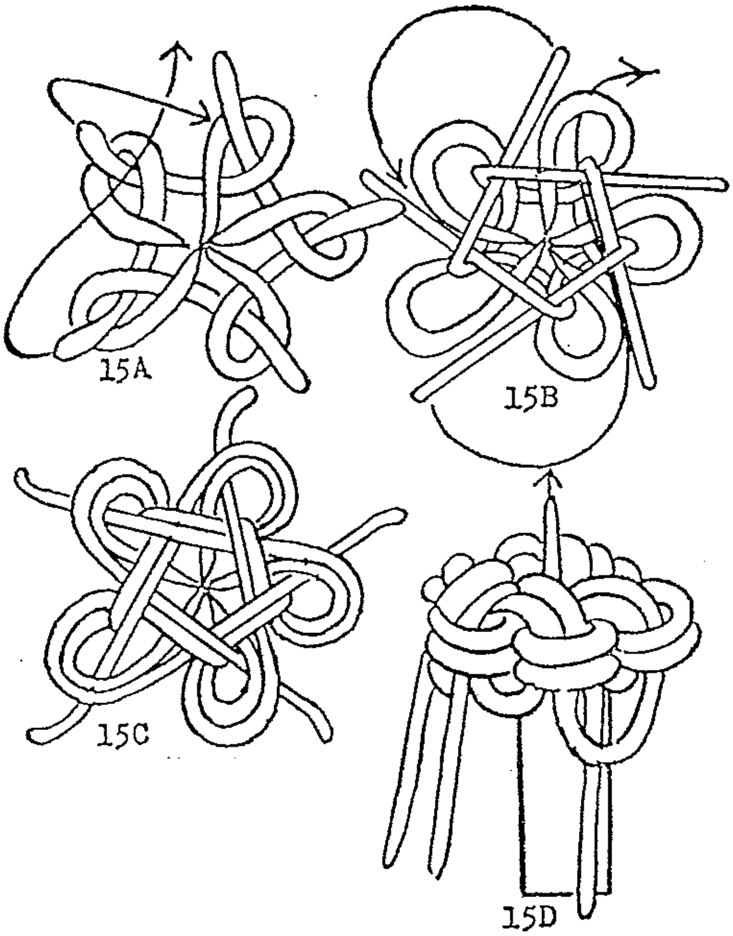


Fig. 1160 — The five pointed Standard Star Knot in the Harrison Book of Knots.

Grasp the stem in the left hand. Commence by forming a Half Hitch with each strand and passing the end of the strand through the loop of the next Half Hitch to the right. Work the Half Hitches up snugly to the centre (Figure 15A). Form a Left-hand Crown with all the strands. Figure 15B shows the completed Left-hand Crown.

Tuck each strand back above the loop of the Half Hitch and under its own part. With each strand in turn follow the lead inside the adjoining strand to the right, over one Half Hitch, and down through the loop of the next Half Hitch. Figure 15C shows the knot at this stage, and Figure 15D shows another view of it. Continue to follow the lead underneath the knot and tuck each strand in turn up through the centre. (Figure 15D).†

A better understanding of the string-run of the Standard Star Knot is obtained from its layout as presented in Fig. 1161 (upper diagram depicts the five pointed Standard Star Knot and the lower diagram depicts the eight pointed Standard Star Knot).

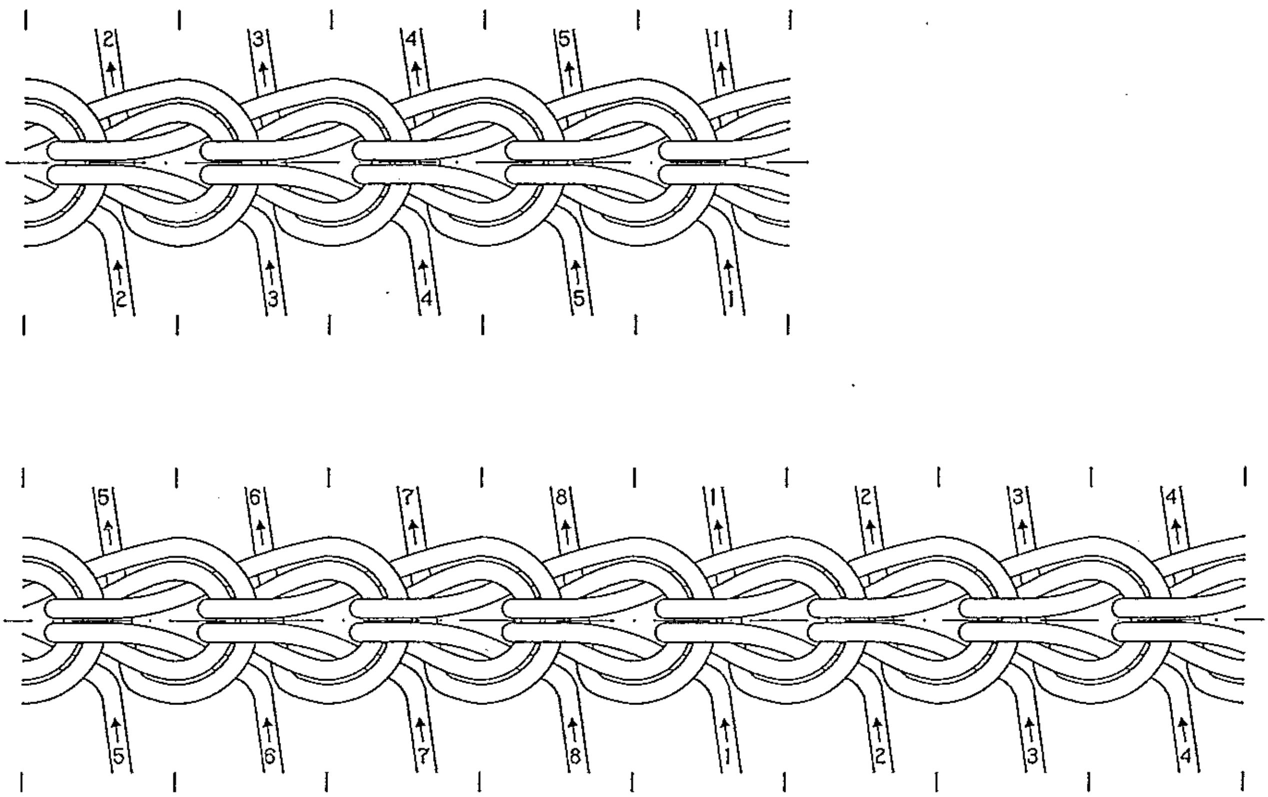


Fig. 1161 — The five and the eight pointed Standard Star Knot.

The string-run clearly shows that an n pointed Standard Star Knot requires n strings.

Replacing the exit-loop of string $|a+1|_n$ by string a and joining it to the entry of string $|a+1|_n$, where $1 \leq a \leq n$, leads to the single string n pointed Star Knot (the n pointed Perfect Star Knot). In Fig. 1162 is shown this procedure for obtaining the five and the eight pointed Perfect Star Knots from the five and the eight pointed Standard Star Knots. Hence the string-run layout of the n pointed Standard Star Knot as depicted in Fig. 1161 leads automatically to the string-run of the n pointed single string Star Knot (the n pointed Perfect Star Knot) in Fig. 1162 and similarly the mirror-imaged string-run of the n pointed Standard Star Knot in Fig. 1161 leads automatically to the mirror-imaged string-run of the n pointed single string Star Knot in Fig. 1162.

† Note that in Harrison's book the right black arrowed line in Figure 15B contains an error which has been rectified here in Fig. 1160. Also note that this error in Harrison's book has been repeated there on pg. 36 in Figure 73B.

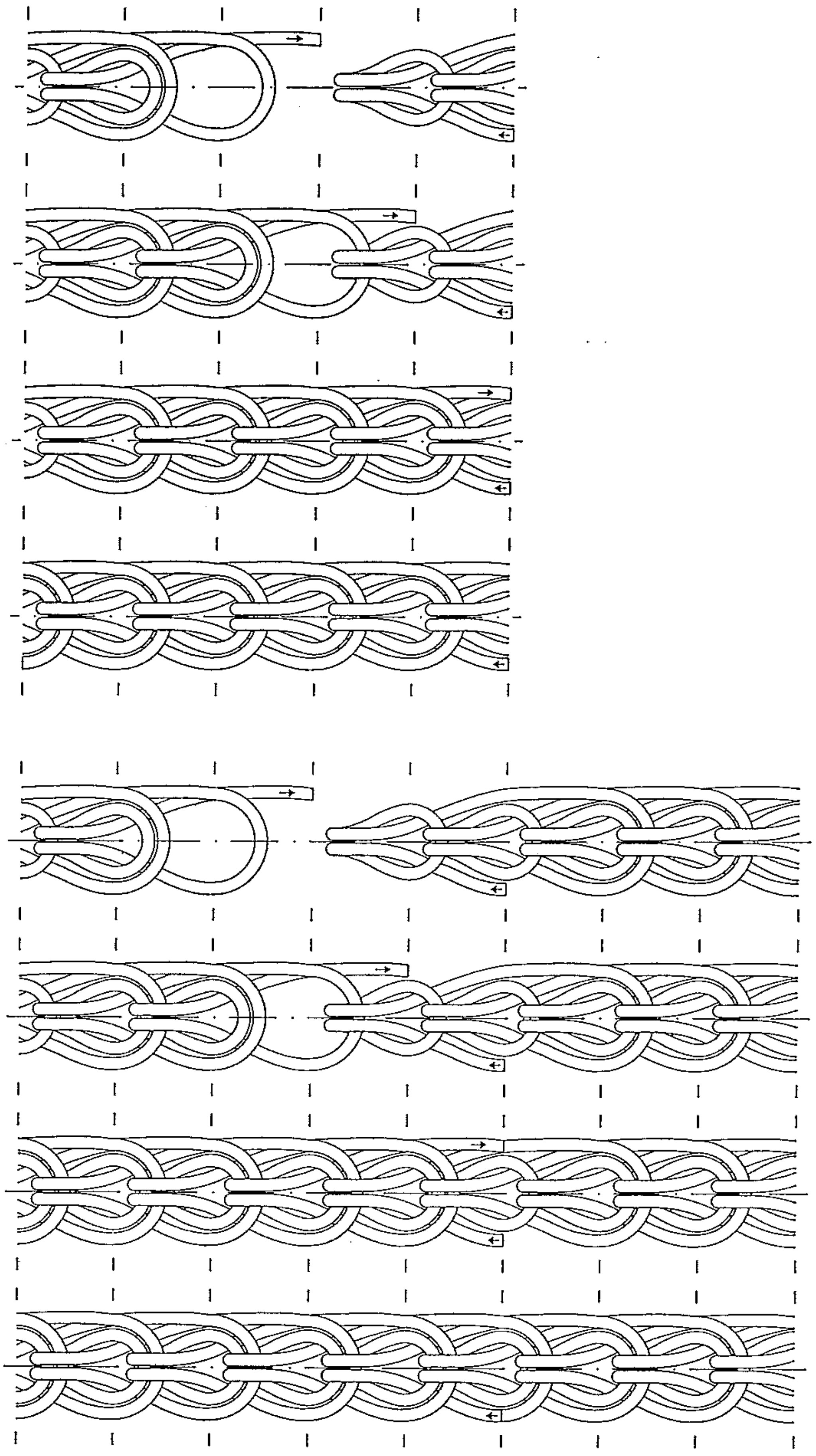


Fig. 1162 — The five and the eight pointed Perfect Star Knot.

Consequently the development of the single string Star Knot by P. P. O. Harrison in Fig. 1163, which has the mirror-imaged string-run layout of the *n* pointed single string Star Knot in Figs. 1162 and 1164, cannot be regarded as an achievement of some importance (refer to the Harrison Book of Knots pg. 22, No. 39; pg. 27, No. 55; pg. 93, No. 7; pp. 108-109).

