THE BRAIDER

No.46 **MAY 2006.**

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A quarterly publication
for
the braiding artisan

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Solution to the question in Issue No. 45

Question on pg. 1068.

In general only two essential strings are required in an A-pass Standard Herringbone Grant Knot when the following two conditions are fulfilled:

1. There should only be two first-return string-runs.
2. Let the first-return string-run cover a distance of N nests. Then $\text{g.c.d.}(N, B^*) = 1$.

The number of nests N covered by a first-return string-run of a Standard Herringbone Grant Knot can readily be calculated from its associated Standard Herringbone Pineapple Knot. Let the associated Standard Herringbone Pineapple Knot have the half-cycle $1 \longrightarrow k$ from lower-left to upper-right. Then $x = 2k + 2 + A(2n-1)$, where n is a whole number, in the associated Standard Herringbone Pineapple Knot.[†] Let in a first-return string-run the number of half-cycles associated with the associated Standard Herringbone Pineapple Knot be equal to H. Then:

$$N = \frac{Hx + 4HA - 2\sum(l_i + r_i)}{2A}\,. \ddagger$$

For the string-run of the associated Standard Herringbone Pineapple Knot with $A = 5$ and $k = 4$ Fig. 826 shows the four sets of the two first-return string-runs with their associated Standard Herringbone Pineapple Knot half-cycles.

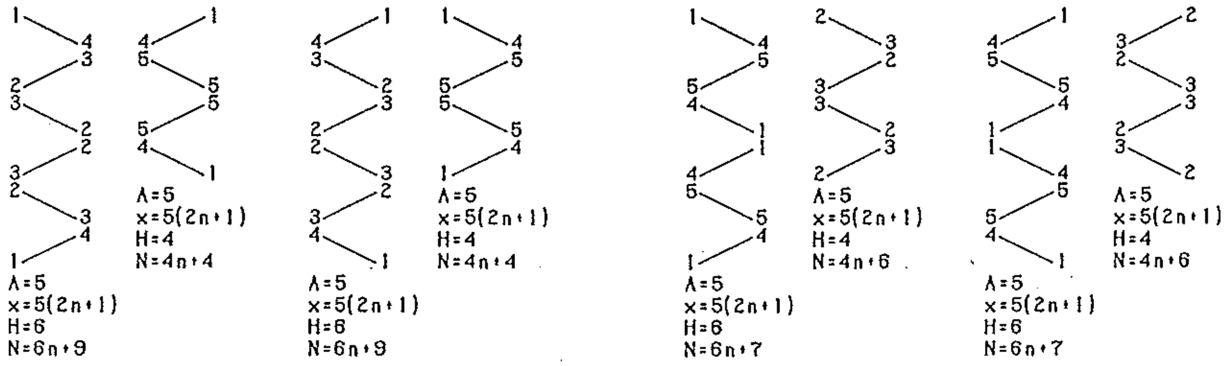


Fig. 826 — The sets of two first-return string-runs with their associated Standard Herringbone Pineapple Knot half-cycles.

The left two sets as well as the right two sets of the two first-return string-runs are each others mirror-image. Note that in the right two sets the first-return string-runs with $H = 6$ are identical.

The left two sets require that $\text{g.c.d.}(N, B^*) = 1$, hence $B^* = 7, 11, 13, 17, 19, \cdots$ for $n = 1$ and $B^* = 5, 11, 13, 17, 19, \cdots$ for $n = 2$.

The right two sets require that $\text{g.c.d.}(N, B^*) = 1$, hence $B^* = 3, 7, 9, 11, 17, 19, \cdots$ for $n = 1$ and $B^* = 3, 5, 9, 11, 13, 15, 17, \cdots$ for $n = 2$.

Only the right two sets give us a symmetric colour-pattern. They are shown in Fig. 827 for $n = 2$ (hence $x_{\text{Pineapple}} = 25$) and $B^* = 5$.

[†] See *The Braider*, Issue No. 23, pg. 521.

[‡] Note that $N = P_c$ for the component associated with a first-return string-run in the Standard Herringbone Grant Knot ($H = 2\alpha$ and $Hx_{\text{Pineapple}} + 4HA - 2\sum(l_i + r_i) = 2\left[\alpha x_{\text{Grant}} + \sum(\Delta_{l_i} + \Delta_{r_i})\right]$, while $A^{**} = A$ and $B^{**} = B^*$. See *The Braider*, Issue No. 19, pg. 417 and Issue No. 23, pg. 532.).

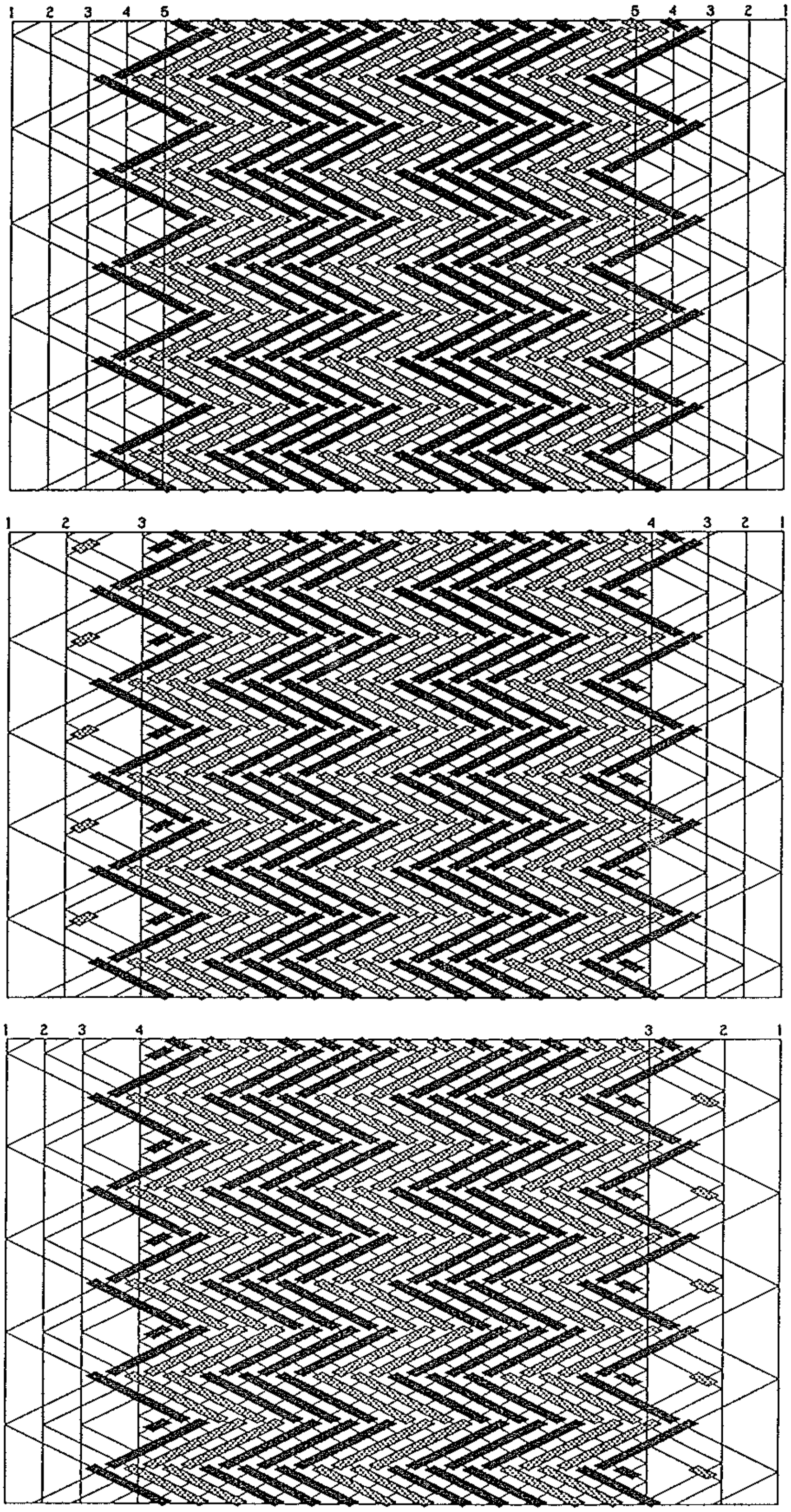


Fig. 827 — A two string Standard Herringbone Grant Knot with $A = 5$, $B^* = 5$, $x = 27$ and symmetric colour-pattern.

For two essential strings in general, the sets of the two first-return string-runs are shown in Figs. 828, 829, 830, 831, 832. Each of these sets has a mirror-imaged set; the right-hand set in Fig. 830 and the left-hand set in Fig. 831 are identical to their respective mirror-image. Note that for $k = A$ the right-hand set in Fig. 828 and that for $k = 2$ the right-hand set in Fig. 829 each reduce to one first-return string-run, in which case g.c.d.$(N, B^*) = 2$ is required for two essential strings.

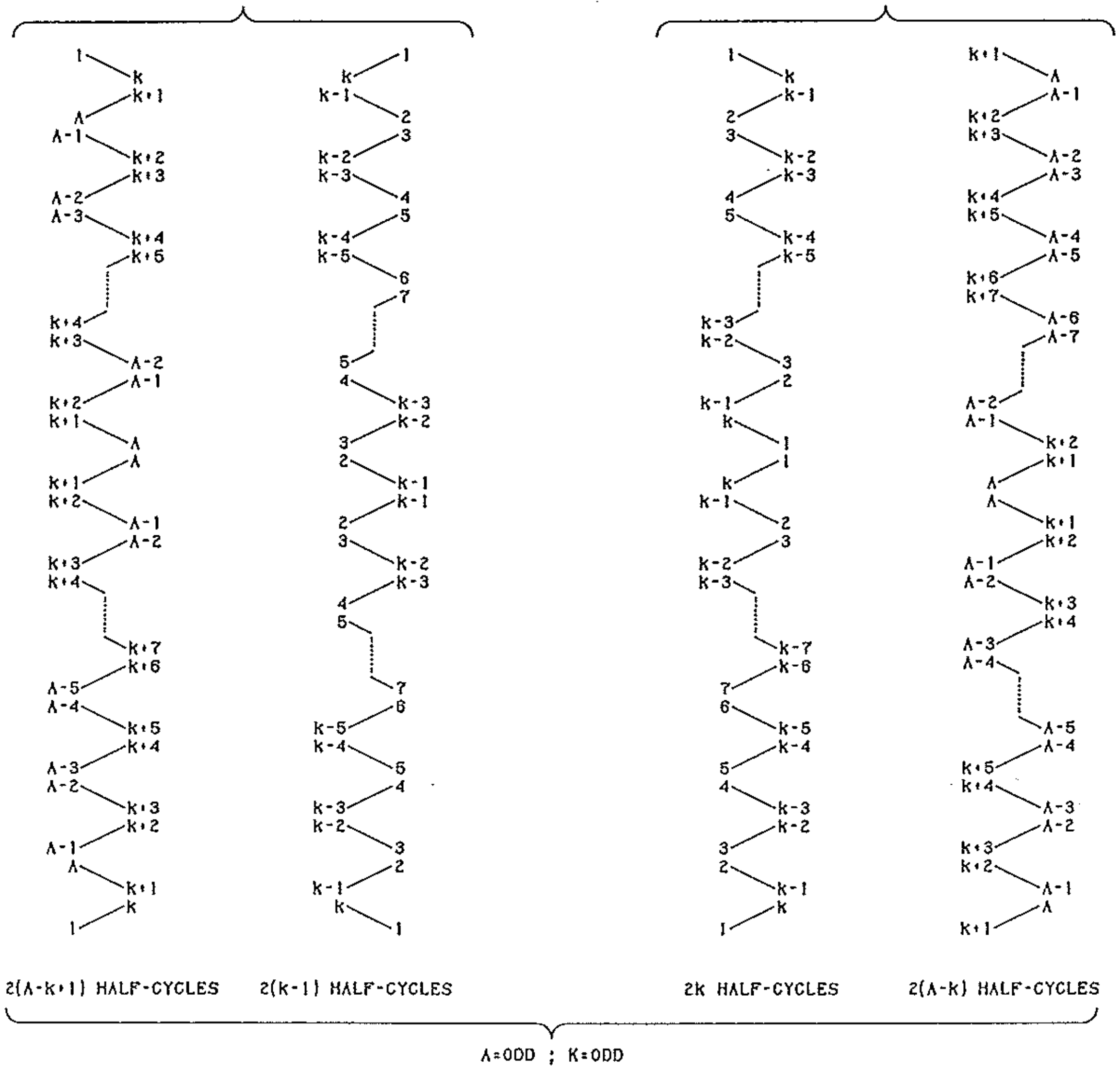
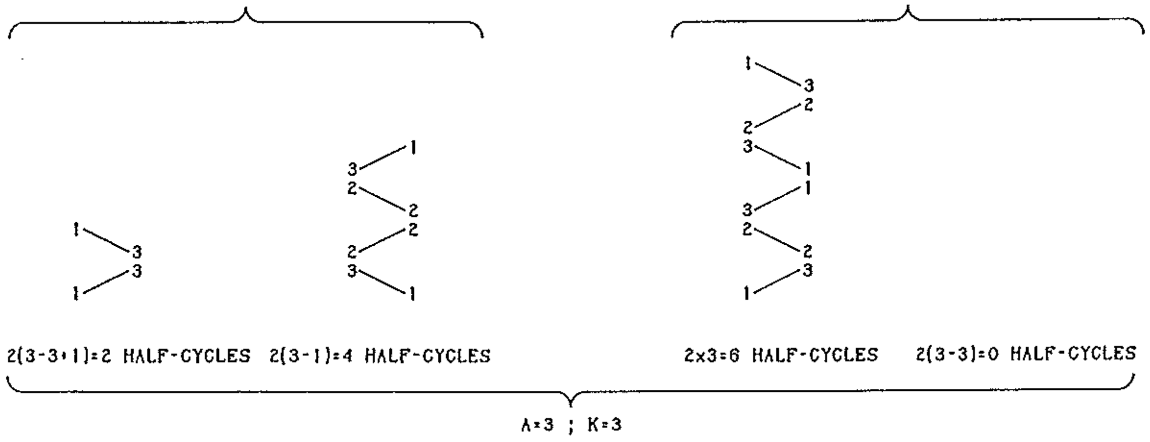


Fig. 828 — A=odd, k=odd.

