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for
the braiding artisan

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The Glen Vandy Knots

In *The Braider*, Issue No. 44 we discussed the Checkered Pineapple Knots and saw that for the types 2 and 3 the value of y is 0 (see pg. 1042). Although the value of n is an odd integer for these two types, hence the minimum n-value is equal to 1, we could relax this minimum n-value condition to $n = 0$ while retaining the y-value of 0. From the coding associated with the types 2 and 3 it will be observed that $2A+1 \leq x \leq 3A+2$ for $n = 0$. Since $y = |2(A-1)+x|_{2A}$ has to be zero, it follows that $x = 2A+2$. The knots of type 2 and type 3 with $n = 0$, $y = 0$ and $x = 2A+2$ are called **the Glen Vandy Knots.** An A-pass Glen Vandy Knot thus consists of A interbraided Regular Cylindrical Braids, each with $P_c = 4$ ($P_c = \frac{P_{total}}{A} = \frac{2A+x-2}{A}$), hence its string-run is the string-run of the Regular Nested Cylindrical Braid

$$(\underbrace{222\cdots2}_{(A-1)\text{ elements}}/2A+2/\underbrace{222\cdots2}_{(A-1)\text{ elements}})\{1(A)(A-1)(A-2)\cdots2/(A)123\cdots(A-1)\}B\,,$$

The upper row of diagrams in Fig. 817 show the coding-patterns of type 2 while the lower row of diagrams in Fig. 817 show the coding-patterns of type 3 Glen Vandy Knots.

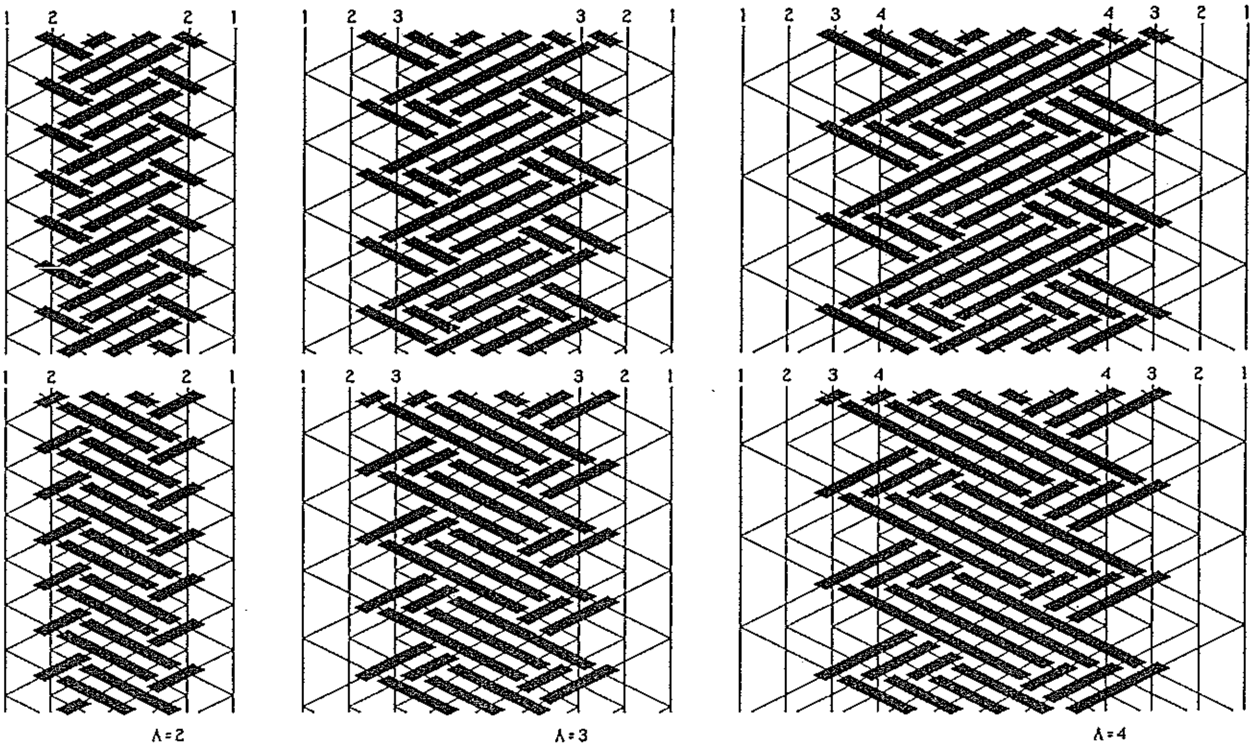


Fig. 817 — The coding-patterns of Glen Vandy Knots.

From the diagrams in Fig. 817 it can be seen that the best way to braid an A-pass Glen Vandy Knot is to braid first a 2-pass Glen Vandy Knot and then progressively to enlarge it to a 3-pass, 4-pass, ..., $(A-1)$-pass, A-pass Glen Vandy Knot by respectively interbraiding the Regular Cylindrical Braid component associated with the left to right bight-boundary pair $2—2$, a left to right bight-boundary pair which belongs to the set $2—3$ to $3—2$, ..., a left to right bight-boundary pair which belongs to the set $2—(A-2)$ to $(A-2)—2$, a left to right bight-boundary pair which belongs to the set $2—(A-1)$ to $(A-1)—2$.

Let P_c and B^* be coprime, hence g.c.d. $(P_c, B^*) =$ g.c.d. $(4, B^*) = 1$ (consequently $B^* = odd$), thus the interbraided Regular Cylindrical Braids are Regular Knots.

Type 2:

For a 2-pass type 2 Glen Vandy Knot the algorithm diagram for the foundation knot between left bight-boundary 1 and right bight-boundary 2 is:

Δ^*	$\lvert 2\Delta^*\rvert_{B*}$	$\lvert 3\Delta^*\rvert_{B*}$
0	0	0
u	o	o
.	.	.
o	u	u
0	0	0
$\lvert 3\Delta^*\rvert_{B*}$	$\lvert 2\Delta^*\rvert_{B*}$	Δ^*

For the half-cycles from lower left to upper right the first line gives the i-values, the second line the reference quantities, the third line the coding and the fourth line the intersection columns. For the half-cycles from lower right to upper left the fourth line gives the intersection columns, the fifth line the coding, the sixth line the reference quantities and the seventh line the i-values. Each reference quantity is increased by 1 when its associated i-value is applicable for the half-cycle concerned.

For a 2-pass type 2 Glen Vandy Knot the algorithm diagram for the interbraided component between left bight-boundary 2 and right bight-boundary 1 is:

	Δ^*	$\lvert 2\Delta^*\rvert_{B*}$	$\lvert 3\Delta^*\rvert_{B*}$
1	1	1	0
u	o	o	u
.	.	.	.
	u	u	o
	1	1	1
	$\lvert 3\Delta^*\rvert_{B*}$	$\lvert 2\Delta^*\rvert_{B*}$	Δ^*

For the half-cycles from lower left to upper right the first line gives the i-values where applicable, the second line the reference quantities, the third line the coding and the fourth line the intersection sets. For the half-cycles from lower right to upper left the fourth line gives the intersection sets, the fifth line the coding, the sixth line the reference quantities and the seventh line the i-values where applicable. Each reference quantity is increased by 1 when its associated i-value is applicable for the half-cycle concerned.

An A-pass type 2 Glen Vandy Knot ($A \geq 3$) is enlarged from a 2-pass type 2 Glen Vandy Knot by interbraiding the Regular Knot components between the left and right bight-boundary pairs $2—(A-1)$ to and including $(A-1)—2$. The algorithm diagram for these interbraids is:

	Δ^*	$\lvert 2\Delta^*\rvert_{B*}$	$\lvert 3\Delta^*\rvert_{B*}$	
	$l-1$	$2A-2$	$A-l$	
	u	o	u	
.	.	.	.	.
u	o	u	o	u
$A-r$	$r-1$	$A-1$	$A-r$	$r-1$
	$\lvert 3\Delta^*\rvert_{B*}$	$\lvert 2\Delta^*\rvert_{B*}$	Δ^*	

$$A \geq 3\,;\; 2 \leq l \leq A-1\,;\; 2 \leq r \leq A-1\,;\; r = A+1-l\,;\; l = A+1-r\,.$$

For the half-cycles from lower left to upper right the first line gives the i-values where applicable, the second line gives the reference quantities, the third line the coding and the fourth line the intersection sets. For the half-cycles from lower right to upper left the fourth line gives the intersection sets, the fifth line the coding, the sixth line the reference quantities and the seventh line gives the i-values where applicable. Each reference quantity is increased by 1 when its associated i-value is applicable for the half-cycle concerned.

Type 3:

For a 2-pass type 3 Glen Vandy Knot the algorithm diagram for the foundation knot between left bight-boundary 1 and right bight-boundary 2 is:

Δ^*	$\lvert 2\Delta^* \rvert_{B*}$	$\lvert 3\Delta^* \rvert_{B*}$
0	0	0
o	u	u
.	.	.
u	o	o
0	0	0
$\lvert 3\Delta^* \rvert_{B*}$	$\lvert 2\Delta^* \rvert_{B*}$	Δ^*

For the half-cycles from lower left to upper right the first line gives the i-values, the second line gives the reference quantities of the coding associated with the intersection columns, the third line the coding of the intersection columns and the fourth line the intersection columns. For the half-cycles from lower right to upper left the fourth line gives the intersection columns, the fifth line the coding of the intersection columns, the sixth line the reference quantities of the coding associated with the intersection columns and the seventh line the i-values. Each reference quantity is increased by 1 when its associated i-value is applicable for the half-cycle concerned.

For a 2-pass type 3 Glen Vandy Knot the algorithm diagram for the interbraided component between left bight-boundary 2 and right bight-boundary 1 is:

	Δ^*	$\lvert 2\Delta^* \rvert_{B*}$	$\lvert 3\Delta^* \rvert_{B*}$
	1	1	1
	u	u	o
.	.	.	.
u	o	o	u
1	1	1	0
	$\lvert 3\Delta^* \rvert_{B*}$	$\lvert 2\Delta^* \rvert_{B*}$	Δ^*

For the half-cycles from lower left to upper right the first line gives the i-values where applicable, the second line gives the reference quantities of the coding associated with the intersection sets, the third line the coding associated with the intersection sets and the fourth line the intersection sets. For the half-cycles from lower right to upper left the fourth line gives the intersection sets, the fifth line the coding associated with the intersection sets, the sixth line the reference quantities of the coding associated with the intersection sets and the seventh line the i-values where applicable. Each reference quantity is increased by 1 when its associated i-value is applicable for the half-cycle concerned.

An A-pass type 3 Glen Vandy Knot ($A \geq 3$) is enlarged from a 2-pass type 3 Glen Vandy Knot by interbraiding the Regular Knot components between the left and right bight-boundary pairs $2 — (A-1)$ to and including $(A-1) — 2$. The algorithm diagram for these interbraids is:

$$\begin{array}{ccccc}
 & \Delta^* & |2\Delta^*|_{B^*} & |3\Delta^*|_{B^*} & \\
l-1 & A-l & A-1 & l-1 & A-l \\
u & o & u & o & u \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
 & u & o & u & \\
 & A-r & 2A-2 & r-1 & \\
 & |3\Delta^*|_{B^*} & |2\Delta^*|_{B^*} & \Delta^* &
\end{array}$$

$$A \geq 3\,;\ 2 \leq l \leq A-1\,;\ 2 \leq r \leq A-1\,;\ r = A+1-l\,;\ l = A+1-r\,.$$

For the half-cycles from lower left to upper right the first line gives the i-values where applicable, the second line gives the reference quantities, the third line the coding and the fourth line the intersection sets. For the half-cycles from lower right to upper left the fourth line gives the intersection sets, the fifth line the coding, the sixth line the reference quantities and the seventh line gives the i-values where applicable. Each reference quantity is increased by 1 when its associated i-value is applicable for the half-cycle concerned.