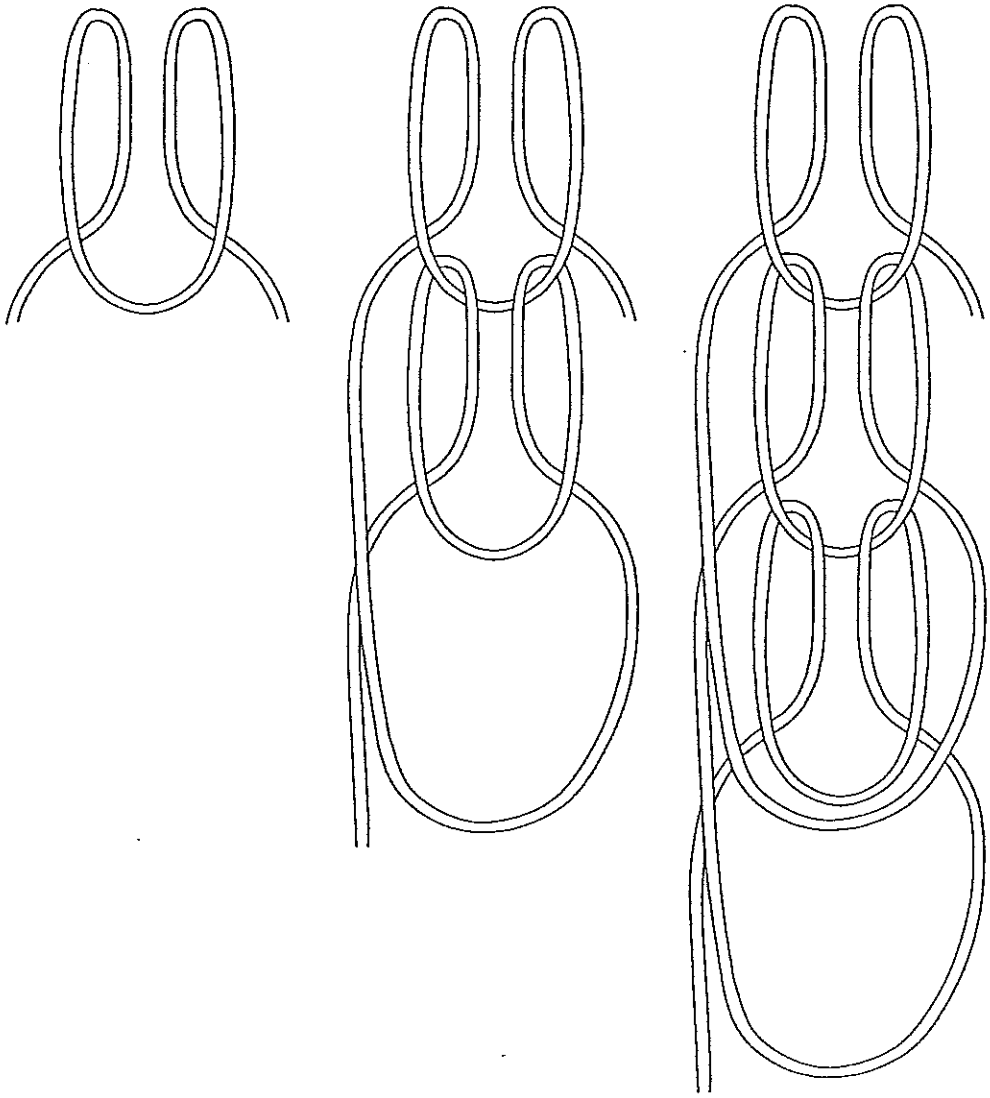


Fig. 1163 — String-run layout of the single string Star Knot by P. P. O. Harrison.

Note that the Perfect Star Knot should not be confused with Solly's single strand Star Knot, nor with any of the various variations of it.†

The string-runs of the five pointed Standard Star Knot and of the eight pointed Standard Star Knot are depicted once again in Fig. 1165 while their mirror-imaged string-runs are shown in Fig. 1166.

The first two braiding stages of these respective string-runs are shown in Fig. 1167 and Fig. 1168.

The last two braiding stages of the five pointed Standard Star Knot and of the eight pointed Standard Star Knot are shown in respectively Fig. 1169 and Fig. 1170.

† Refer to **Solly's Single Strand Star & variations on the theme** by Stuart Grainger, published by the International Guild of Knot Tyers.

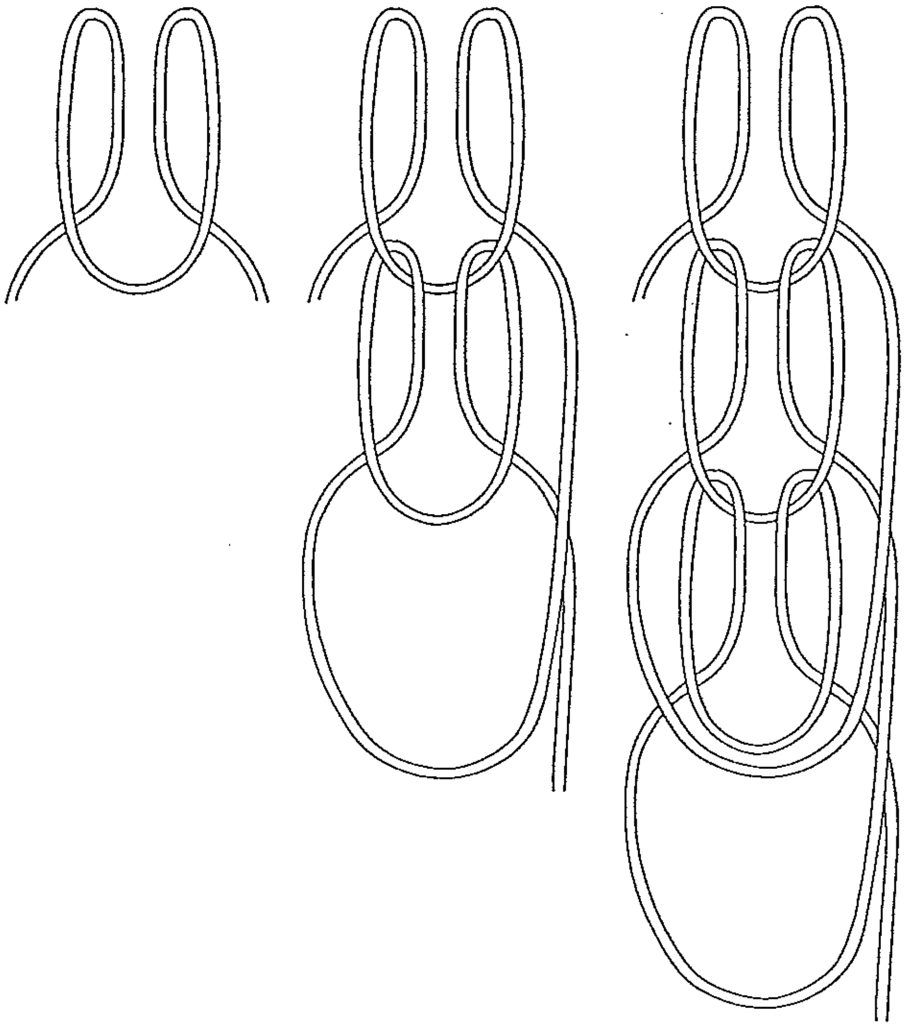


Fig. 1164 — The mirror-imaged string-run of Fig. 1163.

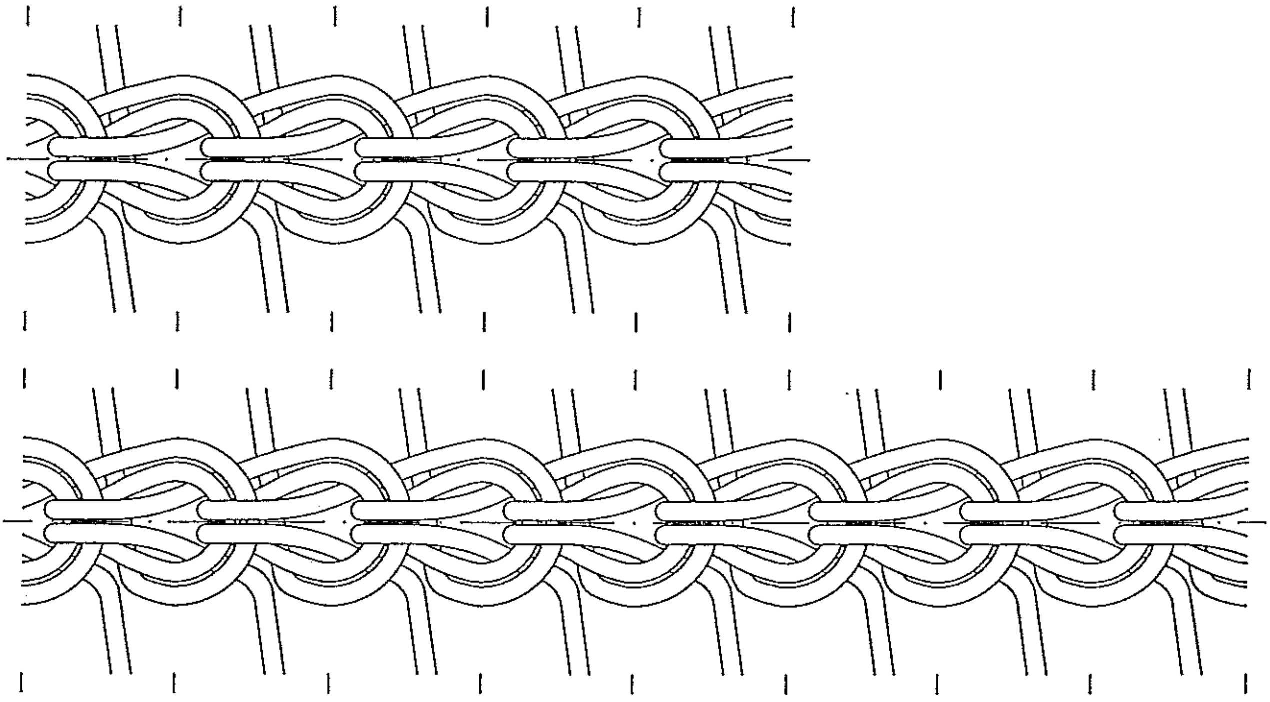


Fig. 1165 — The five and the eight pointed Standard Star Knot.

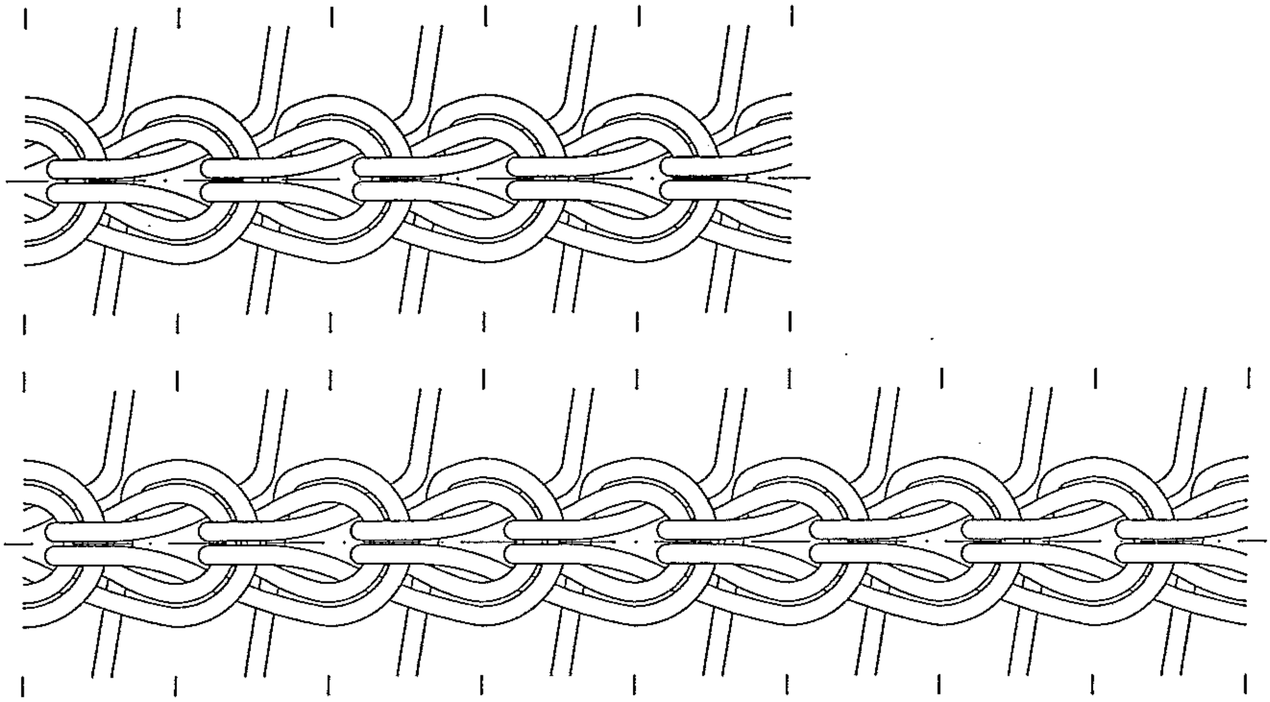


Fig. 1166 — The mirror-imaged string-runs of Fig. 1165.

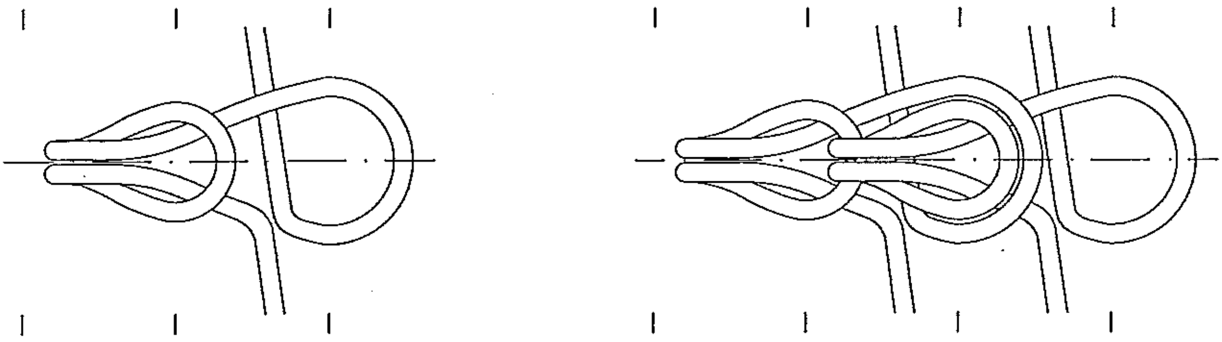


Fig. 1167 — The first two braiding stages of the string-runs in Fig. 1165.

