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A quarterly publication
for
the braiding artisan

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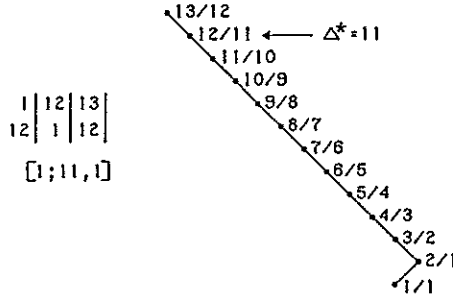
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Solutions to the Questions in Issue No. 40

Question on pg. 931.

For both Flores Knots in Fig. 731 $p/b = 13/12$, and hence both have the same path in the RKT.



For the Flores Knot depicted by the left-hand grid-diagram in Fig. 731 the algorithm diagram is as follows :

i	→	11	10	9	8	7	6	5	4	3	2	1	0	
$h=4n+3$	→	u	o	u	o	o	u	u	o	o	u	o	u	
$h=4n+1; h \neq 1$	→	o	o	o	u	u	o	o	u	u	o	o	o	
	┌	└
		u	u	u	u	o	o	u	u	o	o	u	o	← $h=4n+2$
		o	u	o	o	u	u	o	o	u	u	u	u	← $h=4n+4$
		0	1	2	3	4	5	6	7	8	9	10	11	← i

In the above algorithm diagram $0 \leq n \leq 5$. Note that the first half-cycle is always a free run.

From this algorithm diagram we read the following half-cycle braiding algorithms:

- half-cycle 1 : $L \rightarrow R$ Free Run.
- half-cycle 2 $i = 0$: $L \leftarrow R$ u .
- half-cycle 3 $i = 0$: $L \rightarrow R$ u .
- half-cycle 4 $i \leq 1$: $L \leftarrow R$ $u - o$.
- half-cycle 5 $i \leq 1$: $L \rightarrow R$ $2o$.
- half-cycle 6 $i \leq 2$: $L \leftarrow R$ $3u$.
- half-cycle 7 $i \leq 2$: $L \rightarrow R$ $u - o - u$.
- half-cycle 8 $i \leq 3$: $L \leftarrow R$ $2o - u - o$.
- half-cycle 9 $i \leq 3$: $L \rightarrow R$ $u - 3o$.
- half-cycle 10 $i \leq 4$: $L \leftarrow R$ $o - 4u$.
- half-cycle 11 $i \leq 4$: $L \rightarrow R$ $2o - u - o - u$.
- half-cycle 12 $i \leq 5$: $L \leftarrow R$ $2u - 2o - u - o$.
- half-cycle 13 $i \leq 5$: $L \rightarrow R$ $o - 2u - 3o$.
- half-cycle 14 $i \leq 6$: $L \leftarrow R$ $u - 2o - 4u$.
- half-cycle 15 $i \leq 6$: $L \rightarrow R$ $2u - 2o - u - o - u$.
- half-cycle 16 $i \leq 7$: $L \leftarrow R$ $2o - 2u - 2o - u - o$.
- half-cycle 17 $i \leq 7$: $L \rightarrow R$ $u - 2o - 2u - 3o$.
- half-cycle 18 $i \leq 8$: $L \leftarrow R$ $o - 2u - 2o - 4u$.
- half-cycle 19 $i \leq 8$: $L \rightarrow R$ $2o - 2u - 2o - u - o - u$.
- half-cycle 20 $i \leq 9$: $L \leftarrow R$ $2u - 2o - 2u - 2o - u - o$.
- half-cycle 21 $i \leq 9$: $L \rightarrow R$ $o - 2u - 2o - 2u - 3o$.

- half-cycle 22 $i \leq 10$: $L \leftarrow R$ $u - 2o - 2u - 2o - 4u$.
- half-cycle 23 $i \leq 10$: $L \rightarrow R$ $o - u - 2o - 2u - 2o - u - o - u$.
- half-cycle 24 $i \leq 11$: $L \leftarrow R$ $4u - 2o - 2u - 2o - u - o$.

For the Flores Knot depicted by the right-hand grid-diagram in Fig. 731 the algorithm diagram is as follows:

i	\rightarrow	11	10	9	8	7	6	5	4	3	2	1	0	
$h=4n+3$	\rightarrow	u	u	u	o	o	u	u	o	o	u	u	u	
$h=4n+1; h \neq 1$	\rightarrow	o	o	o	u	u	o	o	u	u	o	o	o	
	\vdash	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\dashv
		u	o	u	u	o	o	u	u	o	o	u	o	$\leftarrow h=4n+2$
		o	u	o	o	u	u	o	o	u	u	o	u	$\leftarrow h=4n+4$
		0	1	2	3	4	5	6	7	8	9	10	11	$\leftarrow i$

In the above algorithm diagram $0 \leq n \leq 5$. Note that the first half-cycle is always a free run.

From this algorithm diagram we read the following half-cycle braiding algorithms:

- half-cycle 1 : $L \rightarrow R$ Free Run.
- half-cycle 2 $i = 0$: $L \leftarrow R$ u .
- half-cycle 3 $i = 0$: $L \rightarrow R$ u .
- half-cycle 4 $i \leq 1$: $L \leftarrow R$ $u - o$.
- half-cycle 5 $i \leq 1$: $L \rightarrow R$ $2o$.
- half-cycle 6 $i \leq 2$: $L \leftarrow R$ $u - o - u$.
- half-cycle 7 $i \leq 2$: $L \rightarrow R$ $3u$.
- half-cycle 8 $i \leq 3$: $L \leftarrow R$ $2o - u - o$.
- half-cycle 9 $i \leq 3$: $L \rightarrow R$ $u - 3o$.
- half-cycle 10 $i \leq 4$: $L \leftarrow R$ $o - 2u - o - u$.
- half-cycle 11 $i \leq 4$: $L \rightarrow R$ $2o - 3u$.
- half-cycle 12 $i \leq 5$: $L \leftarrow R$ $2u - 2o - u - o$.
- half-cycle 13 $i \leq 5$: $L \rightarrow R$ $o - 2u - 3o$.
- half-cycle 14 $i \leq 6$: $L \leftarrow R$ $u - 2o - 2u - o - u$.
- half-cycle 15 $i \leq 6$: $L \rightarrow R$ $2u - 2o - 3u$.
- half-cycle 16 $i \leq 7$: $L \leftarrow R$ $2o - 2u - 2o - u - o$.
- half-cycle 17 $i \leq 7$: $L \rightarrow R$ $u - 2o - 2u - 3o$.
- half-cycle 18 $i \leq 8$: $L \leftarrow R$ $o - 2u - 2o - 2u - o - u$.
- half-cycle 19 $i \leq 8$: $L \rightarrow R$ $2o - 2u - 2o - 3u$.
- half-cycle 20 $i \leq 9$: $L \leftarrow R$ $2u - 2o - 2u - 2o - u - o$.
- half-cycle 21 $i \leq 9$: $L \rightarrow R$ $o - 2u - 2o - 2u - 3o$.
- half-cycle 22 $i \leq 10$: $L \leftarrow R$ $u - 2o - 2u - 2o - 2u - o - u$.
- half-cycle 23 $i \leq 10$: $L \rightarrow R$ $2u - 2o - 2u - 2o - 3u$.
- half-cycle 24 $i \leq 11$: $L \leftarrow R$ $4u - 2o - 2u - 2o - u - o$.