In *The Braider*, Issue No. 16, pg. 367, an example of a 2-pass type 2 and of a 2-pass type 3 Glen Vandy Knot is shown.

For the upper knot the algorithm diagram of the foundation knot is:

$$\begin{array}{ccc}
1 & 2 & 3 \\
0 & 0 & 0 \\
u & o & o \\
\cdot & \cdot & \cdot \\
o & u & u \\
0 & 0 & 0 \\
3 & 2 & 1
\end{array}$$

while the algorithm diagram of the interwoven knot is:

$$\begin{array}{cccc}
 & 1 & 2 & 3 \\
1 & 1 & 1 & 0 \\
u & o & o & u \\
\cdot & \cdot & \cdot & \cdot \\
 & u & u & o \\
 & 1 & 1 & 1 \\
 & 3 & 2 & 1
\end{array}$$

For the lower knot the algorithm diagram of the foundation knot is:

$$\begin{array}{ccc}
1 & 2 & 3 \\
0 & 0 & 0 \\
o & u & u \\
\cdot & \cdot & \cdot \\
u & o & o \\
0 & 0 & 0 \\
3 & 2 & 1
\end{array}$$

while the algorithm diagram of the interwoven knot is:

	1	2	3
	1	1	1
	u	u	o
.	.	.	.
u	o	o	u
1	1	1	0
	3	2	1

Braid Design

Let's return to the 12-lead tubular flat braid in *The Braider*, Issue No. 43 (see the lower diagrams of Fig. 796), and form from this braid a cylindrical braid as depicted by the diagrams of Fig. 818. This cylindrical braid is then a **flat tubular torus braid**.

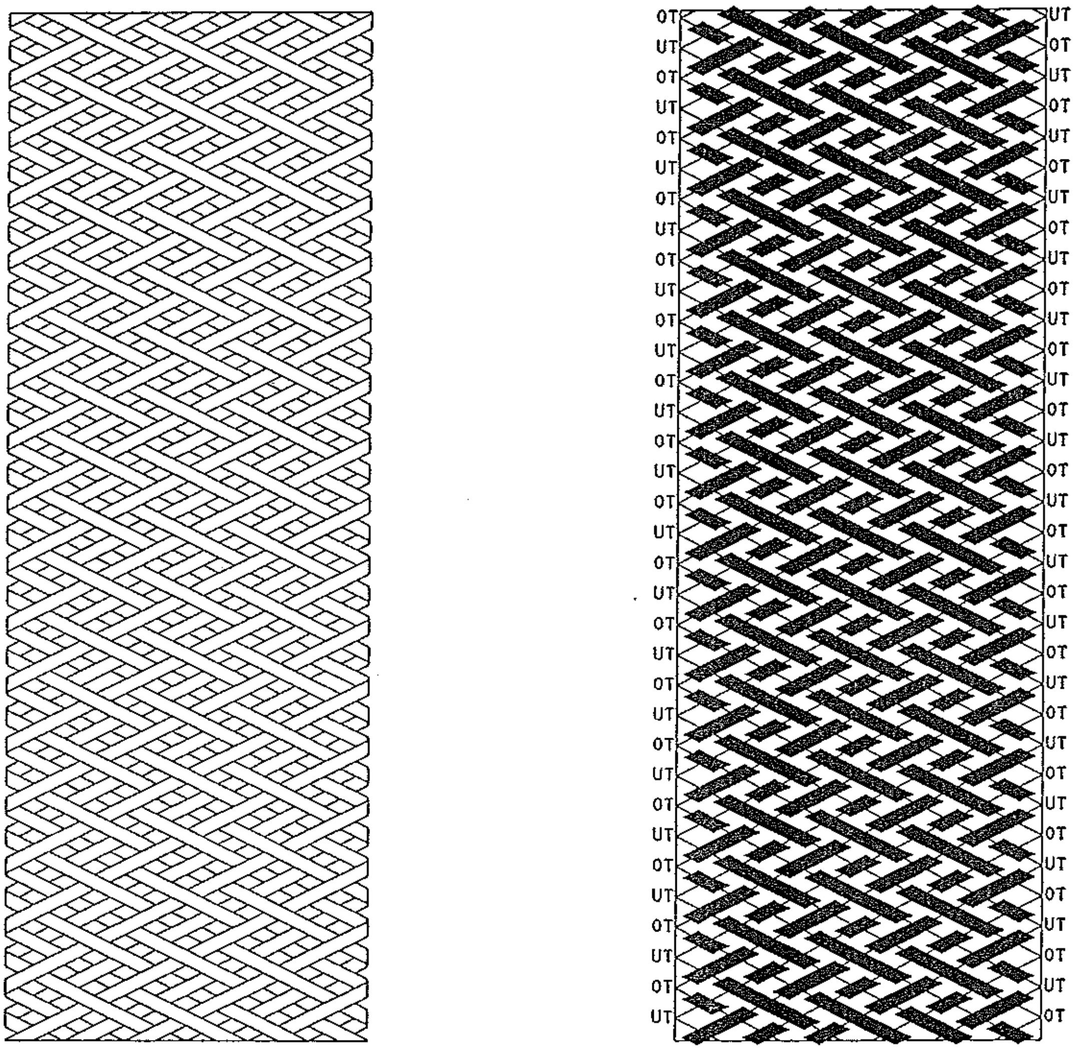


Fig. 818 — A $p/b = 12/34$ flat tubular torus braid.

The braid consists of two interbraided primitive flat torus braids: one having $p/b = 6/17$ with a right helix and one having $p/b = 6/17$ with a left helix. The one with a right helix has OT's on the left 'bight' boundary and UT's on the right 'bight' boundary, while the one with the left helix has UT's on the left 'bight' boundary and OT's on the right 'bight' boundary.† Say we first braid the primitive flat torus braid with the right helix as the foundation knot. The path-formula of a $p/b = 6/17$ Regular Knot is $[0; 2, 1, 4, 1]$ (obtained with the aid of Euclid's algorithm) and hence $\Delta^* = 14$.‡ Hence the algorithm diagram for the foundation knot (the primitive flat torus braid with the right helix; odd numbered half-cycles from lower-left to upper-right start with OT, even numbered half-cycles from lower-right to upper-left start with UT) is:

14	11	8	5	2
o	o	u	o	o
.	.	.	.	.
u	u	o	u	u
2	5	8	11	14

From this algorithm diagram we read the following half-cycle braiding algorithms:

half-cycle 1:			Free run.	half-cycle 18:	$i \leq 8$	:	$o - 2u$.
half-cycle 2:	$i = 0$	:	Free run.	half-cycle 19:	$i \leq 8$	:	$u - 2o$.
half-cycle 3:	$i = 0$	:	Free run.	half-cycle 20:	$i \leq 9$	:	$o - 2u$
half-cycle 4:	$i \leq 1$	:	Free run.	half-cycle 21:	$i \leq 9$	:	$u - 2o$.
half-cycle 5:	$i \leq 1$	:	Free run.	half-cycle 22:	$i \leq 10$	:	$o - 2u$.
half-cycle 6:	$i \leq 2$	:	u.	half-cycle 23:	$i \leq 10$	:	$u - 2o$.
half-cycle 7:	$i \leq 2$	:	o.	half-cycle 24:	$i \leq 11$	:	$u - o - 2u$.
half-cycle 8:	$i \leq 3$	:	u.	half-cycle 25:	$i \leq 11$	:	$o - u - 2o$.
half-cycle 9:	$i \leq 3$	:	o.	half-cycle 26:	$i \leq 12$	:	$u - o - 2u$.
half-cycle 10:	$i \leq 4$	:	u.	half-cycle 27:	$i \leq 12$	:	$o - u - 2o$.
half-cycle 11:	$i \leq 4$	:	o.	half-cycle 28:	$i \leq 13$	:	$u - o - 2u$.
half-cycle 12:	$i \leq 5$	:	$2u$.	half-cycle 29:	$i \leq 13$	:	$o - u - 2o$.
half-cycle 13:	$i \leq 5$	:	$2o$.	half-cycle 30:	$i \leq 14$	:	$2u - o - 2u$.
half-cycle 14:	$i \leq 6$	:	$2u$.	half-cycle 31:	$i \leq 14$	:	$2o - u - 2o$.
half-cycle 15:	$i \leq 6$	:	$2o$.	half-cycle 32:	$i \leq 15$	:	$2u - o - 2u$.
half-cycle 16:	$i \leq 7$	:	$2u$.	half-cycle 33:	$i \leq 15$	:	$2o - u - 2o$.
half-cycle 17:	$i \leq 7$	:	$2o$.	half-cycle 34:	$i \leq 16$	:	$2u - o - 2u$.

The algorithm diagram for the interbraided knot (the primitive flat torus braid with the left helix; odd numbered half-cycles from lower-left to upper-right start with UT, even numbered half-cycles from lower-right to upper-left start with OT) is:

	14		11		8		5		2	
u	u	o	u	u	o	u	u	o	u	u
.	.	.	.	.	.	.	.	.	.	.
u	o	o	o	u	u	u	o	o	o	u
	2		5		8		11		14	

From this algorithm diagram we read the following half-cycle braiding algorithms:

† Note that the OT's and UT's have changed place in the Figs. 818 & 796 since in Fig. 818 the braiding is from lower-left to upper-right and from lower-right to upper-left. Also note that the contact crossings in a primitive flat torus braid are **not** contact crossings in a primitive torus braid.