Example:



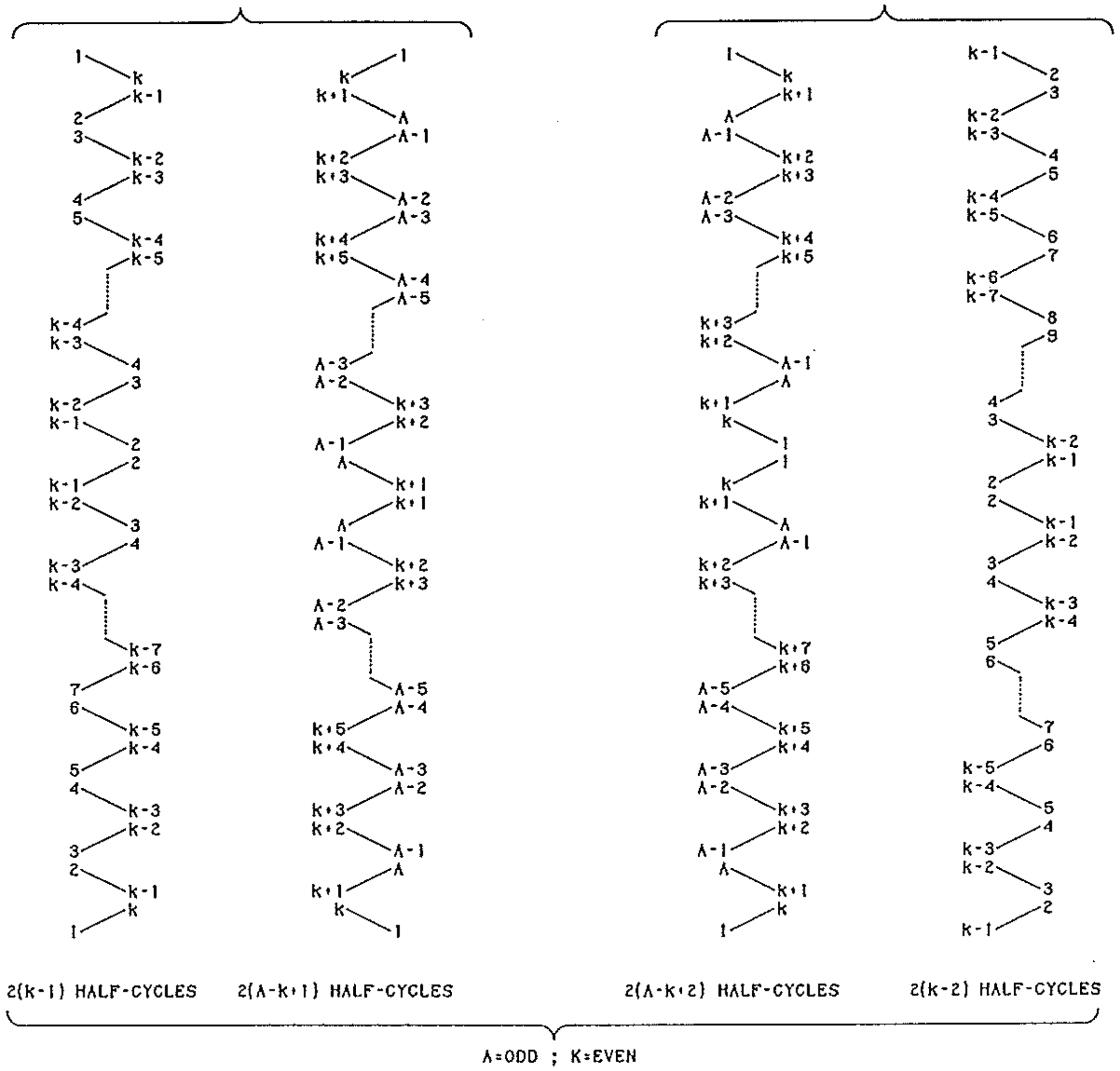
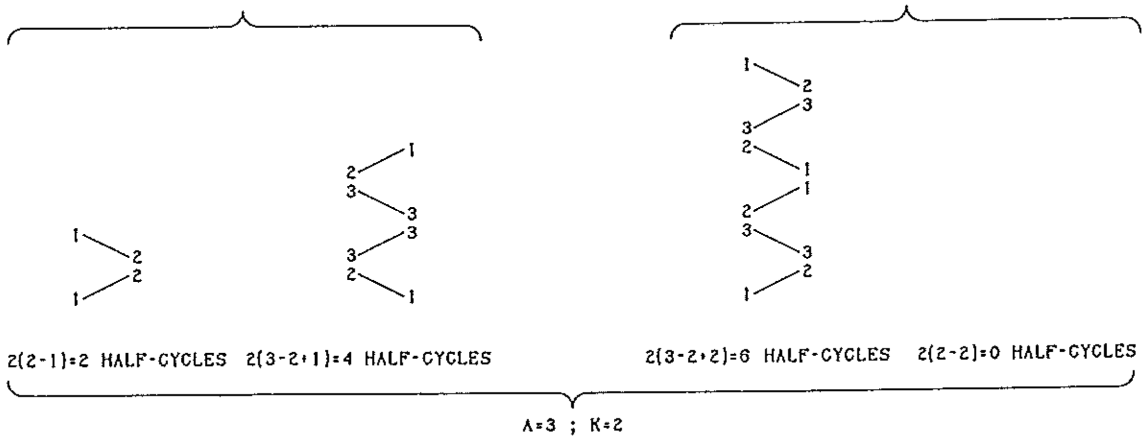


Fig. 829 — A=odd, k=even.

Example :



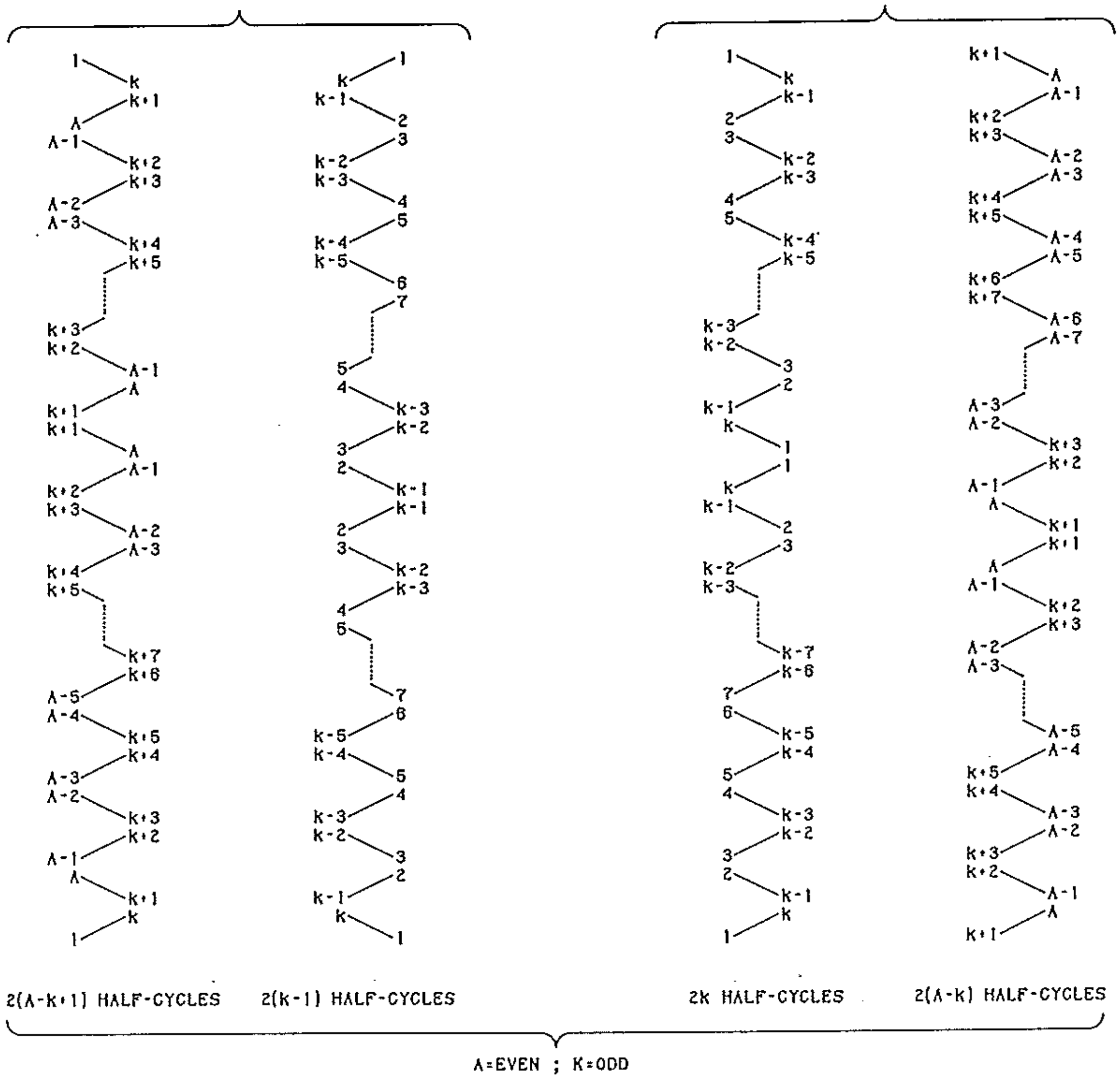
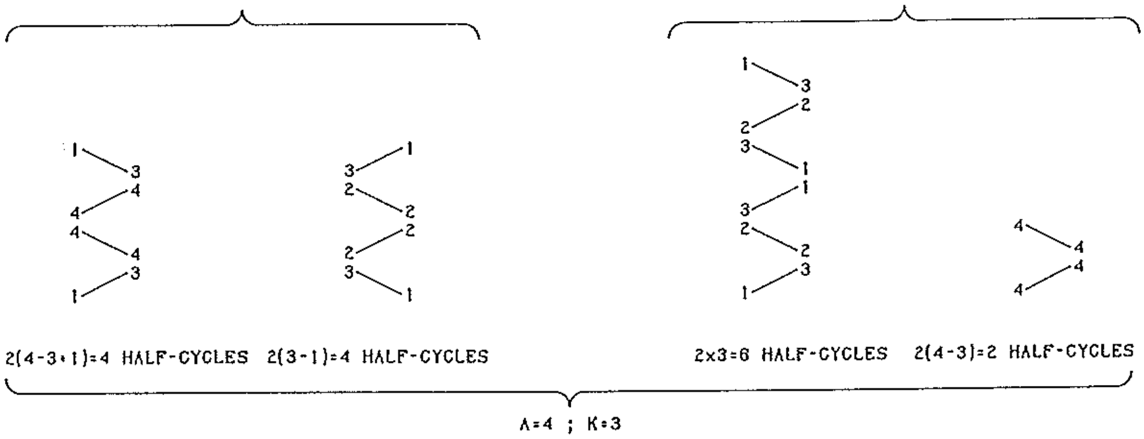


Fig. 830 — A=even, k=odd.

