The Braider

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A Braiding Project — A walking stick with knots

Many forms of walking sticks can be decorated with braided knots, but for this project we assume that the actual cane of the walking stick is of a slightly tapered cylindrical form with an upper diameter of approximately 25 mm. and lower diameter of approximately 19 mm. over a length of approximately 82 cm.. The walking stick we envisage here consists of four parts which screw together: the handle and a three section cane; this to enable easy transportation when required. The braiding material of various colours is braided synthetic round cord of 2 mm. diameter.

In this initial walking stick project we will use knots which can readily be braided to their weaving-pattern. There are four sets of Column-coded Regular Knots which lend themselves especially to weaving-pattern braiding:

- **i.** The string-run is typified by $p = nb + 1$, where $nb = \alpha m$ (α-pass Regular Knot); n, α and , m are natural numbers.
- **ii.** The string-run is typified by $p = nb - 1$, where $nb = \alpha m$ (α-pass Regular Knot); n, α and m are natural numbers.
- **iii.** The string-run is typified by $p = nm + 1$, $b = nm - n + 1$ (n-pass Regular Knot); $n = \alpha$ and m are natural numbers.
- **iv.** The string-run is typified by $p = nm + 1$, $b = nm + n + 1$ (n-pass Regular Knot); $n = \alpha$ and m are natural numbers.

set i:

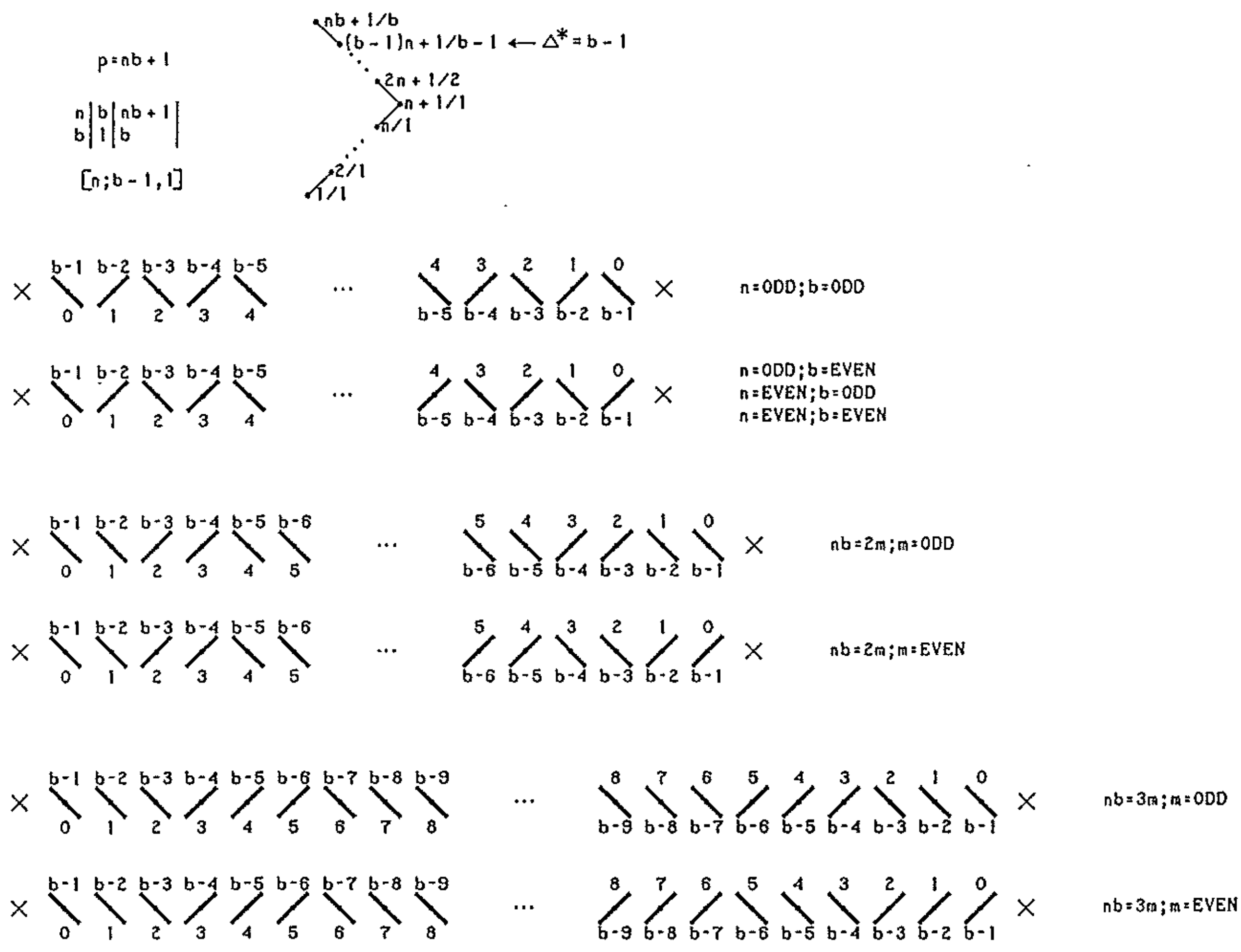

Fig. 1088 — Set **i** with $p = nb + 1$.

Most used by pattern-braiders is $n = 1$, thus $p = b + 1$. Since $nb = \alpha m$, it follows that $b = \alpha m$. The braiding sequence for $\alpha = 1$ is depicted in Fig. 1089; left of the vertical line for $b = m = odd$, and right of the vertical line for $b = m = even$. It shows that the pattern gets built up from the two bight-boundaries towards the centre.

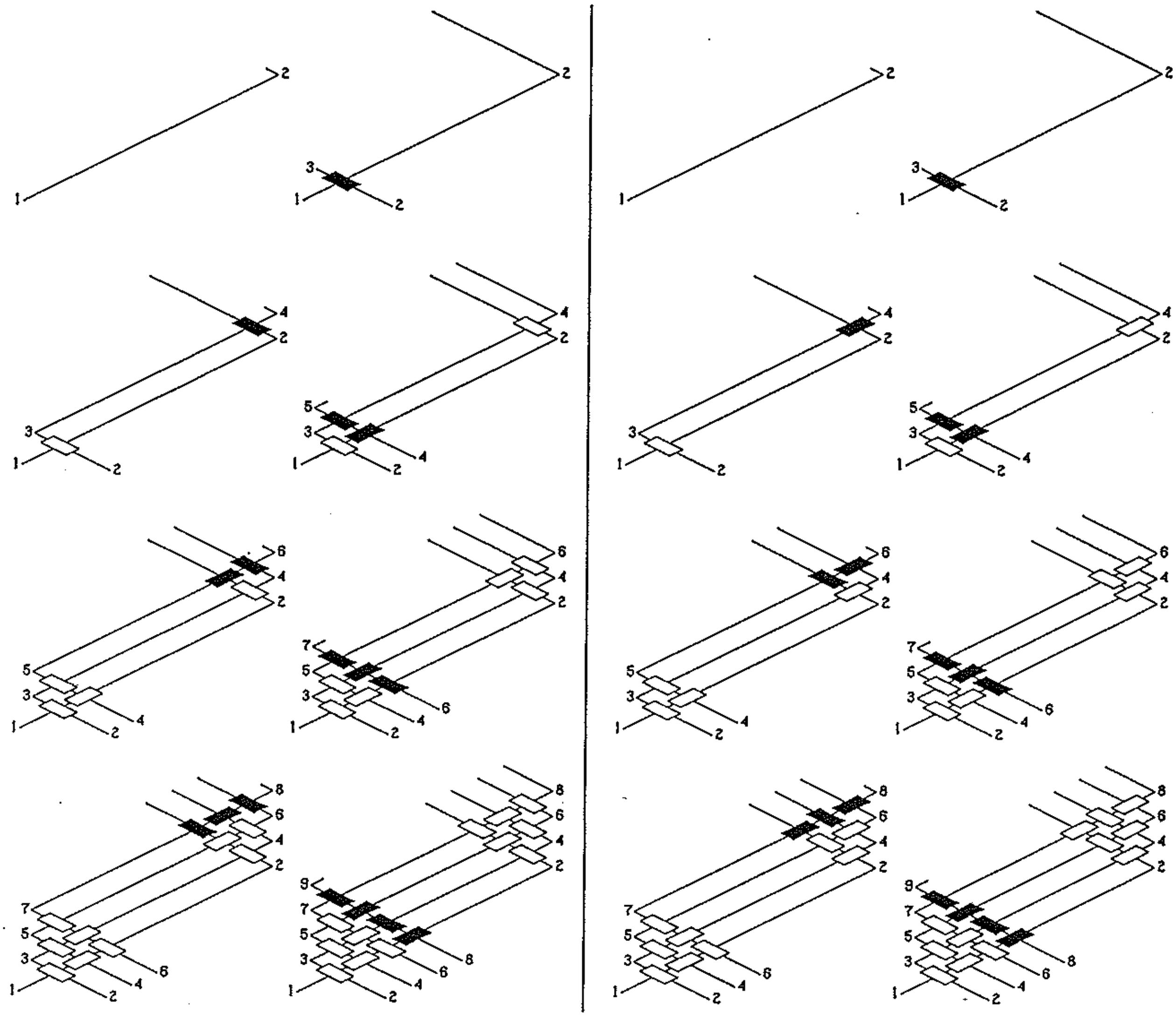

Fig. 1089 — Braiding sequences for $p = b + 1 = \alpha m + 1$, where $\alpha = 1$.

In addition to the pattern-building procedure from the two bight-boundaries towards the centre, there are, when $n > 1$, a further $n - 1$ equispaced positions between the two bight-boundaries from where pattern-building starts. These further $n - 1$ starting positions are the further crossings encountered by half-cycle 2. In order to get the codings at these further $n - 1$ pattern-building start-position correct we can make use of the tables in Fig. 1091 when $\alpha = 1$, the tables in Fig. 1092 when $\alpha = 2$, the tables in Fig. 1093 when $\alpha = 3$, and the tables in Fig. 1094 when $\alpha = 4$. Further tables can then be employed for further values of α. The general algorithm diagram, which displays one of such further $n - 1$ equispaced positions between the two bight-boundaries from where pattern-building starts, is depicted in Fig. 1090.

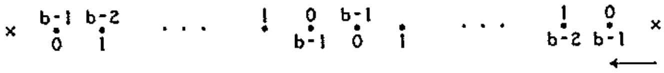

Fig. 1090 — One of the further $n - 1$ pattern-building centres.

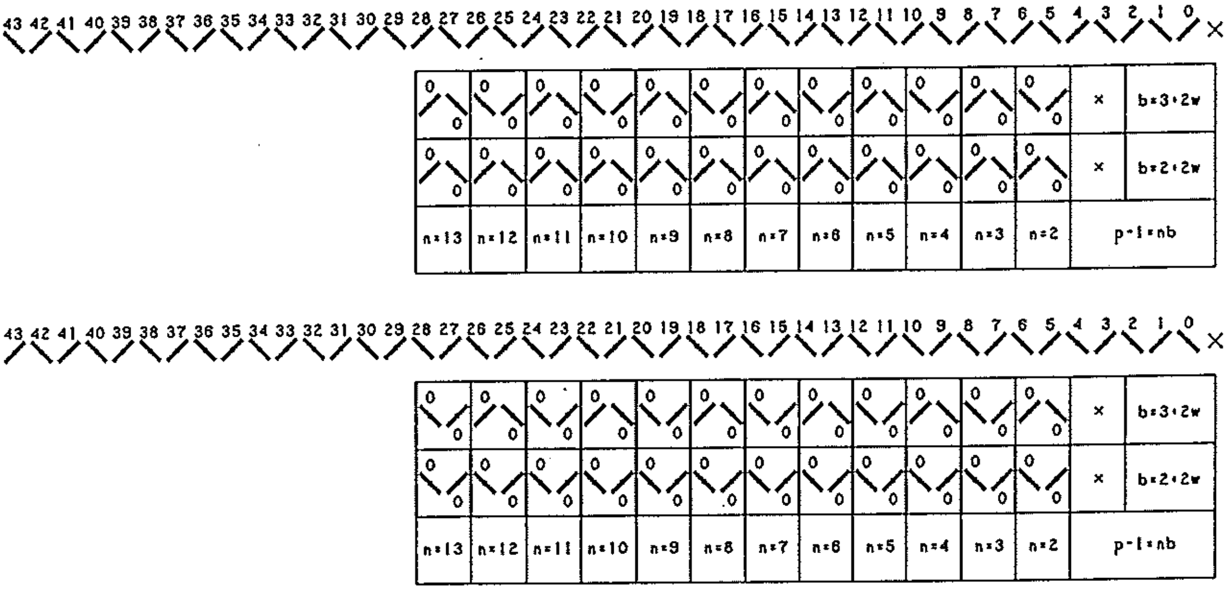

Fig. 1091 — Tables associated with $\alpha = 1$.

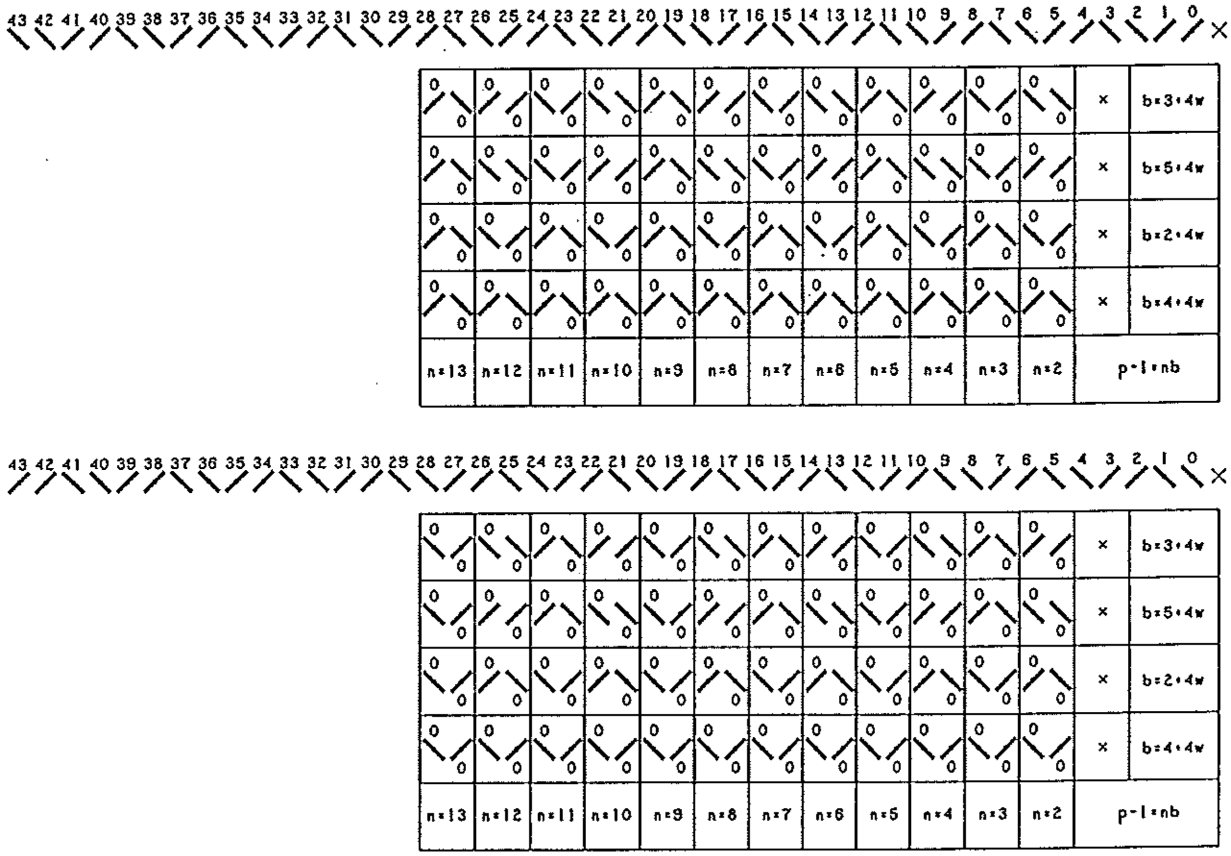

Fig. 1092 — Tables associated with $\alpha = 2$.