‡ Note that each of the two interbraided primitive flat torus braids behaves as a Regular Knot.

half-cycle 1:			$u - o - 2u - o - u$.
half-cycle 2:	$i = 0$	:	$u - o - 2u - o - u$.
half-cycle 3:	$i = 0$	:	$u - o - 2u - o - u$.
half-cycle 4:	$i \leq 1$	:	$u - o - 2u - o - u$.
half-cycle 5:	$i \leq 1$	:	$u - o - 2u - o - u$.
half-cycle 6:	$i \leq 2$	:	$u - o - 2u - 2o - u$.
half-cycle 7:	$i \leq 2$	:	$u - o - 2u - o - 2u$.
half-cycle 8:	$i \leq 3$	:	$u - o - 2u - 2o - u$.
half-cycle 9:	$i \leq 3$	:	$u - o - 2u - o - 2u$.
half-cycle 10:	$i \leq 4$	:	$u - o - 2u - 2o - u$.
half-cycle 11:	$i \leq 4$	:	$u - o - 2u - o - 2u$.
half-cycle 12:	$i \leq 5$	:	$u - o - 2u - 3o - u$.
half-cycle 13:	$i \leq 5$	:	$u - o - 3u - o - 2u$.
half-cycle 14:	$i \leq 6$	:	$u - o - 2u - 3o - u$.
half-cycle 15:	$i \leq 6$	:	$u - o - 3u - o - 2u$.
half-cycle 16:	$i \leq 7$	:	$u - o - 2u - 3o - u$.
half-cycle 17:	$i \leq 7$	:	$u - o - 3u - o - 2u$.
half-cycle 18:	$i \leq 8$	:	$u - o - 3u - 3o - u$.
half-cycle 19:	$i \leq 8$	:	$u - o - u - o - 2u - o - 2u$.
half-cycle 20:	$i \leq 9$	:	$u - o - 3u - 3o - u$.
half-cycle 21:	$i \leq 9$	:	$u - o - u - o - 2u - o - 2u$.
half-cycle 22:	$i \leq 10$	:	$u - o - 3u - 3o - u$.
half-cycle 23:	$i \leq 10$	:	$u - o - u - o - 2u - o - 2u$.
half-cycle 24:	$i \leq 11$	:	$u - 2o - 3u - 3o - u$.
half-cycle 25:	$i \leq 11$	:	$u - o - 2u - o - 2u - o - 2u$.
half-cycle 26:	$i \leq 12$	:	$u - 2o - 3u - 3o - u$.
half-cycle 27:	$i \leq 12$	:	$u - o - 2u - o - 2u - o - 2u$.
half-cycle 28:	$i \leq 13$	:	$u - 2o - 3u - 3o - u$.
half-cycle 29:	$i \leq 13$	:	$u - o - 2u - o - 2u - o - 2u$.
half-cycle 30:	$i \leq 14$	:	$u - 3o - 3u - 3o - u$.
half-cycle 31:	$i \leq 14$	:	$2u - o - 2u - o - 2u - o - 2u$.
half-cycle 32:	$i \leq 15$	:	$u - 3o - 3u - 3o - u$.
half-cycle 33:	$i \leq 15$	:	$2u - o - 2u - o - 2u - o - 2u$.
half-cycle 34:	$i \leq 16$	:	$u - 3o - 3u - 3o - u$.

Similarly we can form a cylindrical braid as depicted in Fig. 819 from the 'tubular' braid in Fig. 807 (see *The Braider*, Issue No. 44, pg. 1035). This cylindrical braid consists of three components a primitive toroidal foundation knot with $p/b = 4/11$ and two primitive torus knots, each with $p/b = 2/11$ (one with a left-helix and one with a right-helix).

With the braiding direction for the odd-numbered half-cycles from lower-left to upper-right, the primitive toroidal foundation knot has OT bights, and with its Δ^*-value of 8, its algorithm diagram is:

$$\begin{array}{ccc} 8 & 5 & 2 \\ o & u & u \\ \cdot & \cdot & \cdot \\ u & o & o \\ 2 & 5 & 8 \end{array}$$

From this algorithm diagram we read the following half-cycle braiding algorithms for the foundation knot:

half-cycle 1:		Free run.	half-cycle 12:	$i \leq 5$	: $o - u$.
half-cycle 2:	$i = 0$	: Free run.	half-cycle 13:	$i \leq 5$	: $2u$.
half-cycle 3:	$i = 0$	: Free run.	half-cycle 14:	$i \leq 6$	: $o - u$
half-cycle 4:	$i \leq 1$	: Free run.	half-cycle 15:	$i \leq 6$	: $2u$.
half-cycle 5:	$i \leq 1$	: Free run.	half-cycle 16:	$i \leq 7$	: $o - u$.
half-cycle 6:	$i \leq 2$	: u.	half-cycle 17:	$i \leq 7$	: $2u$.
half-cycle 7:	$i \leq 2$	: u.	half-cycle 18:	$i \leq 8$	: $2o - u$.
half-cycle 8:	$i \leq 3$	: u.	half-cycle 19:	$i \leq 8$	: $o - 2u$.
half-cycle 9:	$i \leq 3$	: u.	half-cycle 20:	$i \leq 9$	: $2o - u$.
half-cycle 10:	$i \leq 4$	: u.	half-cycle 21:	$i \leq 9$	: $o - 2u$.
half-cycle 11:	$i \leq 4$	: u.	half-cycle 22:	$i \leq 10$	: $2o - u$.

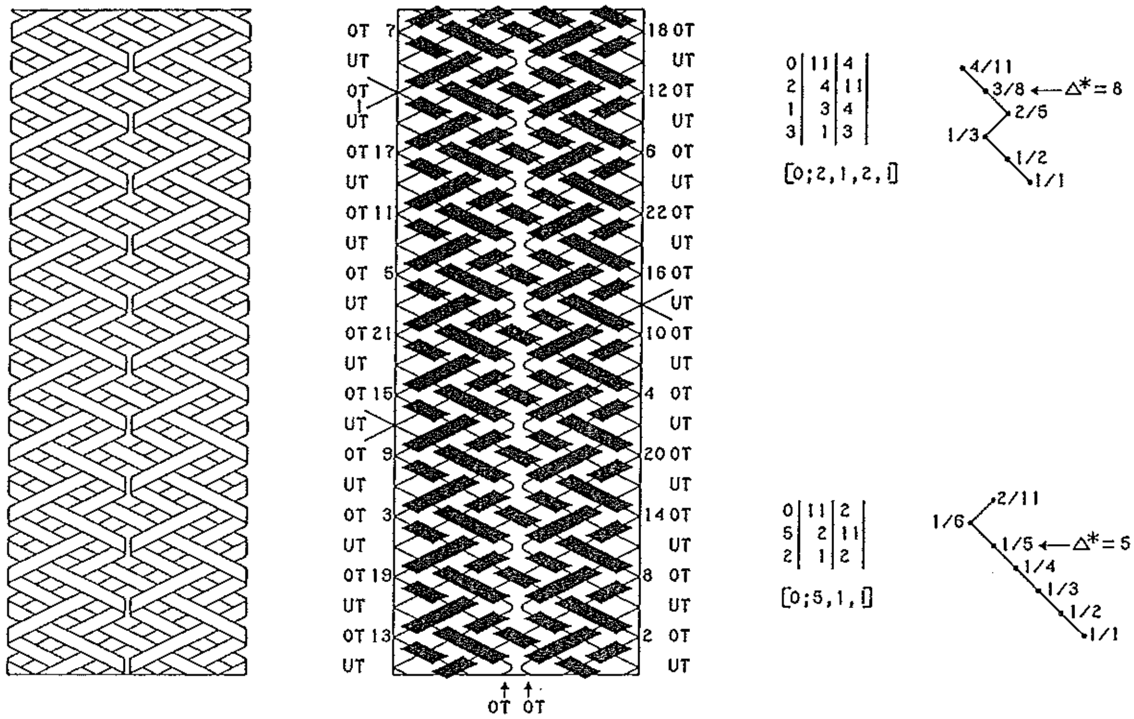


Fig. 819 — A toroidal braid.

For the interbraided left-hand primitive torus knot (left-helix) with $p/b = 2/11$ and hence $\Delta^* = 5$, the algorithm diagram is:

$$\begin{array}{ccc} & 5 & \\ u & u & o \\ \cdot & \cdot & \cdot \\ u & o & o \\ & 5 & \end{array}$$

From this algorithm diagram we read the following half-cycle braiding algorithms (odd numbered half-cycles from lower-left to upper-right and even numbered half-cycles from lower-right to upper-left):

half-cycle 1:		: $u - o$.	half-cycle 12:	$i \leq 5$	: $2o - u$.
half-cycle 2:	$i = 0$	: $o - u$.	half-cycle 13:	$i \leq 5$	: $2u - o$.
half-cycle 3:	$i = 0$	: $u - o$.	half-cycle 14:	$i \leq 6$	: $2o - u$
half-cycle 4:	$i \leq 1$	: $o - u$.	half-cycle 15:	$i \leq 6$	: $2u - o$.
half-cycle 5:	$i \leq 1$	: $u - o$.	half-cycle 16:	$i \leq 7$	: $2o - u$.

half-cycle 6:	$i \leq 2$	: $o - u$.	half-cycle 17:	$i \leq 7$	: $2u - o$.
half-cycle 7:	$i \leq 2$	: $u - o$.	half-cycle 18:	$i \leq 8$	: $2o - u$.
half-cycle 8:	$i \leq 3$	: $o - u$.	half-cycle 19:	$i \leq 8$	: $2u - o$.
half-cycle 9:	$i \leq 3$	: $u - o$.	half-cycle 20:	$i \leq 9$	: $2o - u$.
half-cycle 10:	$i \leq 4$	: $o - u$.	half-cycle 21:	$i \leq 9$	: $2u - o$.
half-cycle 11:	$i \leq 4$	: $u - o$.	half-cycle 22:	$i \leq 10$	: $2o - u$.

For the interbraided right-hand primitive torus knot (right-helix) with $p/b = 2/11$ and hence $\Delta^* = 5$, the algorithm diagram is:

	5	
o	o	u
.	.	.
o	u	u
	5	

From this algorithm diagram we read the following half-cycle braiding algorithms (odd numbered half-cycles from lower-right to upper-left and even numbered half-cycles from lower-left to upper-right):

half-cycle 1:		: $u - o$.	half-cycle 12:	$i \leq 5$	: $2o - u$.
half-cycle 2:	$i = 0$	: $o - u$.	half-cycle 13:	$i \leq 5$	: $2u - o$.
half-cycle 3:	$i = 0$	: $u - o$.	half-cycle 14:	$i \leq 6$	: $2o - u$
half-cycle 4:	$i \leq 1$	: $o - u$.	half-cycle 15:	$i \leq 6$	: $2u - o$.
half-cycle 5:	$i \leq 1$	: $u - o$.	half-cycle 16:	$i \leq 7$	: $2o - u$.
half-cycle 6:	$i \leq 2$	: $o - u$.	half-cycle 17:	$i \leq 7$	: $2u - o$.
half-cycle 7:	$i \leq 2$	: $u - o$.	half-cycle 18:	$i \leq 8$	: $2o - u$.
half-cycle 8:	$i \leq 3$	: $o - u$.	half-cycle 19:	$i \leq 8$	: $2u - o$.
half-cycle 9:	$i \leq 3$	: $u - o$.	half-cycle 20:	$i \leq 9$	: $2o - u$.
half-cycle 10:	$i \leq 4$	: $o - u$.	half-cycle 21:	$i \leq 9$	: $2u - o$.
half-cycle 11:	$i \leq 4$	: $u - o$.	half-cycle 22:	$i \leq 10$	: $2o - u$.