Example:



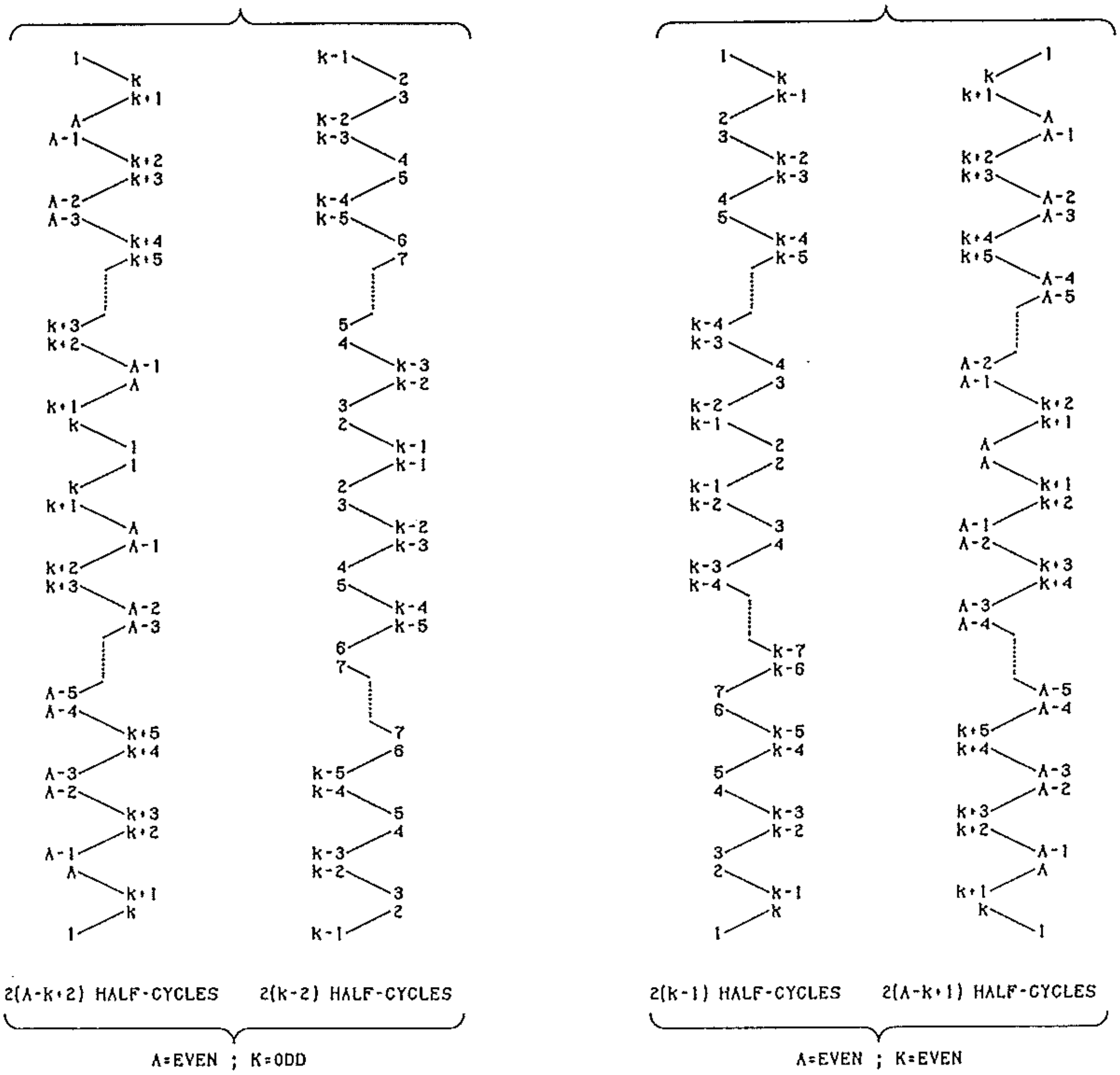
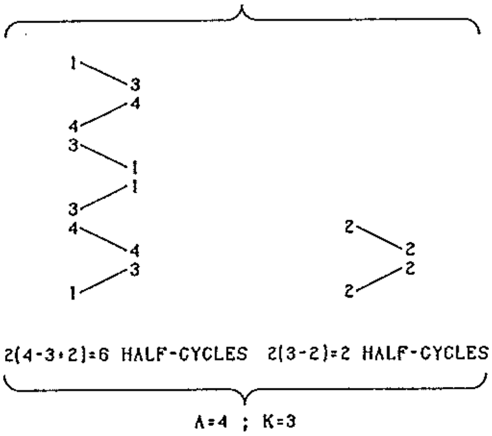
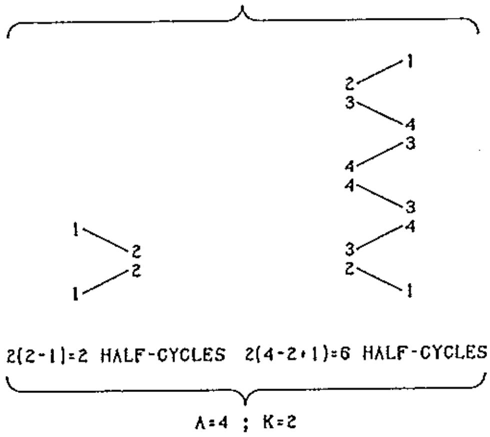


Fig. 831 — A=even , k=odd and A=even , k=even.

Example :





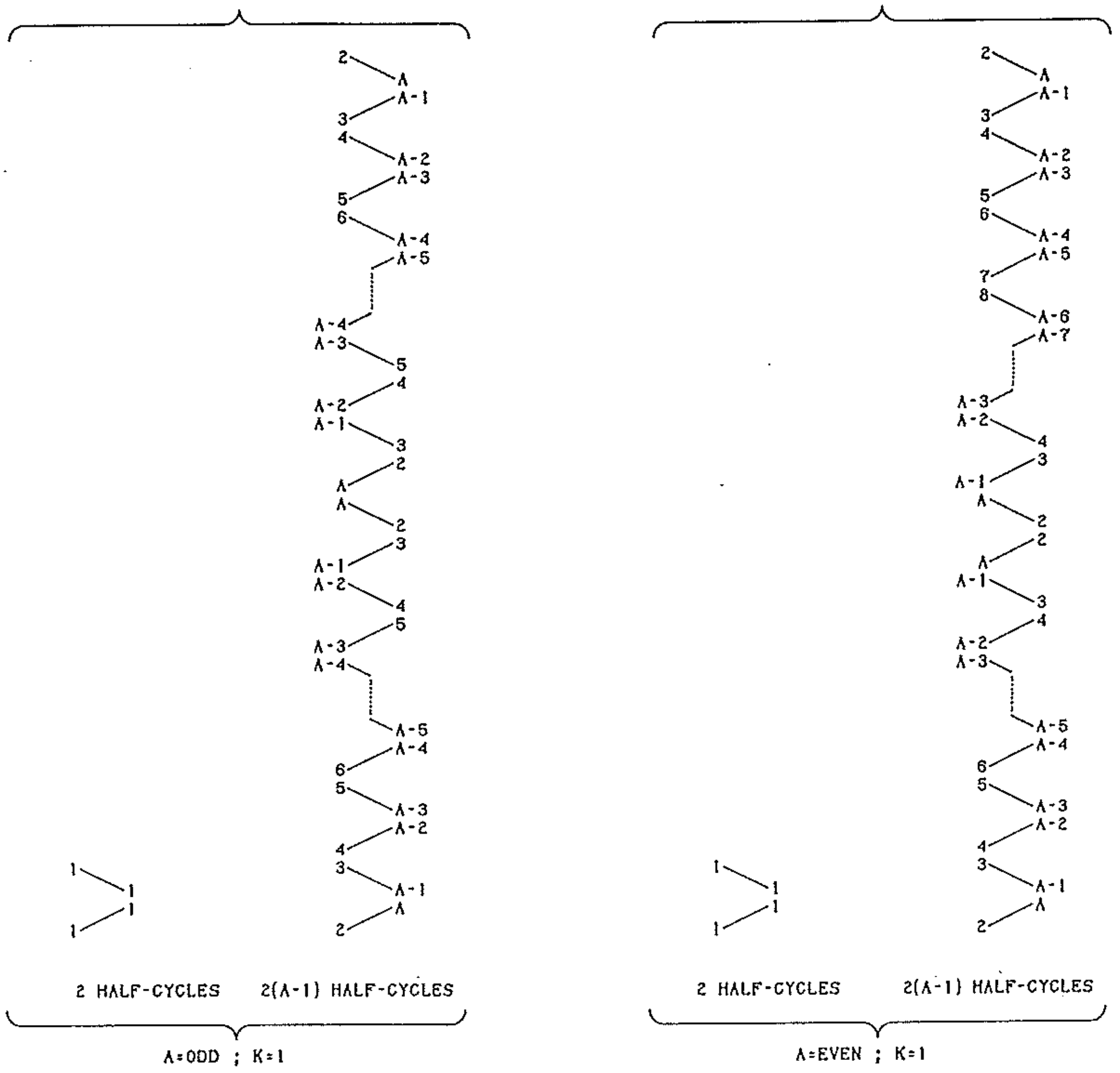
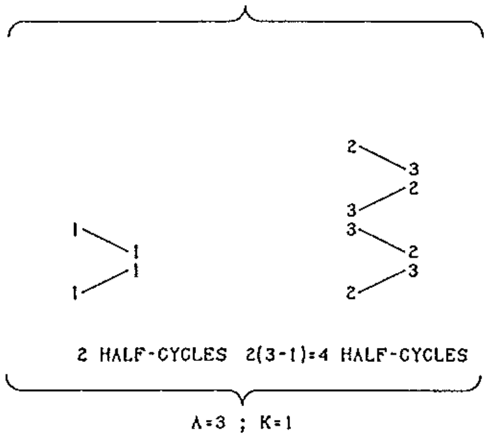
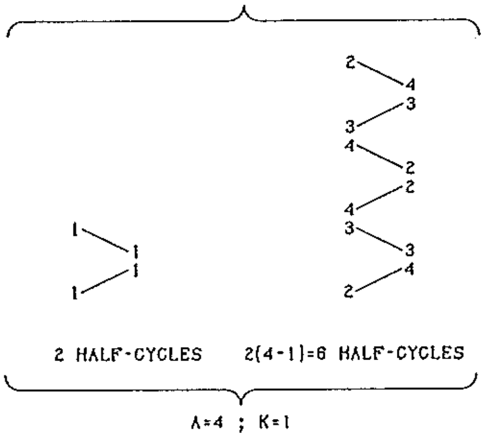


Fig. 832 — A=odd , k=1 and A=even , k=1.

Example :





Grant Knots

On pg. 1066 we have seen that the string-runs of **Standard Herringbone Grant Knots** are characterised by $y = A$. To be more explicit:

The string-run of a **Standard Herringbone Grant Knot** is characterised by $y = A$ and the total number of essential strings equals the total number of components (hence equals the number of first-return string-runs, thus when $\text{g.c.d.}(P_c, B^*) = 1$ for each component).

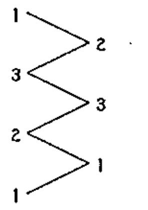
The string-run of a **Semi-Standard Herringbone Grant Knot** is characterised by $y = A$ and the total number of essential strings is greater than the total number of components (hence greater than the number of first-return string-runs, thus when for at least one component $\text{g.c.d.}(P_c, B^*) \neq 1$).

Note that the string-run of a Standard or Semi-Standard Herringbone Grant Knot can consist of one component when $A = odd$ (see pg. 1079), then $P_c = P_{total}$.

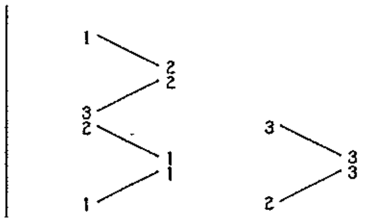
The string-run of a **Perfect Herringbone Grant Knot** is characterised by $y = A - 1$ or $y = A + 1$ and the total number of essential strings equals the total number of components (hence equals the number of first-return string-runs, thus when $\text{g.c.d.}(P_c, B^*) = 1$ for each component).

The string-run of a **Semi-Perfect Herringbone Grant Knot** is characterised by $y = A$ or $y = A + 1$ and the total number of essential strings is greater than the total number of components (hence greater than the number of first-return string-runs, thus when for at least one component $\text{g.c.d.}(P_c, B^*) \neq 1$).

In *The Braider*, Issue No. 23, pg. 531 we have already met a Perfect Herringbone Grant Knot which was derived from, and hence associated with, a Semi-Perfect Herringbone Pineapple Knot. The first-return string-run of this Herringbone Pineapple Knot is shown at the left below, and since $B^* = 3$ with $P = 2A + x - 2 = 18$, $\text{g.c.d.}(P_c, B^*) = 3$, hence $\neq 1$, this Herringbone Pineapple Knot with $y = A + 1$ is a Semi-Perfect Herringbone Pineapple Knot (it requires three essential strings).



$A=3$, $x=14$, $y=A+1$



$A=3$, $x=14$, $y=A+1$. $x_{GRANT}=15$.

For the Perfect Herringbone Grant Knot (with $x = 15$), the two first-return string-runs associated with its associated Semi-Perfect Herringbone Pineapple Knot are shown at right. The leftmost one of these two first-return string-runs has $A = 3$, $x = 14$, $H = 4$, $\sum(l_i + r_i) = 7 + 6 = 13$; hence $N = \frac{Hx + 4HA - 2\sum(l_i + r_i)}{2A} = 13 = P_c$. For the rightmost one of these two first-return string-runs we have $A = 3$, $x = 14$, $H = 2$, $\sum(l_i + r_i) = 5 + 6 = 11$; hence $N = \frac{Hx + 4HA - 2\sum(l_i + r_i)}{2A} = 5 = P_c$. Since $\text{g.c.d.}(N, B^*) = \text{g.c.d.}(13, 3) = 1$ and $\text{g.c.d.}(N, B^*) = \text{g.c.d.}(5, 3) = 1$, the Perfect Herringbone Grant Knot requires two essential strings.