First question on pg. 933.

For both Flores Knots in Fig. 733 $p/b = 13/12$, and hence both have the same path in the RKT as the two Flores Knots in Fig. 731.

For the Flores Knot depicted by the left-hand grid-diagram in Fig. 733 the algorithm diagram is as follows:

i	\rightarrow	11	10	9	8	7	6	5	4	3	2	1	0	
$h=4n+3$	\rightarrow	u	u	u	u	o	o	u	u	o	o	u	o	
$h=4n+1; h \neq 1$	\rightarrow	o	u	o	o	u	u	o	o	u	u	u	u	
	\vdash	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\dashv
		u	o	u	o	o	u	u	o	o	u	o	u	\leftarrow $h=4n+2$
		o	o	o	u	u	o	o	u	u	o	o	o	\leftarrow $h=4n+4$
		0	1	2	3	4	5	6	7	8	9	10	11	\leftarrow i

In the above algorithm diagram $0 \leq n \leq 5$. Note that the first half-cycle is always a free run.

From this algorithm diagram we read the following half-cycle braiding algorithms:

half-cycle 1		:	$L \rightarrow R$	Free Run.
half-cycle 2	$i = 0$:	$L \leftarrow R$	u .
half-cycle 3	$i = 0$:	$L \rightarrow R$	o .
half-cycle 4	$i \leq 1$:	$L \leftarrow R$	$2o$.
half-cycle 5	$i \leq 1$:	$L \rightarrow R$	$2u$.
half-cycle 6	$i \leq 2$:	$L \leftarrow R$	$u - o - u$.
half-cycle 7	$i \leq 2$:	$L \rightarrow R$	$o - u - o$.
half-cycle 8	$i \leq 3$:	$L \leftarrow R$	$u - 3o$.
half-cycle 9	$i \leq 3$:	$L \rightarrow R$	$4u$.
half-cycle 10	$i \leq 4$:	$L \leftarrow R$	$2o - u - o - u$.
half-cycle 11	$i \leq 4$:	$L \rightarrow R$	$u - 2o - u - o$.
half-cycle 12	$i \leq 5$:	$L \leftarrow R$	$o - 2u - 3o$.
half-cycle 13	$i \leq 5$:	$L \rightarrow R$	$2o - 4u$.
half-cycle 14	$i \leq 6$:	$L \leftarrow R$	$2u - 2o - u - o - u$.
half-cycle 15	$i \leq 6$:	$L \rightarrow R$	$o - 2u - 2o - u - o$.
half-cycle 16	$i \leq 7$:	$L \leftarrow R$	$u - 2o - 2u - 3o$.
half-cycle 17	$i \leq 7$:	$L \rightarrow R$	$2u - 2o - 4u$.
half-cycle 18	$i \leq 8$:	$L \leftarrow R$	$2o - 2u - 2o - u - o - u$.
half-cycle 19	$i \leq 8$:	$L \rightarrow R$	$u - 2o - 2u - 2o - u - o$.
half-cycle 20	$i \leq 9$:	$L \leftarrow R$	$o - 2u - 2o - 2u - 3o$.
half-cycle 21	$i \leq 9$:	$L \rightarrow R$	$2o - 2u - 2o - 4u$.
half-cycle 22	$i \leq 10$:	$L \leftarrow R$	$o - u - 2o - 2u - 2o - u - o - u$.
half-cycle 23	$i \leq 10$:	$L \rightarrow R$	$3u - 2o - 2u - 2o - u - o$.
half-cycle 24	$i \leq 11$:	$L \leftarrow R$	$3o - 2u - 2o - 2u - 3o$.

For the Flores Knot depicted by the right-hand grid-diagram in Fig. 733 the algorithm diagram is as follows:

i	\rightarrow	11	10	9	8	7	6	5	4	3	2	1	0	
$h=4n+3$	\rightarrow	u	o	u	u	o	o	u	u	o	o	u	o	
$h=4n+1; h \neq 1$	\rightarrow	o	u	o	o	u	u	o	o	u	u	o	u	
	\vdash	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\dashv
		u	u	u	o	o	u	u	o	o	u	u	u	\leftarrow $h=4n+2$
		o	o	o	u	u	o	o	u	u	o	o	o	\leftarrow $h=4n+4$
		0	1	2	3	4	5	6	7	8	9	10	11	\leftarrow i

In the above algorithm diagram $0 \leq n \leq 5$. Note that the first half-cycle is always a free run.

From this algorithm diagram we read the following half-cycle braiding algorithms:

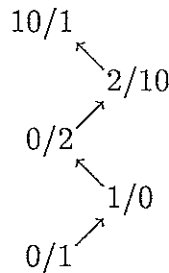
half-cycle 1	:	$L \longrightarrow R$	Free Run.
half-cycle 2	$i = 0$:	$L \longleftarrow R$	u .
half-cycle 3	$i = 0$:	$L \longrightarrow R$	o .
half-cycle 4	$i \leq 1$:	$L \longleftarrow R$	$2o$.
half-cycle 5	$i \leq 1$:	$L \longrightarrow R$	$o - u$.
half-cycle 6	$i \leq 2$:	$L \longleftarrow R$	$3u$.
half-cycle 7	$i \leq 2$:	$L \longrightarrow R$	$o - u - o$.
half-cycle 8	$i \leq 3$:	$L \longleftarrow R$	$u - 3o$.
half-cycle 9	$i \leq 3$:	$L \longrightarrow R$	$2u - o - u$.
half-cycle 10	$i \leq 4$:	$L \longleftarrow R$	$2o - 3u$.
half-cycle 11	$i \leq 4$:	$L \longrightarrow R$	$u - 2o - u - o$.
half-cycle 12	$i \leq 5$:	$L \longleftarrow R$	$o - 2u - 3o$.
half-cycle 13	$i \leq 5$:	$L \longrightarrow R$	$2o - 2u - o - u$.
half-cycle 14	$i \leq 6$:	$L \longleftarrow R$	$2u - 2o - 3u$.
half-cycle 15	$i \leq 6$:	$L \longrightarrow R$	$o - 2u - 2o - u - o$.
half-cycle 16	$i \leq 7$:	$L \longleftarrow R$	$u - 2o - 2u - 3o$.
half-cycle 17	$i \leq 7$:	$L \longrightarrow R$	$2u - 2o - 2u - o - u$.
half-cycle 18	$i \leq 8$:	$L \longleftarrow R$	$2o - 2u - 2o - 3u$.
half-cycle 19	$i \leq 8$:	$L \longrightarrow R$	$u - 2o - 2u - 2o - u - o$.
half-cycle 20	$i \leq 9$:	$L \longleftarrow R$	$o - 2u - 2o - 2u - 3o$.
half-cycle 21	$i \leq 9$:	$L \longrightarrow R$	$2o - 2u - 2o - 2u - o - u$.
half-cycle 22	$i \leq 10$:	$L \longleftarrow R$	$2u - 2o - 2u - 2o - 3u$.
half-cycle 23	$i \leq 10$:	$L \longrightarrow R$	$o - 2u - 2o - 2u - 2o - u - o$.
half-cycle 24	$i \leq 11$:	$L \longleftarrow R$	$3o - 2u - 2o - 2u - 3o$.