Fig. 820 depicts a toroidal braid where four 4-lead under-over coded torus braids have been 'combined'. With the braiding direction for the odd-numbered half-cycles from lower-left to upper-right, the primitive toroidal foundation knot ($p/b = 8/11$) has OT bights, and with its Δ^*-value of 4, its algorithm diagram is:

4	8	1	5	9	2	6
o	u	u	o	o	u	u
.	.	.	.	.	.	.
u	o	o	u	u	o	o
6	2	9	5	1	8	4

From this algorithm diagram we read the following half-cycle braiding algorithms for the foundation knot:

half-cycle 1:		Free run.	half-cycle 12:	$i \leq 5$	: $o - 2u - o$.
half-cycle 2:	$i = 0$	: Free run.	half-cycle 13:	$i \leq 5$	: $o - u - o - u$.
half-cycle 3:	$i = 0$	: Free run.	half-cycle 14:	$i \leq 6$	: $o - 2u - o - u$
half-cycle 4:	$i \leq 1$	: u.	half-cycle 15:	$i \leq 6$	: $o - u - o - 2u$.
half-cycle 5:	$i \leq 1$	: u.	half-cycle 16:	$i \leq 7$	: $o - 2u - o - u$.
half-cycle 6:	$i \leq 2$	: $u - o$.	half-cycle 17:	$i \leq 7$	: $o - u - o - 2u$.
half-cycle 7:	$i \leq 2$	: $2u$.	half-cycle 18:	$i \leq 8$	: $2o - 2u - o - u$.
half-cycle 8:	$i \leq 3$	: $u - o$.	half-cycle 19:	$i \leq 8$	: $o - 2u - o - 2u$.
half-cycle 9:	$i \leq 3$	: $2u$.	half-cycle 20:	$i \leq 9$	: $2o - 2u - 2o - u$.

half-cycle 10 : $i \leq 4$: $o - u - o$.
half-cycle 11 : $i \leq 4$: $o - 2u$.

half-cycle 21 : $i \leq 9$: $o - 2u - 2o - 2u$.
half-cycle 22 : $i \leq 10$: $2o - 2u - 2o - u$.

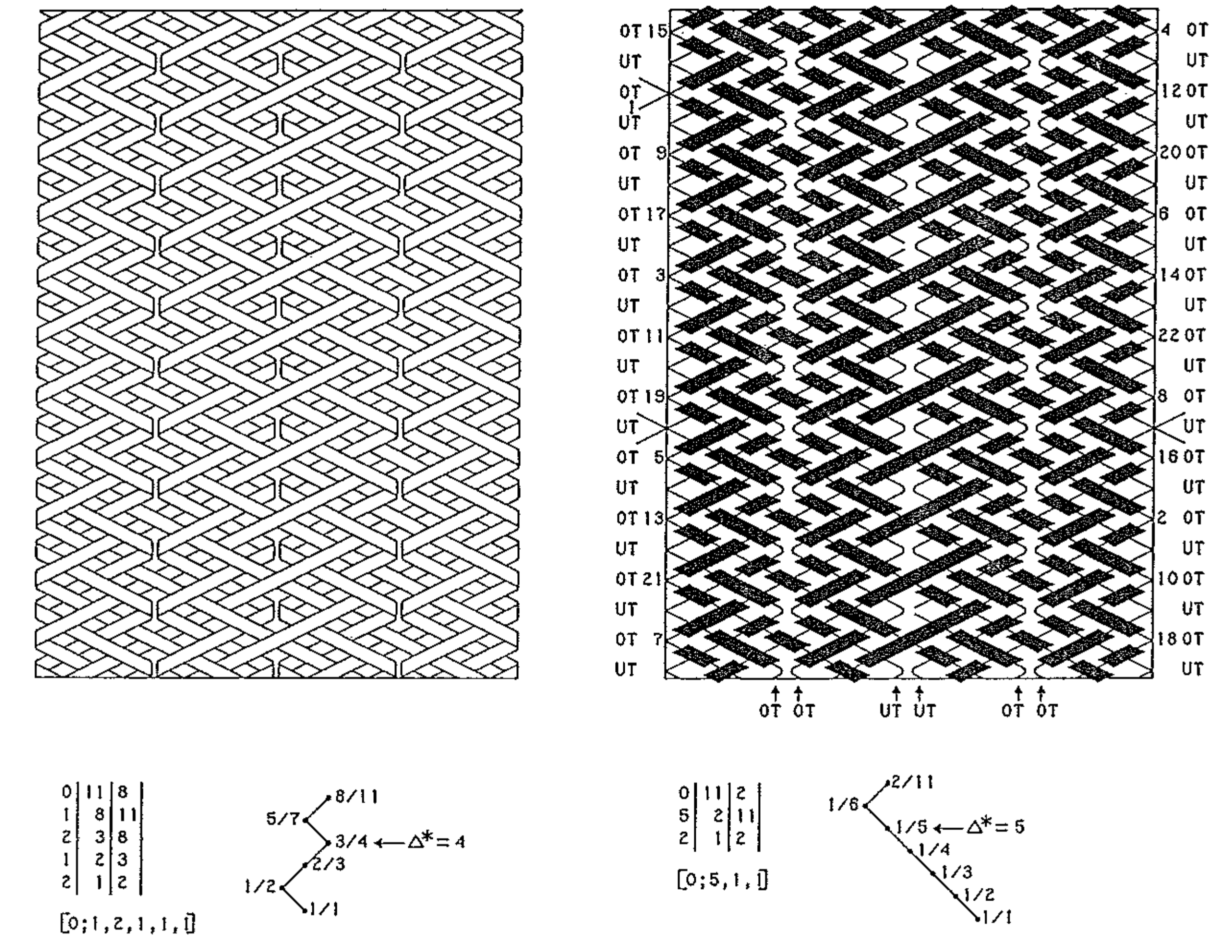


Fig. 820 — A toroidal braid formed from four 4-lead under-over coded torus braids.

The two interbraided left-helix primitive torus knots, each with $p/b = 2/11$ and hence $\Delta^* = 5$, are interbraided as the left-helix primitive torus knot in the toroidal foundation knot $p/b = 4/11$ we discussed earlier. Similarly, the two interbraided right-helix primitive torus knots, each with $p/b = 2/11$ and hence $\Delta^* = 5$, are interbraided as the right-helix primitive torus knot in the toroidal foundation knot $p/b = 4/11$ we discussed earlier.

A multitude of various toroidal braids can be designed by combining in similar ways various different torus braids. Furthermore, various toroidal braids can be designed by 'combining' torus braids and round braids; an example of such a braid is depicted in Fig. 821. With the braiding direction for the odd-numbered half-cycles from lower-left to upper-right, the toroidal foundation knot ($p/b = 15/11$) of that braid has OT bights. With its Δ^*-value of 8, its algorithm diagram is:

8	5	2	10	7	4	1	9	6	3	0	8	5	2
o	u	u	o	o	u	u	o	o	u	u	o	o	u
.	.	.	.	.	.	.	.	.	.	.	.	.	.
u	o	o	u	u	o	o	u	u	o	o	u	u	o
2	5	8	0	3	6	9	1	4	7	10	2	5	8

The interbraided left-helix primitive torus knot with $p/b = 2/11$ and the interbraided right-helix primitive torus knot with $p/b = 2/11$ are interbraided as in the earlier discussed toroidal foundation knot $p/b = 4/11$.

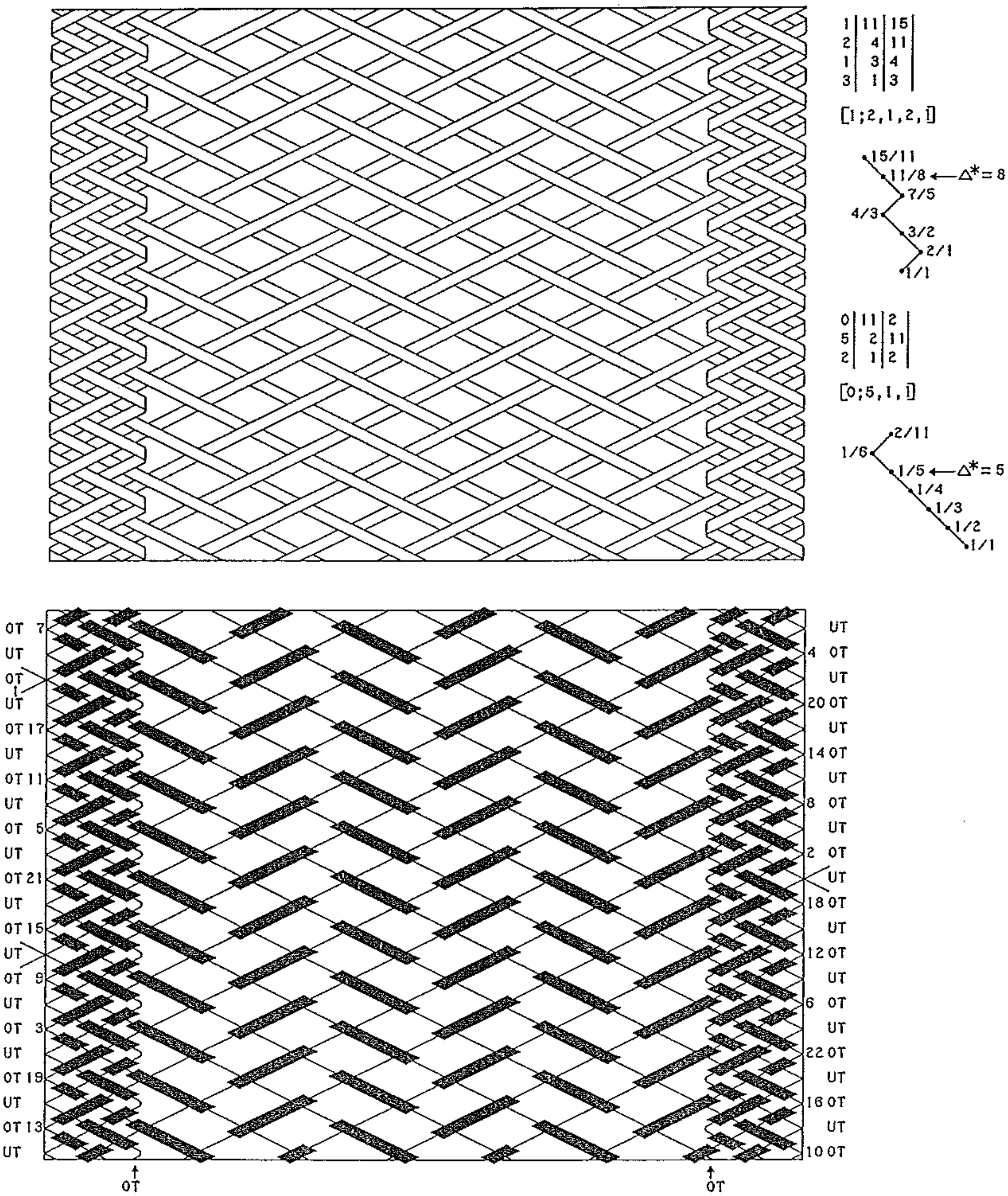


Fig. 821 — A toroidal braid.

From the algorithm diagram of the toroidal foundation knot ($p/b = 15/11$) we read the following half-cycle braiding algorithms for this foundation knot:

half-cycle 1:			Free run.
half-cycle 2:	$i = 0$	:	u.
half-cycle 3:	$i = 0$	:	u.
half-cycle 4:	$i \leq 1$	:	$2u$.
half-cycle 5:	$i \leq 1$	:	$2u$.
half-cycle 6:	$i \leq 2$	:	$4u$.

half-cycle 7:	$i \leq 2$	:	$4u$.
half-cycle 8:	$i \leq 3$	:	$5u$.
half-cycle 9:	$i \leq 3$	:	$5u$.
half-cycle 10:	$i \leq 4$	:	$6u$.
half-cycle 11:	$i \leq 4$	:	$6u$.
half-cycle 12:	$i \leq 5$	:	$6u - o - u$.
half-cycle 13:	$i \leq 5$	:	$6u - o - u$.
half-cycle 14:	$i \leq 6$	:	$4u - o - 2u - o - u$
half-cycle 15:	$i \leq 6$	:	$4u - o - 2u - o - u$.
half-cycle 16:	$i \leq 7$	:	$2u - o - 2u - o - 2u - o - u$.
half-cycle 17:	$i \leq 7$	:	$2u - o - 2u - o - 2u - o - u$.
half-cycle 18:	$i \leq 8$	:	$o - 2u - o - 2u - o - 2u - 2o - u$.
half-cycle 19:	$i \leq 8$	:	$o - 2u - o - 2u - o - 2u - 2o - u$.
half-cycle 20:	$i \leq 9$	:	$o - 2u - o - 2u - 2o - 2u - 2o - u$.
half-cycle 21:	$i \leq 9$	:	$o - 2u - o - 2u - 2o - 2u - 2o - u$.
half-cycle 22:	$i \leq 10$	:	$o - 2u - 2o - 2u - 2o - 2u - 2o - u$.