There is no symmetry in the colour-pattern, although it has orientation and balance. However, in general we desire Perfect and Semi-Perfect Herringbone Grant Knots with either symmetry in the colour-pattern or with a better balanced one. A few of the following examples will show that this can readily be achieved.

Example 1:

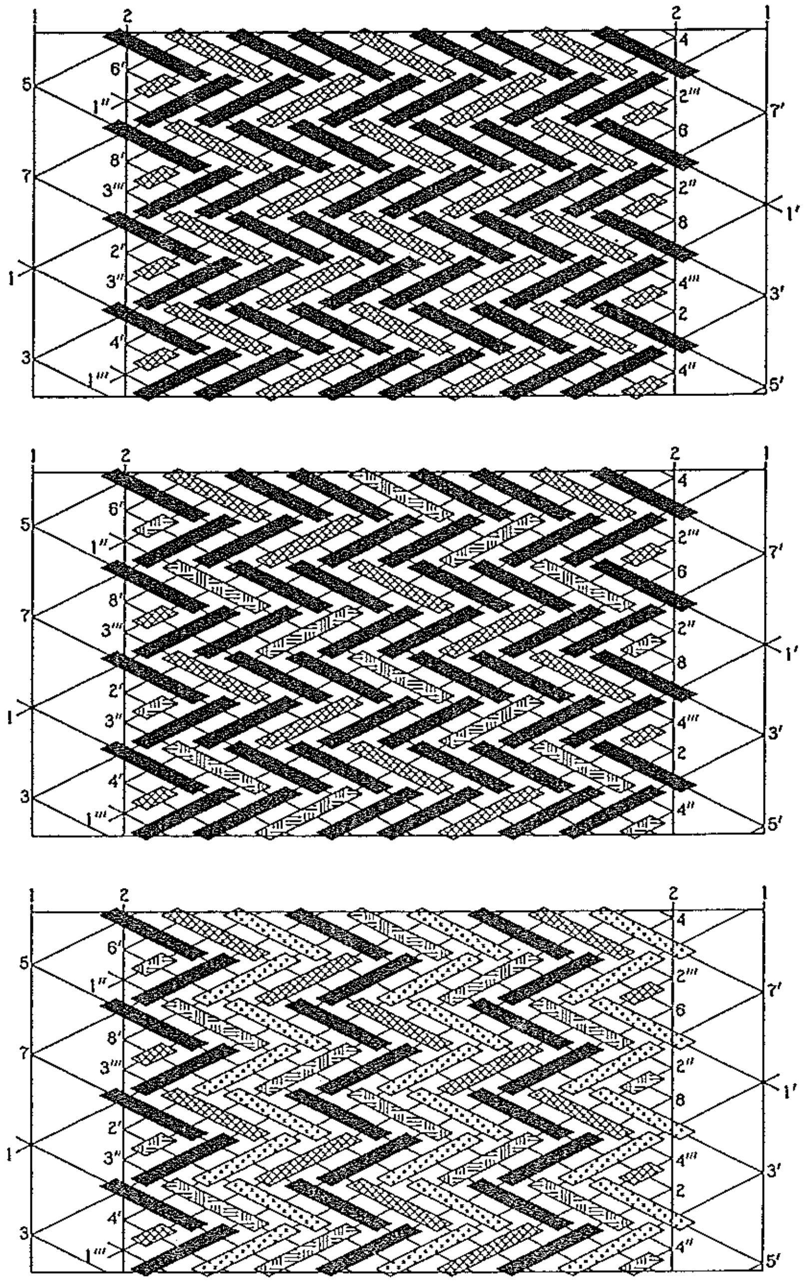
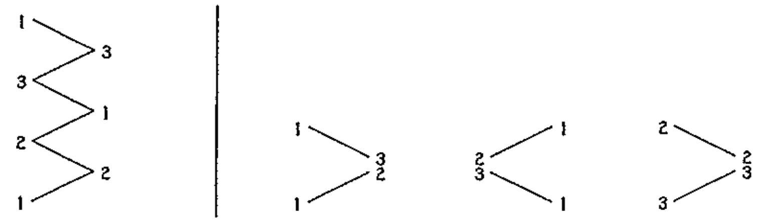


Fig. 833 — Different colour-patterns for identical string-runs.

For the Semi-Perfect Herringbone Grant Knots in Fig. 833 (with $x = 18$), the three first-return string-runs associated with their associated Semi-Perfect Herringbone Pineapple Knot are shown below at right.



$\Lambda=3,\ x=16,\ y=\Lambda+1$ $\Lambda=3,\ x=16,\ y=\Lambda+1.\ x_{count}=18.$

For the leftmost one of these three first-return string-runs we have $A = 3$, $x = 16$, $H = 2$, $\sum(l_i + r_i) = 2 + 5 = 7$; hence $N = \frac{Hx+4HA-2\sum(l_i+r_i)}{2A} = 7 = P_c$. For the next first-return string-run to the right we have $A = 3$, $x = 16$, $H = 2$, $\sum(l_i + r_i) = 5 + 2 = 7$; hence $N = \frac{Hx+4HA-2\sum(l_i+r_i)}{2A} = 7 = P_c$. For the rightmost one of these three first-return string-runs we have $A = 3$, $x = 16$, $H = 2$, $\sum(l_i + r_i) = 5 + 5 = 10$; hence $N = \frac{Hx+4HA-2\sum(l_i+r_i)}{2A} = 6 = P_c$. Since g.c.d.$(N, B^*) =$ g.c.d.$(7,4) = 1$, g.c.d.$(N, B^*) =$ g.c.d.$(7,4) = 1$ and g.c.d.$(N, B^*) =$ g.c.d.$(6,4) = 2$, the Semi-Perfect Herringbone Grant Knot requires four essential strings.

Example 2:

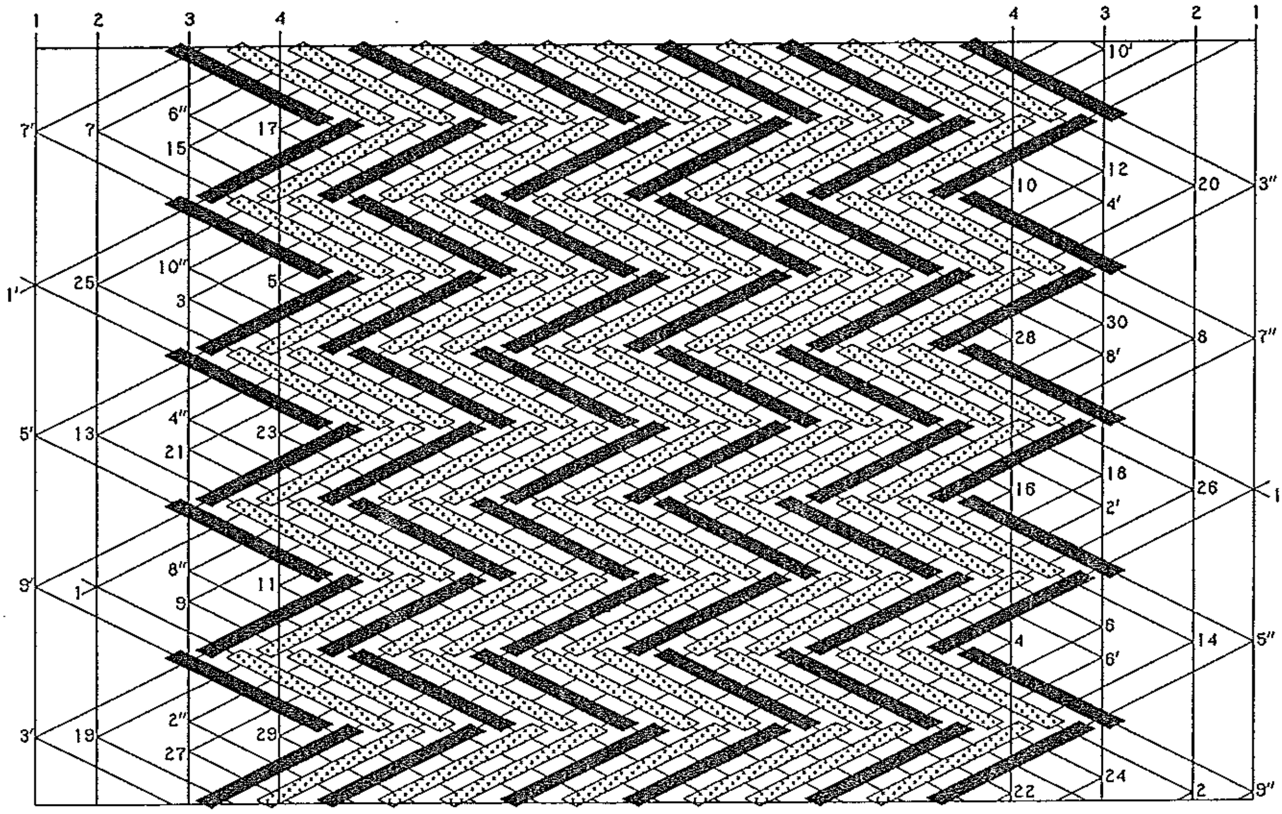
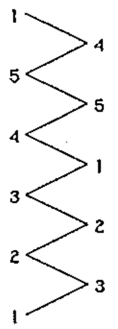
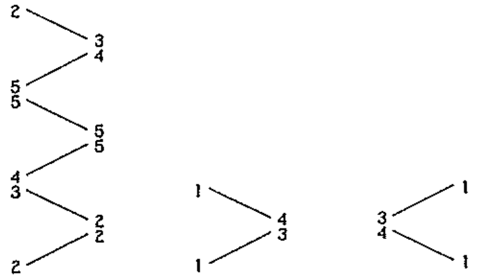


Fig. 834 — $A = 5$, $B^* = 5$, $x = 24$, $y = 6$.

For the Perfect Herringbone Grant Knots in Fig. 834 and the Semi-Perfect Herringbone Grant Knot in Fig. 835 (both with $x = 24$), the three first-return string-runs associated with their associated respective Perfect and Semi-Perfect Herringbone Pineapple Knot are shown below at right.



A=5, x=24, y=A+1



A=5, x=24, y=A+1. x_{GRANT}=24.

For the leftmost one of these three first-return string-runs we have $A = 5$, $x = 24$, $H = 6$, $\sum(l_i + r_i) = 21 + 21 = 42$; hence $N = \frac{Hx+4HA-2\sum(l_i+r_i)}{2A} = 18 = P_c$. For the next first-return string-run to the right we have $A = 5$, $x = 24$, $H = 2$, $\sum(l_i + r_i) = 2 + 7 = 9$; hence $N = \frac{Hx+4HA-2\sum(l_i+r_i)}{2A} = 7 = P_c$. For the rightmost one of these three first-return string-runs we have $A = 5$, $x = 24$, $H = 2$, $\sum(l_i + r_i) = 7 + 2 = 9$; hence $N = \frac{Hx+4HA-2\sum(l_i+r_i)}{2A} = 7 = P_c$. Since for the Perfect Herringbone Grant Knot in Fig. 834 g.c.d.(N, B^*) = g.c.d.$(18, 5) = 1$, g.c.d.(N, B^*) = g.c.d.$(7, 5) = 1$ and g.c.d.(N, B^*) = g.c.d.$(7, 5) = 1$, this Perfect Herringbone Grant Knot requires three essential strings.

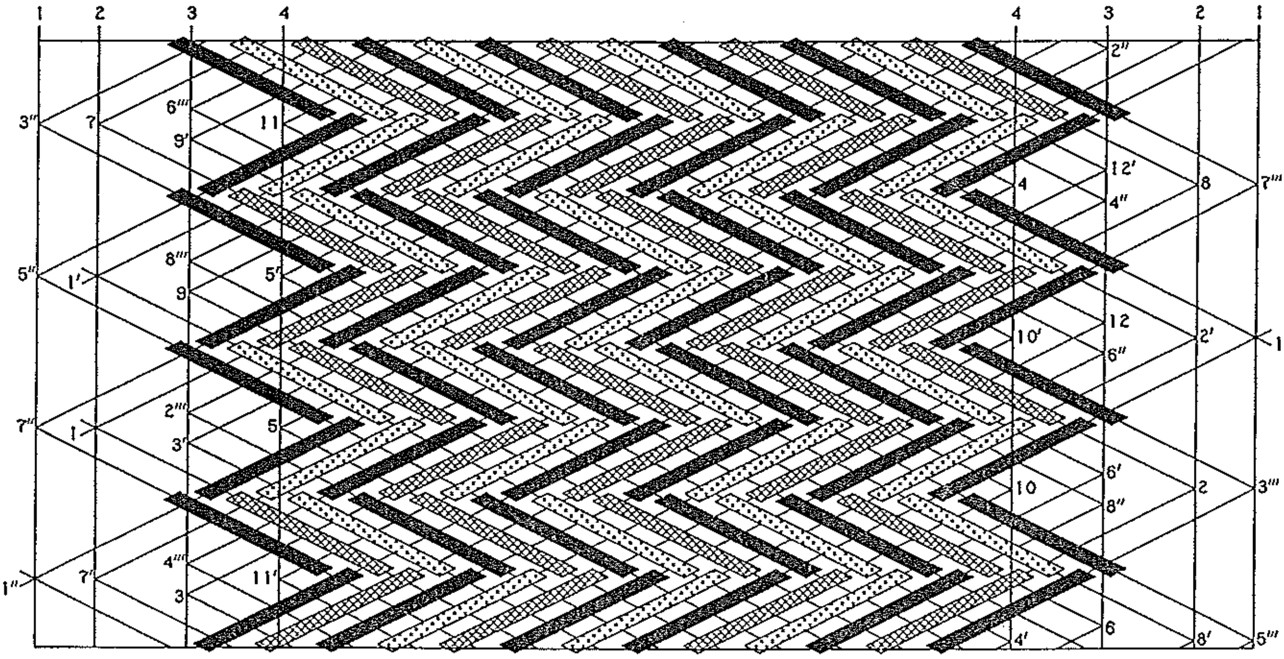
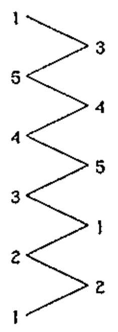


Fig. 835 — $A = 5$, $B^* = 4$, $x = 24$, $y = 6$.