Second question on pg. 933.

For the Perfect Herringbone Pineapple Knot in Fig. 730:

$A = 2; B^* = 6; x_P = 9; y_P = A - 1 = 2 - 1 = 1; k = \left\lfloor \frac{x_P - A - 1}{2} \right\rfloor_A = \left\lfloor \frac{9 - 2 - 1}{2} \right\rfloor_2 = 1$,
 hence $1 \longrightarrow 1$ is the first lower-left to upper-right half-cycle; $\Delta = |y|_A = |1|_2 = 1$;
 $P_P = 2A + x_P - 2 = 4 + 9 - 2 = 11$; $\text{g.c.d.}(P_P, B^*) = \text{g.c.d.}(11, 6) = 1$.

Its first-return string-run is thus as follows:



For calculating the nest-index numbers we use the formulae:[†]

$$I_{L_1} = 0 \quad ; \quad I_{L_{n+1}} = \left| I_{L_n} + 4A + x_P - (l_n + l_{n+1} + 2r_n) \right|_B.$$

$$I_{R_1} = 0 \quad ; \quad I_{R_{n+1}} = \left| I_{R_n} + 4A + x_P - (r_n + r_{n+1} + 2l_{n+1}) \right|_B.$$

The half-cycle pattern arrangement of this Perfect Herringbone Pineapple Knot is then as depicted in Fig. 746.

[†] See *The Braider*, Issue No. 26, pg. 593.

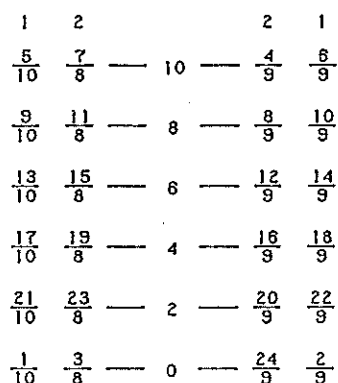


Fig. 746 — The half-cycle pattern of the Perfect Herringbone Pineapple Knot.

From this half-cycle pattern in Fig. 746 we derive the half-cycle table for the lower-left to upper-right half-cycles (the left table in Fig. 747) and the half-cycle table for the lower-right to upper-left half-cycles (the right table in Fig. 747).

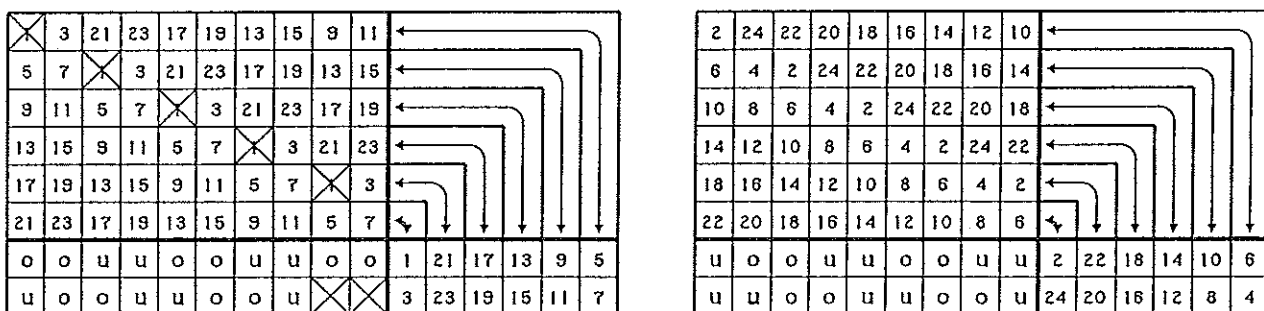


Fig. 747 — The half-cycle tables for the Perfect Herringbone Pineapple Knot.

From the half-cycle tables in Fig. 747 we read the half-cycle braiding algorithms for the Perfect Herringbone Pineapple Knot :

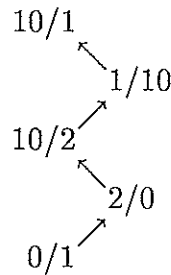
- half-cycle 1 : 1 → 1 Free Run.
- half-cycle 2 : 2 ← 1 Free Run.
- half-cycle 3 : 2 → 2 Free Run.
- half-cycle 4 : 1 ← 2 u .
- half-cycle 5 : 1 → 1 o .
- half-cycle 6 : 2 ← 1 u .
- half-cycle 7 : 2 → 2 o .
- half-cycle 8 : 1 ← 2 2u - o .
- half-cycle 9 : 1 → 1 2o - u .
- half-cycle 10 : 2 ← 1 u - 2o .
- half-cycle 11 : 2 → 2 u - o - u .
- half-cycle 12 : 1 ← 2 2u - 2o - u .
- half-cycle 13 : 1 → 1 2o - 2u - o .
- half-cycle 14 : 2 ← 1 u - 2o - 2u .
- half-cycle 15 : 2 → 2 u - 2o - u - o .
- half-cycle 16 : 1 ← 2 2u - 2o - 2u - o .
- half-cycle 17 : 1 → 1 2o - 2u - 2o - u .
- half-cycle 18 : 2 ← 1 u - 2o - 2u - 2o .
- half-cycle 19 : 2 → 2 u - 2o - 2u - o - u .
- half-cycle 20 : 1 ← 2 2u - 2o - 2u - 2o - u .