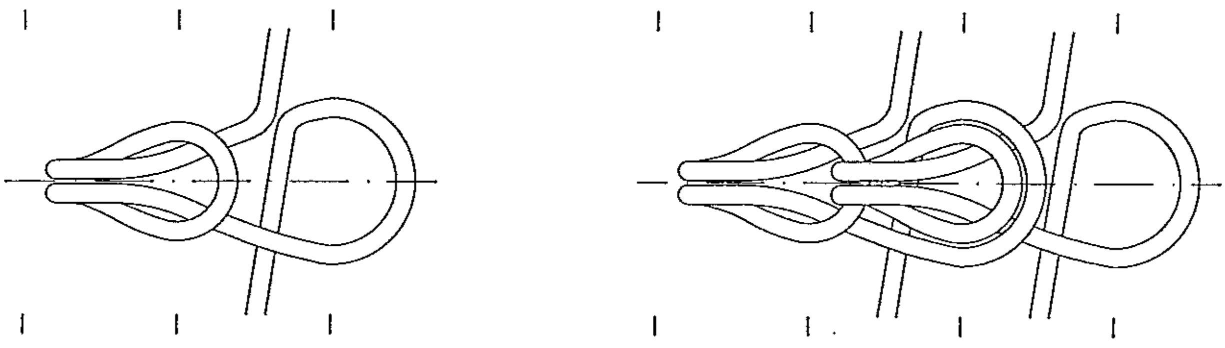
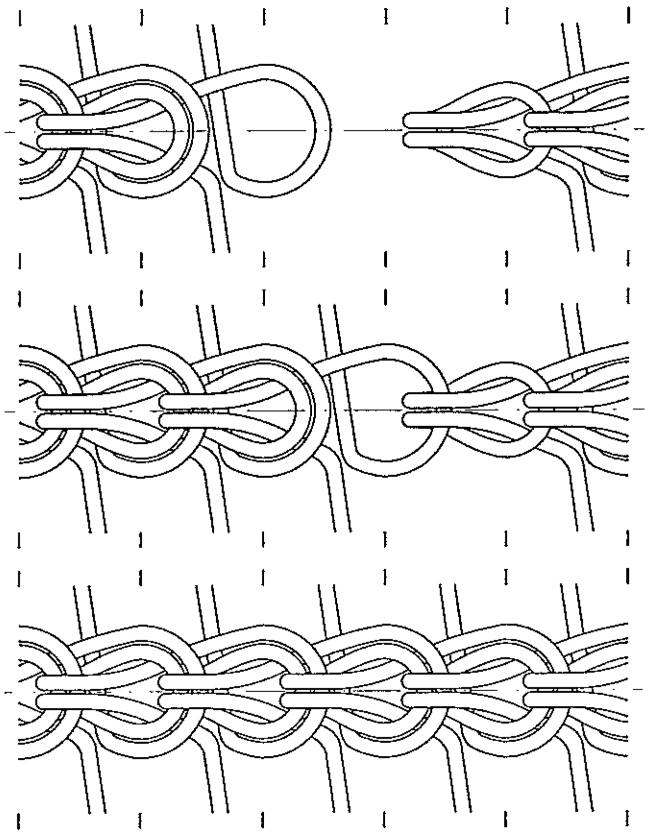


Fig. 1168 — The first two braiding stages of the string-runs in Fig. 1166.

Modifying the string-runs in Figs. 1165, 1166, 1167, 1168, 1169 and 1170 to those in respectively Figs. 1171, 1172, 1173, 1174, 1175 and 1176 results in a greater formfastness of the knots.



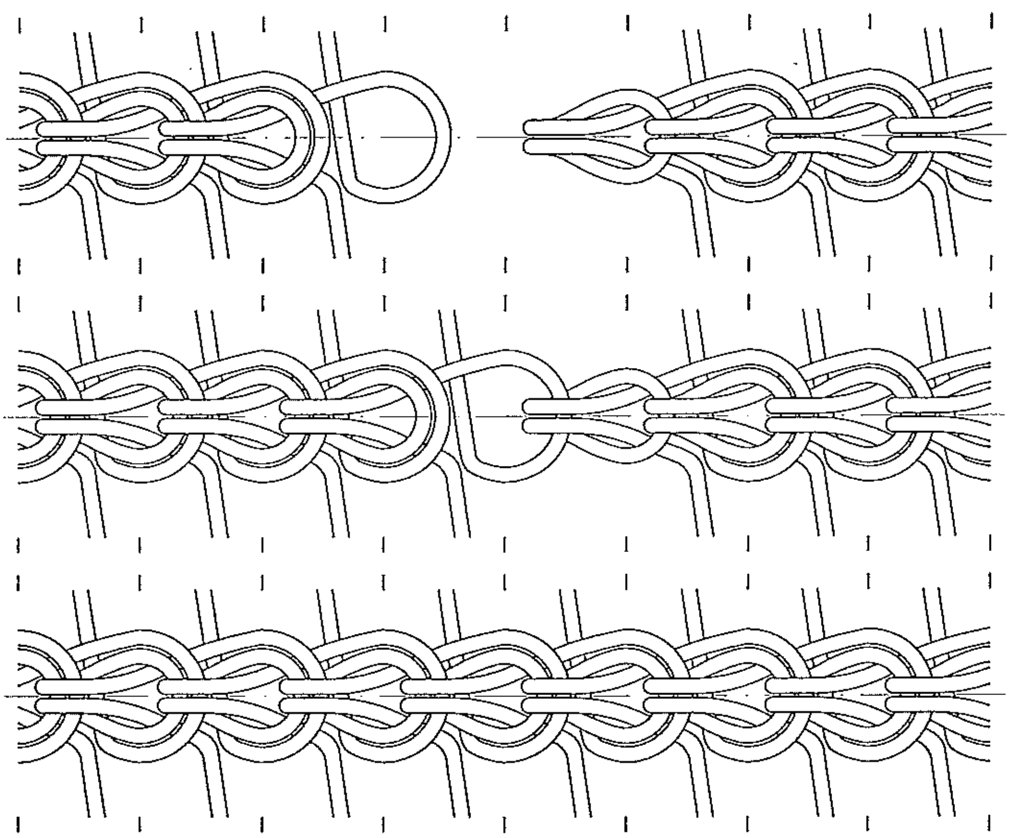


Fig. 1169 — The last two braiding stages of the string-run in Fig. 1165.

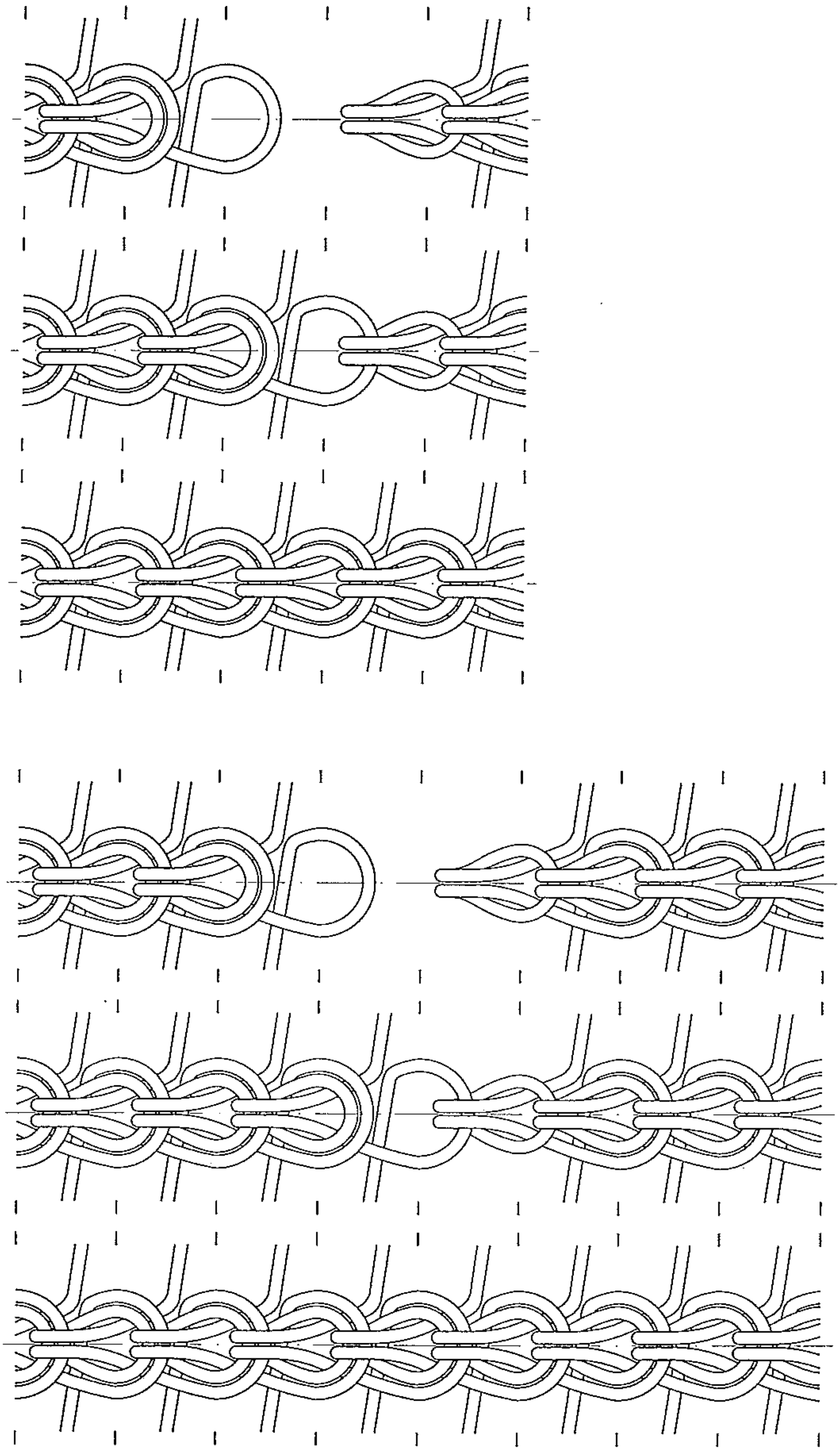


Fig. 1170 — The last two braiding stages of the string-run in Fig. 1166.

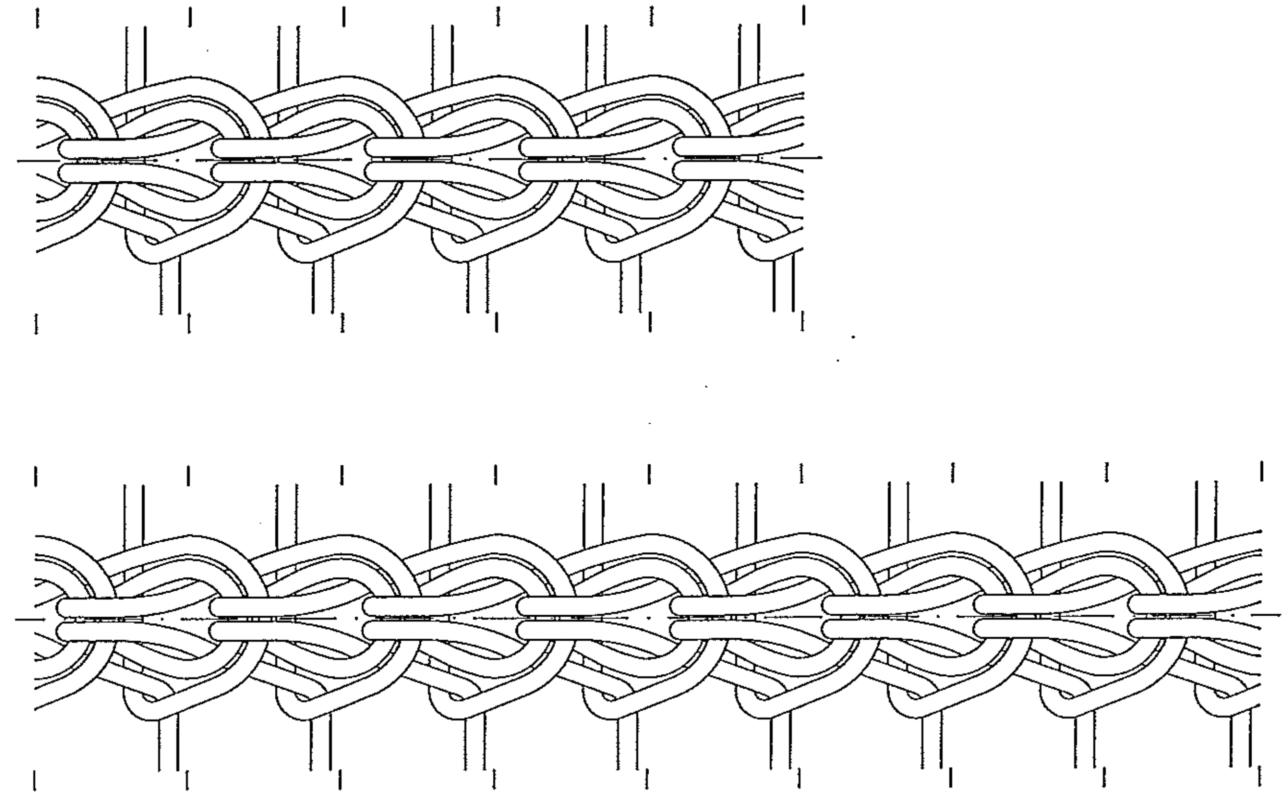


Fig. 1171 — The more formfast five and eight pointed Standard Star Knot.