When $\alpha = 1$ or $\alpha = 2$ we require only the column-codings of the crossings encountered by the half-cycles 2 and 3 ($i = 0$) associated with the pattern-building centres additional to the $n = 1$ pattern-building sites; when $\alpha = 3$ we require not only the column-codings of the crossings encountered by the half-cycles 2 and 3 ($i = 0$), associated with the pattern-building centres additional to the $n = 1$ pattern-building sites, but also the column-codings of the crossings encountered by half-cycle 4 ($i \leq 1$) associated with the pattern-building centres additional to the $n = 1$ pattern-building sites; when $\alpha = 4$ we require the column-codings of the crossings encountered by the half-cycles 2, 3, 4 and 5 ($i \leq 1$) associated with the pattern-building centres additional to the $n = 1$ pattern-building sites; etc.

Fig. 1093 — Tables associated with $\alpha = 3$.

An example for $\alpha = 4$, $b = 12$, $n = 3$, with an over-crossing for the first crossing encountered by half-cycle 2, is shown in Fig. 1095. Since $m = 9 = odd$ ($nb = 3 \times 12 = 36 = \alpha m$), this 4-pass Regular Knot is a 4-pass Headhunter's knot.

We can either memorise the codings associated with the first five half-cycles as obtained from the lower table in Fig. 1094 (the relevant table section is shown at the upper-left in Fig. 1095) and displayed at the upper-right in Fig. 1095, or by braiding the first five half-cycles in accordance with this relevant table section as follows:

1.	$L \longrightarrow R$		:	Free run.
2.	$R \longrightarrow L$	$i = 0$	:	$o - u - o$.
3.	$L \longrightarrow R$	$i = 0$	:	$u - o - u$.
4.	$R \longrightarrow L$	$i = 1$	:	$2o - 2u - 2o$.
5.	$L \longrightarrow R$	$i = 1$	:	$2u - 2o - 2u$.

The further 19 half-cycles can now readily be pattern braided.

Although it sufficient to establish the first three half-cycles in order to braid this knot by means pattern-braiding, in general this will not be sufficient (note from the tables in Fig. 1094 that for pattern-braiding the 4-pass Gaucho and Headhunter's knots with $n > 1$, the first three half-cycles only suffice when $b = 4 + 8w$ or $b = 8 + 8w$.

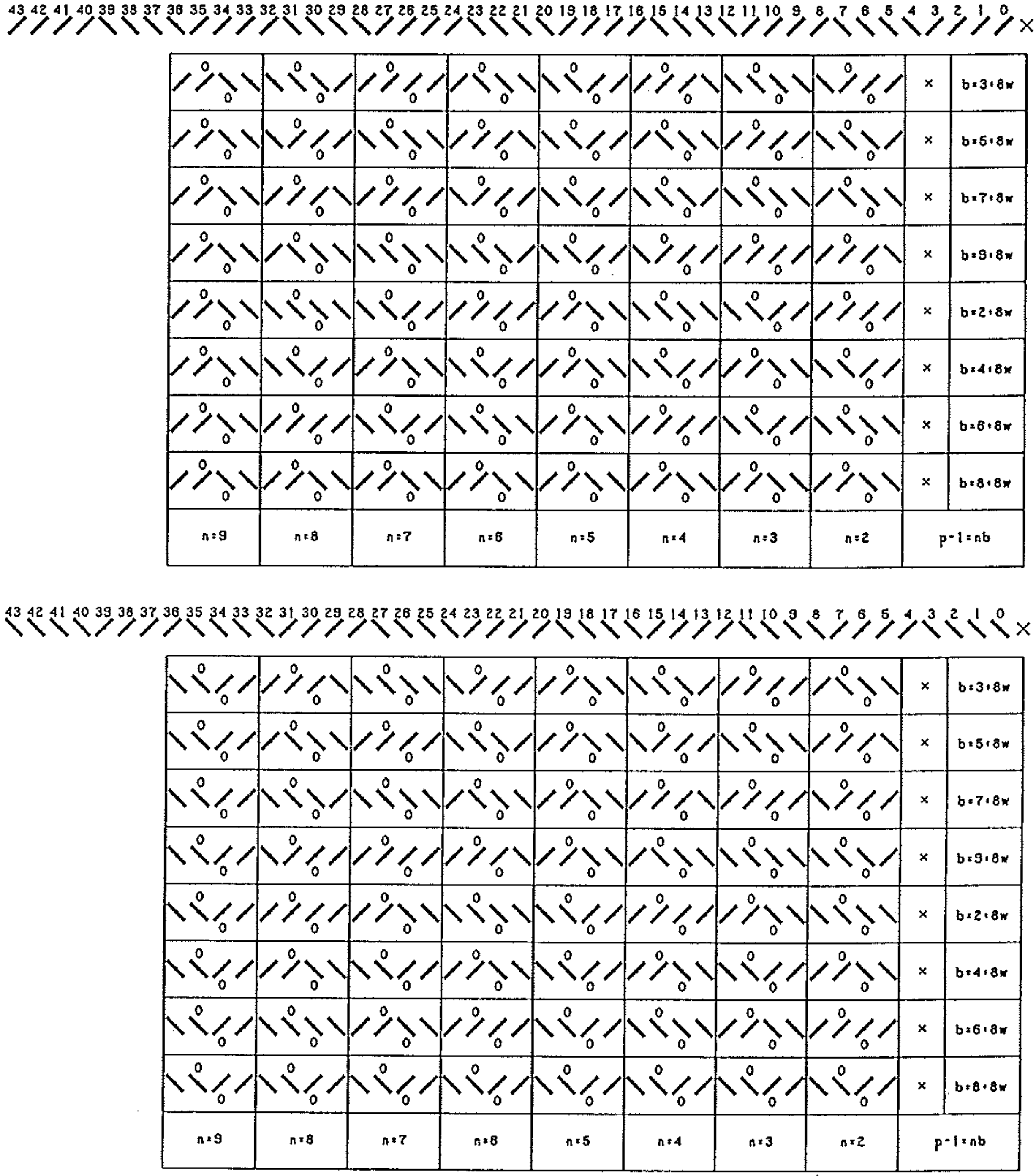

Fig. 1094 — Tables associated with $\alpha = 4$.

Note that the coding patterns of the $n-1$ pattern-building start-positions, additional to the $n = 1$ pattern-building sites, exhibit a periodic sequence. With $b = v + 2\alpha w$, where $2 \leq v \leq 2\alpha$ and w is a whole number, the periodic sequence of pattern-building start-positions, additional to the $n = 1$ pattern-building sites, consists of $\dfrac{2\alpha}{\text{g.c.d.}(v, 2\alpha)}$

consecutive coding patterns.

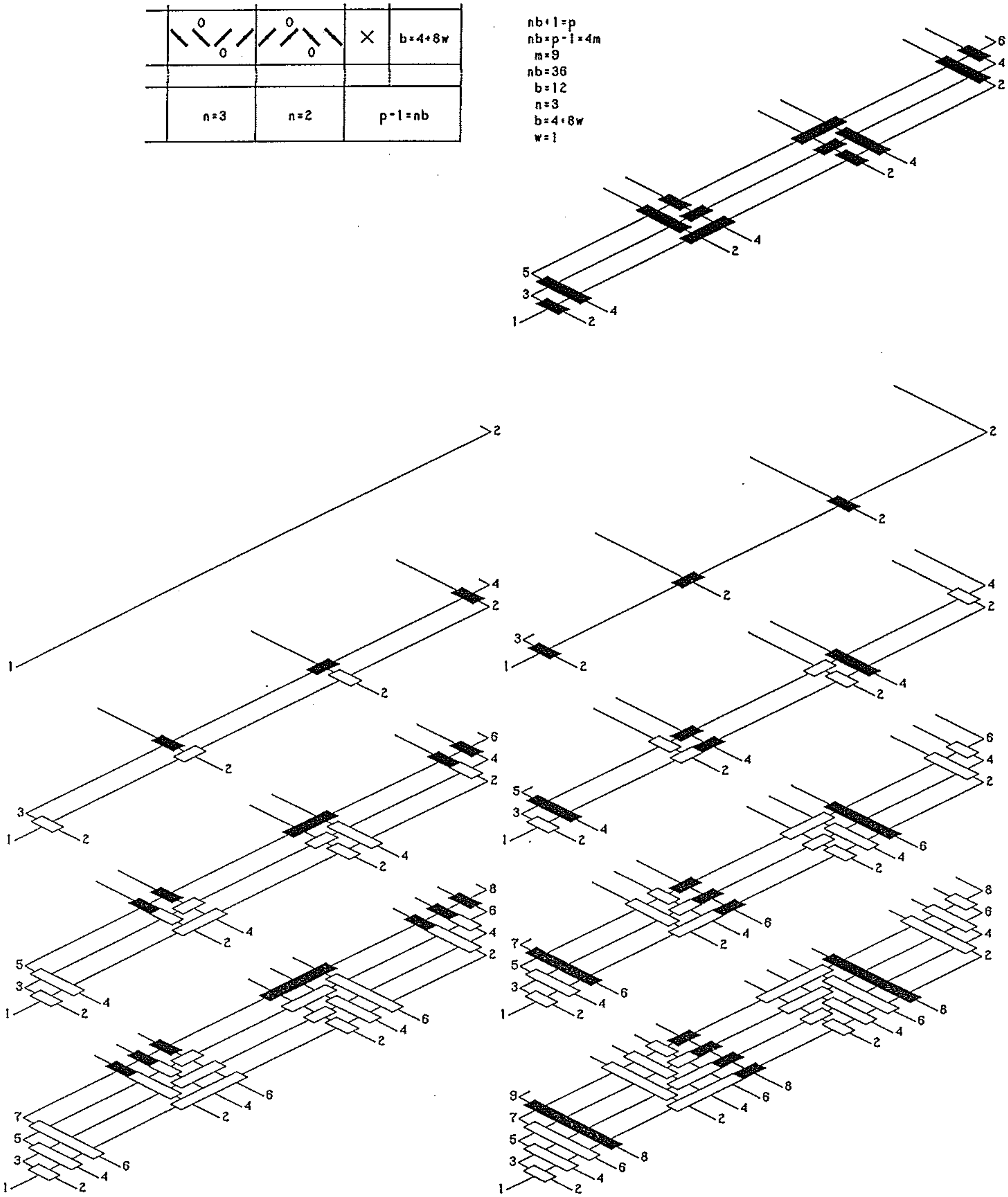

Fig. 1095 — $\alpha = 4$, $b = 12$, $n = 3$, first crossing by half-cycle 2 an over-crossing.

set ii :

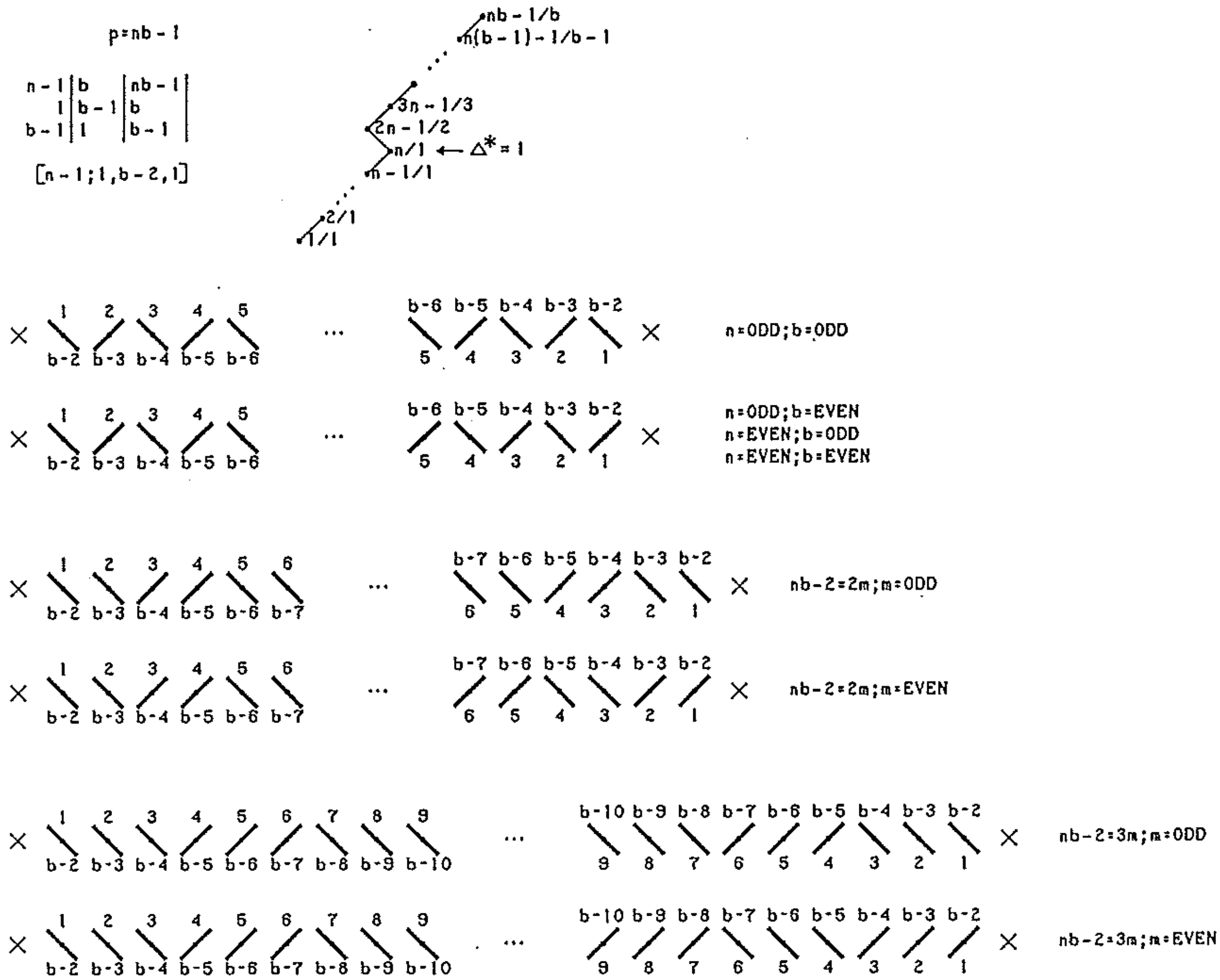

Fig. 1096 — Set **ii** with $p = nb - 1$.