- half-cycle 21 : 1 → 1 2o - 2u - 2o - 2u - o .
- half-cycle 22 : 2 ← 1 u - 2o - 2u - 2o - 2u .
- half-cycle 23 : 2 → 2 u - 2o - 2u - 2o - u .
- half-cycle 24 : 1 ← 2 2u - 2o - 2u - 2o - u .

For the Perfect Herringbone Pineapple Knot in Fig. 732 :

$A = 2 ; B^* = 6 ; x_P = 9 ; y_P = A + 1 = 2 + 1 = 3 ; k = \left| \frac{x_P - A - 3}{2} \right|_A = \left| \frac{9 - 2 - 3}{2} \right|_2 = 2$,
 hence $1 \rightarrow 2$ is the first lower-left to upper-right half-cycle ; $\Delta = |y|_A = |3|_2 = 1 ;$
 $P_P = 2A + x_P - 2 = 4 + 9 - 2 = 11 ; \text{g.c.d.}(P_P, B^*) = \text{g.c.d.}(11, 6) = 1 .$

Its first-return string-run is thus as follows :



For calculating the nest-index numbers we use the formulae :

$$I_{L_1} = 0 \quad ; \quad I_{L_{n+1}} = \left| I_{L_n} + 4A + x_P - (l_n + l_{n+1} + 2r_n) \right|_B .$$

$$I_{R_1} = 0 \quad ; \quad I_{R_{n+1}} = \left| I_{R_n} + 4A + x_P - (r_n + r_{n+1} + 2l_{n+1}) \right|_B .$$

The half-cycle pattern arrangement of this Perfect Herringbone Pineapple Knot is then as depicted in Fig. 748.

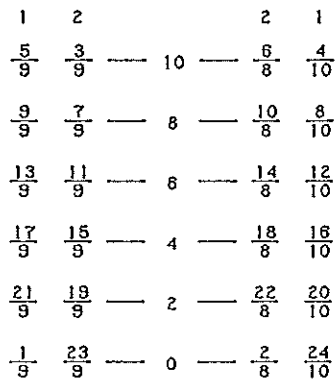


Fig. 748 — The half-cycle pattern of the Perfect Herringbone Pineapple Knot.

From this half-cycle pattern in Fig. 748 we derive the half-cycle table for the lower-left to upper-right half-cycles (the left table in Fig. 749) and the half-cycle table for the lower-right to upper-left half-cycles (the right table in Fig. 749).

×	23	21	19	17	15	13	11	9	←
5	3	×	23	21	19	17	15	13	←
9	7	5	3	×	23	21	19	17	←
13	11	9	7	5	3	×	23	21	←
17	15	13	11	9	7	5	3	×	←
21	19	17	15	13	11	9	7	5	←
u	o	o	u	u	o	o	u	u	1 21 17 13 9 5
u	u	o	o	u	u	o	o	u	23 19 15 11 7 3

4	6	24	2	20	22	16	18	12	14	←
8	10	4	6	24	2	20	22	16	18	←
12	14	8	10	4	6	24	2	20	22	←
16	18	12	14	8	10	4	6	24	2	←
20	22	16	18	12	14	8	10	4	6	←
24	2	20	22	16	18	12	14	8	10	←
o	o	u	u	o	o	u	u	o	o	4 24 20 16 12 8
u	o	o	u	u	o	o	u	×	×	6 2 22 18 14 10

Fig. 749 — The half-cycle tables for the Perfect Herringbone Pineapple Knot.

Note that these half-cycle tables can be obtained directly from the half-cycle tables in Fig. 747 (See *The Braider*, Issue No. 28, pg. 656).

From the half-cycle tables in Fig. 749 we read the half-cycle braiding algorithms for the Perfect Herringbone Pineapple Knot:

half-cycle 1	:	$1 \longrightarrow 2$	Free Run.
half-cycle 2	:	$2 \longleftarrow 2$	Free Run.
half-cycle 3	:	$2 \longrightarrow 1$	Free Run.
half-cycle 4	:	$1 \longleftarrow 1$	o .
half-cycle 5	:	$1 \longrightarrow 2$	Free Run.
half-cycle 6	:	$2 \longleftarrow 2$	o .
half-cycle 7	:	$2 \longrightarrow 1$	$2u$.
half-cycle 8	:	$1 \longleftarrow 1$	$2o - u$.
half-cycle 9	:	$1 \longrightarrow 2$	$u - o$.
half-cycle 10	:	$2 \longleftarrow 2$	$u - o - u$.
half-cycle 11	:	$2 \longrightarrow 1$	$2u - 2o$.
half-cycle 12	:	$1 \longleftarrow 1$	$2o - 2u - o$.
half-cycle 13	:	$1 \longrightarrow 2$	$u - 2o - u$.
half-cycle 14	:	$2 \longleftarrow 2$	$u - 2o - u - o$.
half-cycle 15	:	$2 \longrightarrow 1$	$2u - 2o - 2u$.
half-cycle 16	:	$1 \longleftarrow 1$	$2o - 2u - 2o - u$.
half-cycle 17	:	$1 \longrightarrow 2$	$u - 2o - 2u - o$.
half-cycle 18	:	$2 \longleftarrow 2$	$u - 2o - 2u - o - u$.
half-cycle 19	:	$2 \longrightarrow 1$	$2u - 2o - 2u - 2o$.
half-cycle 20	:	$1 \longleftarrow 1$	$2o - 2u - 2o - 2u - o$.
half-cycle 21	:	$1 \longrightarrow 2$	$u - 2o - 2u - 2o - u$.
half-cycle 22	:	$2 \longleftarrow 2$	$u - 2o - 2u - 2o - u$.
half-cycle 23	:	$2 \longrightarrow 1$	$2u - 2o - 2u - 2o - u$.
half-cycle 24	:	$1 \longleftarrow 1$	$2o - 2u - 2o - 2u - 2o$.

Question on pg. 954.

Since we prefer to employ the usual procedures for calculating the half-cycle pattern and the half-cycle tables as previously discussed, the braiding procedure to be followed should be such that these half-cycle tables can also be used for the derivation of the half-cycle braiding algorithms associated with starting at the centre of the required string-length. We can then employ one of the following two construction procedures:

1. First braid the table half-cycles $1 \leq h_t \leq B$, starting with table half-cycle $h_t = B$, in the reverse direction. Read for h_t the intersection codings from right to left for $h_t < h \leq (B + 1)$. Then braid the table half-cycles $(B + 1) \leq h_t \leq 2B$, starting with table half-cycle $h_t = (B + 1)$, in the normal direction. Read for h_t the intersection codings from left to right for $1 < h \leq h_t$.
2. First braid the table half-cycles $(B + 1) \leq h_t \leq 2B$, starting with table half-cycle $h_t = (B + 1)$, in the normal direction. Read for h_t the intersection codings from left to right for $(B + 1) < h \leq h_t$. Then braid the table half-cycles $1 \leq h_t \leq B$, starting with table half-cycle $h_t = B$, in the reverse direction. Read for h_t the intersection codings from right to left for $h = 1$ and $h_t < h \leq 2B$.