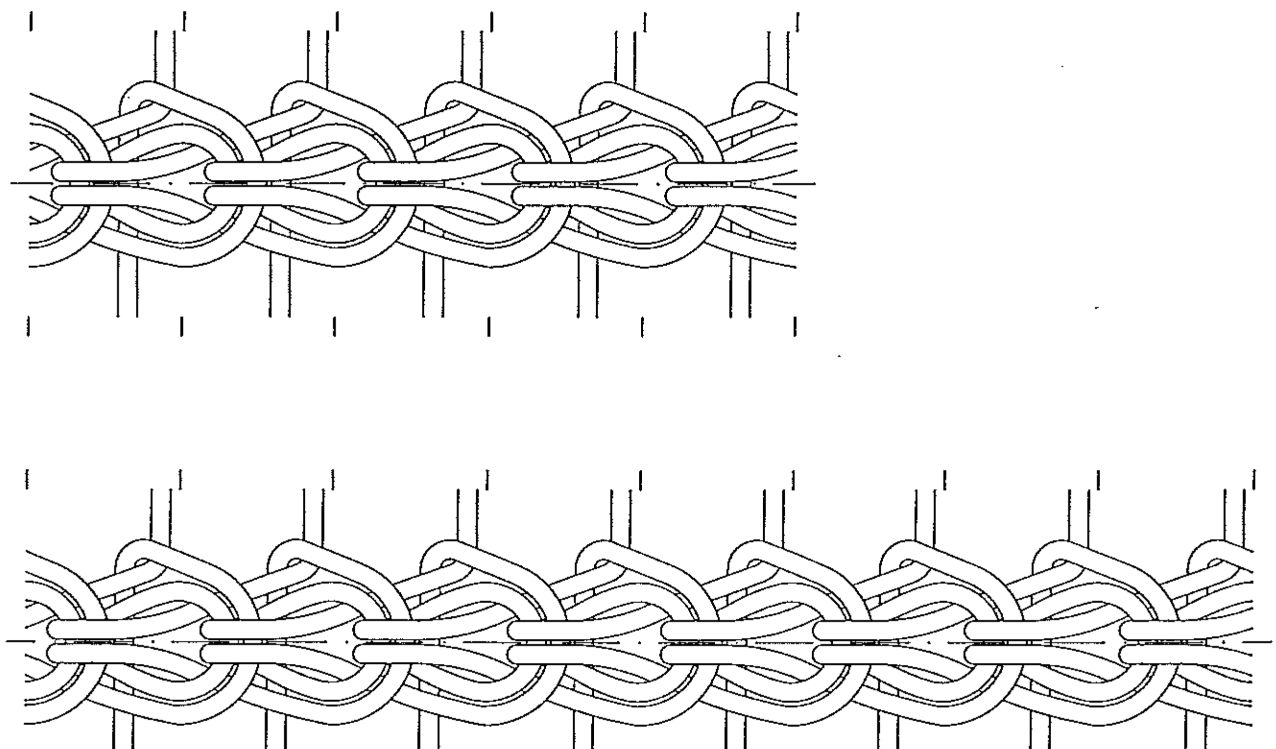


Fig. 1172 — The mirror-imaged string-runs of Fig. 1171.

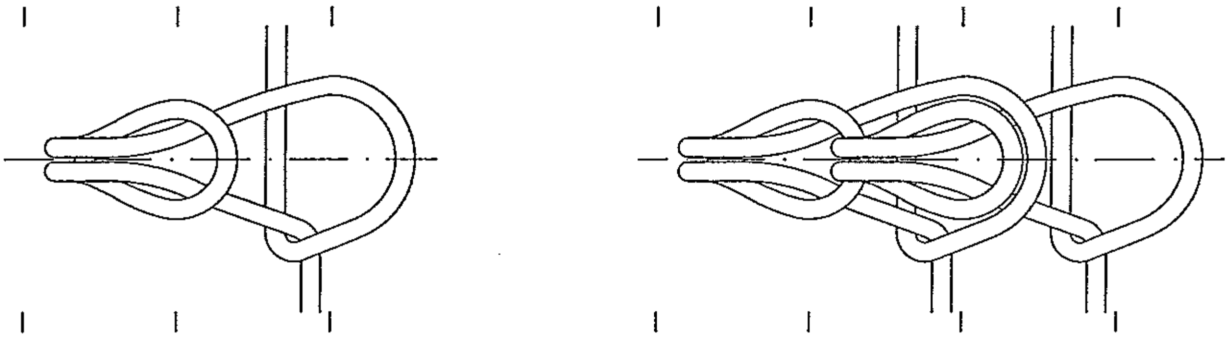


Fig. 1173 — The first two braiding stages of the string-runs in Fig. 1171.

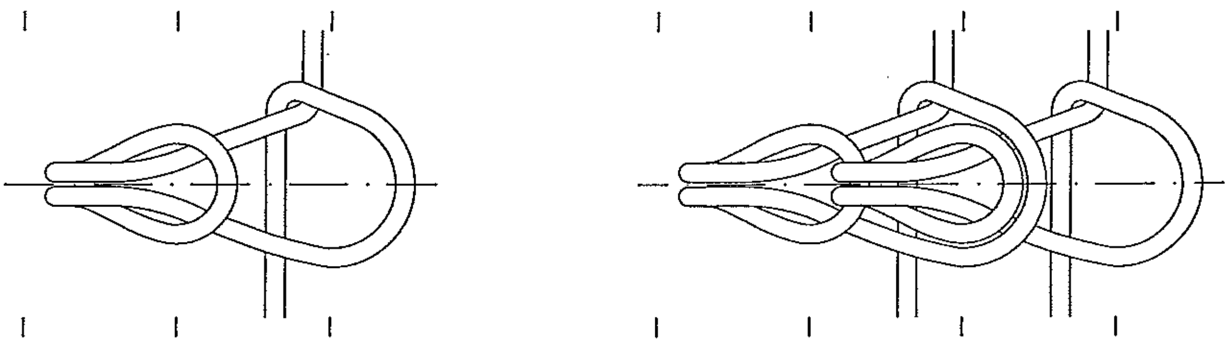


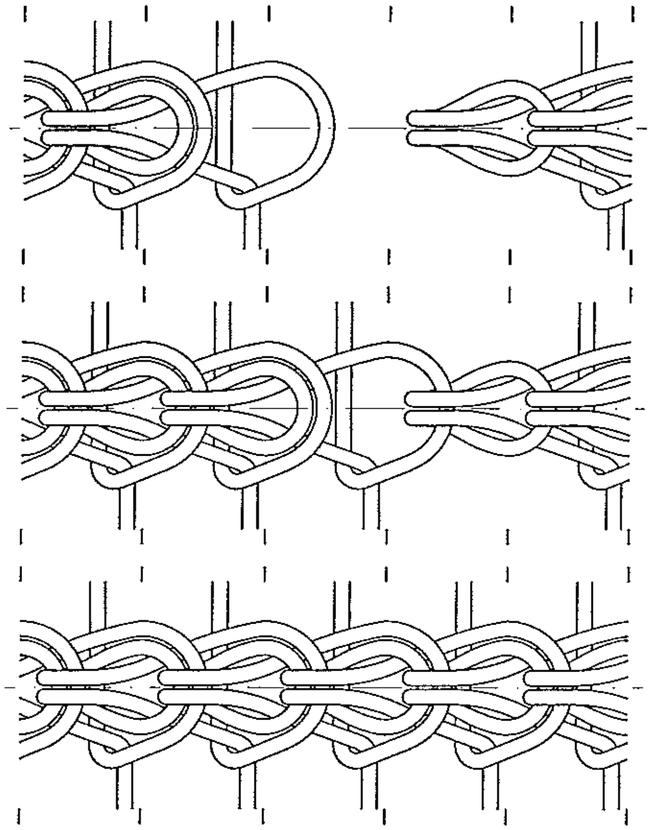
Fig. 1174 — The first two braiding stages of the string-runs in Fig. 1172.

The collar forming multi-pointed Standard Star Knot in a Saturn Knot is an integral part of the Saturn Knot's string-run. Hence a Saturn Knot consists of a foundation knot, where the string-run of this foundation knot is modified by incorporating the string-run of the collar forming multi-pointed Standard Star Knot. To facilitate the braiding process, we can first braid the foundation knot and then replace its string by a new string which incorporates in its string-run the string-run sections of the Standard Star Knot. The easiest to braid Saturn Knots are then those where the incorporated string-run sections of the Standard Star Knot are consecutive; this facilitates in obtaining the desired tightness in the multi-pointed Star Knot.

Example 1.

Let in this simple, and easy to braid Saturn knot, the foundation knot be the upper-left or lower-left Regular Knot with $p/b = 25/24$ in Fig. 1177. Euclid's algorithm with the path-formula in the RKT and the respective algorithm diagrams for these Regular Knots are presented in Fig. 1178. Both algorithm diagrams give us the same half-cycle braiding sequences which are as follows:

half-cycle 1		:	Free Run.
half-cycle 2	$i = 0$	:	o.
half-cycle 3	$i = 0$	:	u.
half-cycle 4	$i \leq 1$	:	$2o$.
half-cycle 5	$i \leq 1$	:	$2u$.
half-cycle 6	$i \leq 2$	:	$3o$.
half-cycle 7	$i \leq 2$	:	$3u$.
half-cycle 8	$i \leq 3$	:	$u - 3o$.



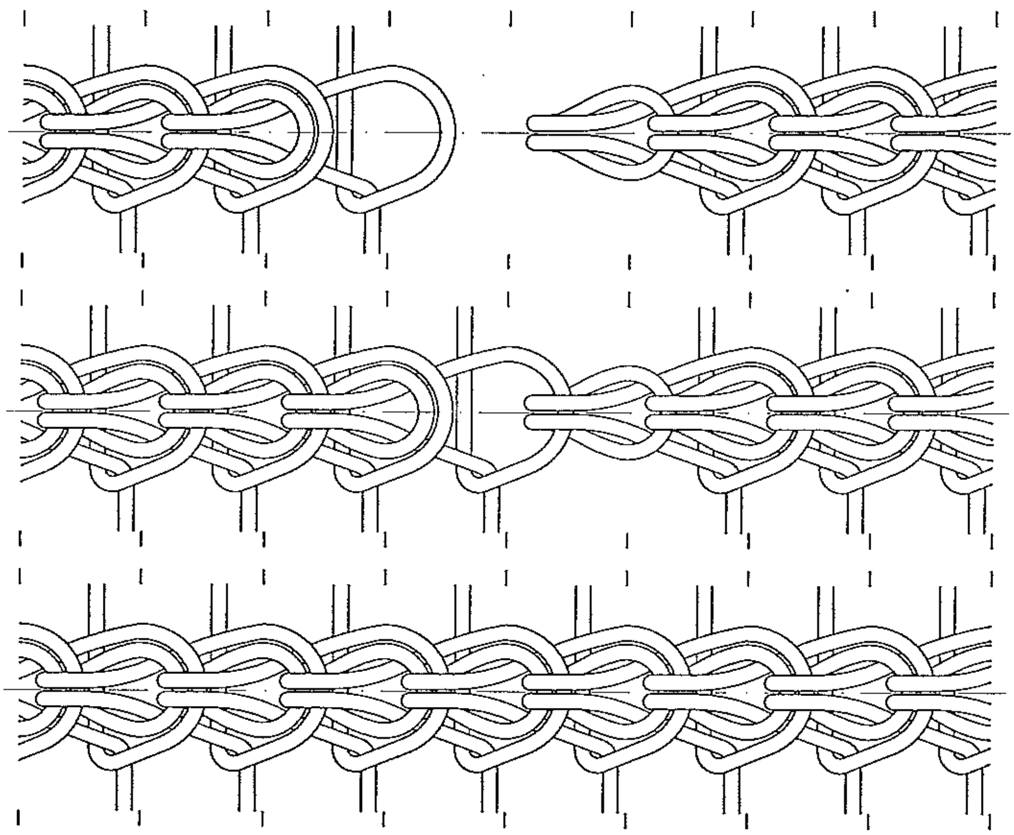


Fig. 1175 — The last two braiding stages of the string-run in Fig. 1171.