Most used by pattern-braiders is $n = 1$, thus $p = b - 1$. Since $nb = \alpha m$, it follows that $b = \alpha m$. The braiding sequence for $\alpha = 1$ is depicted in Fig. 1098; left of the vertical line for $b = m = odd$, and right of the vertical line for $b = m = even$. It shows that the pattern gets built up from the two bight-boundaries towards the centre.

In addition to the pattern-building procedure from the two bight-boundaries towards the centre, there are, when $n > 1$, a further $n - 1$ equispaced positions between the two bight-boundaries from where pattern-building starts. These further $n - 1$ starting positions are the crossings encountered by half-cycle 2. In order to get the codings at these further $n - 1$ pattern-building start-position correct we can make use of the tables in Fig. 1099 when $\alpha = 1$, the tables in Fig. 1100 when $\alpha = 2$, the tables in Fig. 1101 when $\alpha = 3$, and the tables in Fig. 1102 when $\alpha = 4$. Further tables can then be employed for further values of α. The general algorithm diagram, which displays one of such further $n - 1$ equispaced positions between the two bight-boundaries from where pattern-building starts, is depicted in Fig. 1097.

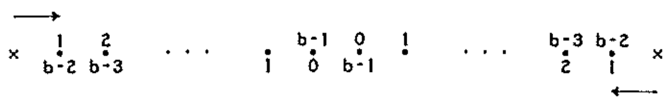

Fig. 1097 — One of the further $n - 1$ pattern-building centres.

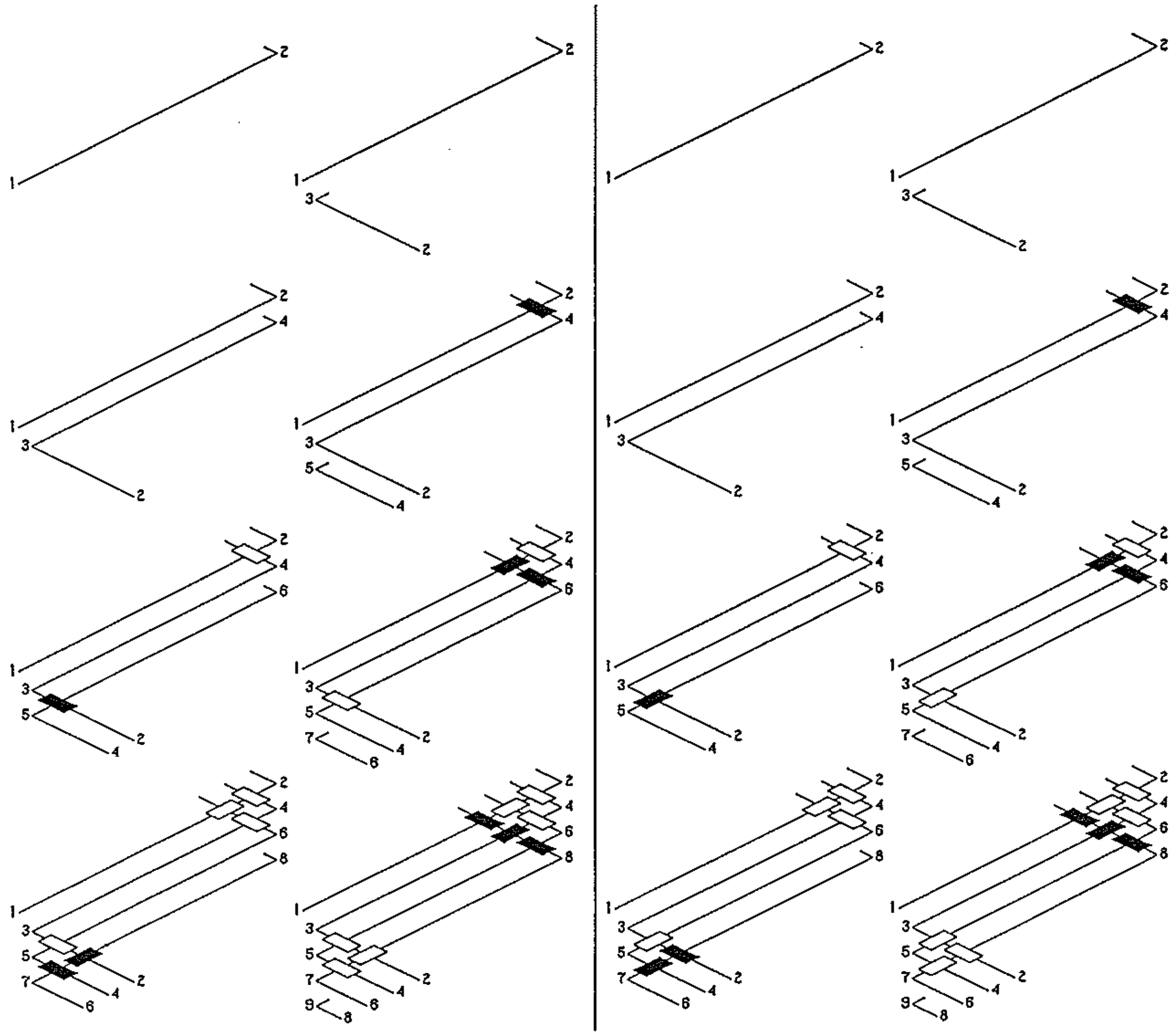

Fig. 1098 — Braiding sequences for $p = b - 1 = \alpha m - 1$, where $\alpha = 1$.

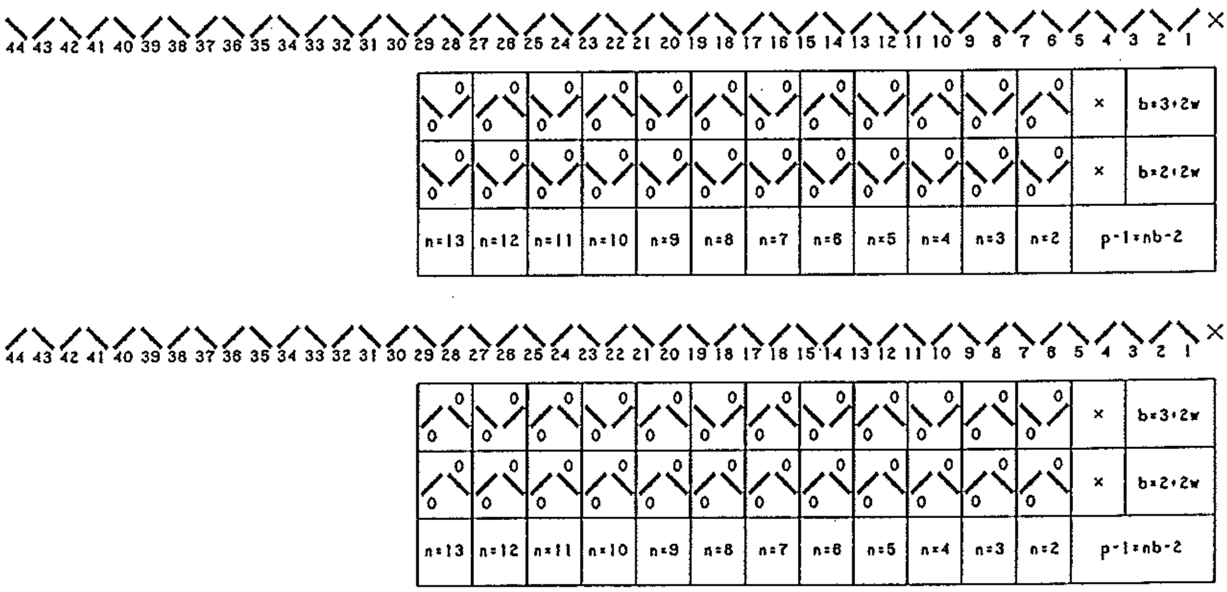

Fig. 1099 — Tables associated with $\alpha = 1$.

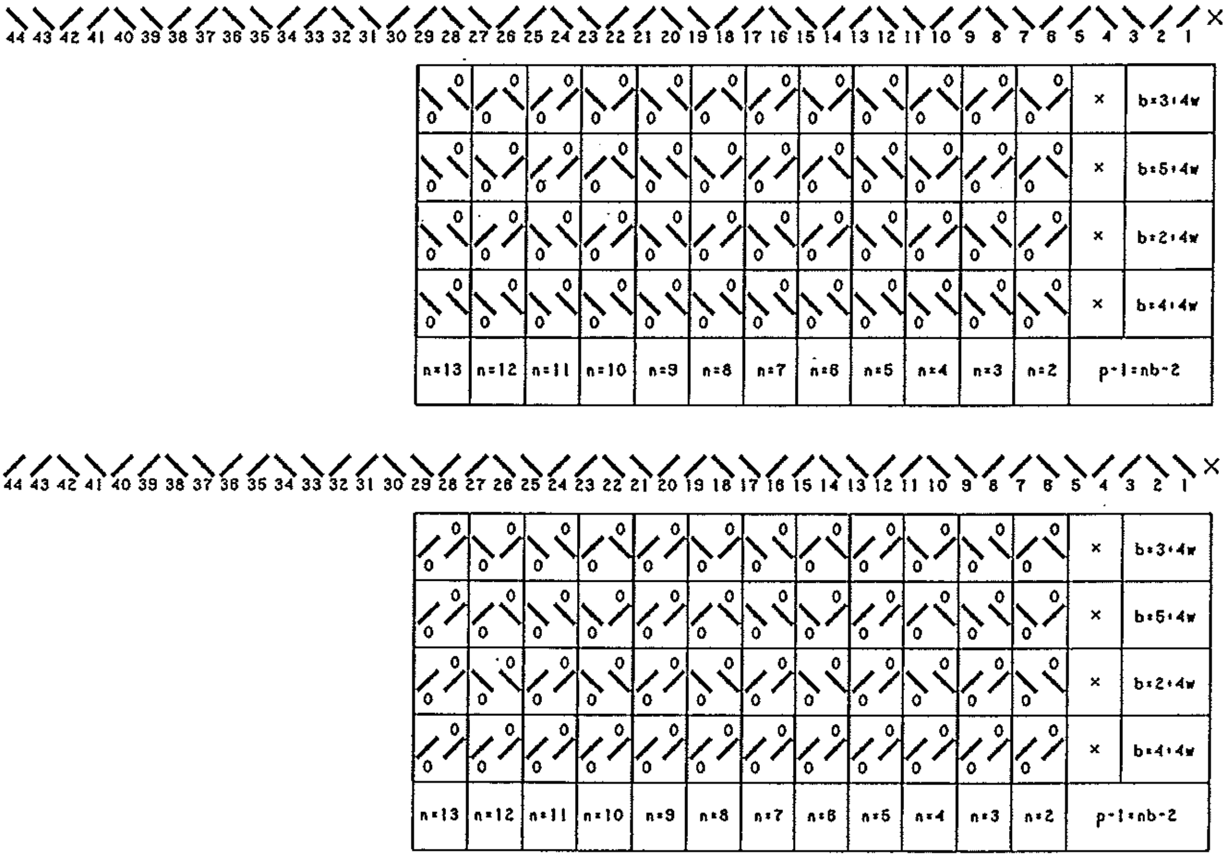

Fig. 1100 — Tables associated with $\alpha = 2$.

When $\alpha = 1$ or $\alpha = 2$ we require only the column-codings of the crossings encountered by the half-cycles 2 and 3 (hence associated with bight-number $i = 0$) associated with the pattern-building centres additional to the $n = 1$ pattern-building sites; when $\alpha = 3$ we require not only the column-codings of the crossings encountered by the half-cycles 2 and 3 (hence associated with bight-number $i = 0$), associated with the pattern-building centres additional to the $n = 1$ pattern-building sites but also the column-codings of the crossings encountered by half-cycle 4 (hence associated with the bight-numbers $i = 0$ and $i = 1$) associated with the pattern-building centres additional to the $n = 1$ pattern-building sites; when $\alpha = 4$ we require the column-codings of the crossings encountered by the half-cycles $2, 3, 4$ and 5 (hence associated with bight-numbers $i = 0$ and $i = 1$) associated with the pattern-building centres additional to the $n = 1$ pattern-building sites; etc.

An example for $\alpha = 4$, $b = 10$, $n = 3$, with an over-crossing for the first crossing encountered by half-cycle 2, is shown in Fig. 1103. Since $m = 7 = odd$ $(nb - 2 = 3 \times 10 - 2 = 30 - 2 = 28 = \alpha m)$, this 4-pass Regular Knot is a 4-pass Headhunter's knot.

We can either again memorise the codings associated with the first five half-cycles as obtained from the lower table in Fig. 1102 (the relevant table section is shown at the upper-left in Fig. 1103) and displayed at the upper-right in Fig. 1103, or by braiding the first five half-cycles in accordance with this relevant table section as follows:

1. $L \longrightarrow R$: Free run.
2. $R \longrightarrow L$ $i = 0$: $2o$.

3. $L \longrightarrow R$ $i = 0$: $2u$.
4. $R \longrightarrow L$ $i = 1$: $4o - u$.
5. $L \longrightarrow R$ $i = 1$: $4u - o$.

The further 15 half-cycles can now readily be pattern braided.

Note that we do require here the first five half-cycles for establishing the braid-pattern at the $n-1$ pattern-braiding centres additional to the $n=1$ pattern-building sites. Note furthermore from the tables in Fig. 1102 that for pattern-braiding the 4-pass Gaucho and Headhunter's knots there are no b-values for which the first three half-cycles are sufficient for establishing the braid-pattern at the $n-1$ pattern-braiding centres, additional to the $n=1$ pattern-building sites, when $n > 2$. For pattern-braiding the 4-pass Gaucho and Headhunter's knots, the first three half-cycles are only sufficient for establishing the braid-pattern at the $n-1=1$ pattern-braiding centre (hence when $n=2$), additional to the $n=1$ pattern-building sites, for $b = 5 + 8w$ and $b = 9 + 8w$, where w is a whole number.