Construction procedure 1. is to be preferred when we already have for the normal braiding direction the half-cycle braiding algorithms for the half-cycles $1 \leq h_t \leq 2B$.

In order to show both construction procedures we shall follow for **Example 1.** construction procedure 1., and for **Example 2.** construction procedure 2..

For the Flores Knot in **Example 1.** we first braid the table half-cycles $1 \leq h_t \leq 25$, starting with $h_t = B = 25$, in the reverse direction. We read for h_t the intersection codings from right to left for $h_t < h \leq 26$. Then we braid the table half-cycles $26 \leq h_t \leq 50$, starting with table half-cycle $h_t = B + 1 = 26$, in the normal direction. We read for h_t the intersection codings from left to right for $1 < h \leq h_t$. This results in the following half-cycle braiding algorithms:

1. ($h_t = 25$ rev.) $5_4 \leftarrow 4_3$: Free run.
 2. ($h_t = 24$ rev.) $5_4 \rightarrow 2_2$: Free Run.
 3. ($h_t = 23$ rev.) $3_5 \leftarrow 2_2$: o .
 4. ($h_t = 22$ rev.) $3_5 \rightarrow 1_1$: $2o$.
 5. ($h_t = 21$ rev.) $1_1 \leftarrow 1_1$: $o - u$.
 6. ($h_t = 20$ rev.) $1_1 \rightarrow 3_5$: $2u - o$.
 7. ($h_t = 19$ rev.) $2_2 \leftarrow 3_5$: $u - o - u$.
 8. ($h_t = 18$ rev.) $2_2 \rightarrow 5_4$: $2o - 2u$.
 9. ($h_t = 17$ rev.) $4_3 \leftarrow 5_4$: $u - o - u$.
 10. ($h_t = 16$ rev.) $4_3 \rightarrow 4_3$: $2o - 2u$.
 11. ($h_t = 15$ rev.) $5_4 \leftarrow 4_3$: $u - o - u$.
 12. ($h_t = 14$ rev.) $5_4 \rightarrow 2_2$: $u - o - 2u$.
 13. ($h_t = 13$ rev.) $3_5 \leftarrow 2_2$: $6o$.
 14. ($h_t = 12$ rev.) $3_5 \rightarrow 1_1$: $3o - u - 3o$.
 15. ($h_t = 11$ rev.) $1_1 \leftarrow 1_1$: $2o - u - 3o - u$.
 16. ($h_t = 10$ rev.) $1_1 \rightarrow 3_5$: $6u - 3o$.
 17. ($h_t = 9$ rev.) $2_2 \leftarrow 3_5$: $o - 4u - 2o - 2u$.
 18. ($h_t = 8$ rev.) $2_2 \rightarrow 5_4$: $2u - 3o - 4u$.
 19. ($h_t = 7$ rev.) $4_3 \leftarrow 5_4$: $3u - 3o - 2u$.
 20. ($h_t = 6$ rev.) $4_3 \rightarrow 4_3$: $3u - 2o - u - o - 2u$.
 21. ($h_t = 5$ rev.) $5_4 \leftarrow 4_3$: $2u - 3o - 3u$.
 22. ($h_t = 4$ rev.) $5_4 \rightarrow 2_2$: $4u - o - u - 2o - u$.
 23. ($h_t = 3$ rev.) $3_5 \leftarrow 2_2$: $u - 10o$.
 24. ($h_t = 2$ rev.) $3_5 \rightarrow 1_1$: $8o - u - 4o$.
 25. ($h_t = 1$ rev.) $1_1 \leftarrow 1_1$: $u - 4o - 2u - 4o - 2u$.
-
26. ($h_t = 26$) $4_3 \leftarrow 4_3$: $3u - 3o - 2u - 2o - 2u$.
 27. ($h_t = 27$) $4_3 \rightarrow 5_4$: $2u - 3o - 6u$.
 28. ($h_t = 28$) $2_2 \leftarrow 5_4$: $4u - 2o - 2u - 3o - u$.
 29. ($h_t = 29$) $2_2 \rightarrow 3_5$: $u - 4o - u - 8o$.
 30. ($h_t = 30$) $1_1 \leftarrow 3_5$: $5o - u - 5o - u - 4o$.
 31. ($h_t = 31$) $1_1 \rightarrow 1_1$: $u - 5o - 4u - 4o - 2u$.
 32. ($h_t = 32$) $3_5 \leftarrow 1_1$: $u - o - u - 4o - 5u - o - u - 4o$.
 33. ($h_t = 33$) $3_5 \rightarrow 2_2$: $5o - 5u - o - u - 2o - 3u$.
 34. ($h_t = 34$) $5_4 \leftarrow 2_2$: $u - 3o - 2u - 4o - 2u - o - 4u$.
 35. ($h_t = 35$) $5_4 \rightarrow 4_3$: $8u - 2o - u - o - 4u$.
 36. ($h_t = 36$) $4_3 \leftarrow 4_3$: $2u - 2o - 3u - 3o - 2u - 2o - 3u$.
 37. ($h_t = 37$) $4_3 \rightarrow 5_4$: $3u - o - 3u - 3o - 6u$.
 38. ($h_t = 38$) $2_2 \leftarrow 5_4$: $3u - o - 4u - 2o - 2u - 3o - 2u$.
 39. ($h_t = 39$) $2_2 \rightarrow 3_5$: $2u - 2o - 2u - 4o - u - 9o$.

40. $(h_t = 40)$ $1_1 \leftarrow 3_5$: $4o - u - 4o - u - 5o - 2u - 4o - u$.
41. $(h_t = 41)$ $1_1 \rightarrow 1_1$: $2u - 5o - 4u - 5o - u - 3o - 2u$.
42. $(h_t = 42)$ $3_5 \leftarrow 1_1$: $u - o - u - 5o - 5u - 5o - u - 4o$.
43. $(h_t = 43)$ $3_5 \rightarrow 2_2$: $5o - 5u - 5o - 2u - 2o - 3u$.
44. $(h_t = 44)$ $5_4 \leftarrow 2_2$: $u - 3o - 3u - 5o - 5u - o - 4u$.
45. $(h_t = 45)$ $5_4 \rightarrow 4_3$: $8u - 5o - 3u - o - 4u$.
46. $(h_t = 46)$ $4_3 \leftarrow 4_3$: $2u - 2o - 4u - 5o - 4u - 2o - 3u$.
47. $(h_t = 47)$ $4_3 \rightarrow 5_4$: $3u - o - 3u - 5o - 9u$.
48. $(h_t = 48)$ $2_2 \leftarrow 5_4$: $3u - o - 5u - 5o - 3u - 3o - 2u$.
49. $(h_t = 49)$ $2_2 \rightarrow 3_5$: $2u - 2o - 2u - 5o - 5u - 9o$.
50. $(h_t = 50)$ $1_1 \leftarrow 3_5$: $4o - u - 5o - 5u - 5o - 2u - 4o - u$.