Grant Knots

If the string-run of a **Regular Nested Cylindrical Braid** with the essential coding of a **Pineapple Knot**† is modified in such a way that, at least at one set of two adjacent bight-boundaries with bight-boundary numbers greater than 1, the two consecutive bights $<<$ and/or $>>$ in each nest are respectively transformed to $\substack{<\\<}$ and $\substack{>\\>}$, the modified string-run with that essential Pineapple coding is then the string-run of a **Grant Knot**‡ Similarly to the Herringbone Pineapple Knots with $y = A$, the Herringbone Grant Knots with $y = A$ we shall call the **Standard Herringbone Grant Knots.** Not only can the Standard Herringbone Grant Knots have the same colour-patterns as the Standard Herringbone Pineapple Knots and hence have identical appearances, the Standard Herringbone Grant Knots can have colour patterns which the Standard Herringbone Pineapple Knots cannot. Furthermore, the Standard Herringbone Grant Knots often require fewer essential strings. A few examples are shown in Fig. 822. When the Standard Herringbone Grant Knot has the same colour-pattern and requires the same number of essential strings as its associated Standard Herringbone Pineapple Knot, it will be more convenient to use the Standard Herringbone Pineapple Knot.

† See *The Braider*, Issue No. 44, pg. 1041, Fig. 810.

‡ The best known books on braiding are no doubt the books by Bruce Grant (see *The Braider*, Issue No. 1, pp. 17-18). On pp. 418-419 of the *Encyclopedia of Rawhide and Leather Braiding* Bruce describes two knots he 'invented' (the **Bruce Knot** and the **Catharine Knot**) which he respectively named after himself and his wife, and thought to be small Herringbone-type knots. However, these knots are **Standard Herringbone Pineapple Knots**, hence do not deserve a special name (see the book *Braiding — Standard Herringbone Pineapple Knots — Vol 4/1*, by A.G. Schaake, J.C. Turner and D.A. Sedgwick, pg. 87). Consequently, in order to recognise his valuable contribution to braiding, the naming after him in the early nineteen eighties of a special family of knots, closely associated with Pineapple Knots, was felt to be justified.

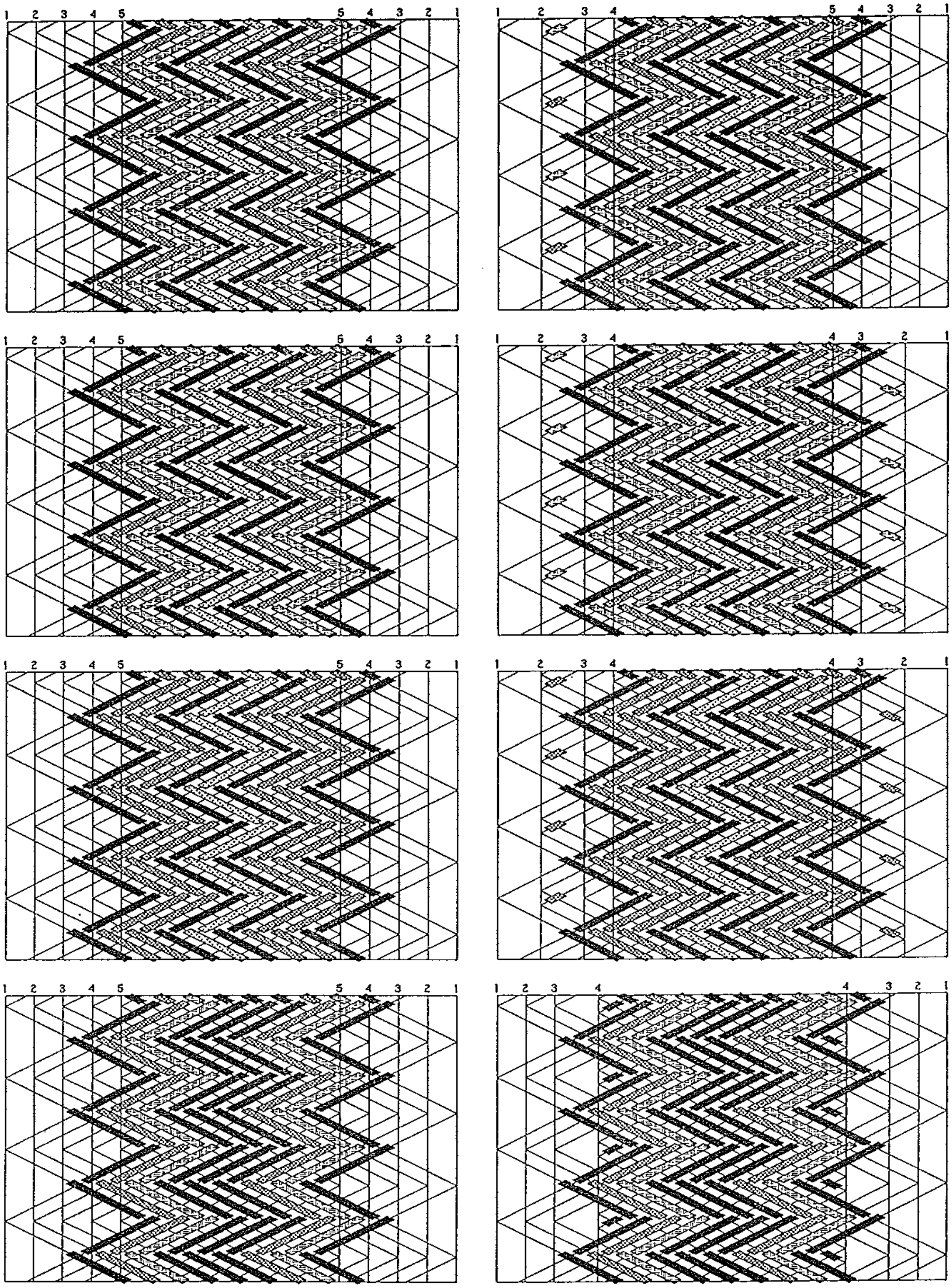


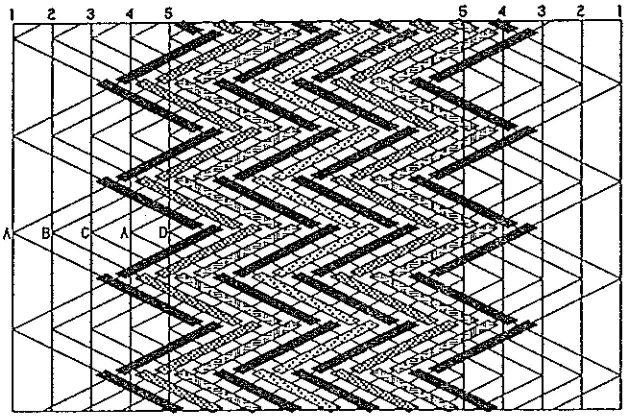
Fig. 822 — Some examples of 5-pass Standard Herringbone Pineapple Knots and their associated 5-pass Standard Herringbone Grant Knots.

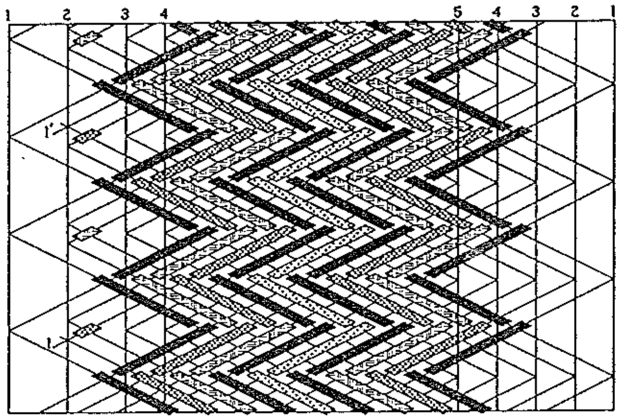
In each row of two diagrams in Fig. 822, the Standard Herringbone Grant Knot is on the right and its associated Standard Herringbone Pineapple Knot is on the left. Each of the Standard Herringbone Pineapple Knots ((2222/15/2222){15432/45123}20) requires five essential strings. The Standard Herringbone Grant Knots in the first three rows also require five essential strings, but those in the first two rows (respectively (332/15/2222)$\{1_1 4_5 3_4 2_2 2_3/4_1 5_2 1_3 2_4 3_5\}$20 & (332/15/233)$\{1_1 4_5 3_4 2_2 2_3/3_1 4_2 1_3 2_4 2_5\}$20) have a colour pattern which is different to that of their associated Standard Herringbone Pineapple Knots. The Standard Herringbone Grant Knot in the third row ((332/15/233)$\{1_1 4_5 3_4 2_2 2_3/3_1 4_2 1_3 2_4 2_5\}$20) has the same colour pattern as its associated Standard Herringbone Pineapple Knot and since both require the same number of essential strings, the latter should be used in an application. The Standard Herringbone Grant Knot in the fourth row ((223/15/322)$\{1_1 4_4 4_5 3_3 2_2/4_1 4_2 1_3 2_4 3_5\}$20) has the same colour pattern as its associated Standard Herringbone Pineapple Knot, but requires only three essential strings.

⋆⋆ *Say we want only two essential strings (of a different colour) in a 5-pass Standard Herringbone Grant Knot. How can we achieve that?*

After the Standard Herringbone Grant Knot has been designed (its grid-diagram), its algorithm diagrams can be drawn up and from them the half-cycle braiding algorithms can be read off.

For the Standard Herringbone Grant Knot depicted by the right-hand grid-diagram below, the string-colours (A, B, C, D) are indicated in the left-hand grid-diagram of its associated Standard Herringbone Pineapple Knot.





First we braid the component starting with half-cycle 1 and colour B, then we braid the component starting with half-cycle 1′ and colour C. Each of these two components has $p/b = 5/4$ and hence $\Delta^* = 3$. After these two components have been braided, the remaining components are braided as if the knot is a Standard Herringbone Pineapple Knot.