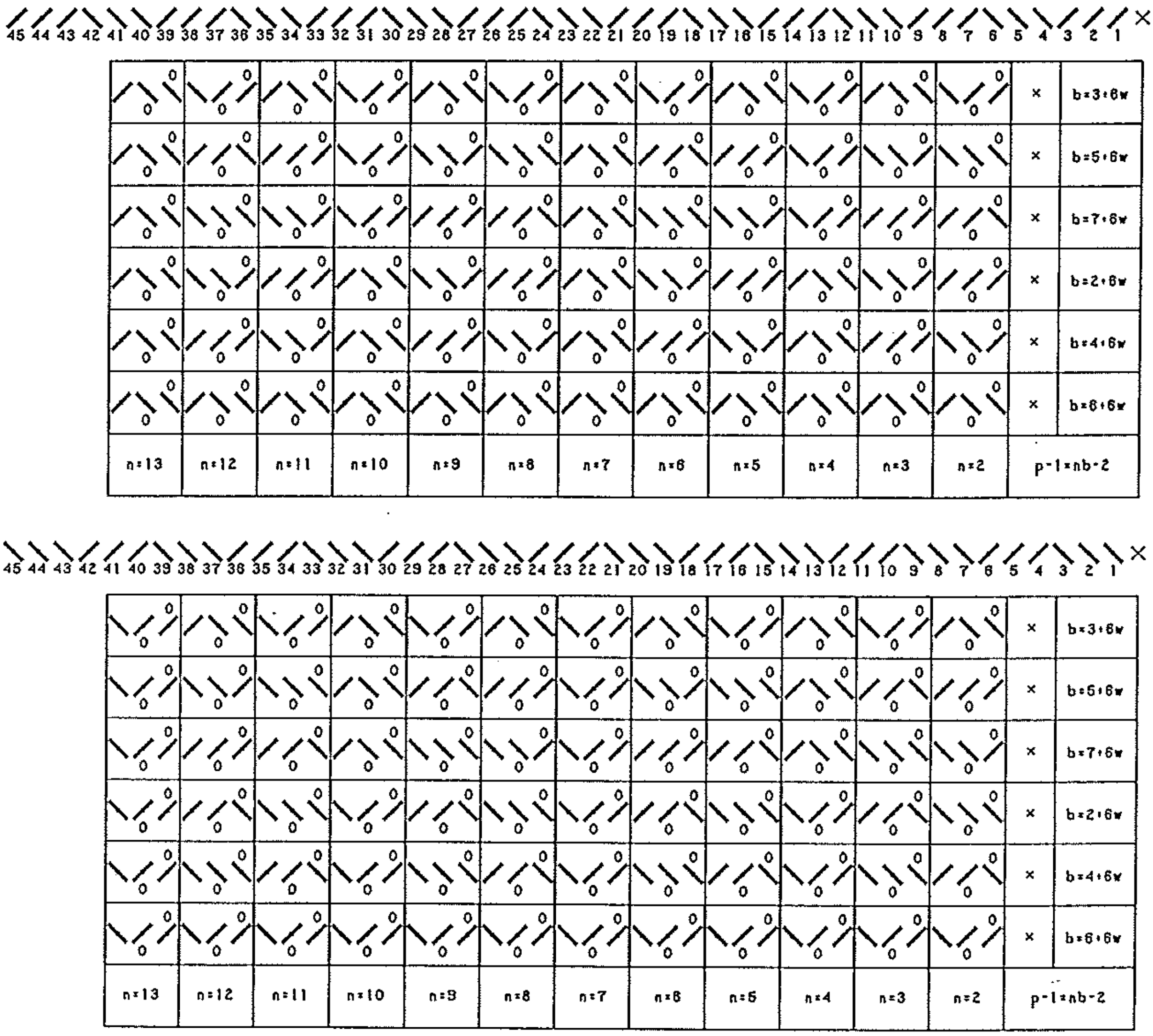

Fig. 11101 — Tables associated with $\alpha = 3$.

The coding patterns of the $n-1$ pattern-building start-positions, additional to the $n=1$ pattern-building sites, exhibit again a periodic sequence. With $b = v + 2\alpha w$, where $2 \leq v \leq 2\alpha$ and w is a whole number, the periodic sequence of pattern-building start-positions, additional to the $n = 1$ pattern-building sites†, consists of $\dfrac{2\alpha}{\text{g.c.d.}(v, 2\alpha)}$ consecutive coding patterns.

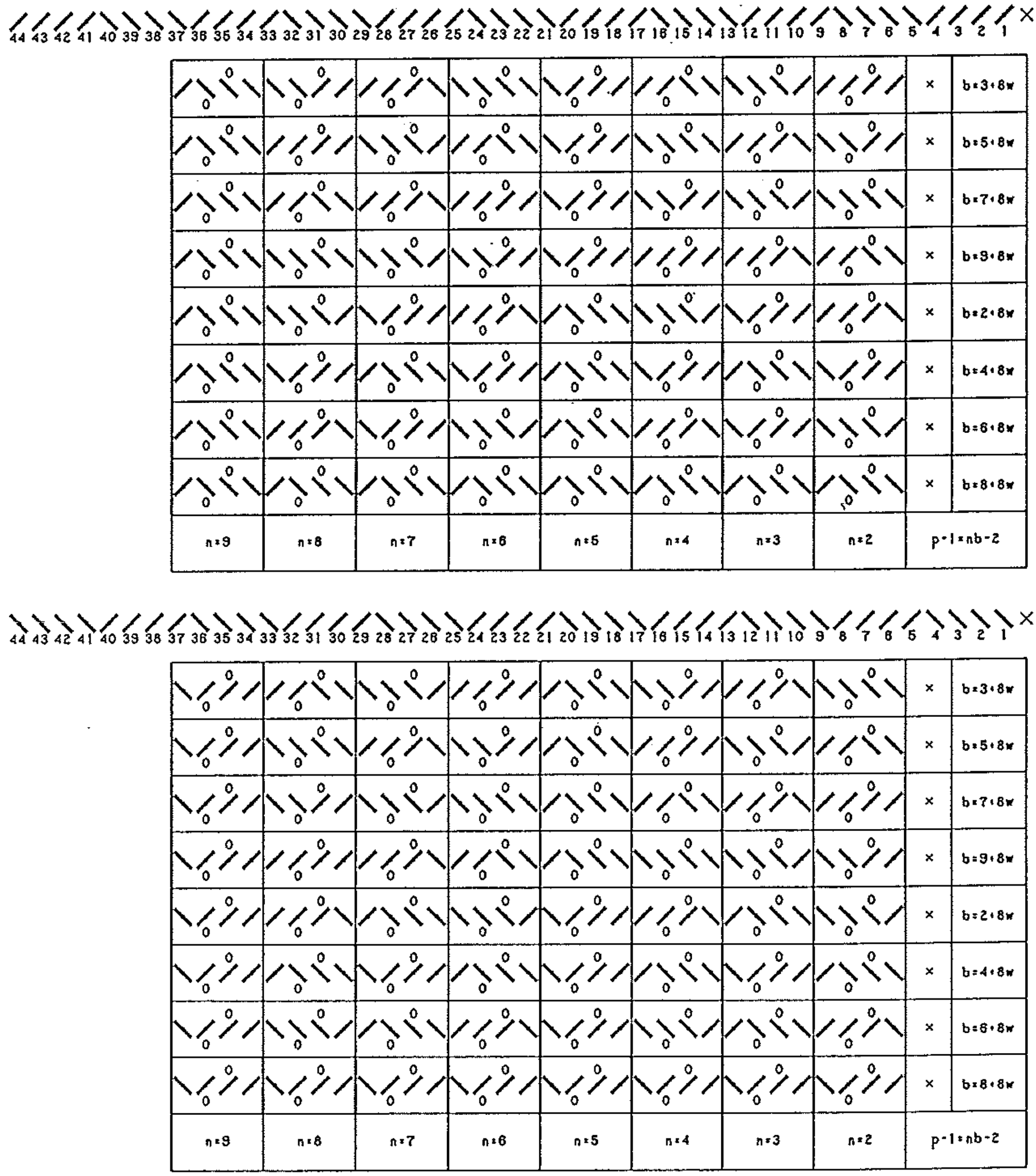

Fig. 1102 — Tables associated with $\alpha = 4$.

An example of a 4-pass Headhunter's knot with $m = 7$, and $n = 3$ hence with $\alpha m = 4 \times 7 = 28$ and $b = \dfrac{\alpha m + 2}{n} = \dfrac{28 + 2}{3} = 10$ is shown in Fig. 1103.

† The $n = 1$ pattern-building sites are the two sites, one at the left-hand bight-boundary and one at the right-hand bight-boundary.

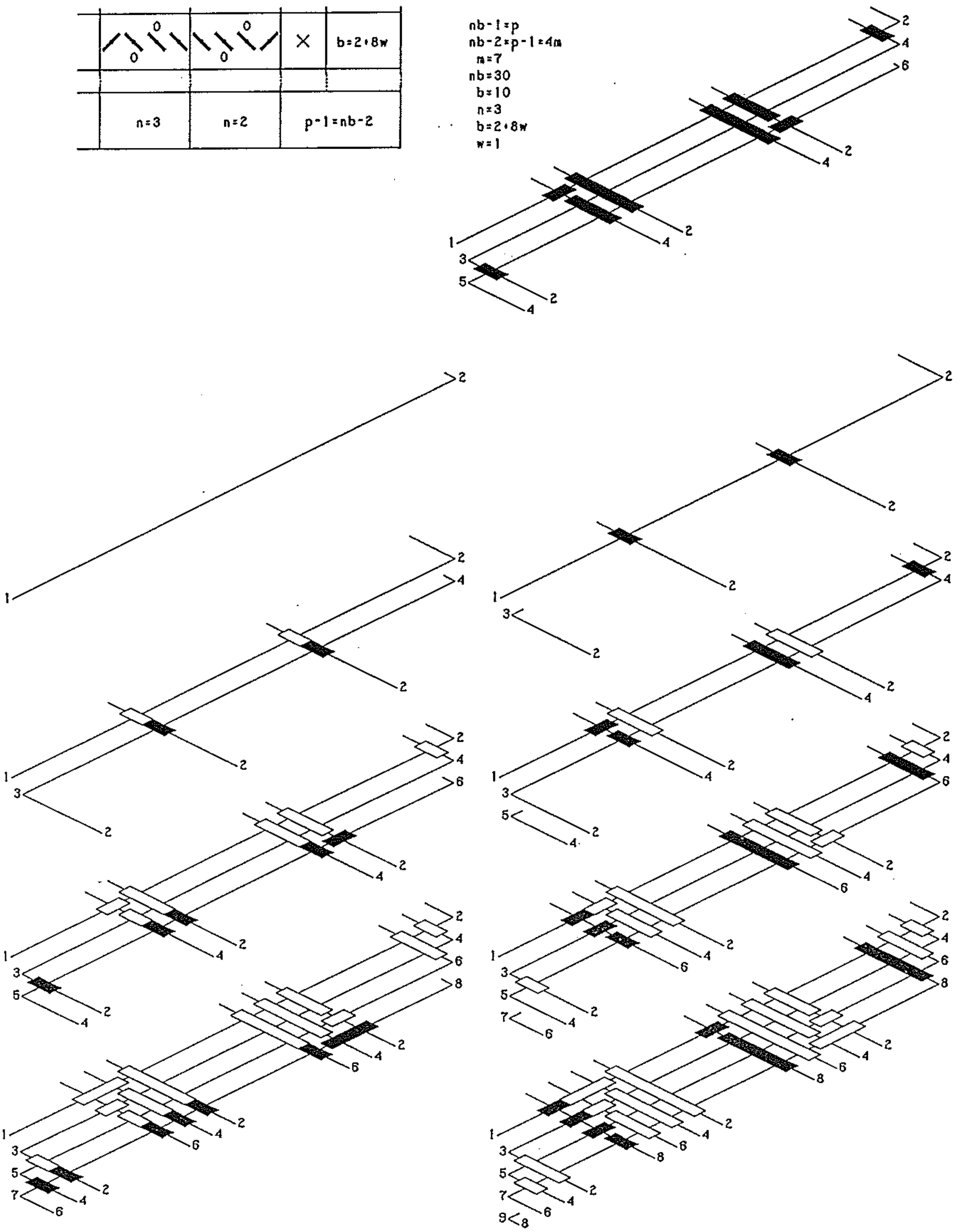

Fig. 1103 — $\alpha = 4$, $b = 10$, $m = 7$, first crossing by half-cycle 2 an over-crossing.

set iii :

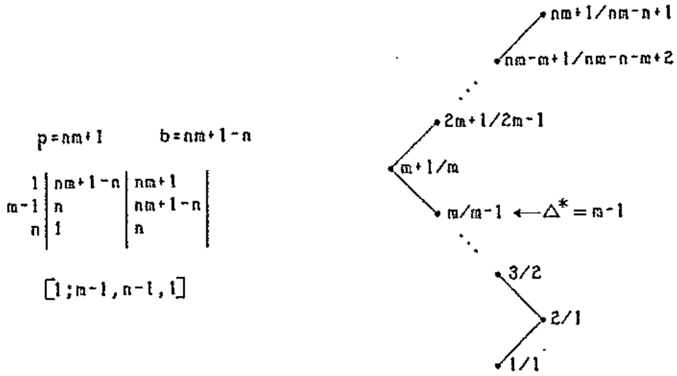

Fig. 1104 — $p = nm + 1$, $b = nm - n + 1$, hence $\Delta^* = m - 1$.

With Euclid's algorithm we obtain the path-formula $[1; m-1, n-1, 1]$ of $p/b = nm + 1/nm - n + 1$ in the RKT and from the path in the RKT we obtain $\Delta^* = m - 1$. See Fig. 1104. Hence $\Delta^* = \dfrac{b-1}{n}$. Consequently $|n\Delta^*|_b = |b-1|_b = b - 1$.

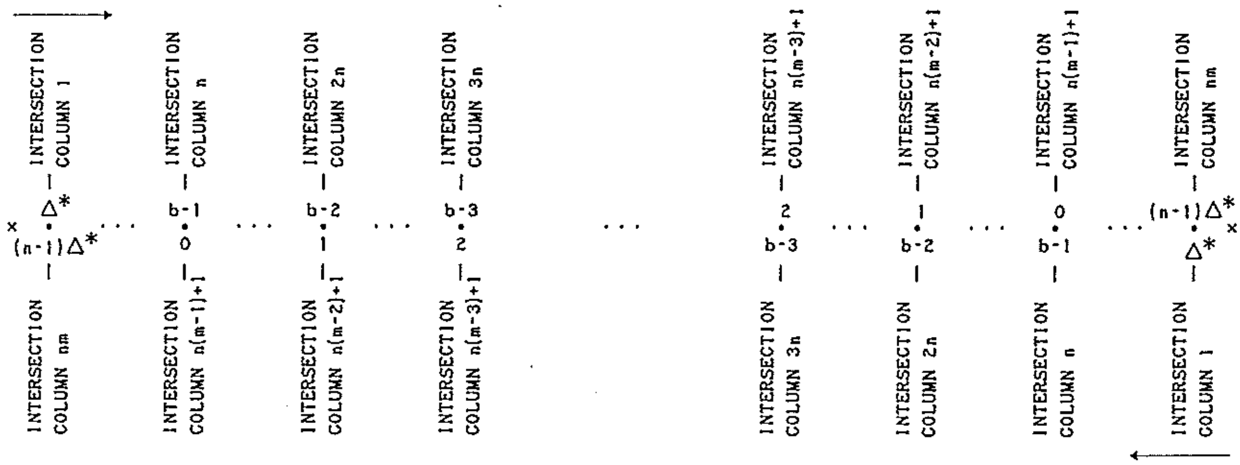

Fig. 1105 — The general layout of the algorithm diagram.