For the Flores Knot in **Example 2**, we first braid the table half-cycles $19 \leq h_t \leq 36$, starting with $h_t = B + 1 = 19$, in the normal direction. We read for h_t the intersection codings from left to right for $19 < h \leq h_t$. Then we braid the table half-cycles $1 \leq h_t \leq 18$, starting with table half-cycle $h_t = B = 18$, in the reverse direction. We read for h_t the intersection codings from right to left for $h = 1$ and $h_t < h \leq 2B$. This results in the following half-cycle braiding algorithms:

1. $(h_t = 19)$ $4_4 \rightarrow 2_4$: Free run.
 2. $(h_t = 20)$ $5_5 \leftarrow 2_4$: u .
 3. $(h_t = 21)$ $5_5 \rightarrow 1_3$: o .
 4. $(h_t = 22)$ $3_6 \leftarrow 1_3$: $o - u - o$.
 5. $(h_t = 23)$ $3_6 \rightarrow 3_2$: $o - u - o$.
 6. $(h_t = 24)$ $1_1 \leftarrow 3_2$: $5o$.
 7. $(h_t = 25)$ $1_1 \rightarrow 5_1$: $u - 2o - 2u$.
 8. $(h_t = 26)$ $2_2 \leftarrow 5_1$: $5u - o$.
 9. $(h_t = 27)$ $2_2 \rightarrow 4_6$: $2u - 2o - 2u$.
 10. $(h_t = 28)$ $3_3 \leftarrow 4_6$: $o - 2u - o - 3u$.
 11. $(h_t = 29)$ $3_3 \rightarrow 3_5$: $2u - 2o - u - o - u$.
 12. $(h_t = 30)$ $4_4 \leftarrow 3_5$: $u - 4o - u - o - u$.
 13. $(h_t = 31)$ $4_4 \rightarrow 2_4$: $2u - o - u - 2o - u - o - u$.
 14. $(h_t = 32)$ $5_5 \leftarrow 2_4$: $2u - 3o - 3u - o - u$.
 15. $(h_t = 33)$ $5_5 \rightarrow 1_3$: $5u - 2o - u - o - u$.
 16. $(h_t = 34)$ $3_6 \leftarrow 1_3$: $2o - u - 2o - 3u - 5o$.
 17. $(h_t = 35)$ $3_6 \rightarrow 3_2$: $5o - 3u - 2o - u - 2o$.
 18. $(h_t = 36)$ $1_1 \leftarrow 3_2$: $5o - 3u - 6o - u$.
-
19. $(h_t = 18 \text{ rev.})$ $4_4 \rightarrow 3_5$: $2u - o - 2u - 2o - 3u - o - 2u$.
 20. $(h_t = 17 \text{ rev.})$ $3_3 \leftarrow 3_5$: $2u - o - 3u - 3o - 2u - o - 3u$.
 21. $(h_t = 16 \text{ rev.})$ $3_3 \rightarrow 4_6$: $u - 2o - u - 4o - 2u - o - 3u$.
 22. $(h_t = 15 \text{ rev.})$ $2_2 \leftarrow 4_6$: $3u - o - 3u - 4o - u - 2o - 2u$.
 23. $(h_t = 14 \text{ rev.})$ $2_2 \rightarrow 5_1$: $2u - 6o - 4u - o - 2u$.
 24. $(h_t = 13 \text{ rev.})$ $1_1 \leftarrow 5_1$: $8u - 4o - 2u - 2o - u$.
 25. $(h_t = 12 \text{ rev.})$ $1_1 \rightarrow 3_2$: $u - 3o - u - 3o - 5u - 7o$.
 26. $(h_t = 11 \text{ rev.})$ $3_6 \leftarrow 3_2$: $4o - u - 4o - 4u - 5o - u - 3o$.
 27. $(h_t = 10 \text{ rev.})$ $3_6 \rightarrow 1_3$: $8o - 4u - 5o - u - 3o - u$.
 28. $(h_t = 9 \text{ rev.})$ $5_5 \leftarrow 1_3$: $2u - 3o - 2u - 4o - 10u$.
 29. $(h_t = 8 \text{ rev.})$ $5_5 \rightarrow 2_4$: $4u - o - 3u - 5o - 2u - 3o - 2u$.
 30. $(h_t = 7 \text{ rev.})$ $4_4 \leftarrow 2_4$: $3u - 2o - 3u - 5o - 4u - o - 4u$.

- 31. ($h_t = 6$ rev.) $4_4 \rightarrow 3_5$: $3u - 2o - 3u - 5o - 3u - 2o - 3u$.
- 32. ($h_t = 5$ rev.) $3_3 \leftarrow 3_5$: $4u - o - 4u - 5o - 4u - 2o - 3u$.
- 33. ($h_t = 4$ rev.) $3_3 \rightarrow 4_6$: $2u - 3o - 2u - 6o - 4u - o - 4u$.
- 34. ($h_t = 3$ rev.) $2_2 \leftarrow 4_6$: $5u - o - 5u - 5o - 3u - 3o - 3u$.
- 35. ($h_t = 2$ rev.) $2_2 \rightarrow 5_1$: $2u - 4o - u - 6o - 5u - o - 5u$.
- 36. ($h_t = 1$ rev.) $1_1 \leftarrow 5_1$: $12u - 6o - 2u - 4o - 2u$.

Transition from two Round Braids to one Round Braid, and vice versa

A transition between two 8-string round braids, each with a 4-string round braid core, and one 12-string round braid with an 8-string round braid outer core over a 4-string round braid inner core.