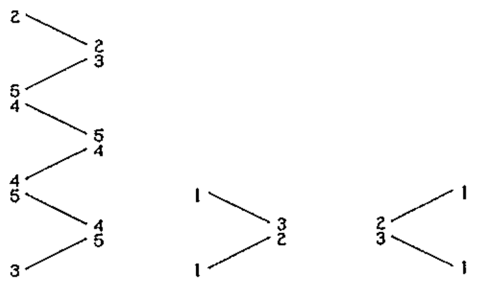
Since g.c.d.(N, B^*) = g.c.d.$(18, 4) = 2$, g.c.d.(N, B^*) = g.c.d.$(7, 4) = 1$ and g.c.d.(N, B^*) = g.c.d.$(7, 4) = 1$ for the Semi-Perfect Herringbone Grant Knot in Fig. 835, this Knot requires four essential strings.

Example 3:

For the Perfect Herringbone Grant Knot in Fig. 836 (with $x = 24$), the three first-return string-runs associated with their associated Semi-Perfect Herringbone Pineapple Knot are shown below at right.



$A=5$, $x=22$, $y=A+1$



$A=5$, $x=22$, $y=A+1$. $x_{\text{COUNT}}=24$.

$A = 5$, $x = 22$, $H = 6$, $\sum(l_i + r_i) = 23+23 = 46$; thus $N = \frac{Hx+4HA-2\sum(l_i+r_i)}{2A} = 16 = P_c$ for the leftmost of these three first-return string-runs. $A = 5$, $x = 22$, $H = 2$, $\sum(l_i + r_i) = 2 + 5 = 7$; thus $N = \frac{Hx+4HA-2\sum(l_i+r_i)}{2A} = 7 = P_c$ for the next first-return string-run to the right. $A = 5$, $x = 22$, $H = 2$, $\sum(l_i + r_i) = 5 + 2 = 7$; thus $N = \frac{Hx+4HA-2\sum(l_i+r_i)}{2A} = 7 = P_c$ for the rightmost one of these three first-return string-runs. Since for the Perfect Herringbone Grant Knot in Fig. 836 g.c.d.$(N, B^*) =$ g.c.d.$(16, 5) = 1$, g.c.d.$(N, B^*) =$ g.c.d.$(7, 5) = 1$ and g.c.d.$(N, B^*) =$ g.c.d.$(7, 5) = 1$, this Perfect Herringbone Grant Knot requires three essential strings.

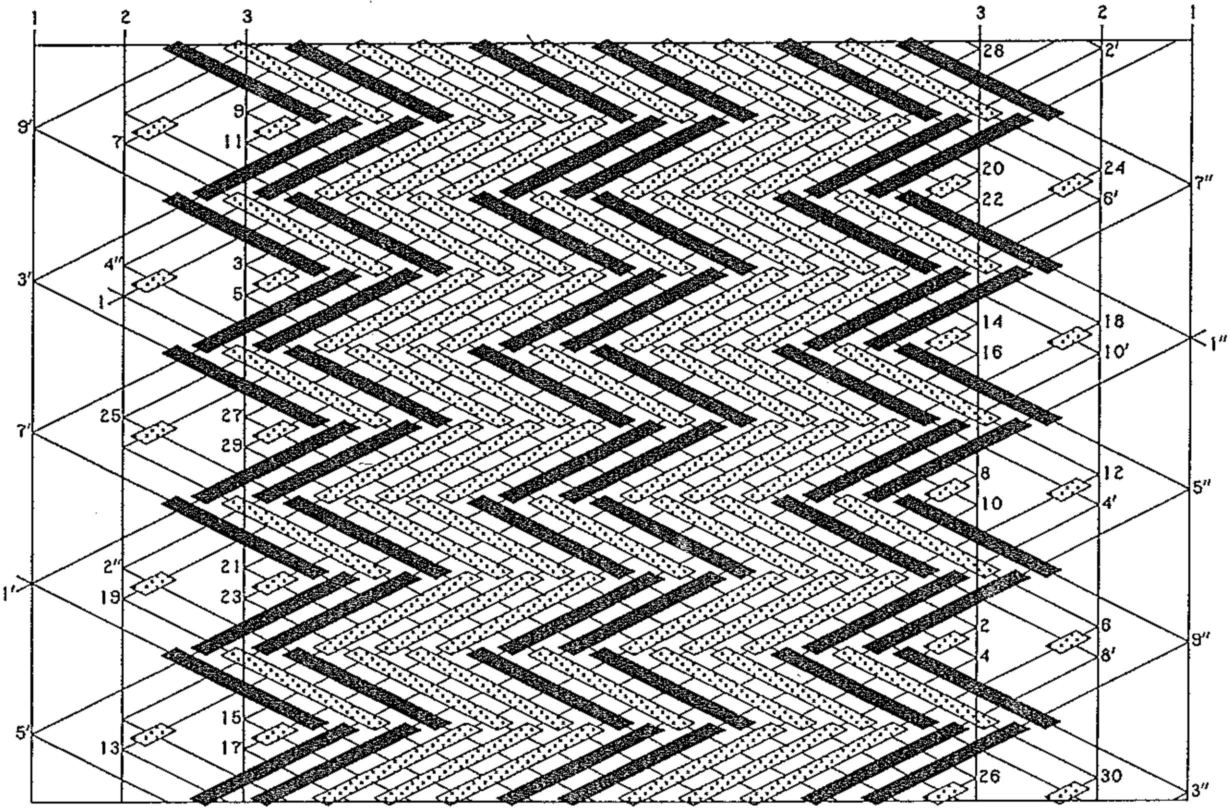
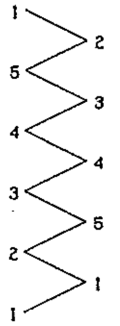


Fig. 836 — $A = 5$, $B^* = 5$, $x = 24$, $y = 6$.

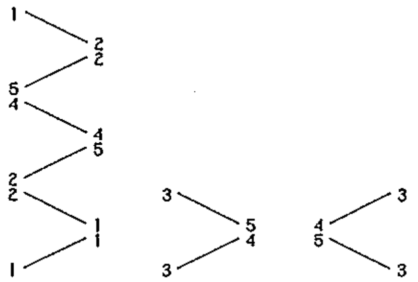
Example 4:

The Perfect Herringbone Grant Knots in Figs. 837 and 838 are all associated with the same Perfect Herringbone Pineapple Knot which has $A = 5$, $x = 10$, $y = 6$, $k = 1$, $B^* = 5$. They show a gradual increase in orientation due to colour and/or coding. The three first-return string-runs for each of the Figs. 837 and 838 associated with their associated Perfect Herringbone Pineapple Knot are shown below.



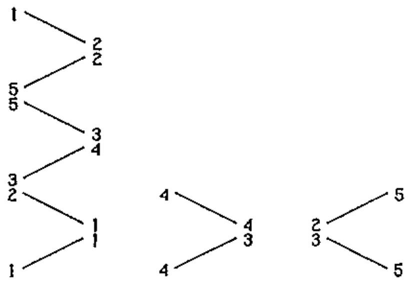
A=5, x=10, y=A+1

Fig.837



A=5, x=10, y=A+1. x_{count}=12.

Fig.838



A=5, x=10, y=A+1. x_{count}=10.

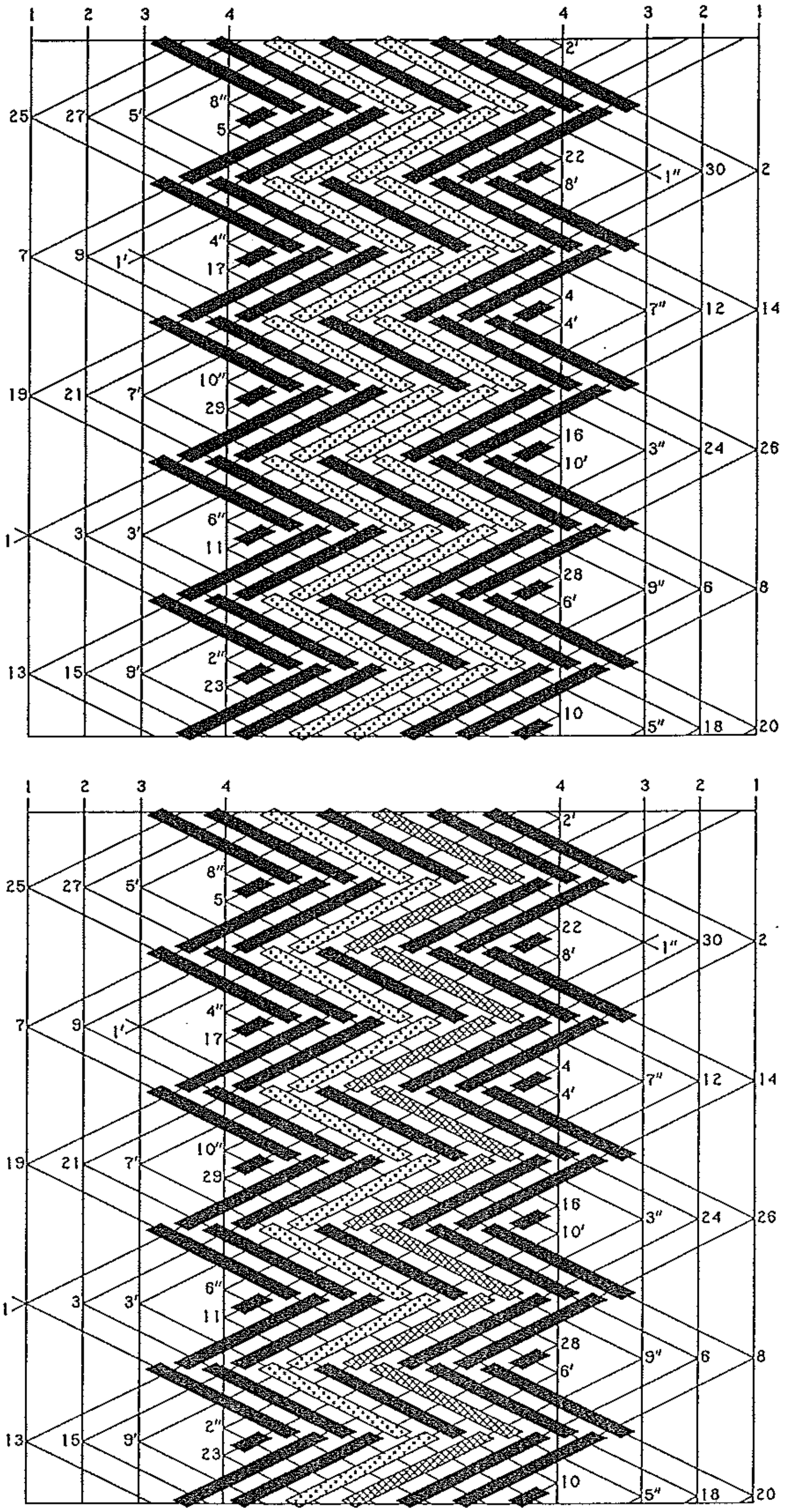


Fig. 837 — $A = 5$, $B^* = 5$, $x = 12$, $y = 6$.