The general layout of the associated algorithm diagram is depicted in Fig. 1105. The intersection-columns, indicated by the dots, can be given an α-pass coding, where $\alpha = n$. Examples for $\alpha = n$ equal 1, 2, 3 and 4 are shown in Fig. 1106.

The braiding of such an α-pass column-coded Regular Knot consists of $n = \alpha$ stages: the first stage produces a $p/b = m + 1/m$ under-over coded Regular Knot (a one-pass Regular Knot), the second stage transforms this $p/b = m + 1/m$ under-over coded Regular Knot into a $p/b = 2m + 1/2m - 1$ two-pass Regular Knot, the third stage transforms this $p/b = 2m+1/2m-1$ two-pass Regular Knot into a $p/b = 3m+1/3m-2$ three-pass Regular Knot, the fourth stage transforms this $p/b = 3m + 1/3m - 2$ three-pass Regular Knot into a $p/b = 4m + 1/4m - 3$ four-pass Regular Knot, and so on.

Note that the first stage is **set i** with $n = 1$ and $nb = b = \alpha m = m$. See the upper two algorithm diagrams in Fig. 1106.

The first stage has been completed after half-cycle $2\Delta^* + 2 = 2m$ has been laid down up to the Standing End (half-cycle $2\Delta^* + 2 = 2m$ reaches the Standing End at the lower left-hand Δ^*-crossing in the algorithm diagrams of Fig. 1107).

The second stage starts at the end of half-cycle $2\Delta^* + 2 = 2m$ with an over-crossing on the Standing End and ends when half-cycle $4\Delta^* + 2 = 4m - 2$ reaches the lower left-hand 2Δ*-crossing. The odd-numbered half-cycles in this stage build the 2-pass pattern sequence from the right-hand bight-boundary towards the left-hand bight-boundary and start from the left-hand bight-boundary with the crossings sequence $u - o - \cdots$ till the 2-pass pattern sequence has been reached. The even-numbered half-cycles in this stage build the 2-pass pattern sequence from the left-hand bight-boundary towards the right-hand bight-boundary and start from the right-hand bight-boundary with the crossings sequence $o - u - \cdots$ when $m = odd$ or $u - o - \cdots$ when $m = even$ till the 2-pass pattern sequence has been reached.

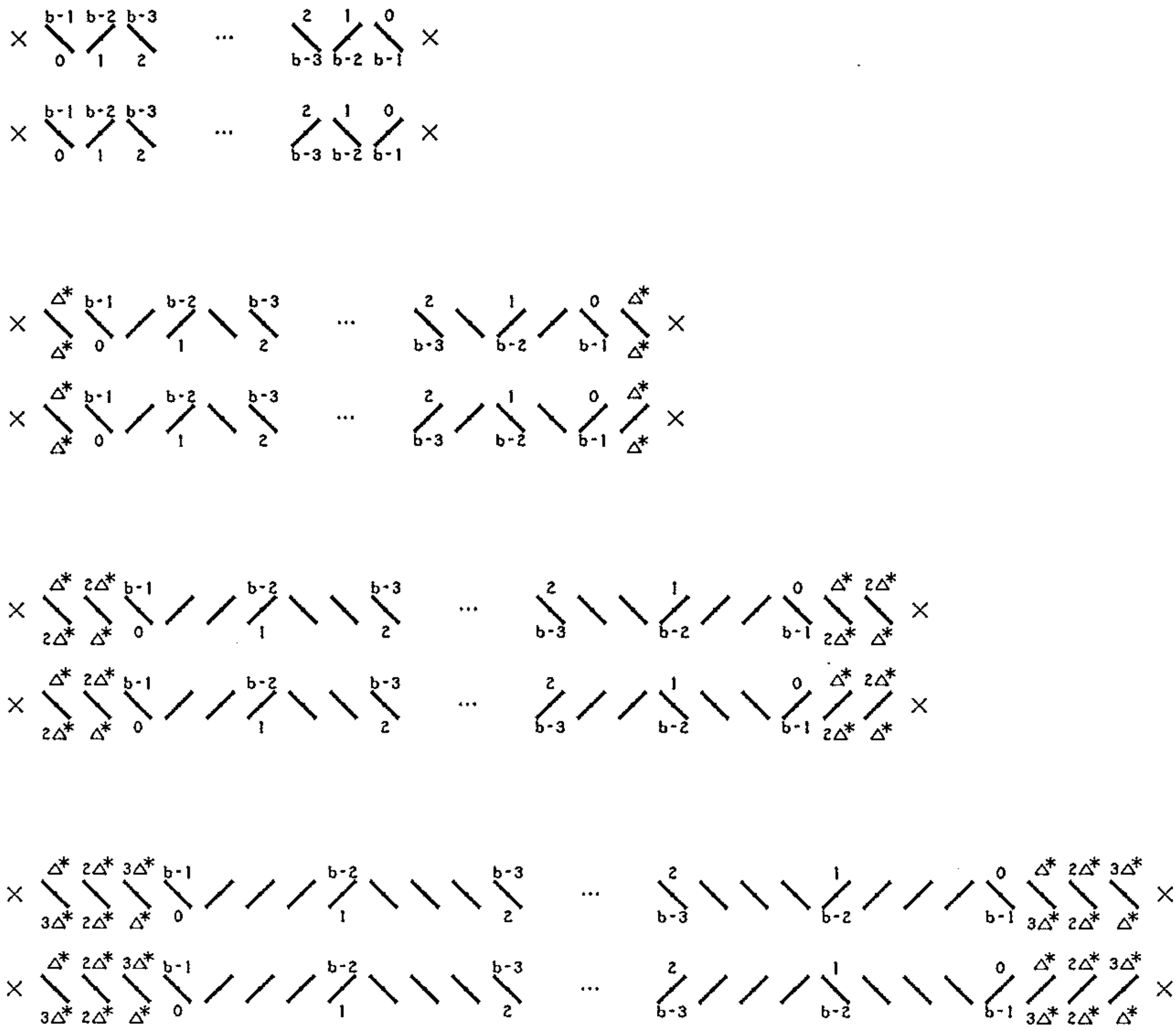

Fig. 1106 — **set iii** with $\alpha = 1, 2, 3$ and 4.

The third stage starts at the end of half-cycle $4\Delta^* + 2 = 4m - 2$ with an over-crossing on the Standing End and ends when half-cycle $6\Delta^* + 2 = 6m - 4$ reaches the lower left-hand 3Δ*-crossing. The odd-numbered half-cycles in this stage build the 3-pass pattern sequence from the right-hand bight-boundary towards the left-hand bight-boundary and start from the left-hand bight-boundary with the crossings sequence $2u - 2o - \cdots$ till the 3-pass pattern sequence has been reached. The even-numbered half-cycles in this stage build the 3-pass pattern sequence from the left-hand bight-boundary towards the right-hand bight-boundary and start from the right-hand bight-boundary with the crossings sequence $2o - 2u - \cdots$ when $m = odd$ or $2u - 2o - \cdots$ when $m = even$ till the 3-pass pattern sequence has been reached.

And so on.

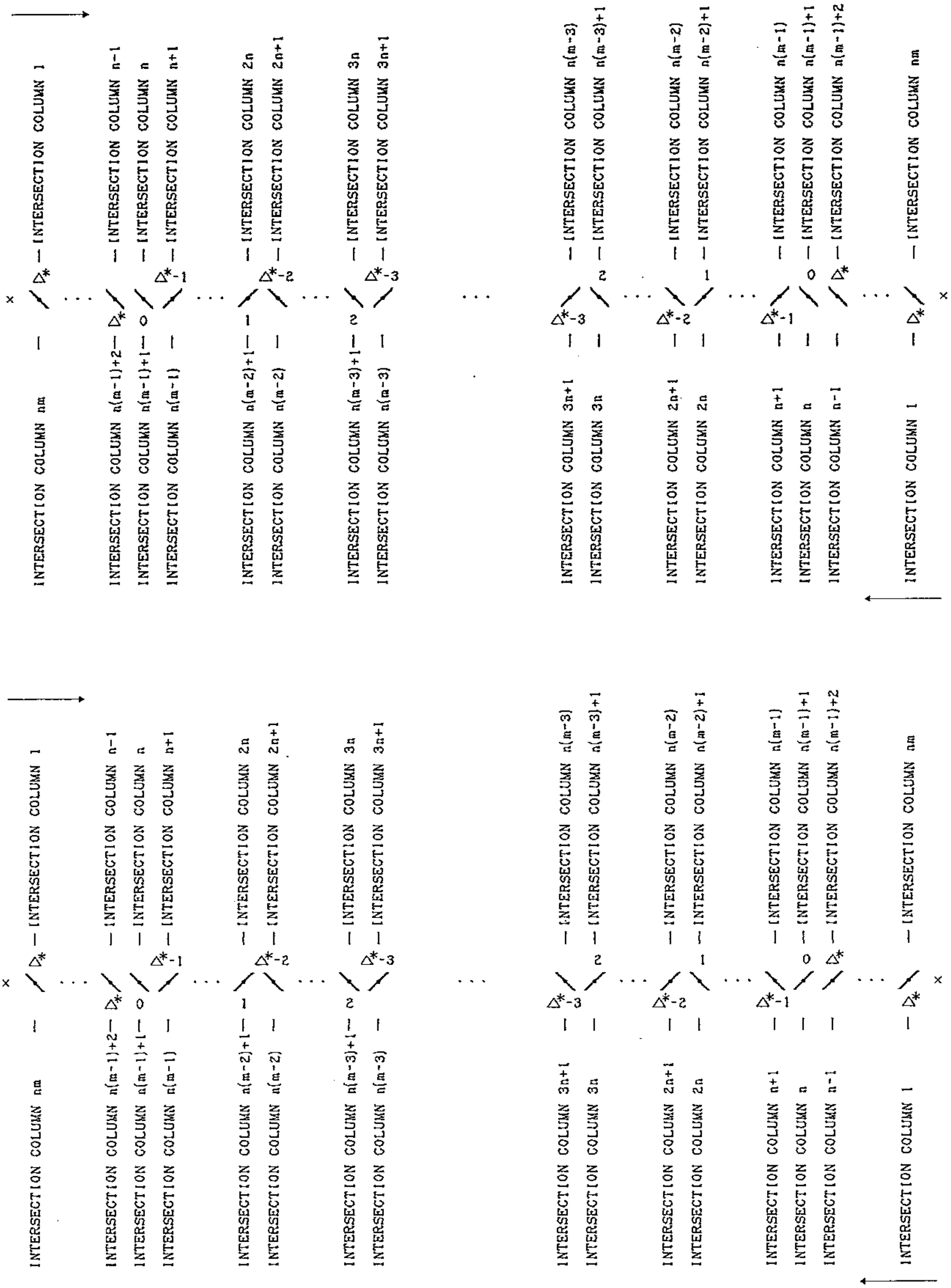

Fig. 1107 — Towards the end of the first stage, start of the second stage.

Fig. 1108 shows an example where $n = 3$, $m = 7$. The bold half-cycles in the upper string-run diagram are the half-cycles in the first stage. The bold half-cycles in the lower string-run diagram are the half-cycles in the first and second stage. The dotted half-cycles in the lower string-run diagram are the further half-cycles in the third stage.

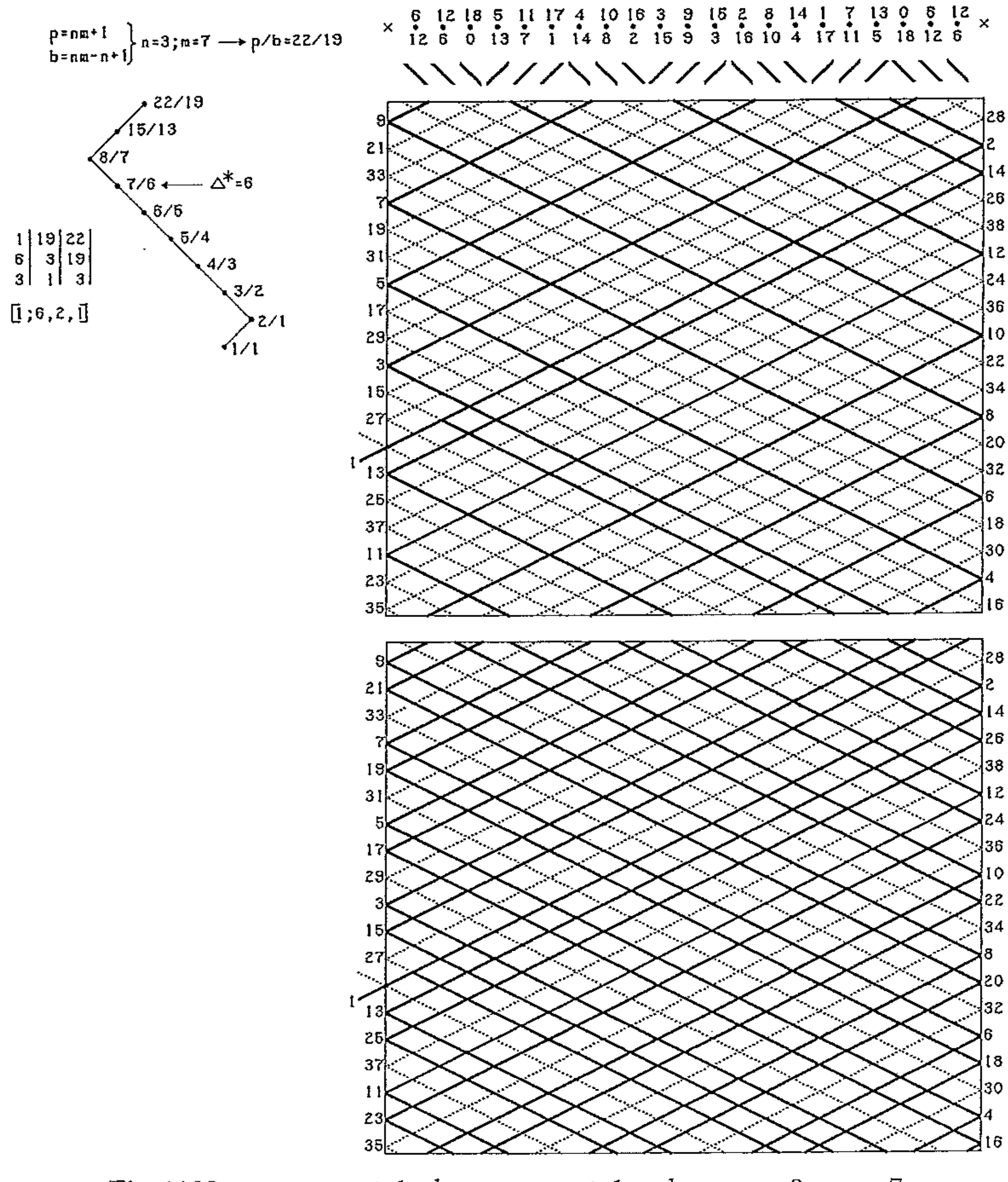

Fig. 1108 — $p = nm + 1$, $b = nm - n + 1$, where $n = 3$, $m = 7$.

set iv :

The path-formula $[0; 1, m, n-1, 1]$ of the path to $p/b = nm + 1/nm + n + 1$ in the RKT is obtained with Euclid's algorithm and from this path in the RKT we obtain $\Delta^* = nm - m + n$. See Fig. 1109. Hence $\Delta^* = b - 1 - m$ and consequently $|n\Delta^*|_b = |(n-1)b + 1|_b = 1$.