BRAIDING TYPE																																															
12 STRING MANTLE												8 STRING OUTER CORE								4 STRING INNER CORE				8 STRING MANTLE								4 STRING CORE															
U	O	U	O	U	O	O	U	O	U	O	U	U	O	U	O	O	U	O	U	U	O	O	U	U	O	O	U	U	O	U	O	O	U	O	U	U	O	O	U								

Figs. 750, 751, 752, 753 and 754:

The 8-string round braid mantle has the strings A, B, C, D, E, F, G, H at one end and the strings $1, 2, 3, 4, 5, 6, 7, 8$ at the other end. This 8-string round braid mantle goes over a 4-string round braid core, which has at one end the strings A^*, B^*, C^*, D^* and at the other end the strings $1^*, 2^*, 3^*, 4^*$. Of the 8-string round braid mantle, the strings $A, B, 1, 2$ go over into a 4-string round braid inner core. Of the 4-string round braid core, the strings A^*, B^*, C^*, D^* and $1^*, 2^*, 3^*, 4^*$ go over into an 8-string round braid outer core. Of the 8-string round braid mantle, the strings $C, D, E, F, G, H, 3, 4, 5, 6, 7, 8$ go over into a 12-string round braid mantle which goes over the 8-string round braid outer core. See Figs. 750 & 751.

The strings $A, B, 1, 2$ go over into the 4-string round braid inner core, the preparation of which is depicted in the upper part of Fig. 750; bring A around the back of the left-hand 8-string round braid mantle from left to right, and bring 2 around the back of the right-hand 8-string round braid mantle from right to left. Note that in the 4-string round braid core the flesh-side of the strings is outermost. The braiding of this core continues as follows:

Bring 2 from the left around the back to the right, then along the front from right to left under 1 , over A .

Bring 1 from the right around the back to the left, then along the front from left to right under B , over 2 .

Bring B from the left around the back to the right, then along the front from right to left under A , over 1 .

Bring A from the right around the back to the left, then along the front from left to right under 2 , over B .

And so on.

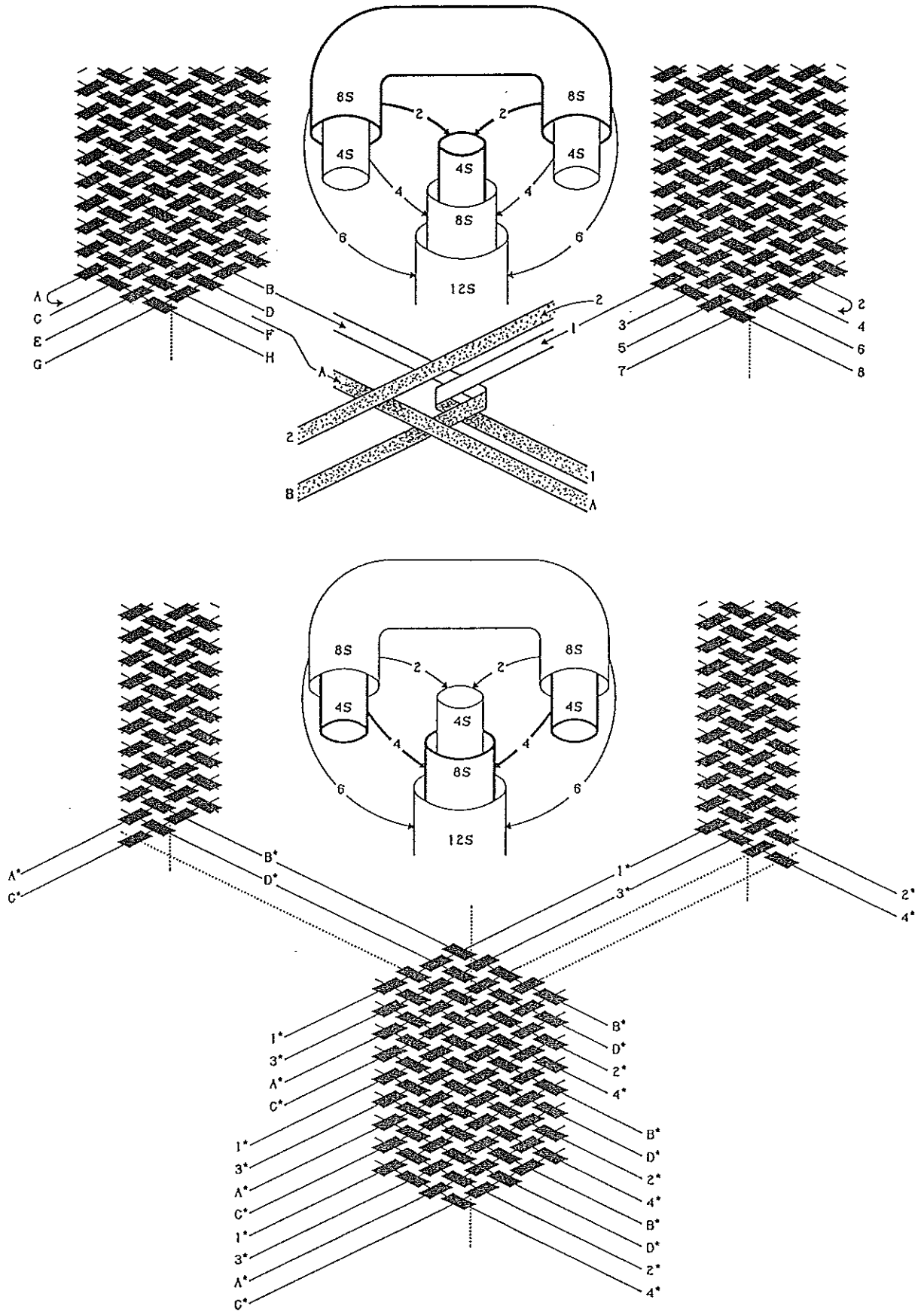


Fig. 750 — Transition from two 8-string over 4-string round braids to a 12-string over 8-string over 4-string round braid.