The algorithm diagram of the first to be braided component is:

		3	2	1	0		
half-cycles 1, 5	→	*u*	*o*	*u*	*o*		
half-cycles 3, 7	→	*u*	*u*	*u*	*u*		
		.	.	.	.		
		o	*o*	*o*	*o*	←	half-cycles 2, 6
		o	*u*	*o*	*u*	←	half-cycles 4, 8
		0	1	2	3		

From this algorithm diagram we read off the half-cycle braiding algorithms for the first to be braided component:

half-cycle 1:		$L_1 \to R_1$:	Free run.
half-cycle 2:	$i=0$:	$R_1 \to L_1$:	o.
half-cycle 3:	$i=0$:	$L_1 \to R_2$:	u.
half-cycle 4:	$i\leq 1$:	$R_2 \to L_1$:	$u-o$.
half-cycle 5:	$i\leq 1$:	$L_1 \to R_1$:	$u-o$.
half-cycle 6:	$i\leq 2$:	$R_1 \to L_1$:	$3o$.
half-cycle 7:	$i\leq 2$:	$L_1 \to R_2$:	$3u$.
half-cycle 8:	$i\leq 3$:	$R_2 \to L_1$:	$u-o-u-o$.

The algorithm diagram of the next to be braided component is:

half-cycles 1′, 5′	→		3			2	1			0		
half-cycles 1′, 5′	→		u	o	o	u	u	o	o	u		
half-cycles 3′, 7′	→			3	2			1	0			
half-cycles 3′, 7′	→	o	u	u	o	o	u	u	o	o		
		.	.	.	.	.	.	.	.	.		
			o	o	u	u	o	o	u	u	←	half-cycles 2′, 6′
			0			1	2			3	←	half-cycles 2′, 6′
		u	u	o	o	u	u	o	o	u	←	half-cycles 4′, 8′
				0	1			2	3		←	half-cycles 4′, 8′

From this algorithm diagram we read off the half-cycle braiding algorithms for the second component:

half-cycle 1′:		$L_1 \to R_2$:	$4o$.
half-cycle 2′:	$i=0$:	$R_2 \to L_1$:	$u-o-u-2o$.
half-cycle 3′:	$i=0$:	$L_1 \to R_1$:	$o-u-o-u-2o$.
half-cycle 4′:	$i\leq 1$:	$R_1 \to L_1$:	$3u-2o-2u$.
half-cycle 5′:	$i\leq 1$:	$L_1 \to R_2$:	$2o-u-2o-u$.
half-cycle 6′:	$i\leq 2$:	$R_2 \to L_1$:	$u-2o-2u-2o$.
half-cycle 7′:	$i\leq 2$:	$L_1 \to R_1$:	$o-u-2o-2u-2o$.
half-cycle 8′:	$i\leq 3$:	$R_1 \to L_1$:	$u-2o-2u-2o-2u$.

As mentioned earlier, the remaining components of the Standard Herringbone Grant Knot are braided as if this Grant Knot is a Standard Herringbone Pineapple Knot, and hence for the remaining components we can use the algorithm diagrams of the equivalent components in its associated Standard Herringbone Pineapple Knot. Left bight-boundary 1 of the Grant Knot is equivalent to left bight-boundary 1 of the Pineapple Knot, left bight-boundary 3 of the Grant Knot is equivalent to left bight-boundary 4 of the Pineapple Knot, and left bight-boundary 4 of the Grant Knot is equivalent to left bight-boundary 5 of the Pineapple Knot.

The algorithm diagram for two components with $p/b = 5/4$, and hence $\Delta^* = 3$, is thus:†

⟶	$l_i - 1$	$A-1$	$A-1$	$A-1$	$r_i - 1$
	3	2	1	0	
/	\	/	\	/	\
	0	1	2	3	
$l_i - 1$	$A-1$	$A-1$	$A-1$	$r_i - 1$	⟵

† Refer to *The Braider*, Issue No. 27, pg. 634.

The algorithm diagram for the component with $p/b = 3/4$, and hence $\Delta^* = 1$, is thus:†

```
    -->   l_i - 1   A - 1   r_i - 1
             1        2
  /          \        /          \
             2        1
l_i - 1   A - 1   r_i - 1   <--
```

The half-cycle braiding algorithms for the component with colour A between left bight-boundary 1 and right bight-boundary 4 in the final Standard Herringbone Grant Knot are:

half-cycle $1''$: $L_1 \to R_3$: $2o - 2u - 2o - 2u$.
half-cycle $2''$: $i = 0$: $R_3 \to L_1$: $2u - 2o - 2u - 3o$.
half-cycle $3''$: $i = 0$: $L_1 \to R_3$: $2o - 2u - 3o - 2u$.
half-cycle $4''$: $i \leq 1$: $R_3 \to L_1$: $2u - 2o - 3u - 3o$.
half-cycle $5''$: $i \leq 1$: $L_1 \to R_3$: $2o - 3u - 3o - 2u$.
half-cycle $6''$: $i \leq 2$: $R_3 \to L_1$: $2u - 3o - 3u - 3o$.
half-cycle $7''$: $i \leq 2$: $L_1 \to R_3$: $3o - 3u - 3o - 2u$.
half-cycle $8''$: $i \leq 3$: $R_3 \to L_1$: $3u - 3o - 3u - 3o$.

Note that for this component $l_i = 1$ and $r_i = 3$. Its string-run is between the bight-boundaries L_1 and R_3. When completed $A = 3$.

The half-cycle braiding algorithms for the component with colour A between left bight-boundary 3 and right bight-boundary 1 in the final Standard Herringbone Grant Knot are:

half-cycle $1'''$: $L_3 \to R_1$: $3u - 3o - 3u - 3o$.
half-cycle $2'''$: $i = 0$: $R_1 \to L_3$: $3o - 3u - 4o - 3u$.
half-cycle $3'''$: $i = 0$: $L_3 \to R_1$: $3u - 3o - 3u - 4o$.
half-cycle $4'''$: $i \leq 1$: $R_1 \to L_3$: $3o - 4u - 4o - 3u$.
half-cycle $5'''$: $i \leq 1$: $L_3 \to R_1$: $3u - 3o - 4u - 4o$.
half-cycle $6'''$: $i \leq 2$: $R_1 \to L_3$: $4o - 4u - 4o - 3u$.
half-cycle $7'''$: $i \leq 2$: $L_3 \to R_1$: $3u - 4o - 4u - 4o$.
half-cycle $8'''$: $i \leq 3$: $R_1 \to L_3$: $u - 4o - 4u - 4o - 3u$.

Note that for this component $l_i = 4$ and $r_i = 1$. Its string-run is between the bight-boundaries L_3 and R_1. When completed $A = 4$.

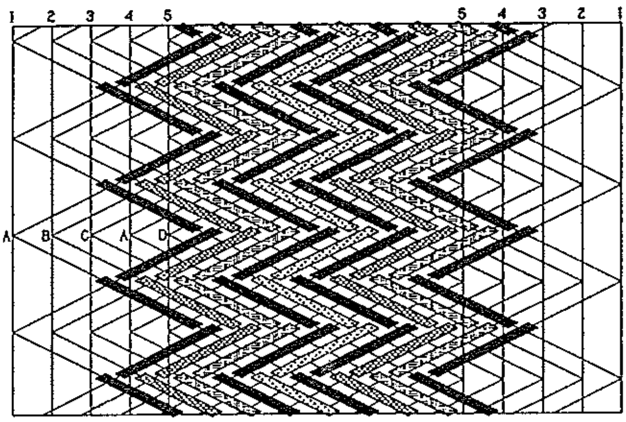
The half-cycle braiding algorithms for the component with colour D between left bight-boundary 4 and right bight-boundary 5 in the final Standard Herringbone Grant Knot are:

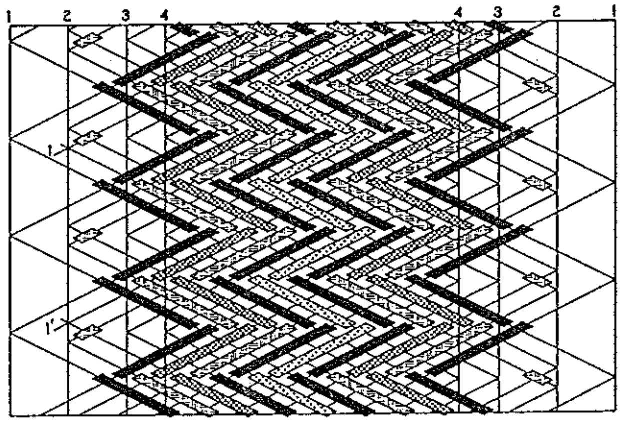
half-cycle $1''''$: $L_4 \to R_5$: $4u - 4o - 4u$.
half-cycle $2''''$: $i = 0$: $R_5 \to L_4$: $4u - 4o - 4u$.
half-cycle $3''''$: $i = 0$: $L_4 \to R_5$: $4u - 4o - 4u$.
half-cycle $4''''$: $i \leq 1$: $R_5 \to L_4$: $5u - 4o - 4u$.
half-cycle $5''''$: $i \leq 1$: $L_4 \to R_5$: $5u - 4o - 4u$.
half-cycle $6''''$: $i \leq 2$: $R_5 \to L_4$: $5u - 5o - 4u$.
half-cycle $7''''$: $i \leq 2$: $L_4 \to R_5$: $5u - 5o - 4u$.
half-cycle $8''''$: $i \leq 3$: $R_5 \to L_4$: $5u - 5o - 5u$.

Note that for this component $l_i = 5$ and $r_i = 5$. Its string-run is between the bight-boundaries L_4 and R_5. When completed $A = 5$.

† Refer to *The Braider*, Issue No. 27, pg. 634.

For the Standard Herringbone Grant Knot (332/15/2222)$\{1_1 4_5 3_4 2_2 2_3/45123\}$20 depicted by the right-hand grid-diagram below, the string-colours (A, B, C, D) are indicated in the left-hand grid-diagram of its associated Standard Herringbone Pineapple Knot.





First we braid the component starting with half-cycle 1 and colour C, then we braid the component starting with half-cycle 1′ and colour B. Each of these two components is a Regular Knot with $p/b = 5/4$ and hence $\Delta^* = 3$. After these two components have been braided, the remaining components are braided as if the knot is a Standard Herringbone Pineapple Knot.

The algorithm diagram of the first to be braided component is:

		3	2	1	0		
half-cycles 1, 3, 5, 7	$\longrightarrow$	u	o	u	o		
		.	.	.	.		
		o	u	o	u	$\longleftarrow$	half-cycles 2, 4, 6, 8
		0	1	2	3		

From this algorithm diagram we read off the half-cycle braiding algorithms for the first to be braided component:

half-cycle 1:		$L_1 \to R_1$:	Free run.
half-cycle 2:	$i = 0$:	$R_1 \to L_1$:	o.
half-cycle 3:	$i = 0$:	$L_1 \to R_1$:	o.
half-cycle 4:	$i \leq 1$:	$R_1 \to L_1$:	$u - o$.
half-cycle 5:	$i \leq 1$:	$L_1 \to R_1$:	$u - o$.
half-cycle 6:	$i \leq 2$:	$R_1 \to L_1$:	$o - u - o$.
half-cycle 7:	$i \leq 2$:	$L_1 \to R_1$:	$o - u - o$.
half-cycle 8:	$i \leq 3$:	$R_1 \to L_1$:	$u - o - u - o$.