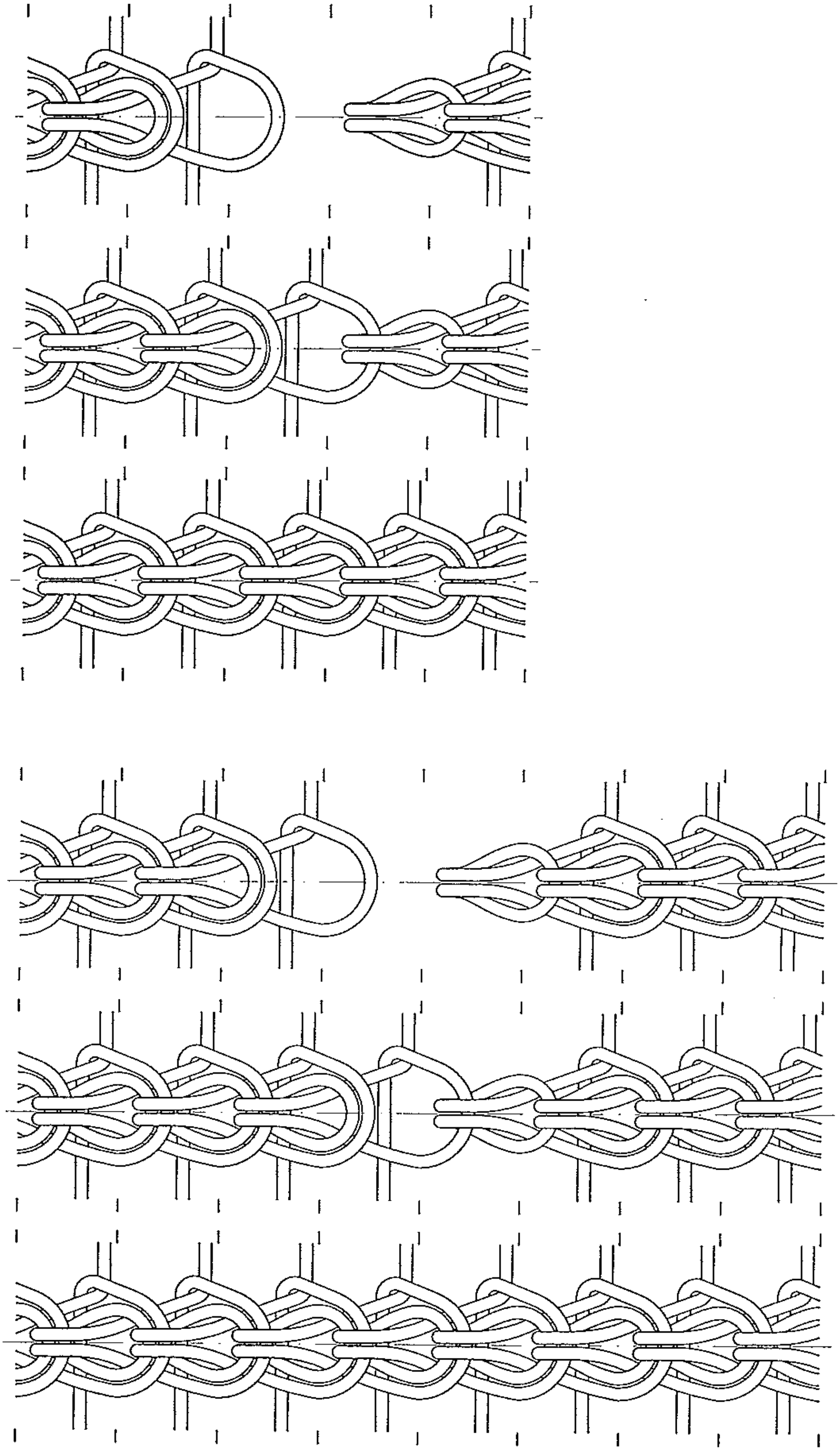


Fig. 1176 — The last two braiding stages of the string-run in Fig. 1172.

half-cycle 9	$i \le 3$	:	$o - 3u$.
half-cycle 10	$i \le 4$	:	$2u - 3o$.
half-cycle 11	$i \le 4$	:	$2o - 3u$.
half-cycle 12	$i \le 5$	:	$o - 2u - 3o$.

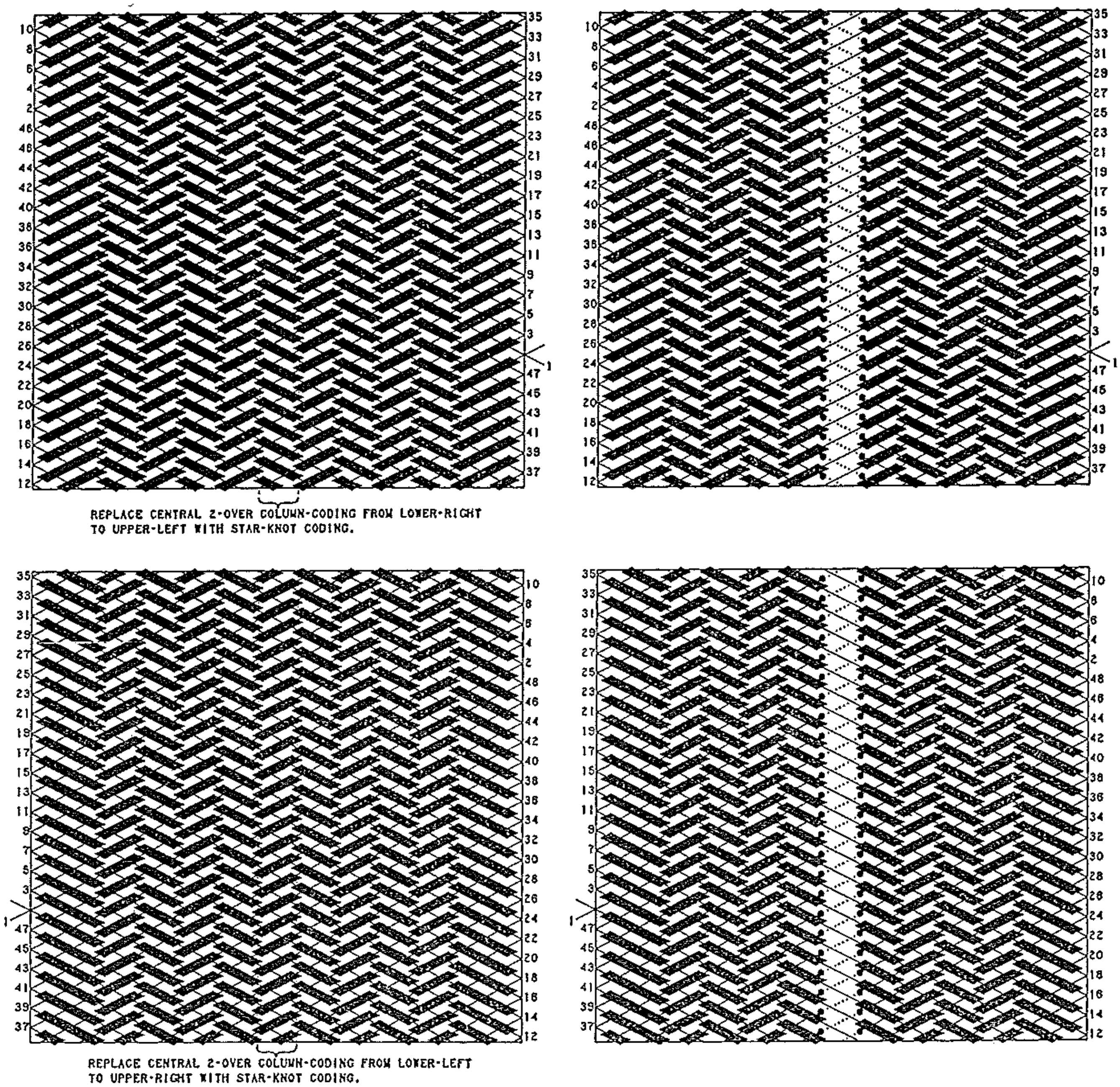


Fig. 1177 — The foundation knot $p/b = 25/24$ in Example 1.

half-cycle 13	$i \le 5$	:	$u - 2o - 3u$.
half-cycle 14	$i \le 6$	:	$2u - 2u - 3o$.
half-cycle 15	$i \le 6$	:	$2u - 2o - 3u$.
half-cycle 16	$i \le 7$	:	$u - 2o - 2u - 3o$.
half-cycle 17	$i \le 7$	:	$o - 2u - 2o - 3u$.
half-cycle 18	$i \le 8$	:	$2u - 2o - 2u - 3o$.
half-cycle 19	$i \le 8$	:	$2o - 2u - 2o - 3u$.
half-cycle 20	$i \le 9$	:	$o - 2u - 2o - 2u - 3o$.
half-cycle 21	$i \le 9$	:	$u - 2o - 2u - 2o - 3u$.

half-cycle 22	$i \le 10$	:	$2o - 2u - 2o - 2u - 3o\,.$
half-cycle 23	$i \le 10$	:	$2u - 2o - 2u - 2o - 3u\,.$
half-cycle 24	$i \le 11$	:	$u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 25	$i \le 11$	:	$o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 26	$i \le 12$	:	$2u - 2o - 2u - 2o - 2u - 3o\,.$

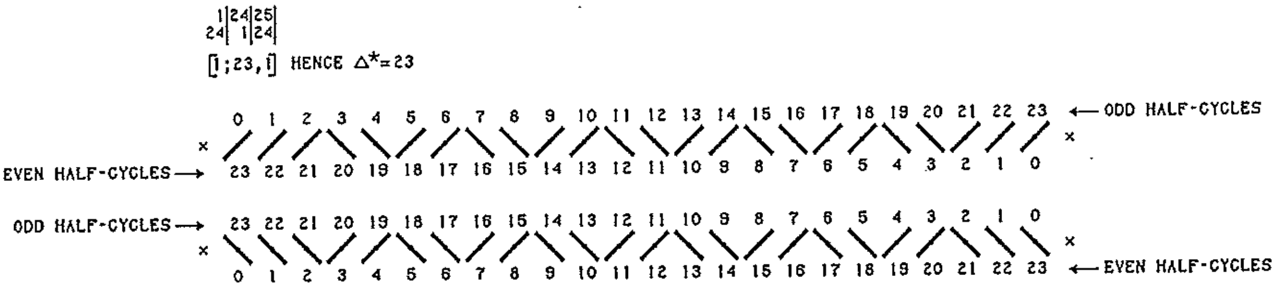


Fig. 1178 — Euclid's algorithm, path formula in RKT and algorithm diagram for the foundation knot.

half-cycle 27	$i \le 12$	:	$2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 28	$i \le 13$	:	$o - 2u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 29	$i \le 13$	:	$u - 2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 30	$i \le 14$	:	$2o - 2u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 31	$i \le 14$	:	$2u - 2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 32	$i \le 15$	:	$u - 2o - 2u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 33	$i \le 15$	:	$o - 2u - 2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 34	$i \le 16$	:	$2u - 2o - 2u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 35	$i \le 16$	:	$2o - 2u - 2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 36	$i \le 17$	:	$o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 37	$i \le 17$	:	$u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 38	$i \le 18$	:	$2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 39	$i \le 18$	:	$2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 40	$i \le 19$	:	$u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 41	$i \le 19$	:	$o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 42	$i \le 20$	:	$2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 43	$i \le 20$	:	$2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 44	$i \le 21$	:	$o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 45	$i \le 21$	:	$u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 46	$i \le 22$	:	$2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 3o\,.$
half-cycle 47	$i \le 22$	:	$2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 3u\,.$
half-cycle 48	$i \le 23$	:	$3o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - 3o\,.$

After the foundation knot has been braided and properly tightened, we replace its string with a longer string which incorporates in its odd half-cycles associated with the upper-left diagram in Fig. 1177 the star knot elements as in Figs. 1166, 1168, 1170 or as in Figs. 1172, 1174, 1176; or which incorporates in its odd half-cycles associated with the lower-left diagram in Fig. 1177 the star knot elements as in Figs. 1165, 1167, 1169 or as in Figs. 1171, 1173, 1175. For 2mm. diameter string this Saturn Knot should fit properly over a cylinder with a diameter of about 25mm.

Example 2.

Let the foundation knot be a single string Regular Nested Knot (hence a Perfect Regular Nested Knot), hence g.c.d. $(\Delta, A) = 1$ and g.c.d. $(P_{\text{total}}, B^*) = 1$).

After the foundation knot has been braided and properly tightened, we replace its string with a longer string that incorporates the star knot elements in its either odd or even half-cycles. In order to make the incorporation of the star knot elements as easy as possible, we ensure that any two consecutive odd or even half-cycles in the foundation knot are adjacent. Hence there are two cases to be considered:

1. The Standing End half-cycle 1 runs from lower-left bight-boundary 1 to upper-right bight-boundary 1. Since half-cycle 3 must be immediately adjacent to half-cycle 1, it follows that half-cycle 2 runs from lower-right bight-boundary 1 to upper-left bight-boundary A. Hence half-cycle 3 runs from lower-left bight-boundary A to upper-right bight-boundary 2. Thus for the first cycle, running from lower-left bight-boundary 1 to upper right bight-boundary 1, then from lower-right bight-boundary 1 to upper-left bight-boundary A we obtain:
$2(A-1)+x+2(A-1)+2(A-1)+x+2(A-A) = 2nAB^*+2A$, hence $x = nAB^*-2A+3$, where n is a natural number.