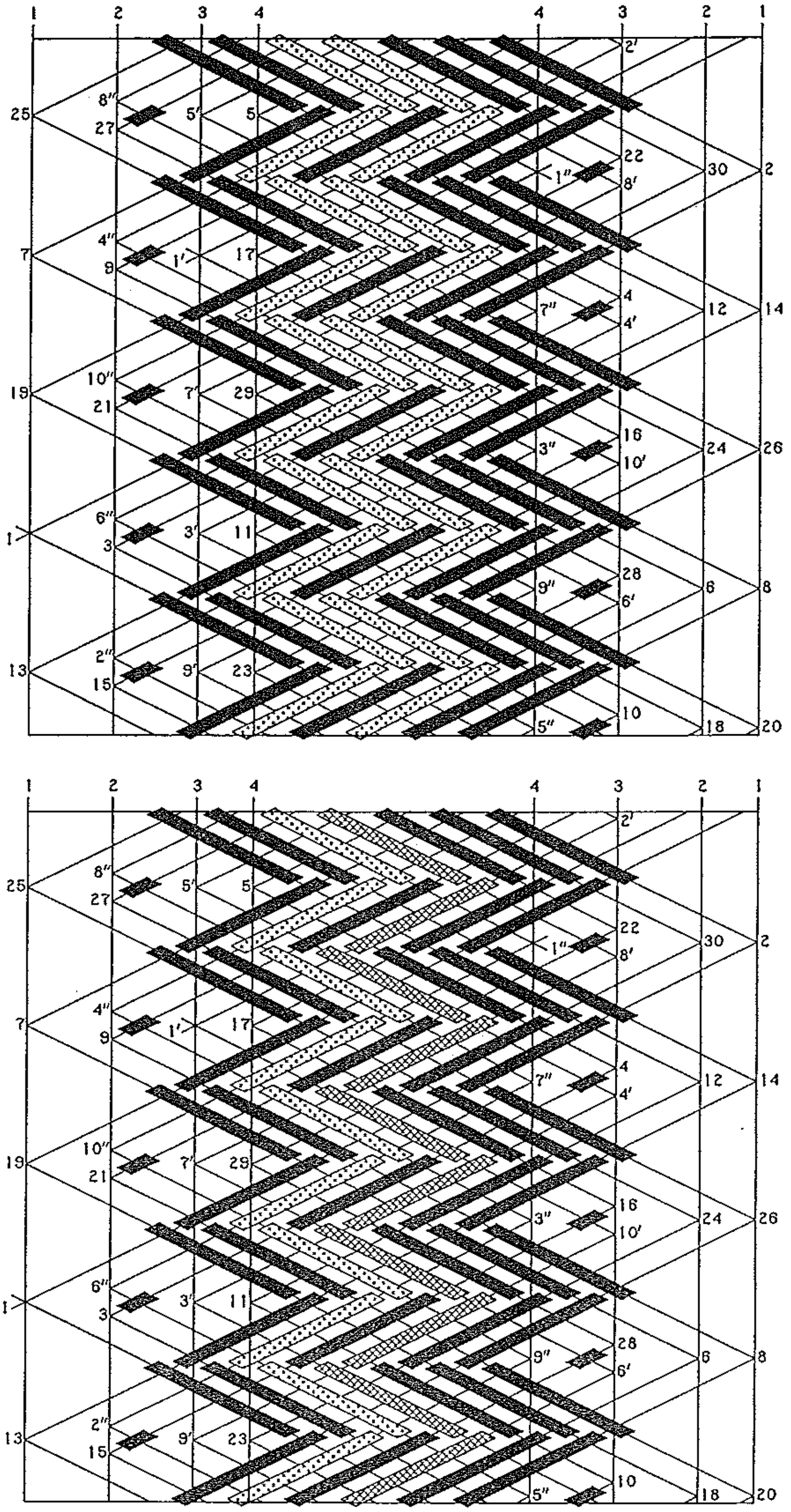


Fig. 838 — $A = 5$, $B^* = 5$, $x = 10$, $y = 6$.

THE BRAIDER'S NOTEBOOK

In *The Braider*, Issue No. 41, we discussed the Regular Cylindrical Braids with an even number of bights of which a string passes diametrically through its supporting object in order to ensure that such a Regular Cylindrical Braid could not be moved from its intended position. At the transition point of two Round Braids into one Round Braid, and especially where two Round Braids are joined, we often cover this transition point or joining point with a knot. In order for this covering knot to rest at all times very firmly against the two Round Braids, one or more strings of this covering knot pass between the two Round Braids directly above the transition point or joining point. Such a string or strings will pull the covering knot hard against the two Round Braids and ensure that the covering knot remains firmly located in that position. Generally such covering knots serve in addition as a decoration and hence often show a suitable colour-pattern. Many suitable colour-patterns can be created with two colours, hence with a minimum of two essential strings. We shall discuss the construction of such covering knots with the aid of an Example.

Fig. 839 shows the string-run of a $p/b = 18/28$ Regular Cylindrical Braid acting as such a covering knot. One string has the initial half-cycles 1 and $1'$, while the other string has the initial half-cycles $1''$ and $1'''$. Note that the half-cycles 1 and $1'$ as well as the half-cycles $1''$ and $1'''$ are $\frac{b}{2}$ bight-units apart. Also note that the half-cycles 1 and $1''$ as well as the half-cycles $1'$ and $1'''$, when produced to the left bight-boundary, intersect on this bight-boundary.

The last half-cycle associated with the initial half-cycle 1 is half-cycle $(\frac{b}{2}+1)$, and simlarly the last half-cycle associated with the initial half-cycle $1'$ is half-cycle $(\frac{b}{2}+1)'$, the last half-cycle associated with the initial half-cycle $1''$ is half-cycle $(\frac{b}{2}+1)''$, the last half-cycle associated with the initial half-cycle $1'''$ is half-cycle $(\frac{b}{2}+1)'''$.

Let's braid the string-run half-cycles $1-2-3-\cdots-13-14-15$ and $1'-2'-2'-\cdots-13'-14'-15'$ by parallel braiding, and do the same with the string-run half-cycles $1''-2''-2''-\cdots-13''-14''-15''$ and $1'''-2'''-2'''-\cdots-13'''-14'''-15'''$. The algorithm diagram associated with the string-run of each of the two $p'/b' = 9/14$ interwoven Regular Knot components[†] is shown at upper-right below the string-run diagram. Since the maximum i-value is $\frac{(b'+1)-3}{2} = \frac{(14+1)-3}{2} = 6$, we can disregard $i > 6$. Hence the upper two rows and the lower two rows of i-values can be replaced by a single row of i-values as shown in the next two string-run algorithm diagrams, the first of which (option 1) is associated with first parallel braiding the string-run half-cycles $1-2-3-\cdots-13-14-15$ and $1'-2'-2'-\cdots-13'-14'-15'$, and the last of which (option 2) is associated with first parallel braiding the string-run half-cycles $1''-2''-2''-\cdots-13''-14''-15''$ and $1'''-2'''-2'''-\cdots-13'''-14'''-15'''$. In case of option 1, the first two half-cycles to be laid down (1 and $1'$ from upper-left to lower-right), or in case of option 2, the first two half-cycles to be laid down ($1''$ and $1'''$ from lower-left to upper-right), are **free runs**. In case of option 1, the last two half-cycles to be laid down ($(b'+1) = 15$ and $(b'+1)' = 15'$ from upper-left to lower-right), or in case of option 2, the last two half-cycles to be laid down ($(b'+1)'' = 15''$ and $(b'+1)''' = 15'''$ from lower-left to upper-right), have **unders to the right** of the X—X line. For the even-numbered half-cycles h_e and $h_e{}'$ from upper-right to lower-left,

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[†] Refer to *The Braider*, Issue No. 41, pp. 973-974.

the intersection associated with $i = \frac{h_e-2}{2} = \frac{h_e'-2}{2}$ is an intersection with half-cycle 1 or $1'$ and when **left** of the X—X line is an **over**. For the even-numbered half-cycles h_e'' and h_e''' from lower-right to upper-left, the intersection associated with $i = \frac{h_e''-2}{2} = \frac{h_e'''-2}{2}$ is an intersection with half-cycle $1''$ or $1'''$ and when **left** of the X—X line is an **over**.

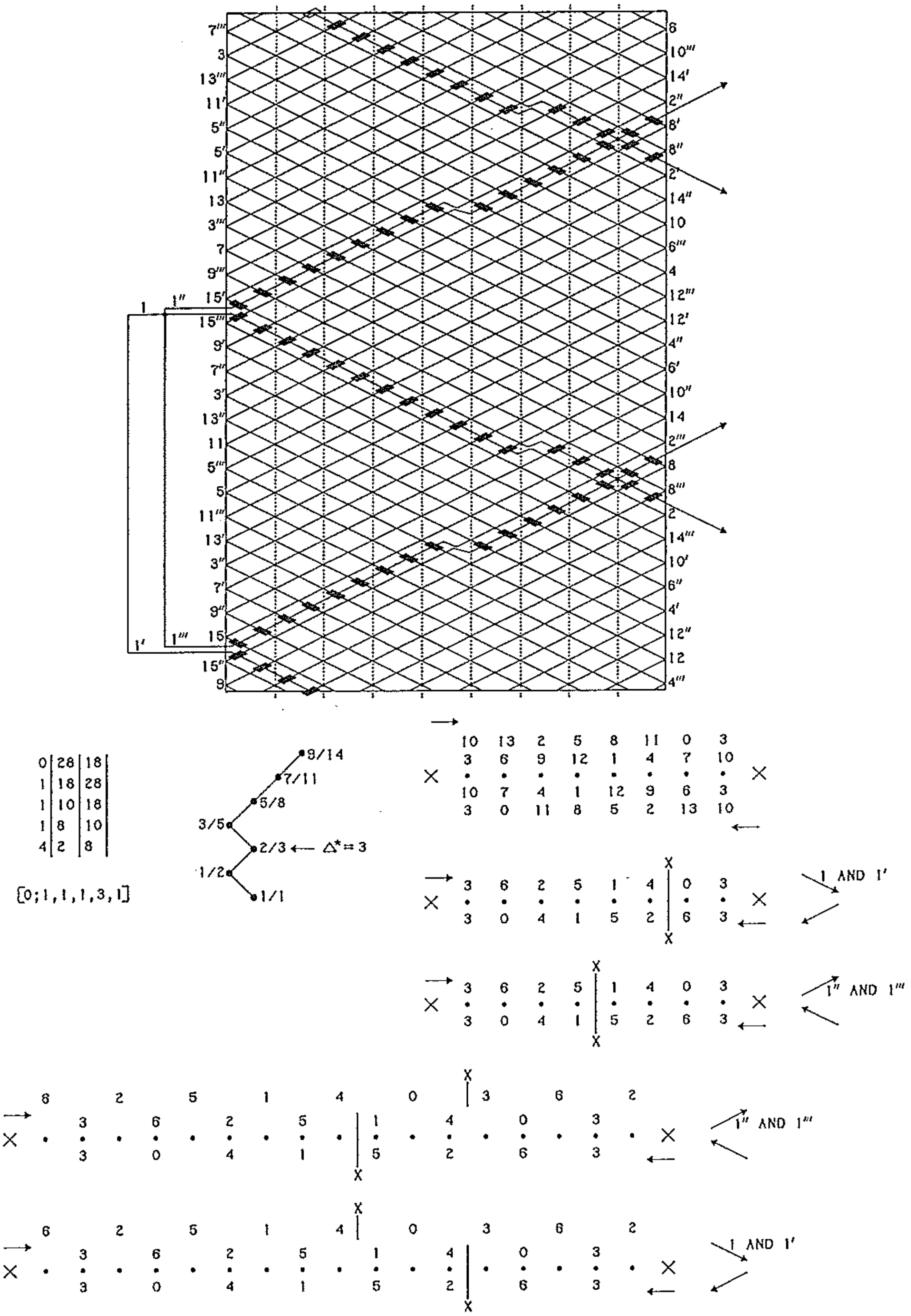

Fig. 839 — A string-run example with its general algorithm diagrams.

In case of option 1, after braiding the component with the initial half-cycles 1 and $1'$ from upper-left to lower-right, we braid the component with the initial half-cycles $1''$ and $1'''$ from lower-left to upper-right. The general algorithm diagram associated with this is the second from the bottom. For the half-cycles $3'' \leq {h_o}'' \leq (b'-1)'' = 13''$ and $3''' \leq {h_o}''' \leq (b'-1)''' = 13'''$ from lower-left to upper-right, the intersection set associated with the i-value $i = \frac{{h_o}''-3}{2} = \frac{{h_o}'''-3}{2}$ is an intersection set with half-cycles 1 and $(b'+1)' = 15'$ or $1'$ and $(b'+1) = 15$; when **left** of the upper X line it is an *over*$-\star$ intersection set, and when **right** of the upper X line it is a $\star -$ *over* intersection set. The associated sequence of i-values is the upper sequence in the general algorithm diagram. This sequence starts with the i-value for the half-cycles $(b'+1)'' = 15''$ and $(b'+1)''' = 15'''$ and of course follows the sequential order of the i-values for the odd-numbered half-cycles ${h_o}''$ and ${h_o}'''$.