The general layout of the associated algorithm diagram is depicted in Fig. 1110. The intersection-columns, indicated by the dots, can be given an α-pass coding, where $\alpha = n$. Examples for $\alpha = n$ equal 1, 2, 3 and 4 are shown in Fig. 1112.

The braiding of such an α-pass column-coded Regular Knot consists of $n = \alpha$ stages:

the first stage produces a $p/b = m+1/m+2$ under-over coded Regular Knot (a one-pass Regular Knot), the second stage transforms this $p/b = m+1/m+2$ under-over coded Regular Knot into a $p/b = 2m+1/2m+3$ two-pass Regular Knot, the third stage transforms this $p/b = 2m+1/2m+3$ two-pass Regular Knot into a $p/b = 3m+1/3m+4$ three-pass Regular Knot, the fourth stage transforms this $p/b = 3m+1/3m+4$ three-pass Regular Knot into a $p/b = 4m+1/4m+5$ four-pass Regular Knot, and so on.

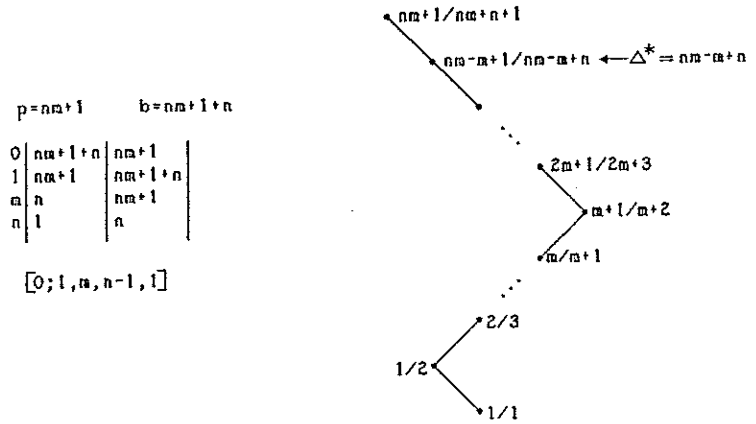

Fig. 1109 — $p = nm+1$, $b = nm+n+1$, hence $\Delta^* = nm - m + n$.

Note that the first stage is **set ii** with $n = 1$ and $nb = b = \alpha m + 2 = m + 2$. See the upper two algorithm diagrams in Fig. 1112.

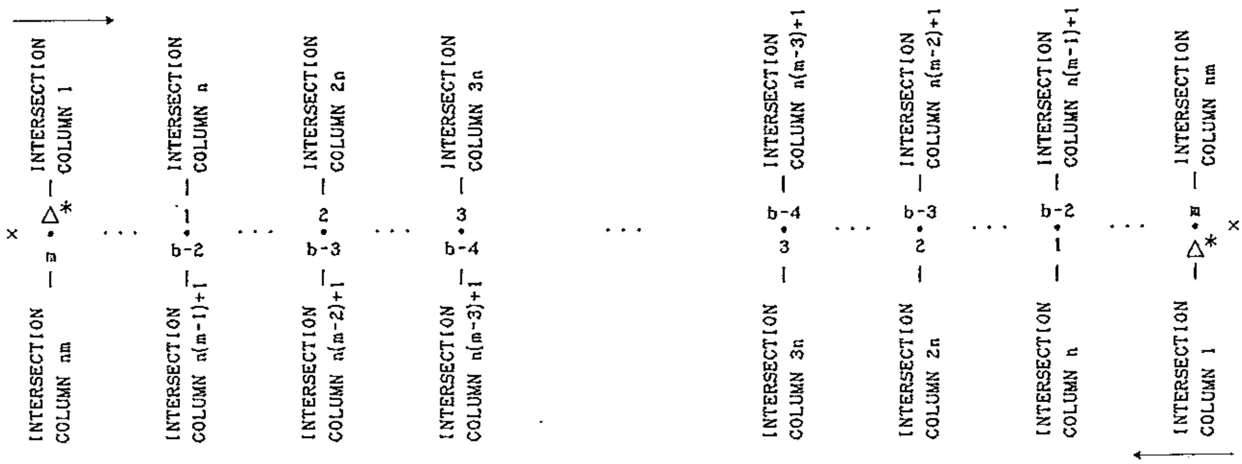

Fig. 1110 — The general layout of the algorithm diagram.

The first stage has been completed after half-cycle $2m+4$ (hence $i \leq m+1$) has been laid down up to the Standing End (half-cycle $2m+4$ reaches the Standing End after the lower left-hand m-crossing in the algorithm diagrams of Fig. 1111).

The crossings sequence associated with the first half-cycle in the second stage (half-cycle $2m+5$, hence $i \leq m+1$) is $\frac{m-1}{2} \times [u-o] - u$ when $m = odd$ or $\frac{m}{2} \times [o-u]$ when $m = even$. The crossings sequence associated with the third half-cycle in the second stage (half-cycle $2m+7$, hence $i \leq m+2$) is $2u - \frac{m-1}{2} \times [o-u]$ when $m = odd$ or $2o - \frac{m-2}{2} \times [u-o] - u$ when $m = even$.

Note that $|(n-1)\Delta^*|_b = |(n-2)b + m + 2|_b = m + 2$.

The crossings sequence associated with the fifth half-cycle in the second stage (half-cycle $2m+9$, hence $i \leq m+3$) is $2u - 2o - \frac{m-3}{2} \times [u-o] - u$ when $m = odd$ or $2o - 2u - \frac{m-2}{2} \times [o-u]$ when $m = even$. And so on.

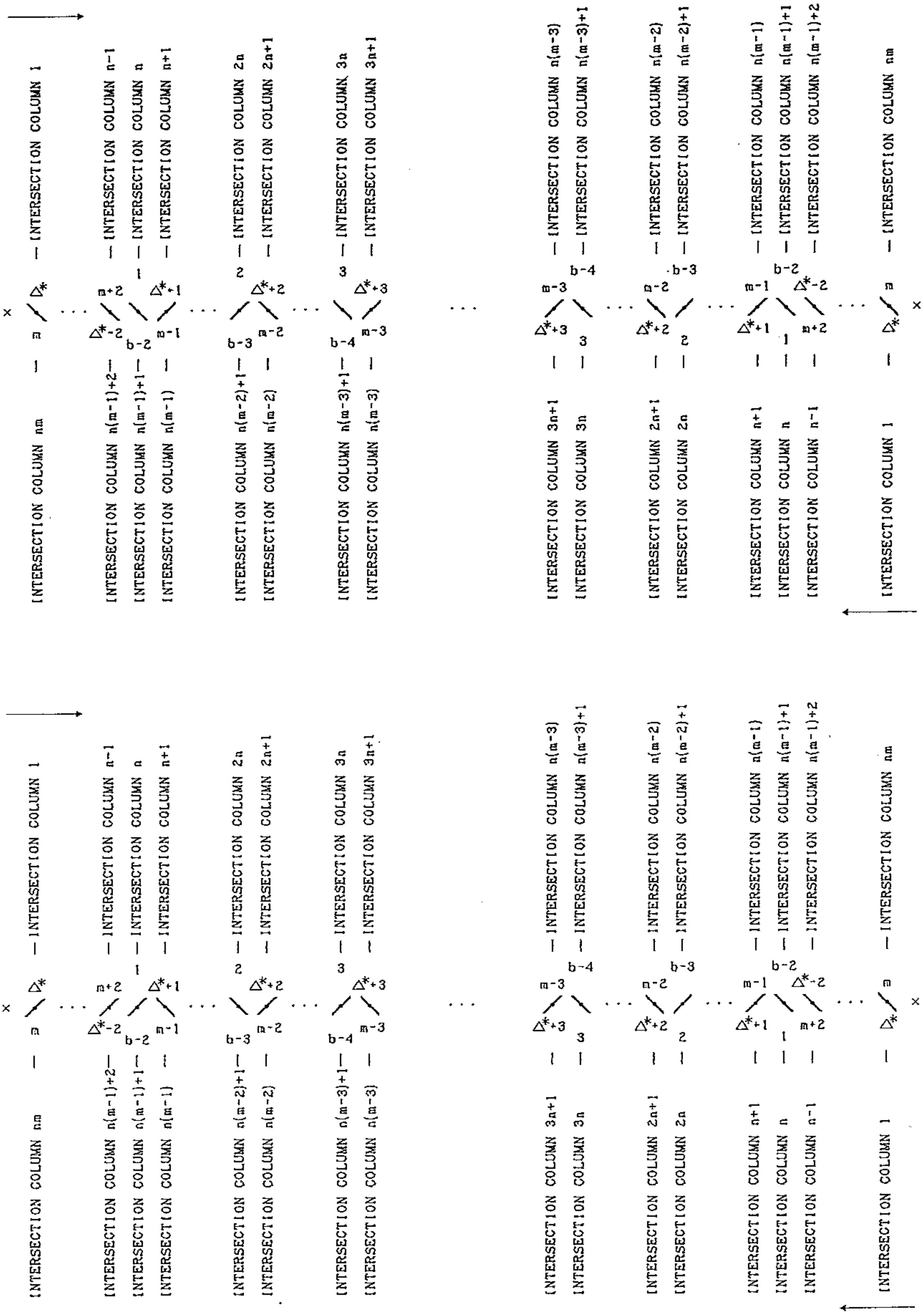

Fig. 1111 — Towards the end of the first stage, start of the second stage.

The crossings sequence associated with the second half-cycle in the second stage (half-cycle $2m+6$, hence $i \le m+2$) is $2o - \frac{m-1}{2} \times [u-o]$ when $m = odd$ or $2o - \frac{m-2}{2} \times [u-o] - u$ when $m = even$. The crossings sequence associated with the fourth half-cycle in the second stage (half-cycle $2m+8$, hence $i \le m+3$) is $2o - 2u - \frac{m-3}{2} \times [o-u] - o$ when $m = odd$ or $2o - 2u - \frac{m-2}{2} \times [o-u]$ when $m = even$. The crossings sequence associated with the sixth half-cycle in the second stage (half-cycle $2m+10$, hence $i \le m+4$) is $2o - 2u - 2o - \frac{m-3}{2} \times [u-o]$ when $m = odd$ or $2o - 2u - 2o - \frac{m-4}{2} \times [u-o] - u$ when $m = even$. And so on.

The second stage has been completed after half-cycle $4m+6$ (hence $i \le 2m+2$) has been laid down up to the Standing End (half-cycle $4m+6$ reaches the Standing End after the lower left-hand m-crossing in the algorithm diagrams of Fig. 1111).

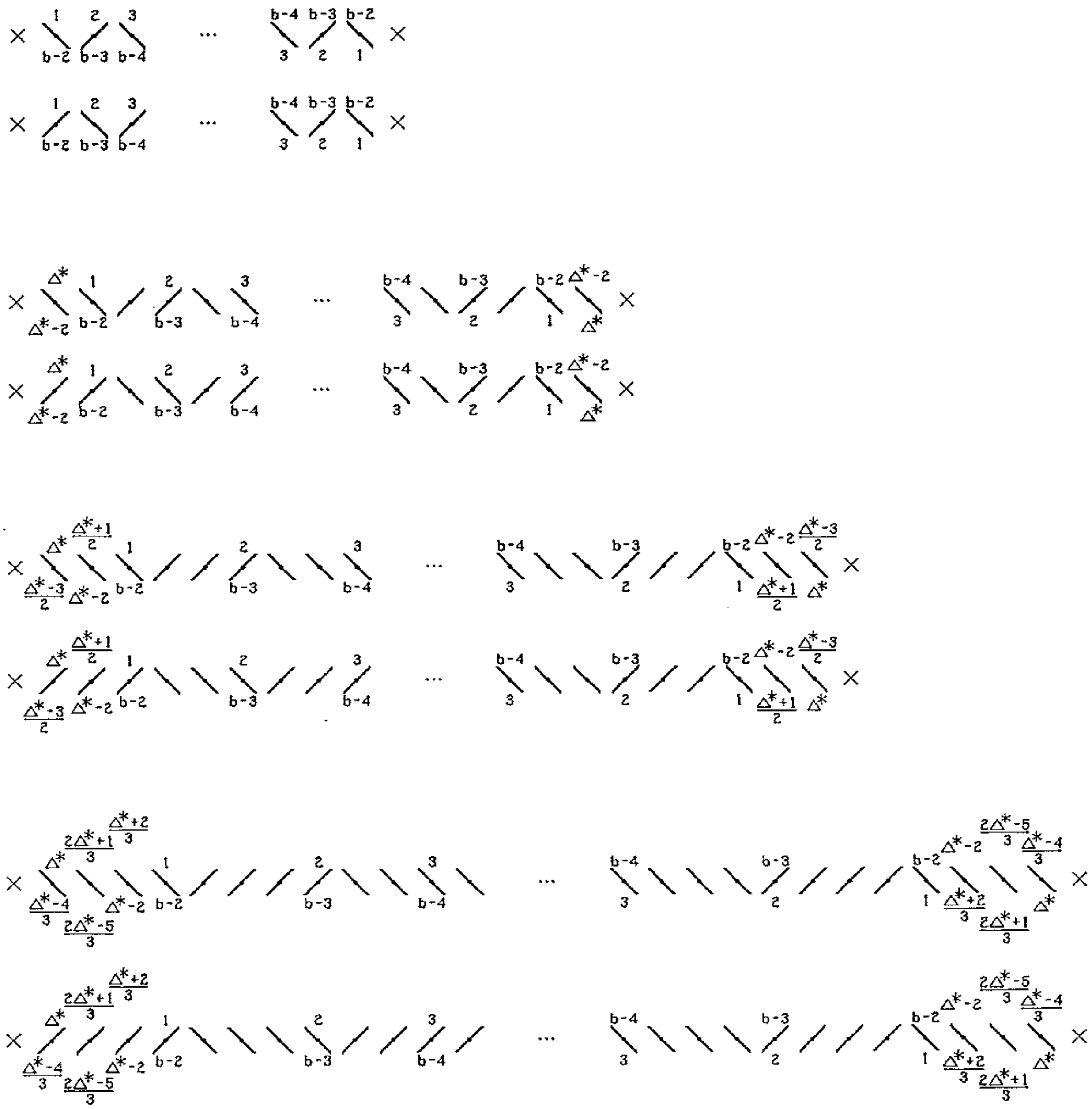

Fig. 1112 — **set iv** with $\alpha = 1, 2, 3$ and 4.