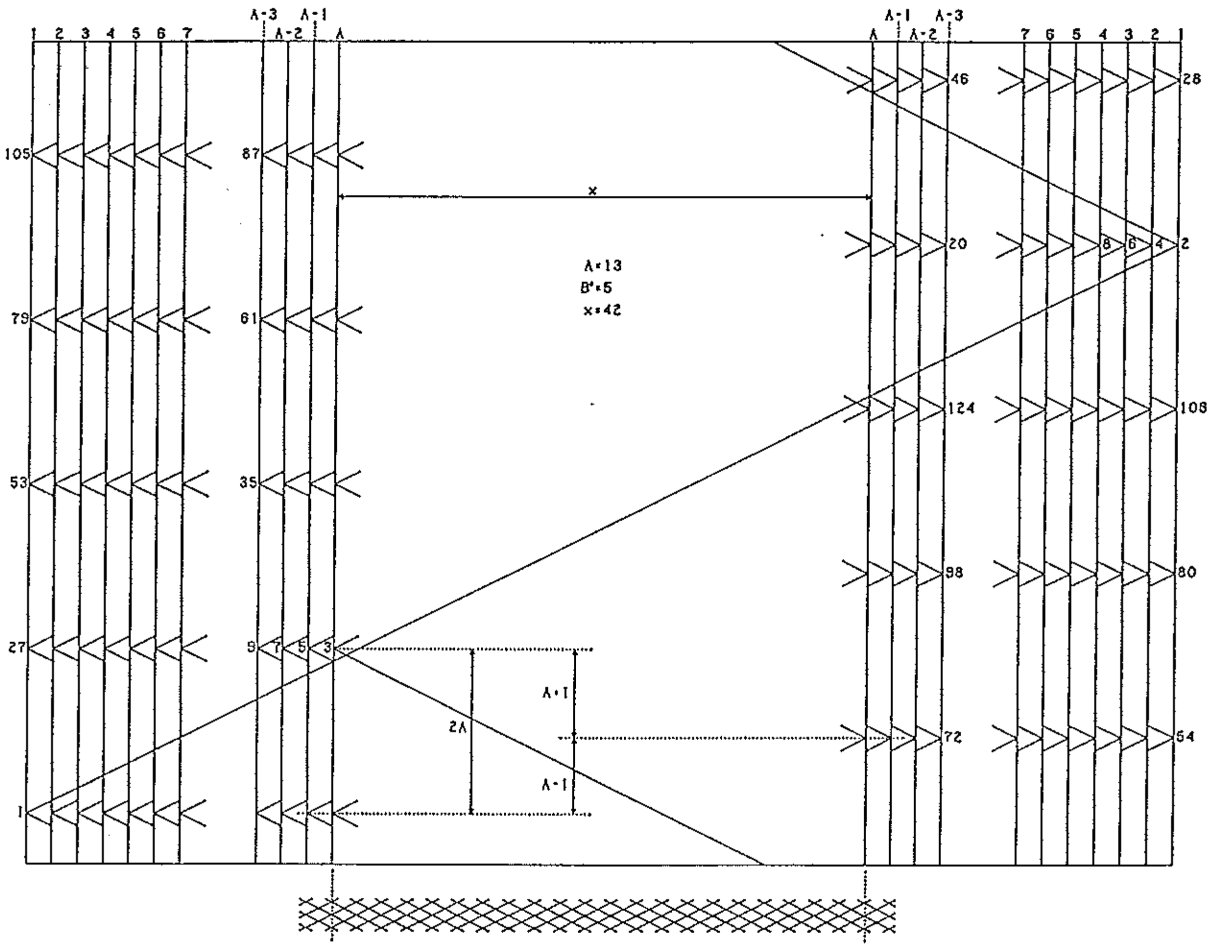


Fig. 1179 — A Perfect Regular Nested Knot with $A = 13$, $B^* = 5$, $x = 42$, $k = 1$.

2. The Standing End half-cycle 1 runs from lower-left bight-boundary 1 to upper-right bight-boundary A. Since half-cycle 3 must be immediately adjacent to half-cycle 1, it follows that half-cycle 2 runs from lower-right bight-boundary A to upper-left bight-boundary 2. Hence half-cycle 3 runs from lower-left bight-boundary 2 to upper-right bight-boundary (A-1). Thus for the first cycle, running from lower-left bight-boundary 1 to upper-right bight-boundary A, then from lower-right bight-boundary A to upper-left bight-boundary 2 we obtain:
$2(A-1)+x+2(A-A)+2(A-A)+x+2(A-2) = 2nAB^*$, hence $x = nAB^* - 2A+3$, where n is a natural number.

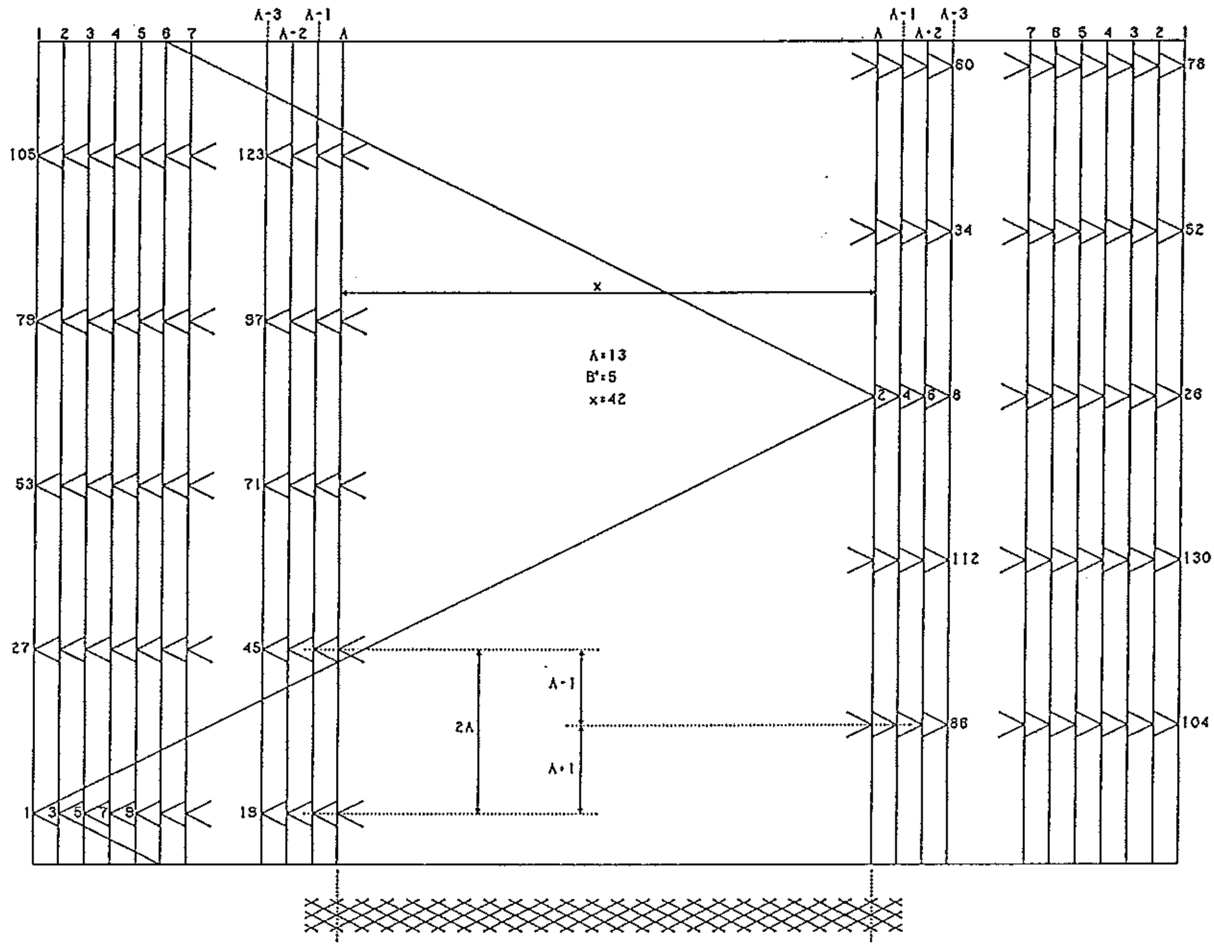


Fig. 1180 — A Perfect Regular Nested Knot with $A = 13$, $B^* = 5$, $x = 42$, $k = A$.

For case 1 the sequentially adjacent half-cycles are the odd-numbered half-cycles (see for example the string-run in Fig.1179 of a Perfect Regular Nested Knot with $A = 13$, $x = 42$, $k = 1$ and hence $y = |x-2(k+1)|_{2A}{}^{\dagger} = |38|_{26} = 12 = A-1$), while for case 2 the sequentially adjacent half-cycles are the even-numbered half-cycles (see for example the string-run in Fig.1180 of a Perfect Regular Nested Knot with $A = 13$, $x = 42$, $k = A = 13$ and hence $y = |x-2(k+1)|_{2A} = |14|_{26} = 14 = A+1$).

Let the foundation knot be spherical ball. For a spherical ball the circumference over the poles is equal to the circumference over the equator. With the odd half-cycles crossing the even half-cycles at the equator at about 90°, the circumference over the poles is about equal to $2 \times (2 \times \frac{A-1}{2} + \frac{x}{2})$ bight units, hence is about equal to

† Refer to *The Braider*, Issue No. 25, pg. 567.

$2A - 2 + x = B$ bight units.† Hence for a single string spherical Regular Nested Knot with $A = 6$ and $B^* = 5$ we require x to be about $B - 2A + 2 = 6 \times 5 - 2 \times 6 + 2 = 20$, while $x = nAB^* - 2A + 3 = 1 \times 6 \times 5 - 2 \times 6 + 3 = 21$ for $n = 1$ in both cases **1** and **2**.

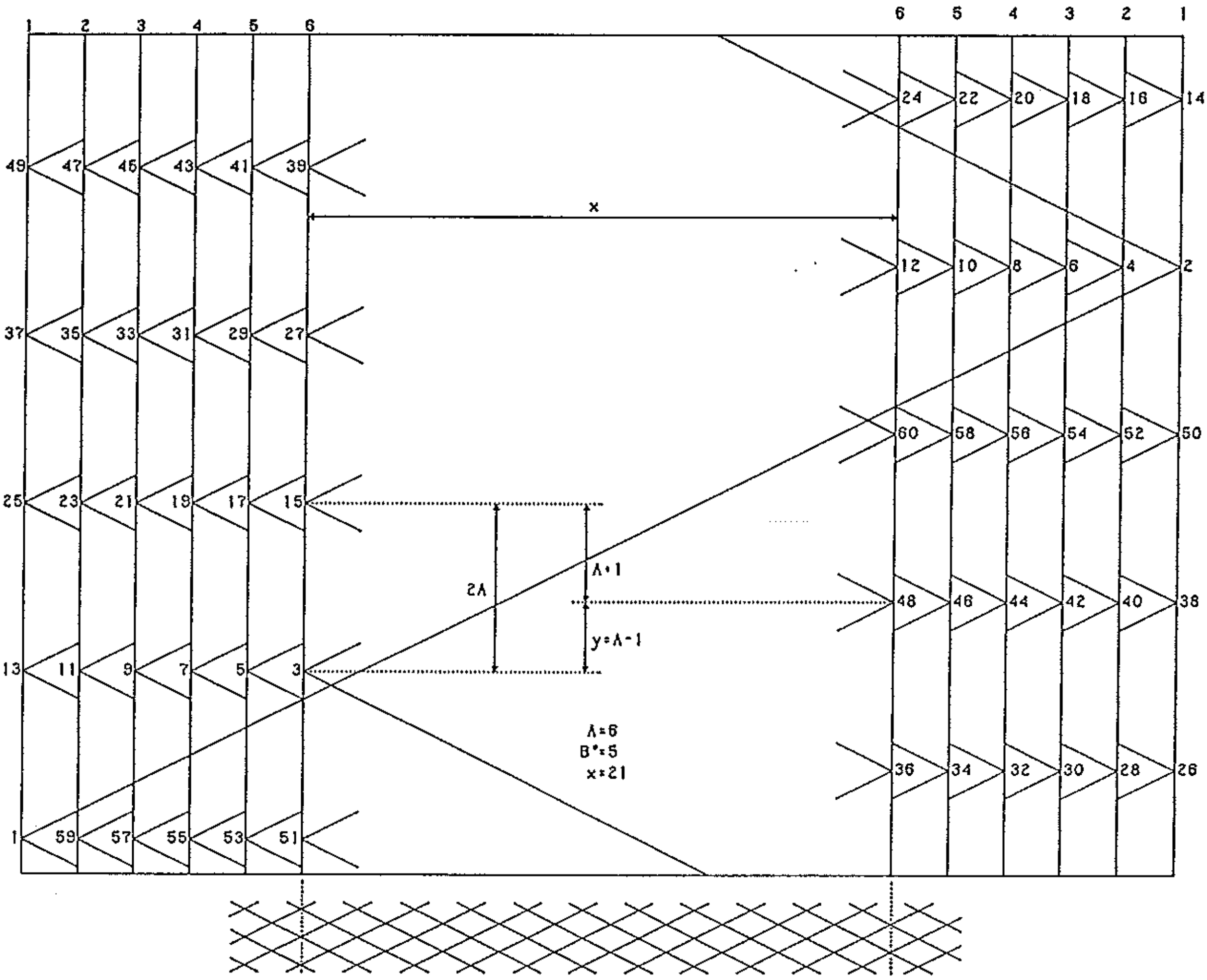


Fig. 1181 — A Perfect Regular Nested Knot with $A = 6$, $B^* = 5$, $x = 21$, $k = 1$.