For the initial half-cycles $1''$ and $1'''$ from lower-left to upper-right, and the final half-cycles $(b'+1)''$ and $(b'+1)'''$ from lower-left to upper-right, there are four string-run configurations to consider (see the left four diagrams in Fig. 840)

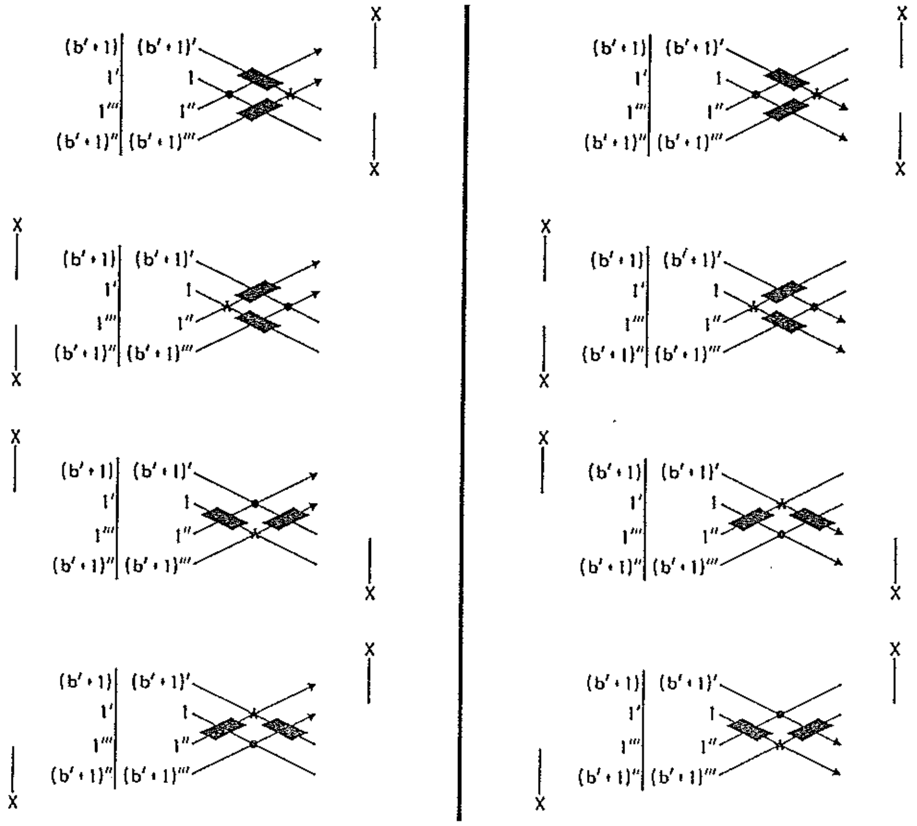

Fig. 840 — The four string-run configurations of the initial and final half-cycles for options 1 and 2.

In these configurations the coding of the intersection with the $\bullet$ can be freely chosen, and the coding of the intersection with the star is its particular coding in the coding-pattern of the braid. For the initial half-cycles $1''$ and $1'''$ they form, in association with the i-value $\frac{(b'+1)-3}{2}$ in the uppermost row of i-values, the intersection set $\bullet -$ *under* or $\star -$ *over* or *under*$-\bullet$ or *over*$-\star$, depending on upper and lower X line positions. For the final half-cycles $(b'+1)''$ and $(b'+1)'''$ they form, in association with the i-value $\frac{(b'+1)-3}{2}$ in the uppermost row of i-values, the intersection set *over*$-\star$ or *under*$-\bullet$

or $\star -$ *over* or $\bullet -$ *under*, depending on upper and lower X line positions. Note that adjacent to the left bight-boundary we have the uppermost arrangement in Fig. 840 without the intersection with the $\bullet$, and that adjacent to the right bight-boundary we may have the second arrangement from the top in Fig. 840 without the intersection with the $\bullet$. Furthermore, for the initial half-cycles $1''$ and $1'''$ from lower-left to upper-right the coding of the remaining intersections below the i-values $i < \frac{(b'+1)-3}{2}$ in the uppermost row left of the lower X line are **unders**, while the coding of the remaining intersections right of the lower X line correspond with the coding associated with the i-values in the uppermost row. For the final half-cycles $(b'+1)''$ and $(b'+1)'''$ from lower-left to upper-right the coding of the remaining intersections below the i-values $i < \frac{(b'+1)-3}{2}$ in the uppermost row right of the lower X line are **unders**, while the coding of the remaining intersections left of the lower X line correspond with the coding associated with the i-values in the uppermost row.

For the even-numbered half-cycles h_e'' and h_e''' from lower-right to upper-left, the intersection associated with $i = \frac{h_e''-2}{2} = \frac{h_e'''-2}{2}$ is an intersection with half-cycle $1''$ or $1'''$ and when **left** of the lower X line is an **over**.

Thus in our Example (Fig. 839), half-cycles $1''$ and $1'''$ from lower-left to upper-right have each **unders** for uppermost $i < 6$ left of lower X line, u for uppermost $i = 6$ adjacent left bight-boundary, and $\star - o$ for uppermost $i = 6$ right of both X lines. Half-cycles $(b'+1)'' = 15''$ and $(b'+1)''' = 15'''$ have each **unders** for uppermost $i < 6$ right of lower X line, $o - \star$ for uppermost $i = 6$ left of both X lines, and $u - \bullet$ for uppermost $i = 6$ right of both X lines. For the even-numbered half-cycles h_e'' and h_e''' from lower-right to upper-left, the intersection associated with $i = \frac{h_e''-2}{2} = \frac{h_e'''-2}{2}$ is an intersection with half-cycle $1''$ or $1'''$ and when **left** of the lower X line is an **over**.

In case of option 2, after braiding the component with the initial half-cycles $1''$ and $1'''$ from lower-left to upper-right, we braid the component with the initial half-cycles 1 and $1'$ from upper-left to lower-right. The general algorithm diagram associated with this is the one at the bottom of Fig. 839. For the half-cycles $3 \leq h_o \leq (b'-1) = 13$ and $3' \leq h_o' \leq (b'-1)' = 13'$ from upper-left to lower-right, the intersection set associated with the i-value $i = \frac{h_o-3}{2} = \frac{h_o'-3}{2}$ is an intersection set with half-cycles $1''$ and $(b'+1)''' = 15'''$ or $1'''$ and $(b'+1)'' = 15''$; when **left** of the upper X line it is an *over*$-\star$ intersection set, and when **right** of the upper X line it is a $\star -$ *over* intersection set. The associated sequence of i-values is the upper sequence in the general algorithm diagram. This sequence starts with the i-value for the half-cycles $(b'+1) = 15$ and $(b'+1)' = 15'$ and of course follows the sequential order of the i-values for the odd-numbered half-cycles h_o and h_o'. For the initial half-cycles 1 and $1'$ from upper-left to lower-right, and the final half-cycles $(b'+1)$ and $(b'+1)'$ from upper-left to lower-right, there are four string-run configurations to consider (see the right four diagrams in Fig. 840) In these configurations the coding of the intersection with the $\bullet$ can be freely chosen, and the coding of the intersection with the star is its particular coding in the coding-pattern of the braid. For the initial half-cycles 1 and $1'$ they form, in association with the i-value $\frac{(b'+1)-3}{2}$ in the uppermost row of i-values, the intersection set $\bullet -$ *under* or $\star -$ *over* or *under* $- \bullet$ or *over* $- \star$, depending on upper and lower X line positions. For the final half-cycles $(b'+1)$ and $(b'+1)'$ they form, in association with the i-value $\frac{(b'+1)-3}{2}$ in the uppermost row of i-values, the intersection set *over* $- \star$ or *under* $- \bullet$ or $\star -$ *over* or $\bullet -$ *under*, depending on upper and lower X line positions. Note that adjacent to the left bight-boundary we have the uppermost arrangement in Fig. 840

without the intersection with the $\bullet$, and that adjacent to the right bight-boundary we may have the second arrangement from the top in Fig. 840 without the intersection with the $\bullet$. Furthermore, for the initial half-cycles 1 and 1′ from upper-left to lower-right the coding of the remaining intersections below the i-values $i < \frac{(b'+1)-3}{2}$ in the uppermost row left of the lower X line are **unders**, while the coding of the remaining intersections right of the lower X line correspond with the coding associated with the i-values in the uppermost row. For the final half-cycles $(b'+1)$ and $(b'+1)'$ from upper-left to lower-right the coding of the remaining intersections below the i-values $i < \frac{(b'+1)-3}{2}$ in the uppermost row right of the lower X line are **unders**, while the coding of the remaining intersections left of the lower X line correspond with the coding associated with the i-values in the uppermost row.

For the even-numbered half-cycles h_e and $h_e{}'$ from upper-right to lower-left, the intersection associated with $i = \frac{h_e-2}{2} = \frac{h_e{}'-2}{2}$ is an intersection with half-cycle 1 or 1′ and when **left** of the lower X line is an **over**.

Thus in our Example (Fig. 839), half-cycles 1 and 1′ have each **unders** for uppermost $i < 6$ left of lower X line, u for uppermost $i = 6$ adjacent left bight-boundary, and $\star - o$ for uppermost $i = 6$ right of both X lines. Half-cycles $(b'+1) = 15$ and $(b'+1)' = 15'$ have each **unders** for uppermost $i < 6$ right of lower X line, $o - \star$ for uppermost $i = 6$ left of both X lines, and $u - \bullet$ for uppermost $i = 6$ right of both X lines. For the even-numbered half-cycles h_e and $h_e{}'$ the intersection associated with $i = \frac{h_e-2}{2} = \frac{h_e{}'-2}{2}$ is an intersection with half-cycle 1 or 1′ and when **left** of the lower X line is an **over**.

A coding we could place on the string-run in Fig. 839 is a two-pass Herringbone coding as shown in Fig. 841. The associated algorithm diagrams are shown in Fig. 842.

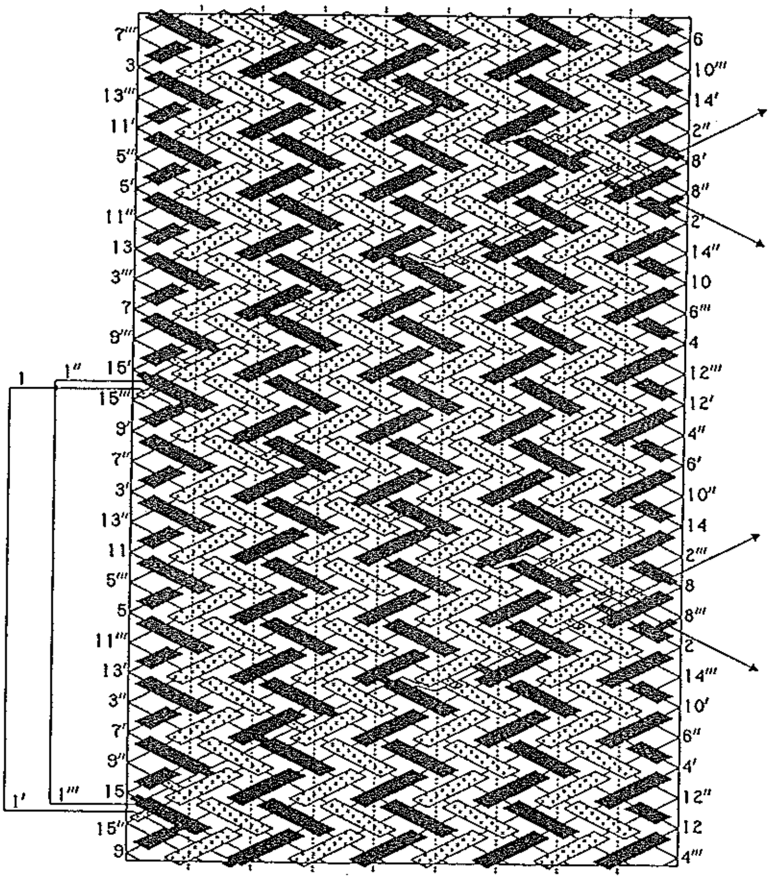

Fig. 841 — A two-pass Herringbone coding on the string-run in Fig. 839.

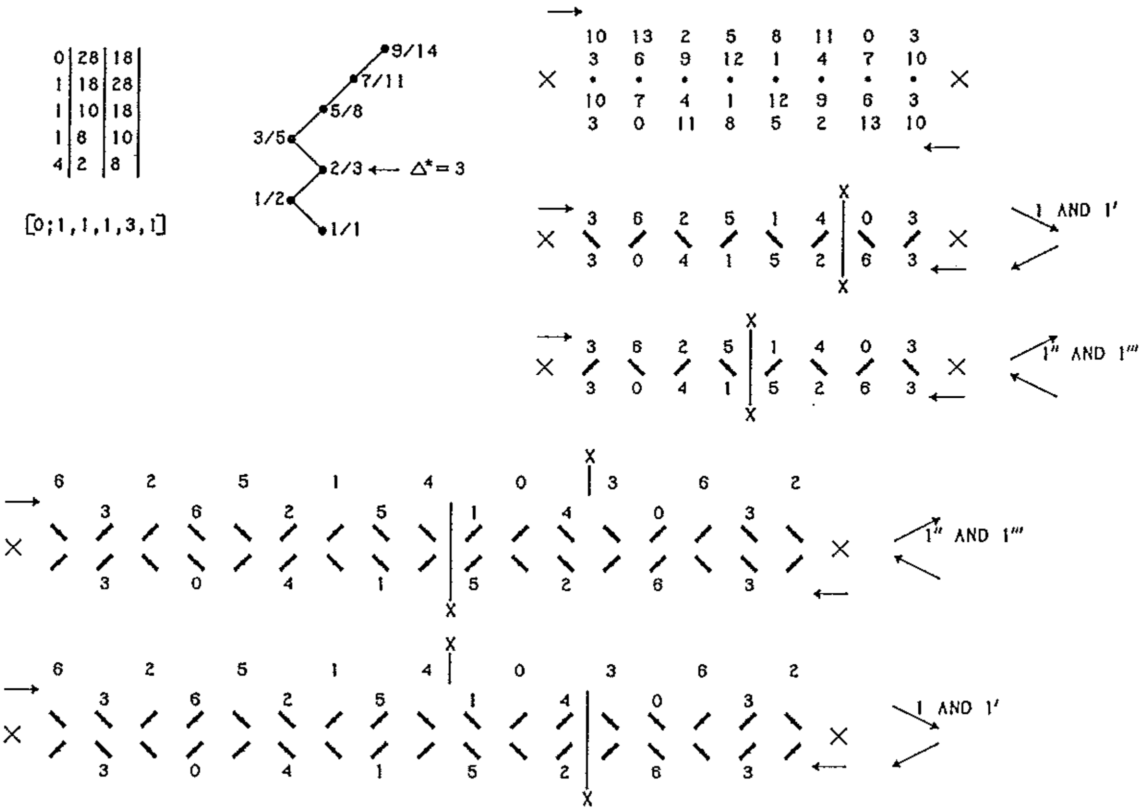

Fig. 842 — The algorithm diagrams associated with Fig. 841.