The algorithm diagram of the next to be braided component is:

			3		2		1		0				
half-cycles 1′, 3′, 5′, 7′	$\longrightarrow$	u	o	o	u	u	o	o	u	u			
		.	.	.	.	.	.	.	.	.			
		u	u	o	o	u	u	o	o	u	$\longleftarrow$	half-cycles 2′, 4′, 6′, 8′	
			0		1		2		3				

From this algorithm diagram we read off the half-cycle braiding algorithms for the second component:

half-cycle 1′:		$L_1 \to R_1$:	$u - o - u - o - u$.
half-cycle 2′:	$i = 0$:	$R_1 \to L_1$:	$u - o - u - o - 2u$.

half-cycle $3'$: $i=0$: $L_1 \to R_1$: $u-o-u-o-2u$.
half-cycle $4'$: $i\leq 1$: $R_1 \to L_1$: $u-o-u-2o-2u$.
half-cycle $5'$: $i\leq 1$: $L_1 \to R_1$: $u-o-u-2o-2u$.
half-cycle $6'$: $i\leq 2$: $R_1 \to L_1$: $u-o-2u-2o-2u$.
half-cycle $7'$: $i\leq 2$: $L_1 \to R_1$: $u-o-2u-2o-2u$.
half-cycle $8'$: $i\leq 3$: $R_1 \to L_1$: $u-2o-2u-2o-2u$.

As mentioned earlier, the remaining components of the Standard Herringbone Grant Knot are braided as if this Grant Knot is a Standard Herringbone Pineapple Knot, and hence for the remaining components we can use the algorithm diagrams of the equivalent components in its associated Standard Herringbone Pineapple Knot. Left bight-boundary 1 of the Grant Knot is equivalent to left bight-boundary 1 of the Pineapple Knot, left bight-boundary 3 of the Grant Knot is equivalent to left bight-boundary 4 of the Pineapple Knot, and left bight-boundary 4 of the Grant Knot is equivalent to left bight-boundary 5 of the Pineapple Knot. Right bight-boundary 1 of the Grant Knot is equivalent to right bight-boundary 1 of the Pineapple Knot, right bight-boundary 3 of the Grant Knot is equivalent to right bight-boundary 4 of the Pineapple Knot, and right bight-boundary 4 of the Grant Knot is equivalent to right bight-boundary 5 of the Pineapple Knot.

The algorithm diagram for two components with $p/b = 5/4$, and hence $\Delta^* = 3$, is thus:

$$\begin{array}{cccccc}
\longrightarrow & l_i-1 & A-1 & A-1 & A-1 & r_i-1 \\
 & 3 & 2 & 1 & 0 & \\
/ & \backslash & / & \backslash & / & \backslash \\
 & 0 & 1 & 2 & 3 & \\
l_i-1 & A-1 & A-1 & A-1 & r_i-1 & \longleftarrow
\end{array}$$

The algorithm diagram for the component with $p/b = 3/4$, and hence $\Delta^* = 1$, is thus:

$$\begin{array}{cccc}
\longrightarrow & l_i-1 & A-1 & r_i-1 \\
 & 1 & 2 & \\
/ & \backslash & / & \backslash \\
 & 2 & 1 & \\
l_i-1 & A-1 & r_i-1 & \longleftarrow
\end{array}$$

The half-cycle braiding algorithms for the component with colour A between left bight-boundary 1 and right bight-boundary 3 in the final Standard Herringbone Grant Knot are:

half-cycle $1''$: $L_1 \to R_2$: $2o-2u-2o-2u$.
half-cycle $2''$: $i=0$: $R_2 \to L_1$: $2u-2o-2u-3o$.
half-cycle $3''$: $i=0$: $L_1 \to R_2$: $2o-2u-3o-2u$.
half-cycle $4''$: $i\leq 1$: $R_2 \to L_1$: $2u-2o-3u-3o$.
half-cycle $5''$: $i\leq 1$: $L_1 \to R_2$: $2o-3u-3o-2u$.
half-cycle $6''$: $i\leq 2$: $R_2 \to L_1$: $2u-3o-3u-3o$.
half-cycle $7''$: $i\leq 2$: $L_1 \to R_2$: $3o-3u-3o-2u$.
half-cycle $8''$: $i\leq 3$: $R_2 \to L_1$: $3u-3o-3u-3o$.

Note that for this component $l_i = 1$ and $r_i = 3$. Its string-run is between the bight-boundaries L_1 and R_2. When completed $A = 3$.

The half-cycle braiding algorithms for the component with colour A between left bight-boundary 3 and right bight-boundary 1 in the final Standard Herringbone Grant Knot are:

half-cycle $1'''$: $L_3 \to R_1$: $3u - 3o - 3u - 3o$.
half-cycle $2'''$: $i = 0$: $R_1 \to L_3$: $3o - 3u - 4o - 3u$.
half-cycle $3'''$: $i = 0$: $L_3 \to R_1$: $3u - 3o - 3u - 4o$.
half-cycle $4'''$: $i \leq 1$: $R_1 \to L_3$: $3o - 4u - 4o - 3u$.
half-cycle $5'''$: $i \leq 1$: $L_3 \to R_1$: $3u - 3o - 4u - 4o$.
half-cycle $6'''$: $i \leq 2$: $R_1 \to L_3$: $4o - 4u - 4o - 3u$.
half-cycle $7'''$: $i \leq 2$: $L_3 \to R_1$: $3u - 4o - 4u - 4o$.
half-cycle $8'''$: $i \leq 3$: $R_1 \to L_3$: $u - 4o - 4u - 4o - 3u$.

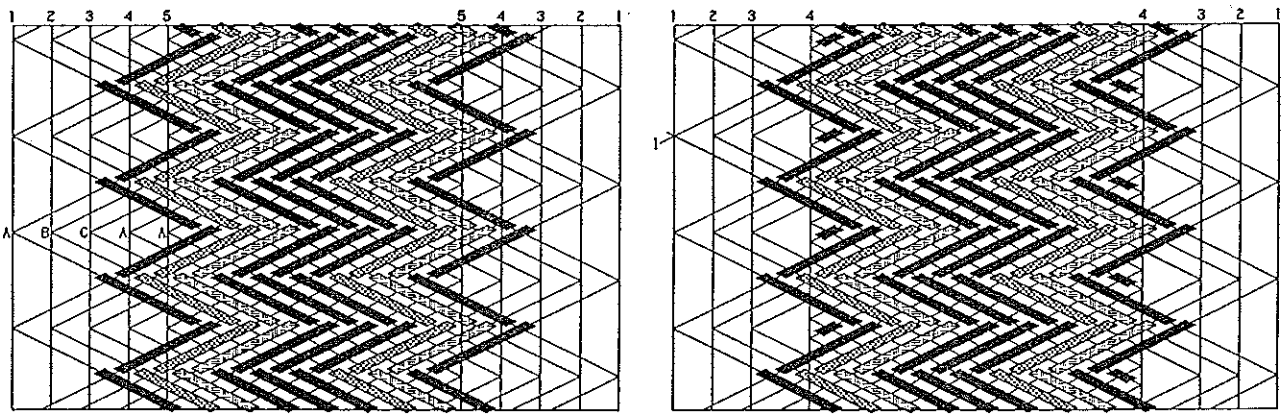
Note that for this component $l_i = 4$ and $r_i = 1$. Its string-run is between the bight-boundaries L_3 and R_1. When completed $A = 4$.

The half-cycle braiding algorithms for the component with colour D between left bight-boundary 4 and right bight-boundary 4 in the final Standard Herringbone Grant Knot are:

half-cycle $1''''$: $L_4 \to R_4$: $4u - 4o - 4u$.
half-cycle $2''''$: $i = 0$: $R_4 \to L_4$: $4u - 4o - 4u$.
half-cycle $3''''$: $i = 0$: $L_4 \to R_4$: $4u - 4o - 4u$.
half-cycle $4''''$: $i \leq 1$: $R_4 \to L_4$: $5u - 4o - 4u$.
half-cycle $5''''$: $i \leq 1$: $L_4 \to R_4$: $5u - 4o - 4u$.
half-cycle $6''''$: $i \leq 2$: $R_4 \to L_4$: $5u - 5o - 4u$.
half-cycle $7''''$: $i \leq 2$: $L_4 \to R_4$: $5u - 5o - 4u$.
half-cycle $8''''$: $i \leq 3$: $R_4 \to L_4$: $5u - 5o - 5u$.

Note that for this component $l_i = 5$ and $r_i = 5$. Its string-run is between the bight-boundaries L_4 and R_4. When completed $A = 5$.

For the Standard Herringbone Grant Knot $(223/17/322)\{1_1 4_4 4_5 3_3 2_2/4_1 4_2 1_3 2_4 3_5\}20$ depicted by the right-hand grid-diagram below, the string-colours (A, B, C) are indicated in the left-hand grid-diagram of its associated Standard Herringbone Pineapple Knot.



First we braid the component $(3/11/3)\{1_1 2_2 2_3/2_1 2_2 1_3\}12$, a 3-pass Standard Herringbone Grant Knot starting with half-cycle 1 and colour A, then we braid the two remaining components (Regular Knots) as if the knot is a Standard Herringbone Pineapple Knot. The first first-return string-run and the half-cycle pattern of the first to be braided component are shown in Fig. 823. From this half-cycle pattern we assemble the half-cycle table for the odd-numbered half-cycles (from lower left to upper right) and the half-cycle table for the even-numbered half-cycles (from lower right to upper left).† See Fig. 824.

† Refer to *The Braider*, Issue No. 28.

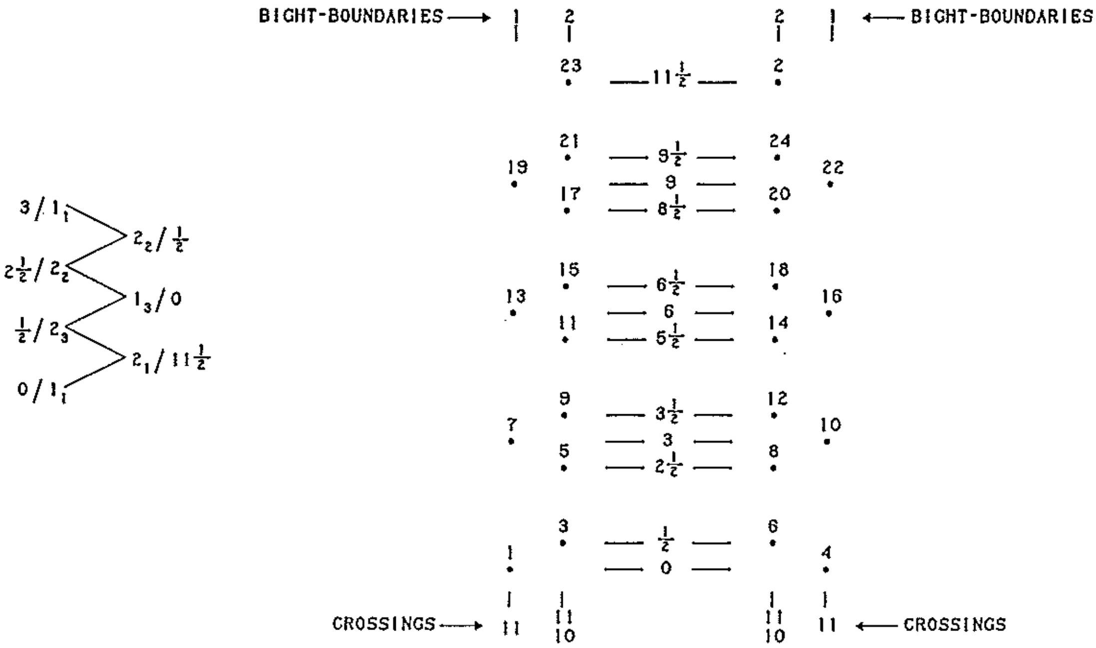


Fig. 823 — First-return string-run and half-cycle pattern.