The full string-run of the single string spherical Regular Nested knot of Fig. 1181 with $A = 6$, $B^* = 5$, $x = 21$, $k = 1$, hence with $y = |x - 2(k+1)|_{2A} = |17|_{12} = 5 = A - 1$, and $\Delta = |x - 2(k+1)|_A = |17|_6 = 5 = |y|_A$†† and $B = A.B^* = 30$, is depicted in Fig. 1182. After superimposing a suitable coding on the string-run in Fig. 1182 we obtain Fig. 1183.

At the upper left-hand side in Fig. 1184 is shown the first-return string-run with its associated nest-index numbers.

At the upper right-hand side in Fig. 1184 are depicted the first-return string-run, the half-cycle numbers of the half-cycles, and the number of crossings which these half-cycles make in the finished knot.

From the half-cycle pattern at the bottom of Fig. 1184 we assemble, in assocition with the superimposed coding, the half-cycle tables in Fig. 1185.‡

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† Recall that also $2A + x - 2 = P_{\text{total}}$.

†† Refer to *The Braider*, Issue No. 25, pg. 567.

‡ Refer to *The Braider*, Issue No. 28.

From the half-cycle tables in Fig. 1185 we then read the half-cycle braiding algorithms for the Perfect Regular Nested Knot in Fig. 1183 as follows:

1. 1 ↗ 1: Free run.
2. 1 ↖ 6: o.
3. 6 ↗ 2: Free run.
4. 2 ↖ 5: u.
5. 5 ↗ 3: Free run.
6. 3 ↖ 4: o.
7. 4 ↗ 4: Free run.
8. 4 ↖ 3: u.
9. 3 ↗ 5: Free run.
10. 5 ↖ 2: o.
11. 2 ↗ 6: Free run.
12. 6 ↖ 1: u.

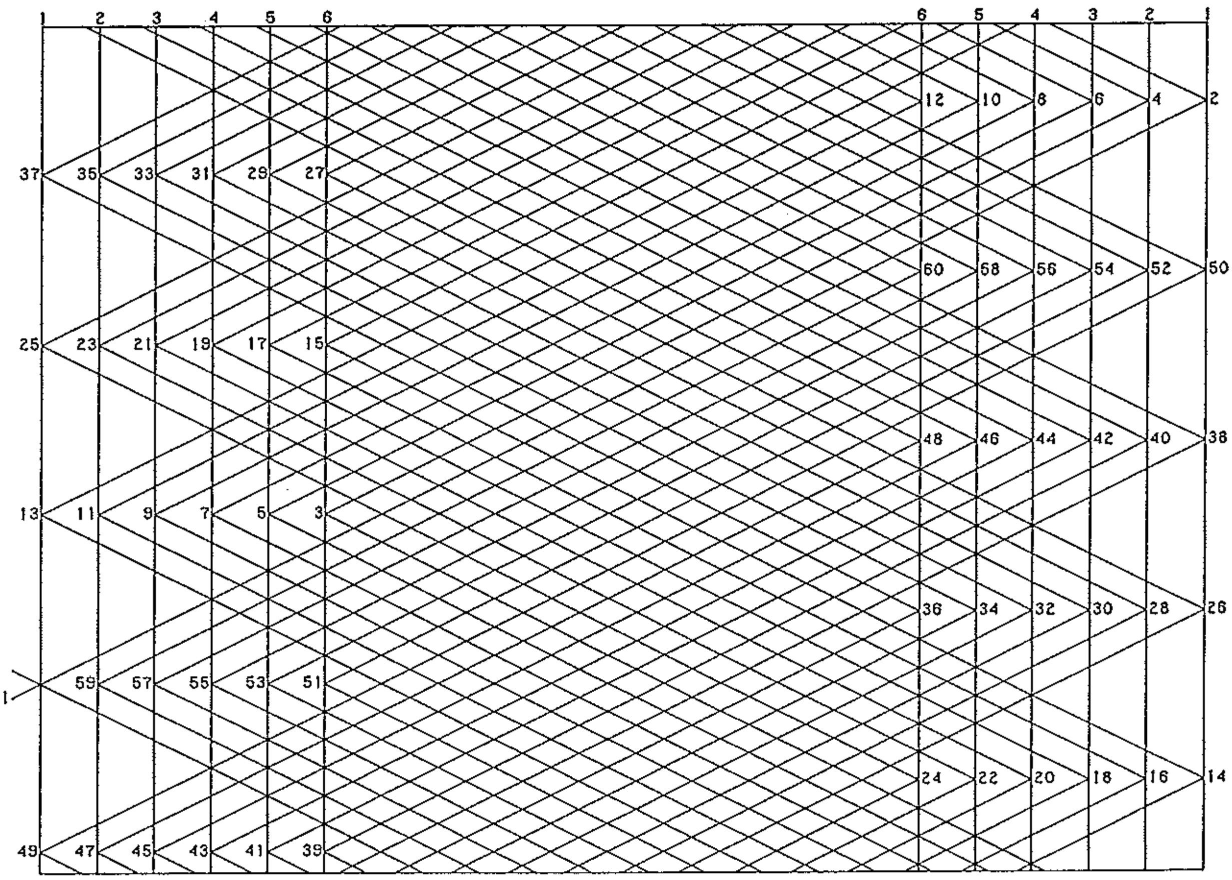

Fig. 1182 — The string-run of the single string spherical Regular Nested Knot with $A = 6$, $B^* = 5$, $x = 21$, $k = 1$.

13. 1 ↗ 1: $u - o - u - o - u - o$.
14. 1 ↖ 6: $o - 3u - 3o$.
15. 6 ↗ 2: $2u - o - u - o - u$.
16. 2 ↖ 5: $3u - 3o - u$.
17. 5 ↗ 3: $3u - o - u - o$.
18. 3 ↖ 4: $2u - 3o - u - o$.
19. 4 ↗ 4: $o - 3u - o - u$.
20. 4 ↖ 3: $u - 3o - u - o - u$.

21. $3 \nearrow 5$: $2o - 3u - o$.
22. $5 \nwarrow 2$: $3o - u - o - u - o$.
23. $2 \nearrow 6$: $3o - 3u$.
24. $6 \nwarrow 1$: $2o - u - o - u - o - u$.
25. $1 \nearrow 1$: $u - 3o - 3u - o - u - o - u - o$.

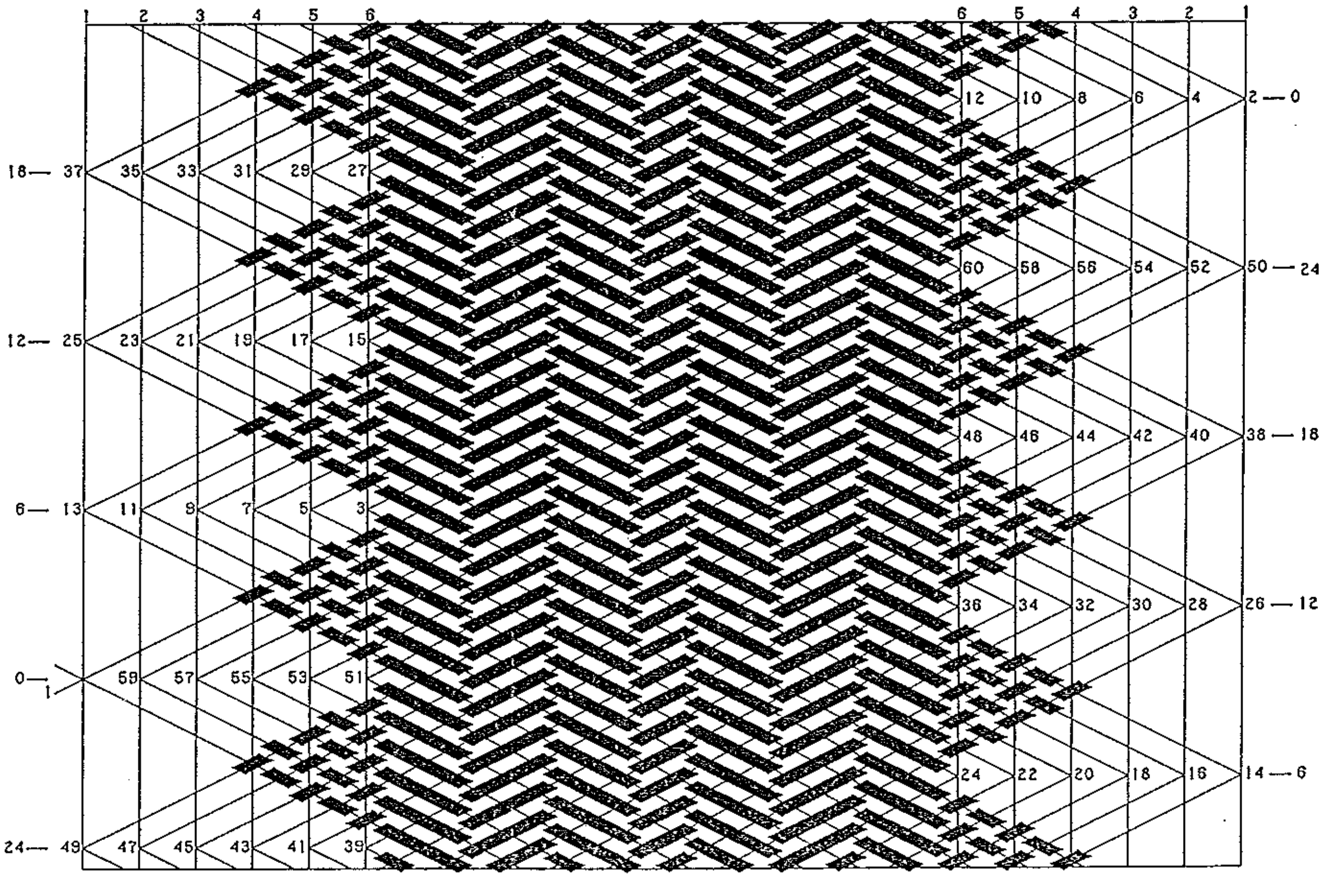

Fig. 1183 — The grid-diagram of the single string spherical Regular Nested Knot with $A = 6$, $B^* = 5$, $x = 21$, $k = 1$.

26. $1 \nwarrow 6$: $2o - 2u - 3o - 3u - 3o$.
27. $6 \nearrow 2$: $2u - 3o - 3u - o - u - o - u$.
28. $2 \nwarrow 5$: $o - 2u - 3o - 3u - 3o - u$.
29. $5 \nearrow 3$: $3u - 3o - 3u - o - u - o$.
30. $3 \nwarrow 4$: $2u - 3o - 3u - 3o - u - o$.
31. $4 \nearrow 4$: $o - 3u - 3o - 3u - o - u$.
32. $4 \nwarrow 3$: $u - 3o - 3u - 3o - u - o - u$.
33. $3 \nearrow 5$: $2o - 3u - 3o - 3u - o$.
34. $5 \nwarrow 2$: $3o - 3u - 3o - u - o - u - o$.
35. $2 \nearrow 6$: $u - 2o - 3u - 3o - 3u$.
36. $6 \nwarrow 1$: $2o - 3u - 3o - u - o - u - o - u$.
37. $1 \nearrow 1$: $2u - 2o - 3u - 3o - 3u - o - u - o - u - o$.
38. $1 \nwarrow 6$: $2o - 3u - 3o - 2u - 3o - 3u - 3o$.
39. $6 \nearrow 2$: $3u - 2o - 3u - 3o - 3u - o - u - o - u$.
40. $2 \nwarrow 5$: $o - 3u - 3o - 2u - 3o - 3u - 3o - u$.
41. $5 \nearrow 3$: $o - 3u - 2o - 3u - 3o - 3u - o - u - o$.
42. $3 \nwarrow 4$: $3u - 3o - 2u - 3o - 3u - 3o - u - o$.

43. 4 ↗ 4: $2o - 3u - 2o - 3u - 3o - 3u - o - u$.
44. 4 ↖ 3: $2u - 3o - 2u - 3o - 3u - 3o - u - o - u$.
45. 3 ↗ 5: $3o - 3u - 2o - 3u - 3o - 3u - o$.
46. 5 ↖ 2: $u - 3o - 2u - 3o - 3u - 3o - u - o - u - o$.
47. 2 ↗ 6: $u - 3o - 3u - 2o - 3u - 3o - 3u$.
48. 6 ↖ 1: $3o - 2u - 3o - 3u - 3o - u - o - u - o - u$.
49. 1 ↗ 1: $2u - 3o - 3u - 2o - 3u - 3o - 3u - o - u - o - u - o$.
50. 1 ↖ 6: $u - o - u - o - u - 3o - 3u - 3o - 2 - 3o - 3u - 3ou - o - u$.