The crossings sequence associated with the first half-cycle in the third stage (half-cycle $4m+7$, hence $i \le 2m+2$) is $\frac{m-1}{2} \times [2u-2o] - 2u$ when $m = odd$ or $\frac{m}{2} \times [2o-2u]$ when $m = even$. The crossings sequence associated with the third half-cycle in the third stage (half-cycle $4m+9$, hence $i \leq 2m+3$) is $3u - \frac{m-1}{2} \times [2o - 2u]$ when $m = odd$ or $3o - \frac{m-2}{2} \times [2u - 2o] - 2u$ when $m = even$.

Note that $|(n-2)\Delta^*|_b = |(n-3)b + 2m + 3|_b = 2m+3$.

The crossings sequence associated with the fifth half-cycle in the third stage (half-cycle $4m+11$, hence $i \leq 2m+4$) is $3u - 3o - \frac{m-3}{2} \times [2u - 2o] - 2u$ when $m = odd$ or $3o - 3u - \frac{m-2}{2} \times [2o - 2u]$ when $m = even$. And so on.

The crossings sequence associated with the second half-cycle in the third stage (half-cycle $4m+8$, hence $i \leq 2m+3$) is $3o - \frac{m-1}{2} \times [2u - 2o]$ when $m = odd$ or $3o - \frac{m-2}{2} \times [2u - 2o] - 2u$ when $m = even$. The crossings sequence associated with the fourth half-cycle in the third stage (half-cycle $4m+10$, hence $i \leq 2m+4$) is $3o - 3u - \frac{m-3}{2} \times [2o - 2u] - 2o$ when $m = odd$ or $3o - 3u - \frac{m-2}{2} \times [2o - 2u]$ when $m = even$. The crossings sequence associated with the sixth half-cycle in the third stage (half-cycle $4m+12$, hence $i \leq 2m+5$) is $3o - 3u - 3o - \frac{m-3}{2} \times [2u - 2o]$ when $m = odd$ or $3o - 3u - 3o - \frac{m-4}{2} \times [2u - 2o] - 2u$ when $m = even$. And so on.

The third stage has been completed after half-cycle $6m+8$ (hence $i \leq 3m+3$) has been laid down up to the Standing End (half-cycle $6m+8$ reaches the Standing End after the lower left-hand m-crossing in the algorithm diagrams of Fig. 1111).

And so on.

An example where $n = 3$, $m = 5$ is shown in Fig. 1113.

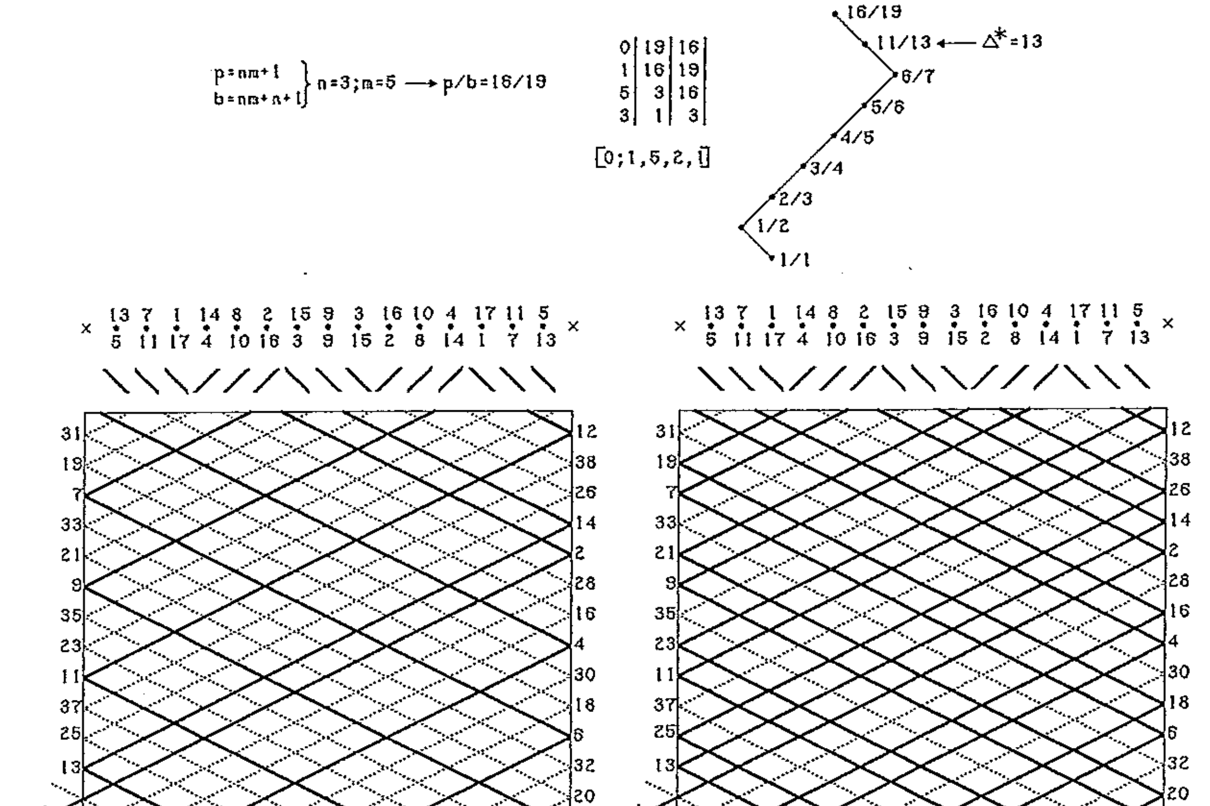

Fig. 1113 — $p = nm+1$, $b = nm+n+1$, where $n = 3$, $m = 5$.

The bold half-cycles in the left-hand string-run diagram are the half-cycles in the first stage. The bold half-cycles in the right-hand string-run diagram are the half-cycles in the first and second stage. The dotted half-cycles in the right-hand string-run diagram are the further half-cycles in the third stage.

Irrespective of the weaving-pattern, the braiding to a weaving-pattern is very simple for the two cases **i** and **ii** when $n = 1$, hence when $p = b+1$ and $p = b-1$ respectively since the weaving-pattern gets built up from the two bight-boundaries towards the centre. Braiding to the weaving-pattern for the two cases **iii** and **iv** is in general not quite so easy and it is in general more convenient to call in the assistance of the algorithm diagram.

Say that we like to braid the knot $p/b = nm + 1/nm + 1 - n$ (case **iii**) with the general column-coding as depicted in Fig. 1114.

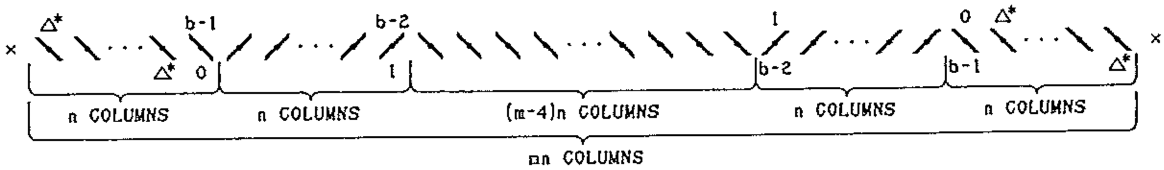

Fig. 1114 — A general column-coding.

The associated algorithm diagrams for three examples are given in Fig. 1115 (with the first crossing encountered an over-crossing).

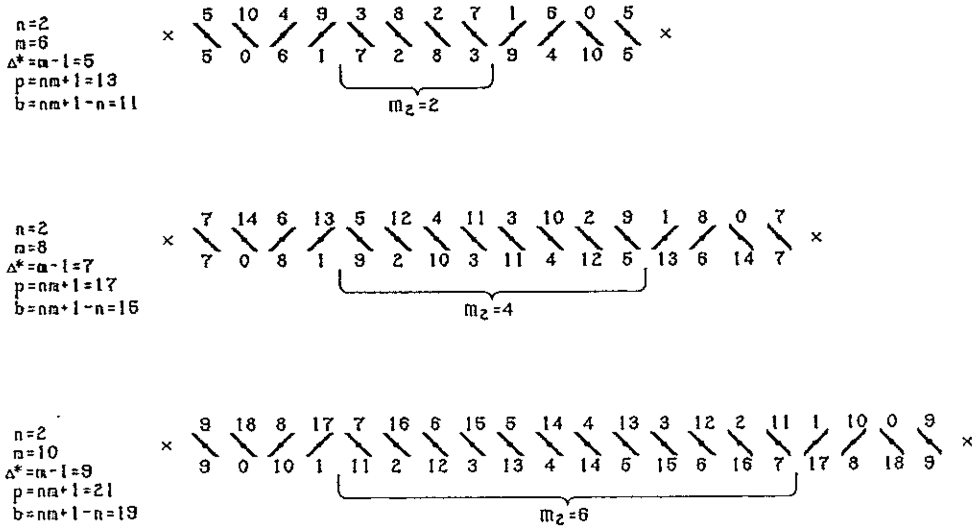

Fig. 1115 — Three examples associated with the general column-coding of Fig. 1114.

The half-cycle braiding algorithms for the second example in Fig. 1115 are:

1. : Free run.
2. $(i = 0)$: $(s)o$.
3. $(i = 0)$: u.
4. $(i \leq 1)$: $(s)u - o$.
5. $(i \leq 1)$: $o - u$.
6. $(i \leq 2)$: $(s)o - u - o$.
7. $(i \leq 2)$: $u - o - u$.
8. $(i \leq 3)$: $(s,1)2o - u - o$.
9. $(i \leq 3)$: $2u - o - u$.
10. $(i \leq 4)$: $(s,2)3o - u - o$.

11.	$(i \leq 4)$	:	$3u - o - u$.
12.	$(i \leq 5)$	:	$(s,3)4o - u - o$.
13.	$(i \leq 5)$	:	$4u - o - u$.
14.	$(i \leq 6)$	:	$(s)u - 4o - u - o$.
15.	$(i \leq 6)$	:	$o - 4u - o - u$.
16.	$(i \leq 7)$	:	$(s)o - u - 4o - u - (1,s)2o$.
17.	$(i \leq 7)$	:	$u - o - 4u - o - 2u$.
18.	$(i \leq 8)$	:	$o - u - 4o - (1,s)2u - 2o$.
19.	$(i \leq 8)$	:	$u - o - 4u - 2o - 2u$.
20.	$(i \leq 9)$	:	$o - u - (4,s)5o - 2u - 2o$.
21.	$(i \leq 9)$	:	$u - o - 5u - 2o - 2u$.
22.	$(i \leq 10)$	:	$o - u - (3,s,2)6o - 2u - 2o$.
23.	$(i \leq 10)$	:	$u - o - 6u - 2o - 2u$.
24.	$(i \leq 11)$	:	$o - u - (2,s,4)7o - 2u - 2o$.
25.	$(i \leq 11)$	:	$u - o - 7u - 2o - 2u$.
26.	$(i \leq 12)$	:	$o - u - (1,s,6)8o - 2u - 2o$.
27.	$(i \leq 12)$	:	$u - o - 8u - 2o - 2u$.
28.	$(i \leq 13)$	:	$o - (1,s)2u - 8o - 2u - 2o$.
29.	$(i \leq 13)$	:	$u - 2o - 8u - 2o - 2u$.
30.	$(i \leq 14)$	:	$(1,s)2o - 2u - 8o - 2u - 2o$.

When $m = even$, the same general arrangement of the column-coding can be obtained by doubling the n-value and halving the m-value. The resulting general column-coding is shown in Fig. 1116 (with the first crossing encountered an over-crossing).

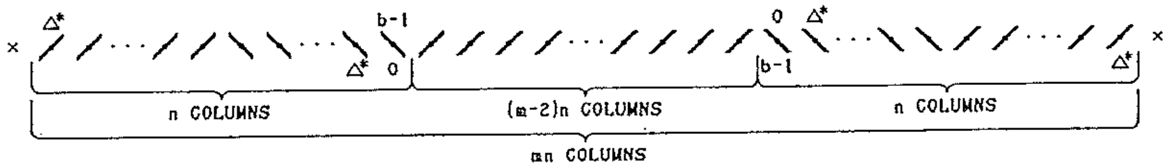

Fig. 1116 — A general column-coding.

The three examples in Fig. 1115 have then their n, m and b-values changed to those in Fig. 1117. Their p-values do of course not change.

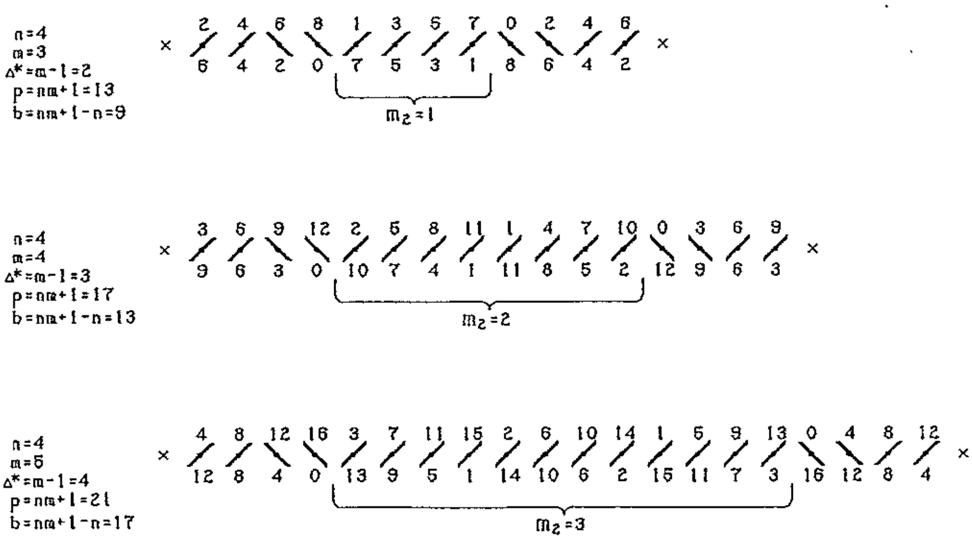

Fig. 1117 — Three examples associated with the general column-coding of Fig. 1116.