L → R

3	7	5	9	13	11	15	19	17	21	X								
X	23	3	7	5	9	13	11	15	19	17								
21	X	23	3	7	5	9	13	11	15	19								
19	17	21	X	23	3	7	5	9	13	11								
15	19	17	21	X	23	3	7	5	9	13								
13	11	15	19	17	21	X	23	3	7	5								
9	13	11	15	19	17	21	X	23	3	7								
7	5	9	13	11	15	19	17	21	X	23								
u	o	o	o	u	u	u	o	o	o	u	1		7		13		19	
u	u	o	o	o	u	u	u	o	o	o	3		9		15		21	
o	u	u	u	o	o	o	u	u	u			5		11		17		23

R → L

6	10	8	12	16	14	18	22	20	24	4								
4	2	6	10	8	12	16	14	18	22	20								
24	4	2	6	10	8	12	16	14	18	22								
22	20	24	4	2	6	10	8	12	16	14								
18	22	20	24	4	2	6	10	8	12	16								
16	14	18	22	20	24	4	2	6	10	8								
12	16	14	18	22	20	24	4	2	6	10								
10	8	12	16	14	18	22	20	24	4	2								
u	o	o	o	u	u	u	o	o	o	u	4		10		16		22	
u	u	o	o	o	u	u	u	o	o	o	6		12		18		24	
o	u	u	u	o	o	o	u	u	u			8		14		20		2

Fig. 824 — The half-cycle tables.

From these half-cycle tables we read the following half-cycle braiding algorithms:

half-cycle 1: $L_1 \to R_2$: Free run.
half-cycle 2: $R_2 \to L_2$: Free run.
half-cycle 3: $L_2 \to R_1$: Free run.
half-cycle 4: $R_1 \to L_2$: $o - u$.
half-cycle 5: $L_2 \to R_2$: u.
half-cycle 6: $R_2 \to L_1$: $2o$.
half-cycle 7: $L_1 \to R_2$: $2o - u$.
half-cycle 8: $R_2 \to L_2$: $2u$.
half-cycle 9: $L_2 \to R_1$: $3o$.
half-cycle 10: $R_1 \to L_2$: $u - 3o - u$.
half-cycle 11: $L_2 \to R_2$: $o - 3u$.
half-cycle 12: $R_2 \to L_1$: $2u - 3o$.
half-cycle 13: $L_1 \to R_2$: $2u - 3o - u$.
half-cycle 14: $R_2 \to L_2$: $3o - 3u$.

half-cycle 15:	$L_2 \to R_1$:	$3u - 3o$.
half-cycle 16:	$R_1 \to L_2$:	$o - 3u - 3o - u$.
half-cycle 17:	$L_2 \to R_2$:	$u - 3o - 3u$.
half-cycle 18:	$R_2 \to L_1$:	$2o - 3u - 3o$.
half-cycle 19:	$L_1 \to R_2$:	$2o - 3u - 3o - u$.
half-cycle 20:	$R_2 \to L_2$:	$3u - 3o - 3u$.
half-cycle 21:	$L_2 \to R_1$:	$3o - 3u - 3o$.
half-cycle 22:	$R_1 \to L_2$:	$u - 3o - 3u - 3o - u$.
half-cycle 23:	$L_2 \to R_2$:	$o - 3u - 3o - 3u$.
half-cycle 24:	$R_2 \to L_1$:	$2u - 3o - 3u - 3o$.

The remaining components of the Standard Herringbone Grant Knot are braided as if this Grant Knot is a Standard Herringbone Pineapple Knot, and hence for the remaining two components we can use the algorithm diagrams of the equivalent components in its associated Standard Herringbone Pineapple Knot. Left bight-boundary 2 of the Grant Knot is equivalent to left bight-boundary 2 of the Pineapple Knot, and left bight-boundary 3 of the Grant Knot is equivalent to left bight-boundary 3 of the Pineapple Knot. Right bight-boundary 2 of the Grant Knot is equivalent to right bight-boundary 2 of the Pineapple Knot, and right bight-boundary 3 of the Grant Knot is equivalent to right bight-boundary 3 of the Pineapple Knot.

The algorithm diagram for two components with $p/b = 5/4$, and hence $\Delta^* = 3$, is thus:

$\longrightarrow$	$l_i - 1$	$A - 1$	$A - 1$	$A - 1$	$r_i - 1$
	3	2	1	0	
/	\	/	\	/	\
	0	1	2	3	
$l_i - 1$	$A - 1$	$A - 1$	$A - 1$	$r_i - 1$	$\longleftarrow$

The half-cycle braiding algorithms for the component with colour B between left bight-boundary 2 and right bight-boundary 3 in the final Standard Herringbone Grant Knot are:

half-cycle $1'$:		$L_2 \to R_2$:	$u - 3o - 3u - 3o - u$.
half-cycle $2'$:	$i = 0$:	$R_2 \to L_2$:	$u - 3o - 3u - 4o - u$.
half-cycle $3'$:	$i = 0$:	$L_2 \to R_2$:	$u - 3o - 3u - 4o - u$.
half-cycle $4'$:	$i \leq 1$:	$R_2 \to L_2$:	$u - 3o - 4u - 4o - u$.
half-cycle $5'$:	$i \leq 1$:	$L_2 \to R_2$:	$u - 3o - 4u - 4o - u$.
half-cycle $6'$:	$i \leq 2$:	$R_2 \to L_2$:	$u - 4o - 4u - 4o - u$.
half-cycle $7'$:	$i \leq 2$:	$L_2 \to R_2$:	$u - 4o - 4u - 4o - u$.
half-cycle $8'$:	$i \leq 3$:	$R_2 \to L_2$:	$2u - 4o - 4u - 4o - u$.

Note that for this component $l_i = 2$ and $r_i = 2$. Its string-run is between the bight-boundaries L_2 and R_2. When completed $A = 4$.

The half-cycle braiding algorithms for the component with colour C between left bight-boundary 3 and right bight-boundary 2 in the final Standard Herringbone Grant Knot are:

half-cycle $1''$:		$L_3 \to R_2$:	$2u - 4o - 4u - 4o - u$.
half-cycle $2''$:	$i = 0$:	$R_2 \to L_3$:	$u - 4o - 4u - 5o - 2u$.
half-cycle $3''$:	$i = 0$:	$L_3 \to R_2$:	$2u - 4o - 4u - 5o - u$.
half-cycle $4''$:	$i \leq 1$:	$R_2 \to L_3$:	$u - 4o - 5u - 5o - 2u$.
half-cycle $5''$:	$i \leq 1$:	$L_3 \to R_2$:	$2u - 4o - 5u - 5o - u$.
half-cycle $6''$:	$i \leq 2$:	$R_2 \to L_3$:	$u - 5o - 5u - 5o - 2u$.

half-cycle 7″: $i \leq 2$: $L_3 \to R_2$: $2u - 5o - 5u - 5o - u$.
half-cycle 8″: $i \leq 3$: $R_2 \to L_3$: $2u - 5o - 5u - 5o - 2u$.

Note that for this component $l_i = 3$ and $r_i = 2$. Its string-run is between the bight-boundaries L_3 and R_2. When completed $A = 5$.

The Single Bugler's Braid

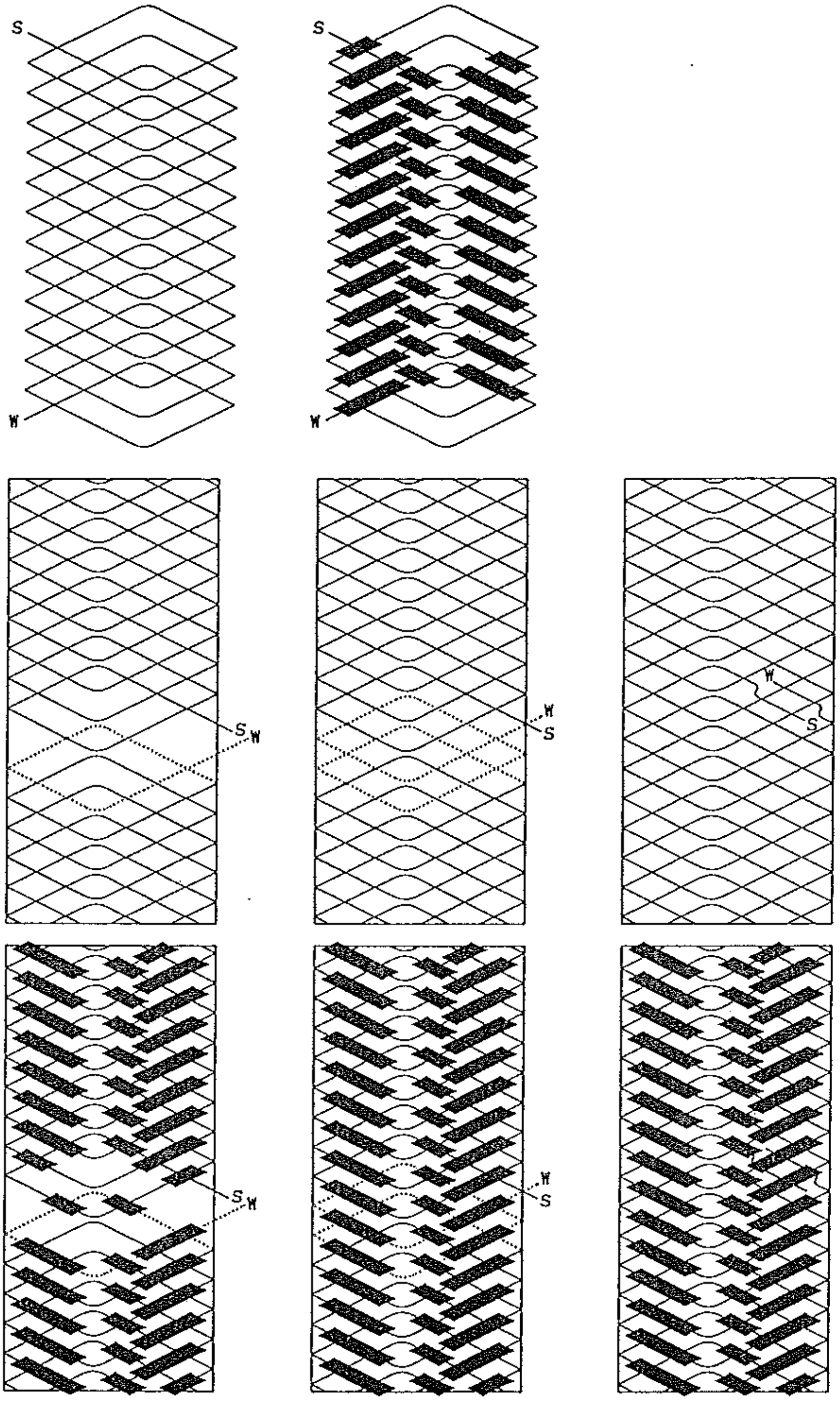


Fig. 825 — The Single Bugler's Braid.