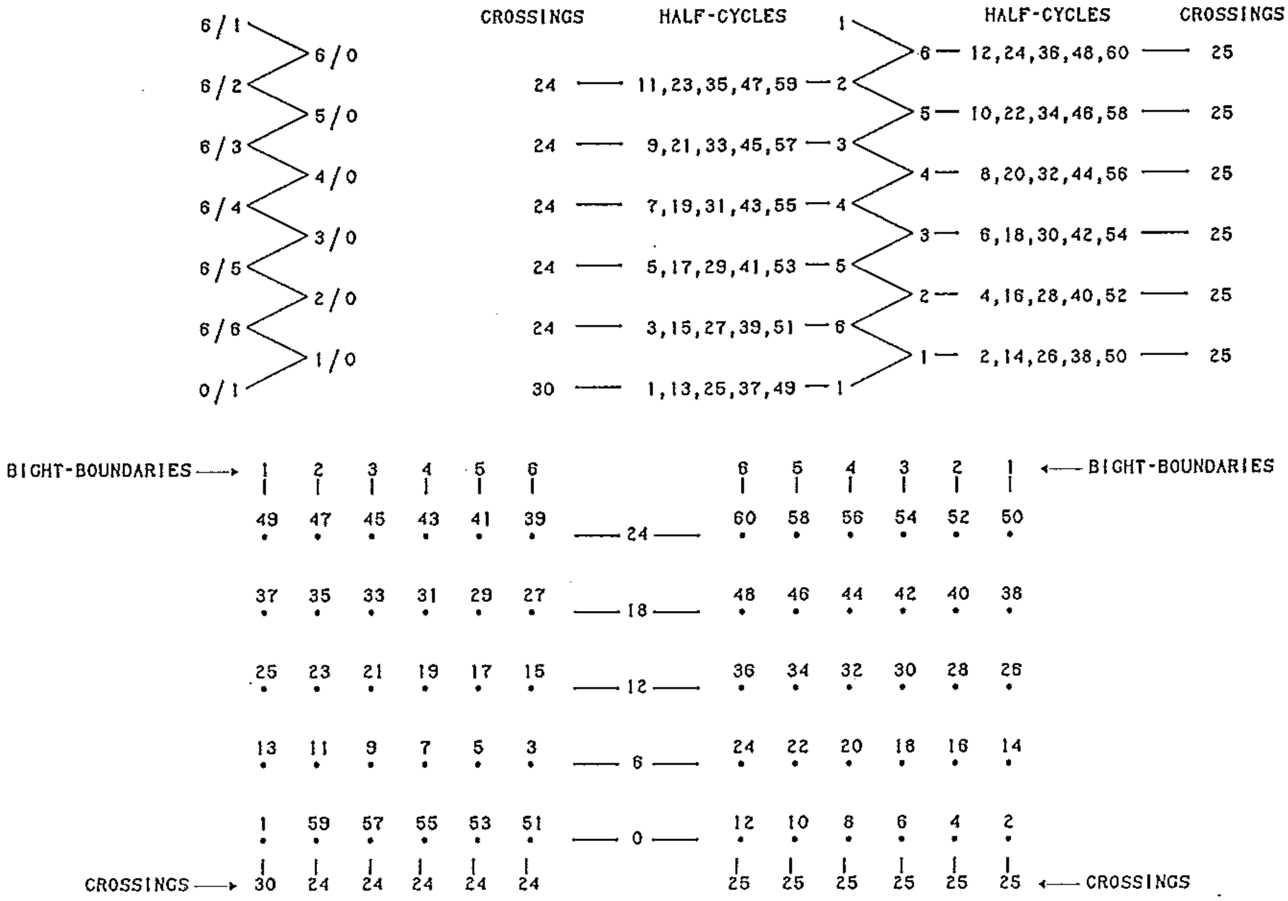

Fig. 1184 — First-return string-run and half-cycle pattern of the single string spherical Regular Nested Knot with $A = 6$, $B^* = 5$, $x = 21$, $k = 1$.

51. 6 ↗ 2: $3u - 3o - 3u - 2o - 3u - 3o - 3u - o - u - o - u$.
52. 2 ↖ 5: $o - u - o - u - 3o - 3u - 3o - 2u - 3o - 3u - 3o - u$.
53. 5 ↗ 3: $o - 3u - 3o - 3u - 2o - 3u - 3o - 3u - o - u - o$.
54. 3 ↖ 4: $u - o - u - 3o - 3u - 3o - 2u - 3o - 3u - 3o - u - o$.
55. 4 ↗ 4: $u - o - 3u - 3o - 3u - 2o - 3u - 3o - 3u - o - u$.
56. 4 ↖ 3: $o - u - 3o - 3u - 3o - 2u - 3o - 3u - 3o - u - o - u$.
57. 3 ↗ 5: $o - u - o - 3u - 3o - 3u - 2o - 3u - 3o - 3u - o$.
58. 5 ↖ 2: $u - 3o - 3u - 3o - 2u - 3o - 3u - 3o - u - o - u - o$.
59. 2 ↗ 6: $u - o - u - o - 3u - 3o - 3u - 2o - 3u - 3o - 3u$.
60. 6 ↖ 1: $3o - 3u - 3o - 2u - 3o - 3u - 3o - u - o - u - o - u$.

After the foundation knot has been braided and properly tightened, we replace its string with a longer string which incorporates in its odd half-cycles associated with the grid-diagrams in Fig. 1186 the star knot elements as in Figs. 1165, 1167, 1169 or as in Figs. 1171, 1173, 1175.

L → R

	59	57	55	53	51	13	11	9	7	5	3	25	23	21	19	17	15	37	35	33	31	29	27	49	47	45	43	41	39					
49	47	45	43	41	39		59	57	55	53	51	13	11	9	7	5	3	25	23	21	19	17	15	37	35	33	31	29	27					
37	35	33	31	29	27	49	47	45	43	41	39		59	57	55	53	51	13	11	9	7	5	3	25	23	21	19	17	15					
25	23	21	19	17	15	37	35	33	31	29	27	49	47	45	43	41	39		59	57	55	53	51	13	11	9	7	5	3					
13	11	9	7	5	3	25	23	21	19	17	15	37	35	33	31	29	27	49	47	45	43	41	39		59	57	55	53	51					
o	u	o	u	o	u	u	u	o	o	o	u	u	u	o	o	u	u	u	o	o	o	u	u	u	o	u	o	u	o	1	13	25	37	49
u	o	u	o	u	u	u	o	o	o	u	u	u	o	o	u	u	u	o	o	o	u	u	u							59	11	23	35	47
o	u	o	u	u	u	o	o	o	u	u	u	o	o	u	u	u	o	o	o	u	u	u	o							57	9	21	33	45
u	o	u	u	u	o	o	o	u	u	u	o	o	u	u	u	o	o	o	u	u	u	o	u							55	7	19	31	43
o	u	u	u	o	o	o	u	u	u	o	o	u	u	u	o	o	o	u	u	u	o	u	o							53	5	17	29	41
u	u	u	o	o	o	u	u	u	o	o	u	u	u	o	o	o	u	u	u	o	u	o	u							51	3	15	27	39

R → L

2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50					
50	52	54	56	58	60	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38					
38	40	42	44	46	48	50	52	54	56	58	60	2	4	6	8	10	12	14	16	18	20	22	24	26					
26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	2	4	6	8	10	12	14					
14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	2					
o	o	o	u	u	u	o	o	o	u	u	o	o	o	u	u	u	o	o	o	u	o	u	o	u	12	24	36	48	60
u	o	o	o	u	u	u	o	o	o	u	u	o	o	o	u	u	u	o	o	o	u	o	u	o	10	22	34	46	58
o	u	o	o	o	u	u	u	o	o	o	u	u	o	o	o	u	u	u	o	o	o	u	o	u	8	20	32	44	56
u	o	u	o	o	o	u	u	u	o	o	o	u	u	o	o	o	u	u	u	o	o	o	u	o	6	18	30	42	54
o	u	o	u	o	o	o	u	u	u	o	o	o	u	u	o	o	o	u	u	u	o	o	o	u	4	16	28	40	52
u	o	u	o	u	o	o	o	u	u	u	o	o	o	u	u	o	o	o	u	u	u	o	o	o	2	14	26	38	50

Fig. 1185 — The half-cycle tables of the single string spherical Regular Nested Knot in Fig. 1183 with $A = 6$, $B^* = 5$, $x = 21$, $k = 1$.

Similar as in Example 1, the star knot elements occupy the position of the dotted line segments in the odd-numbered half-cycles. The solid line segments in the even-numbered half-cycles in these positions follow the coding of the foundation knot.

When we mirror (hence laterally inverse or sideways reverse) the coding and the string-run in Fig. 1183, followed by replacing this string-run with a string-run in which the first cycle, running from lower-left bight-boundary 1 to upper-right bight-boundary A, then from lower-right bight-boundary A to upper-left bight-boundary 2 (hence a string-run with $k = A$), the even-numbered half-cycles in this thus resulting grid-diagram are then sequentially adjacent.†

In the now central 2-overs column on the even-numbered half-cycles we replace the local solid string-run line-segments with dotted string-run line-segments.

After the foundation knot has been braided and properly tightened, we replace its string with a longer string which incorporates in its even-numbered half-cycles, at the dotted line-segments, the star knot elements as in Figs. 1166, 1168, 1170 or as in Figs. 1172, 1174, 1176.

† Refer to *The Braider*, pg. 1428.

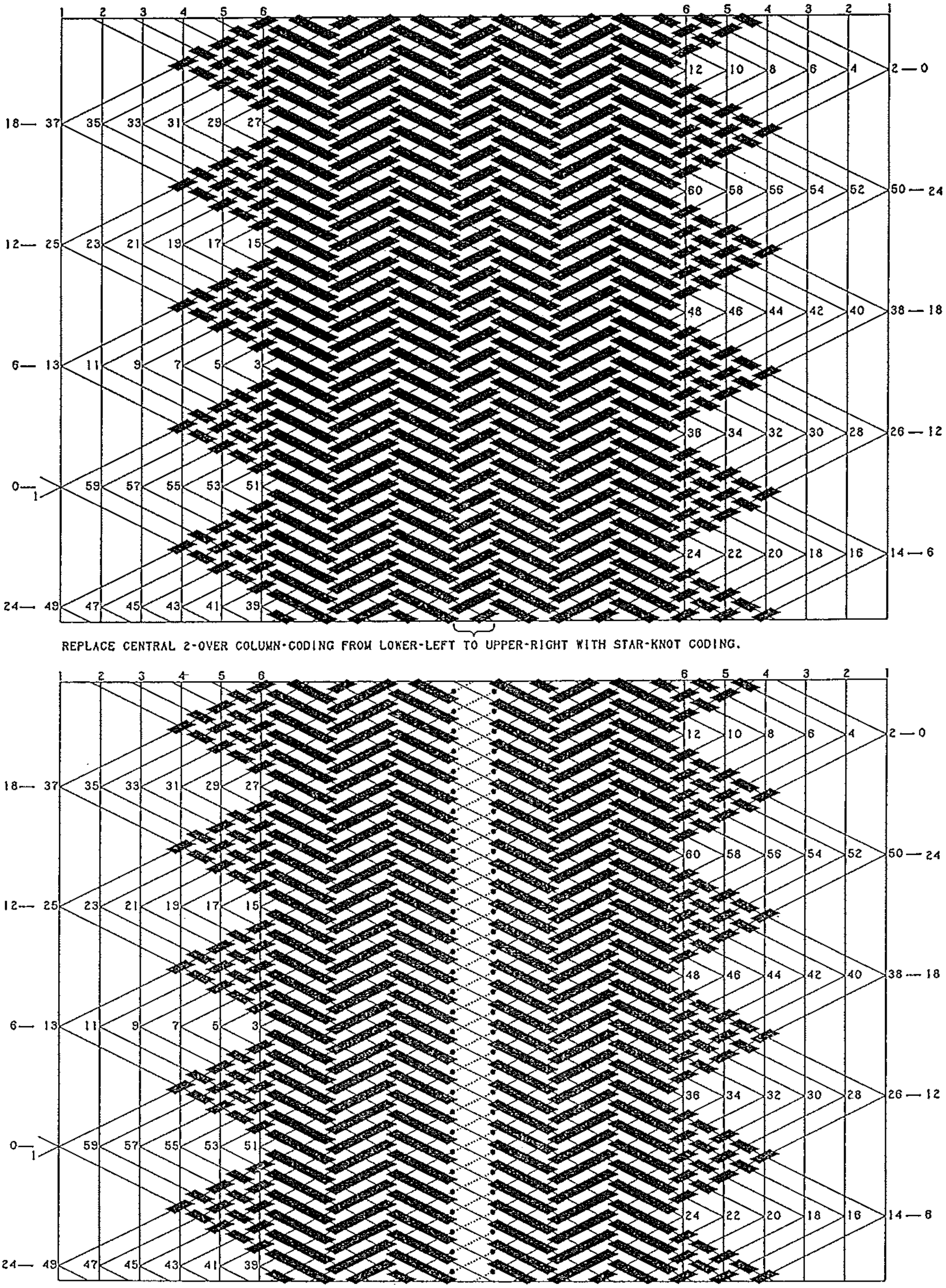

Fig. 1186 — The grid-diagram of the Saturn Knot in Example 2.