The half-cycle braiding algorithms for the second example in Fig. 1117 are:

1\. : Free run.
2\. $(i = 0)$: $(s)o$.
3\. $(i = 0)$: u.
4\. $(i \leq 1)$: $(s)u - o$.
5\. $(i \leq 1)$: $o - u$.
6\. $(i \leq 2)$: $(s,1)2u - o$.
7\. $(i \leq 2)$: $2o - u$.
8\. $(i \leq 3)$: $(s,2)3u - (1,s)2o$.
9\. $(i \leq 3)$: $3o - 2u$.
10\. $(i \leq 4)$: $(3,s)4u - 2o$.
11\. $(i \leq 4)$: $4o - 2u$.
12\. $(i \leq 5)$: $(2,s,2)5u - 2o$.
13\. $(i \leq 5)$: $5o - 2u$.
14\. $(i \leq 6)$: $(1,s,4)6u - 2o - (s)u$.
15\. $(i \leq 6)$: $6o - 2u - o$.
16\. $(i \leq 7)$: $(6,s)7u - 2o - u$.
17\. $(i \leq 7)$: $7o - 2u - o$.
18\. $(i \leq 8)$: $(4,s,3)8u - 2o - u$.
19\. $(i \leq 8)$: $8o - 2u - o$.
20\. $(i \leq 9)$: $2u - (s)o - 6u - 2o - (1,s)2u$.
21\. $(i \leq 9)$: $2o - u - 6o - 2u - 2o$.
22\. $(i \leq 10)$: $2u - o - (6,s)7u - 2o - 2u$.
23\. $(i \leq 10)$: $2o - u - 7o - 2u - 2o$.
24\. $(i \leq 11)$: $2u - o - (3,s,4)8u - 2o - 2u$.
25\. $(i \leq 11)$: $2o - u - 8o - 2u - 2o$.
26\. $(i \leq 12)$: $2u - (1,s)2o - 8u - 2o - 2u$.

Say that we like to braid the knot $p/b = nm + 1/nm + 1 + n$ (case **iv**) with the general column-coding as depicted in Fig. 1118.

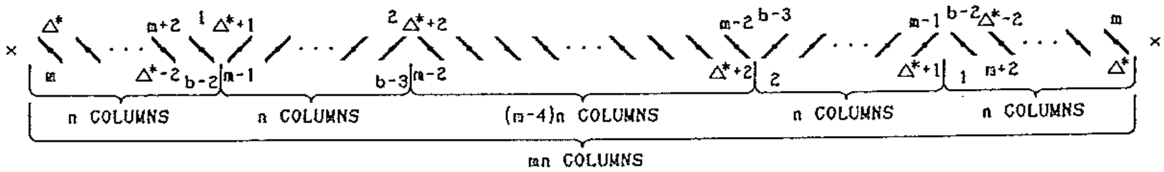

Fig. 1118 — A general column-coding.

The associated algorithm diagrams for three examples are given in Fig. 1119 (with the first crossing encountered an over-crossing). The half-cycle braiding algorithms for the second example in Fig. 1119 are:

1\. : Free run.
2\. $(i = 0)$: Free run.
3\. $(i = 0)$: Free run.
4\. $(i \leq 1)$: $(s)o$.
5\. $(i \leq 1)$: u.
6\. $(i \leq 2)$: $o - (s)u$.
7\. $(i \leq 2)$: $u - o$.
8\. $(i \leq 3)$: $o - u - (s)o$.
9\. $(i \leq 3)$: $u - o - u$.

10. $(i \leq 4)$: $o - u - (1, s)2o$.
11. $(i \leq 4)$: $u - o - 2u$.
12. $(i \leq 5)$: $o - u - (2, s)3o$.
13. $(i \leq 5)$: $u - o - 3u$.
14. $(i \leq 6)$: $o - u - (3, s)4o$.
15. $(i \leq 6)$: $u - o - 4u$.
16. $(i \leq 7)$: $o - u - 4o - (s)u$.
17. $(i \leq 7)$: $u - o - 4u - o$.
18. $(i \leq 8)$: $o - u - 4o - u - (s)o$.
19. $(i \leq 8)$: $u - o - 4u - o - u$.
20. $(i \leq 9)$: $o - u - 4o - u - o$.
21. $(i \leq 9)$: $u - o - 4u - o - u$.
22. $(i \leq 10)$: $(s, 1)2o - u - 4o - u - o$.
23. $(i \leq 10)$: $2u - o - 4u - o - u$.
24. $(i \leq 11)$: $2o - (s, 1)2u - 4o - u - o$.
25. $(i \leq 11)$: $2u - 2o - 4u - o - u$.
26. $(i \leq 12)$: $2o - 2u - (s, 4)5o - u - o$.
27. $(i \leq 12)$: $2u - 2o - 5u - o - u$.
28. $(i \leq 13)$: $2o - 2u - (2, s, 3)6o - u - o$.
29. $(i \leq 13)$: $2u - 2o - 6u - o - u$.
30. $(i \leq 14)$: $2o - 2u - (4, s, 2)7o - u - o$.
31. $(i \leq 14)$: $2u - 2o - 7u - o - u$.
32. $(i \leq 15)$: $2o - 2u - (6, s, 1)8o - u - o$.
33. $(i \leq 15)$: $2u - 2o - 8u - o - u$.
34. $(i \leq 16)$: $2o - 2u - 8o - (s, 1)2u - o$.
35. $(i \leq 16)$: $2u - 2o - 8u - 2o - u$.
36. $(i \leq 17)$: $2o - 2u - 8o - 2u - (s, 1)2o$.
37. $(i \leq 17)$: $2u - 2o - 8u - 2o - 2u$.
38. $(i \leq 18)$: $2o - 2u - 8o - 2u - 2o$.

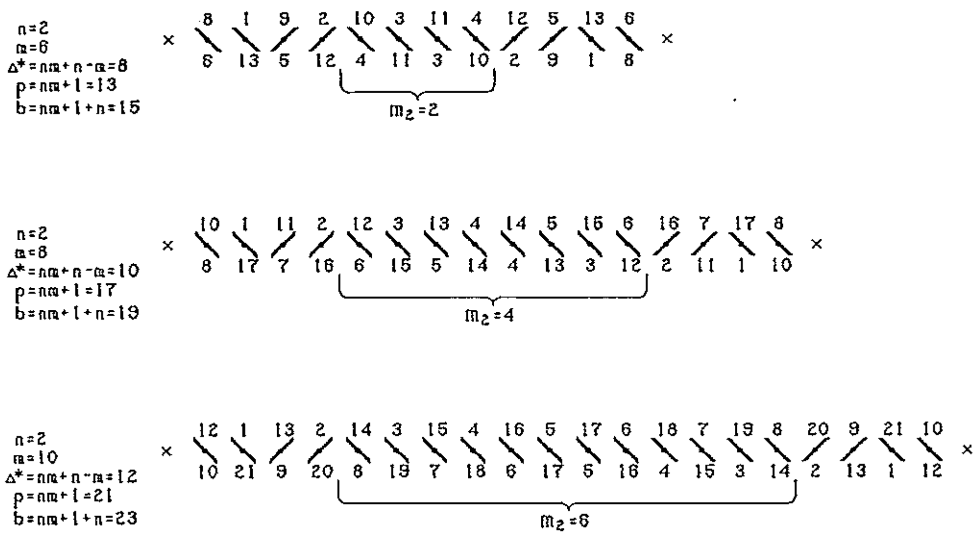

Fig. 1119 — Three examples associated with the general column-coding of Fig. 1118.

When m = *even*, the same general arrangement of the column-coding can be obtained by doubling the n-value and halving the m-value. The resulting general column-coding is shown in Fig. 1120 (with the first crossing encountered an over-crossing). The three examples in Fig. 1119 have then their n, m and b-values changed to those in Fig. 1121. Their p-values do of course not change.

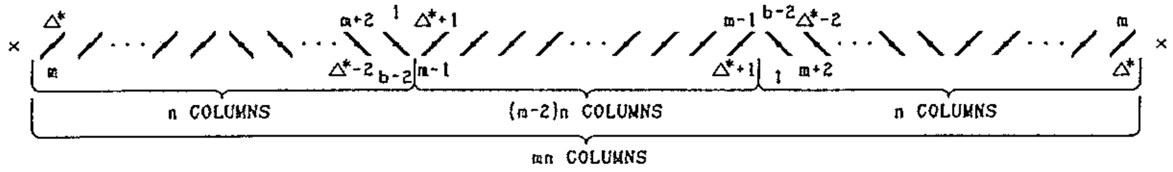

Fig. 1120 — A general column-coding.

The half-cycle braiding algorithms for the second example in Fig. 1121 are:

1.		:	Free run.
2.	$(i = 0)$	:	Free run.
3.	$(i = 0)$	:	Free run.
4.	$(i \leq 1)$	:	$(s)o$.
5.	$(i \leq 1)$	:	u.
6.	$(i \leq 2)$	:	$o - (s)u$.
7.	$(i \leq 2)$	:	$u - o$.
8.	$(i \leq 3)$	:	$o - (1, s)2u$.
9.	$(i \leq 3)$	:	$u - 2o$.
10.	$(i \leq 4)$	:	$o - (2, s)3u$.
11.	$(i \leq 4)$	:	$u - 3o$.
12.	$(i \leq 5)$	:	$o - 3u$.
13.	$(i \leq 5)$	:	$u - 3o$.
14.	$(i \leq 6)$	:	$(s, 1)2o - 3u$.
15.	$(i \leq 6)$	:	$2u - 3o$.
16.	$(i \leq 7)$	:	$2o - (s, 3)4u$.
17.	$(i \leq 7)$	:	$2u - 4o$.
18.	$(i \leq 8)$	:	$2o - (2, s, 2)5u$.
19.	$(i \leq 8)$	:	$2u - 5o$.
20.	$(i \leq 9)$	:	$2o - (4, s, 1)6u$.
21.	$(i \leq 9)$	:	$2u - 6o$.
22.	$(i \leq 10)$	:	$2o - 6u$.
23.	$(i \leq 10)$	:	$2u - 6o$.
24.	$(i \leq 11)$	:	$(s)u - 2o - 6u$.
25.	$(i \leq 11)$	:	$o - 2u - 6o$.
26.	$(i \leq 12)$	:	$u - 2o - (s, 6)7u$.
27.	$(i \leq 12)$	:	$o - 2u - 7o$.
28.	$(i \leq 13)$	:	$u - 2o - (3, s, 4)8u$.
29.	$(i \leq 13)$	:	$o - 2u - 8o$.
30.	$(i \leq 14)$	:	$u - 2o - 6u - (s)o - 2u$.
31.	$(i \leq 14)$	:	$o - 2u - 6o - u - 2o$.
32.	$(i \leq 15)$	:	$u - 2o - 6u - o - 2u$.
33.	$(i \leq 15)$	:	$o - 2u - 6o - u - 2o$.
34.	$(i \leq 16)$	:	$(s, 1)2u - 2o - 6u - o - 2u$.
35.	$(i \leq 16)$	:	$2o - 2u - 6o - u - 2o$.
36.	$(i \leq 17)$	:	$2u - 2o - (s, 6)7u - o - 2u$.
37.	$(i \leq 17)$	:	$2o - 2u - 7o - u - 2o$.
38.	$(i \leq 18)$	:	$2u - 2o - (4, s, 3)8u - o - 2u$.

39. $(i \leq 18)$: $2o - 2u - 8o - u - 2o$.
40. $(i \leq 19)$: $2u - 2o - 8u - (s,1)2o - 2u$.
41. $(i \leq 19)$: $2o - 2u - 8o - 2u - 2o$.
42. $(i \leq 20)$: $2u - 2o - 8u - 2o - 2u$.

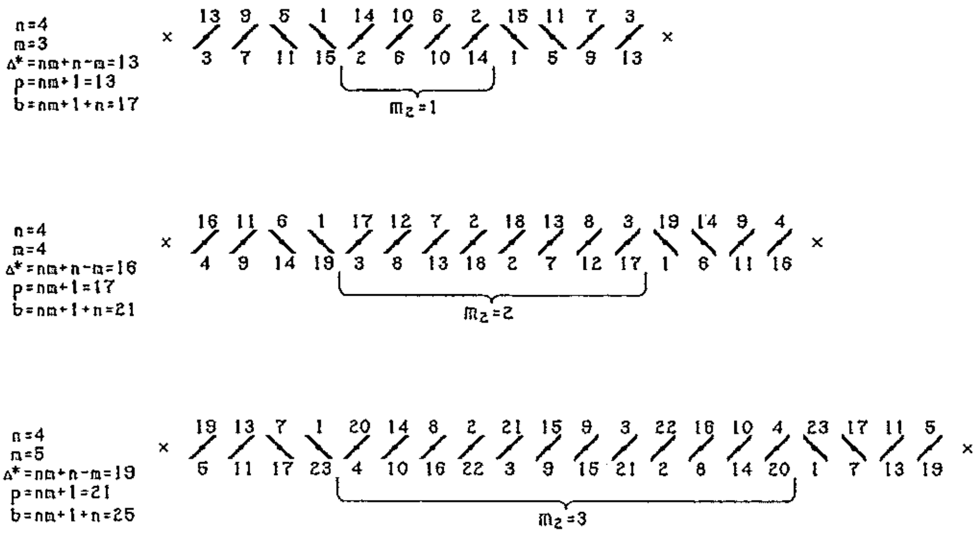

Fig. 1121 — Three examples associated with the general column-coding of Fig. 1120.

When the length of the string required is long, we would of course start braiding from the centre of the string-length. We would thus, for example, braid the second example in Fig. 1121 as follows:

Braid the first twenty two (22) half-cycles as indicated above. Then starting with the Standing End, braid the next twenty (20) half-cycles as follows:

23. $(i \leq 10)$: $2o - 6u$.
24. $(i \leq 11)$: $(s)o - 2u - 6o$.
25. $(i \leq 11)$: $u - 2o - 6u$.
26. $(i \leq 12)$: $o - 2u - (s,6)7o$.
27. $(i \leq 12)$: $u - 2o - 7u$.
28. $(i \leq 13)$: $o - 2u - (3,s,4)8o$.
29. $(i \leq 13)$: $u - 2o - 8u$.
30. $(i \leq 14)$: $o - 2u - 6o - (s)u - 2o$.
31. $(i \leq 14)$: $u - 2o - 6u - o - 2u$.
32. $(i \leq 15)$: $o - 2u - 6o - u - 2o$.
33. $(i \leq 15)$: $u - 2o - 6u - o - 2u$.
34. $(i \leq 16)$: $(s,1)2o - 2u - 6o - u - 2o$.
35. $(i \leq 16)$: $2u - 2o - 6u - o - 2u$.
36. $(i \leq 17)$: $2o - 2u - (s,6)7o - u - 2o$.
37. $(i \leq 17)$: $2u - 2o - 7u - o - 2u$.
38. $(i \leq 18)$: $2o - 2u - (4,s,3)8o - u - 2o$.
39. $(i \leq 18)$: $2u - 2o - 8u - o - 2u$.
40. $(i \leq 19)$: $2o - 2u - 8o - (s,1)2u - 2o$.
41. $(i \leq 19)$: $2u - 2o - 8u - 2o - 2u$.
42. $(i \leq 20)$: $2o - 2u - 8o - 2u - 2o$.

Note that the Standing End for these twenty half-cycles is the end of half-cycle 22.