Option 1 gives for the first component with the initial half-cycles 1 and $1'$ from upper-left to lower-right the following half-cycle braiding algorithms:

half-cycles:

$1 \,\&\, 1'$: : $L \longrightarrow R$: Free run.
$2 \,\&\, 2'$: $i = 0$; o for $i = 0$ left of X–X: $L \longleftarrow R$: o.
$3 \,\&\, 3'$: $i = 0$; : $L \longrightarrow R$: o.
$4 \,\&\, 4'$: $i \leq 1$; o for $i = 1$ left of X–X: $L \longleftarrow R$: $2o$.
$5 \,\&\, 5'$: $i \leq 1$; : $L \longrightarrow R$: $2o$.
$6 \,\&\, 6'$: $i \leq 2$; o for $i = 2$ left of X–X: $L \longleftarrow R$: $3o$.
$7 \,\&\, 7'$: $i \leq 2$; : $L \longrightarrow R$: $3o$.
$8 \,\&\, 8'$: $i \leq 3$; o for $i = 3$ left of X–X: $L \longleftarrow R$: $5o$.
$9 \,\&\, 9'$: $i \leq 3$; : $L \longrightarrow R$: $4o - u$.
$10 \,\&\, 10'$: $i \leq 4$; o for $i = 4$ left of X–X: $L \longleftarrow R$: $5o - u$.
$11 \,\&\, 11'$: $i \leq 4$; : $L \longrightarrow R$: $3o - u - o - u$.
$12 \,\&\, 12'$: $i \leq 5$; o for $i = 5$ left of X–X: $L \longleftarrow R$: $4o - u - o - u$.
$13 \,\&\, 13'$: $i \leq 5$; : $L \longrightarrow R$: $2o - u - o - u - o - u$.
$14 \,\&\, 14'$: $i \leq 6$; o for $i = 6$ left of X–X: $L \longleftarrow R$: $o - u - o - u - o - u - o - u$.
$15 \,\&\, 15'$: $i \leq 6$; unders right of X–X: $L \longrightarrow R$: $o - u - o - u - o - u - 2u$.

Option 1 gives for the second component with the initial half-cycles $1''$ and $1'''$ from lower-left to upper-right the following half-cycle braiding algorithms:

half-cycles:

$1'' \,\&\, 1'''$: u for uppermost $i < 6$ left of lower X line and
u for uppermost $i = 6$ adjacent left bight-boundary and
$\star - o = 2o$ for uppermost $i = 6$ right of both X lines:
$L \longrightarrow R$: $5u - o - u - 2o - u$.

$2'' \,\&\, 2'''$: $i = 0$; o for $i = 0$ left of lower X line:
$L \longleftarrow R$: $u - o - u - o - u - o - u - 2o - u$.
$3'' \,\&\, 3'''$: $i = 0$; $o - \star = 2o$ for uppermost $i = 0$ left of upper X line:
$L \longrightarrow R$: $u - o - u - o - u - 2o - u - 2o - u$.
$4'' \,\&\, 4'''$: $i \leq 1$; o for $i = 1$ left of lower X line:
$L \longleftarrow R$: $u - o - u - o - u - 2o - u - 2o - u$.
$5'' \,\&\, 5'''$: $i \leq 1$; $o - \star = 2o$ for uppermost $i = 1$ left of upper X line:
$L \longrightarrow R$: $u - o - u - 2o - u - 2o - u - 2o - u$.
$6'' \,\&\, 6'''$: $i \leq 2$; o for $i = 2$ left of lower X line:
$L \longleftarrow R$: $u - o - u - 2o - u - 2o - u - 2o - u$.
$7'' \,\&\, 7'''$: $i \leq 2$; $o - \star = 2o$ for uppermost $i = 2$ left of upper X line and
$\star - o = u - o$ for uppermost $i = 2$ right of upper X line:
$L \longrightarrow R$: $u - 2o - u - 2o - u - 2o - u - 2o - u - o$.
$8'' \,\&\, 8'''$: $i \leq 3$; o for $i = 3$ left of lower X line:
$L \longleftarrow R$: $u - 2o - u - 2o - u - 2o - u - 3o - u$.
$9'' \,\&\, 9'''$: $i \leq 3$; $\star - o = u - o$ for uppermost $i = 3$ right of upper X line:
$L \longrightarrow R$: $u - 2o - u - 2o - u - 2o - u - 3o - 2u$.
$10'' \,\&\, 10'''$: $i \leq 4$; o for $i = 4$ left of lower X line:
$L \longleftarrow R$: $u - 2o - u - 2o - u - 3o - u - 2o - 2u$.
$11'' \,\&\, 11'''$: $i \leq 4$; $o - \star = o - u$ for uppermost $i = 4$ left of upper X line:
$L \longrightarrow R$: $u - 2o - u - 3o - u - 2o - 2u - 2o - 2u$.
$12'' \,\&\, 12'''$: $i \leq 5$; o for $i = 5$ left of lower X line:
$L \longleftarrow R$: $u - 2o - u - 2o - 2u - 2o - 2u - 2o - 2u$.
$13'' \,\&\, 13'''$: $i \leq 5$; $o - \star = o - u$ for uppermost $i = 5$ left of upper X line:
$L \longrightarrow R$: $u - 3o - u - 2o - 2u - 2o - 2u - 2o - 2u$.
$14'' \,\&\, 14'''$: $i \leq 6$; o for $i = 6$ left of lower X line:
$L \longleftarrow R$: $u - 2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u$.
$15'' \,\&\, 15'''$: $i \leq 6$; u for uppermost $i < 6$ right of lower X line and
$o - \star = o - u$ for uppermost $i = 6$ left of both X lines and
$u - \bullet$ for uppermost $i = 6$ right of both X lines:
$L \longrightarrow R$: $o - u - 2o - 2u - 2o - 2u - 6u - \bullet - 2u$.

Option 2 gives for the first component with the initial half-cycles $1''$ and $1'''$ from lower-left to upper-right the following half-cycle braiding algorithms:

half-cycles:
$1'' \,\&\, 1'''$: : $L \longrightarrow R$: Free run.
$2'' \,\&\, 2'''$: $i = 0$; o for $i = 0$ left of X–X: $L \longleftarrow R$: o.
$3'' \,\&\, 3'''$: $i = 0$; : $L \longrightarrow R$: o.
$4'' \,\&\, 4'''$: $i \leq 1$; o for $i = 1$ left of X–X: $L \longleftarrow R$: $2o$.
$5'' \,\&\, 5'''$: $i \leq 1$; : $L \longrightarrow R$: $2o$.
$6'' \,\&\, 6'''$: $i \leq 2$; o for $i = 2$ left of X–X: $L \longleftarrow R$: $3o$.
$7'' \,\&\, 7'''$: $i \leq 2$; : $L \longrightarrow R$: $3o$.
$8'' \,\&\, 8'''$: $i \leq 3$; o for $i = 3$ left of X–X: $L \longleftarrow R$: $5o$.
$9'' \,\&\, 9'''$: $i \leq 3$; : $L \longrightarrow R$: $4o - u$.
$10'' \,\&\, 10'''$: $i \leq 4$; o for $i = 4$ left of X–X: $L \longleftarrow R$: $5o - u$.
$11'' \,\&\, 11'''$: $i \leq 4$; : $L \longrightarrow R$: $3o - u - o - u$.
$12'' \,\&\, 12'''$: $i \leq 5$; o for $i = 5$ left of X–X: $L \longleftarrow R$: $2o - u - o - u - o - u$.
$13'' \,\&\, 13'''$: $i \leq 5$; : $L \longrightarrow R$: $2o - u - o - u - o - u$.

$14'' \,\&\, 14'''$: $i \le 6$; o for $i = 6$ left of X–X: $L \longleftarrow R: o - u - o - u - o - u - o - u$.
$15'' \,\&\, 15'''$: $i \le 6$; unders right of X–X: $L \longrightarrow R: o - u - o - 5u$.

Option 2 gives for the second component with the initial half-cycles 1 and $1'$ from upper-left to lower-right the following half-cycle braiding algorithms:

half-cycles:

$1 \,\&\, 1'$: u for uppermost $i < 6$ left of lower X line and
u for uppermost $i = 6$ adjacent left bight-boundary and
$\star - o = u - o$ for uppermost $i = 6$ right of both X lines:
$L \longrightarrow R: 6u - o - u - 2o$.
$2 \,\&\, 2'$: $i = 0$; o for $i = 0$ left of lower X line:
$L \longleftarrow R: o - u - o - u - o - u - 2o - u - o$.
$3 \,\&\, 3'$: $i = 0$; $\star - o = u - o$ for uppermost $i = 0$ right of upper X line:
$L \longrightarrow R: o - u - o - u - o - u - 2o - u - o$.
$4 \,\&\, 4'$: $i \le 1$; o for $i = 1$ left of lower X line:
$L \longleftarrow R: o - u - o - u - 2o - u - 2o - u - o$.
$5 \,\&\, 5'$: $i \le 1$; $o - \star = o - u$ for uppermost $i = 1$ left of upper X line:
$L \longrightarrow R: o - u - 2o - u - 2o - u - 2o - u - o$.
$6 \,\&\, 6'$: $i \le 2$; o for $i = 2$ left of lower X line:
$L \longleftarrow R: o - u - 2o - u - 2o - u - 2o - u - o$.
$7 \,\&\, 7'$: $i \le 2$; $o - \star = o - u$ for uppermost $i = 2$ left of upper X line and
$\star - o = 2o$ for uppermost $i = 2$ right of upper X line:
$L \longrightarrow R: 2o - u - 2o - u - 2o - u - 2o - u - 2o$.
$8 \,\&\, 8'$: $i \le 3$; o for $i = 3$ left of lower X line:
$L \longleftarrow R: 2o - u - 2o - u - 2o - u - 2o - u - 2o$.
$9 \,\&\, 9'$: $i \le 3$; $\star - o = 2o$ for uppermost $i = 3$ right of upper X line:
$L \longrightarrow R: 2o - u - 2o - u - 2o - u - 3o - 2u - o$.
$10 \,\&\, 10'$: $i \le 4$; o for $i = 4$ left of lower X line:
$L \longleftarrow R: 2o - u - 2o - u - 2o - u - 3o - 2u - o$.
$11 \,\&\, 11'$: $i \le 4$; $o - \star = 2o$ for uppermost $i = 4$ left of upper X line:
$L \longrightarrow R: 2o - u - 2o - u - 3o - 2u - 2o - 2u - o$.
$12 \,\&\, 12'$: $i \le 5$; o for $i = 5$ left of lower X line:
$L \longleftarrow R: 2o - u - 2o - u - 3o - 2u - 2o - 2u - o$.
$13 \,\&\, 13'$: $i \le 5$; $o - \star = 2o$ for uppermost $i = 5$ left of upper X line:
$L \longrightarrow R: 2o - u - 3o - 2u - 2o - 2u - 2o - 2u - o$.
$14 \,\&\, 14'$: $i \le 6$; o for $i = 6$ left of lower X line:
$L \longleftarrow R: 2o - 2u - 2o - 2u - 2o - 2u - 2o - 2u - o$.
$15 \,\&\, 15'$: $i \le 6$; u for uppermost $i < 6$ right of lower X line and
$o - \star = 2o$ for uppermost $i = 6$ left of both X lines and
$u - \bullet$ for uppermost $i = 6$ right of both X lines:
$L \longrightarrow R: 3o - 2u - 2o - 2u - 2o - 5u - \bullet - 2u$.

As mentioned earlier, the coding which is associated with the $\bullet$ crossing can be freely chosen, however, it is in most cases convenient to take for this coding the one which simplifies braiding. For example, in option 1 we would braid the half-cycles $15'' \,\&\, 15'''$ as $o - u - 2o - 2u - 2o - 2u - 9u$, and in option 2 we would braid the half-cycles $15 \,\&\, 15'$ as $3o - 2u - 2o - 2u - 2o - 8u$.