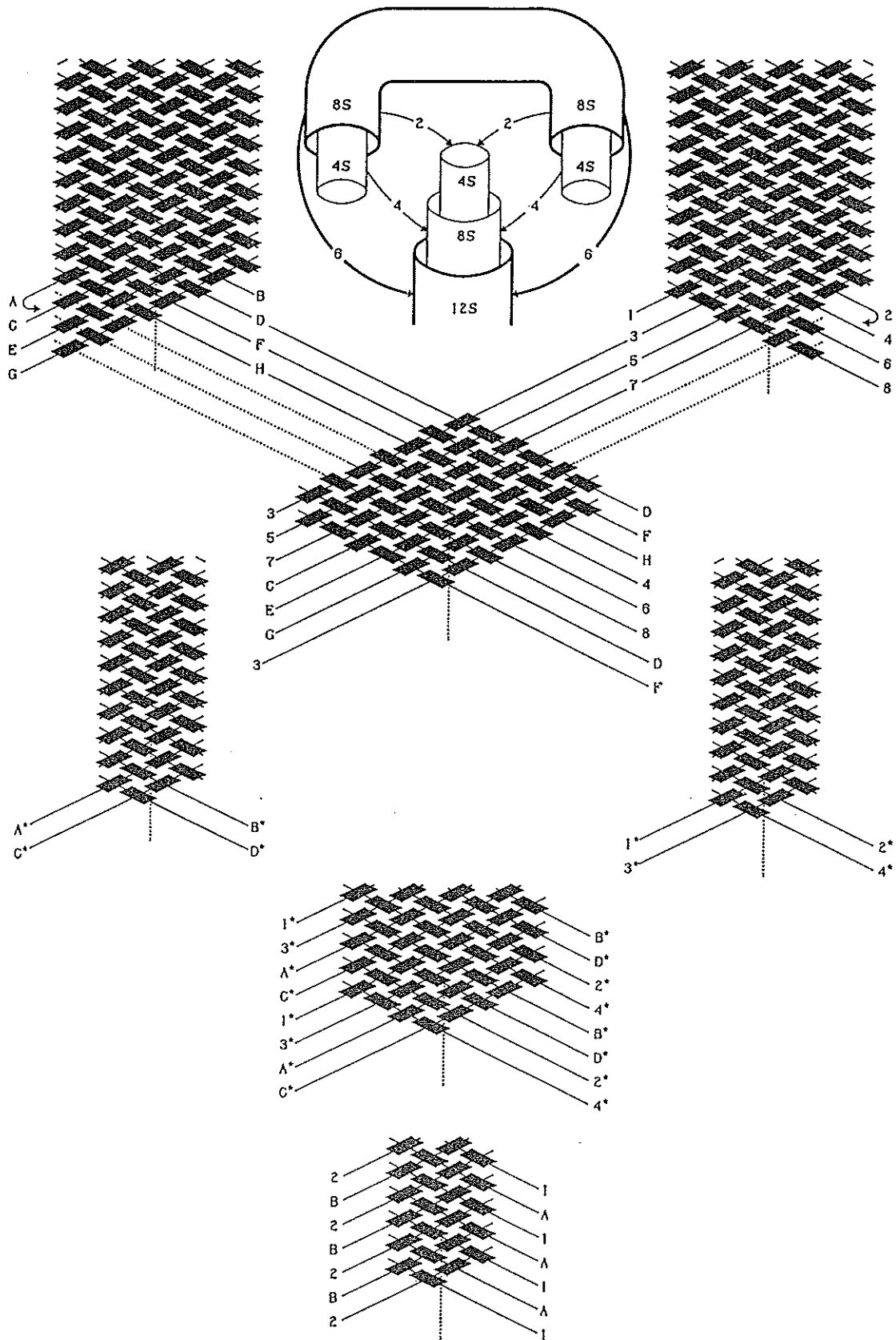


Fig. 751 — Transition from two 8-string over 4-string round braids to a 12-string over 8-string over 4-string round braid.

The strings A^* , B^* , C^* , D^* and 1^* , 2^* , 3^* , 4^* go over into an 8-string round braid outer core.

Form the crossings between the strings B^* , D^* , 1^* , 3^* as depicted in the lower part of Fig. 750.

Next bring A^* from the left around the back to the right, then along the front from right to left under 2^* , over 4^* , under B^* , over D^* .

Bring 2^* from the right around the back to the left, then along the front from left to right under C^* , over 1^* , under 3^* , over A^* .

Bring C^* from the left around the back to the right, then along the front from right to left under 4^* , over B^* , under D^* , over 2^* .

Bring 4^* from the right around the back to the left, then along the front from left to right under 1^* , over 3^* , under A^* , over C^* .

Bring 1^* from the left around the back to the right, then along the front from right to left under B^* , over D^* , under 2^* , over 4^* .

Bring B^* from the right around the back to the left, then along the front from left to right under 3^* , over A^* , under C^* , over 1^* .

Bring 3^* from the left around the back to the right, then along the front from right to left under D^* , over 2^* , under 4^* , over B^* .

Bring D^* from the right around the back to the left, then along the front from left to right under A^* , over C^* , under 1^* , over 3^* .

Bring A^* from the left around the back to the right, then along the front from right to left under 2^* , over 4^* , under B^* , over D^* .

Bring 2^* from the right around the back to the left, then along the front from left to right under C^* , over 1^* , under 3^* , over A^* .

Bring C^* from the left around the back to the right, then along the front from right to left under 4^* , over B^* , under D^* , over 2^* .

Bring 4^* from the right around the back to the left, then along the front from left to right under 1^* , over 3^* , under A^* , over C^* .

And so on.

The strings C , D , E , F , G , H , 3 , 4 , 5 , 6 , 7 , 8 go over into the 12-string round braid mantle.

Form the crossings between the strings D , F , H , 3 , 5 , 7 as depicted in the upper part of Fig. 751.

Next bring 4 from the right around the back to the left, then along the front from left to right under C , over E , under G , over 3 , under 5 , over 7 .

Bring C from the left around the back to the right, then along the front from right to left under 6 , over 8 , under D , over F , under H , over 4 .

Bring 6 from the right around the back to the left, then along the front from left to right under E , over G , under 3 , over 5 , under 7 , over C .

Bring E from the left around the back to the right, then along the front from right to left under 8 , over D , under F , over H , under 4 , over 6 .

Bring 8 from the right around the back to the left, then along the front from left to right under G , over 3 , under 5 , over 7 , under C , over E .

Bring G from the left around the back to the right, then along the front from right to left under D , over F , under H , over 4 , under 6 , over 8 .

Bring D from the right around the back to the left, then along the front from left to right under 3 , over 5 , under 7 , over C , under E , over G .

Bring 3 from the left around the back to the right, then along the front from right to left under *F*, over *H*, under 4, over 6, under 8, over *D*.

Bring *F* from the right around the back to the left, then along the front from left to right under 5, over 7, under *C*, over *E*, under *G*, over 3.

Bring 5 from the left around the back to the right, then along the front from right to left under *H*, over 4, under 6, over 8, under *D*, over *F*.

Bring *H* from the right around the back to the left, then along the front from left to right under 7, over *C*, under *E*, over *G*, under 3, over 5.

Bring 7 from the left around the back to the right, then along the front from right to left under 4, over 6, under 8, over *D*, under *F*, over *H*.

Bring 4 from the right around the back to the left, then along the front from left to right under *C*, over *E*, under *G*, over 3, under 5, over 7.

Bring *C* from the left around the back to the right, then along the front from right to left under 6, over 8, under *D*, over *F*, under *H*, over 4.

Bring 6 from the right around the back to the left, then along the front from left to right under *E*, over *G*, under 3, over 5, under 7, over *C*.

Bring *E* from the left around the back to the right, then along the front from right to left under 8, over *D*, under *F*, over *H*, under 4, over 6.

Bring 8 from the right around the back to the left, then along the front from left to right under *G*, over 3, under 5, over 7, under *C*, over *E*.

Bring *G* from the left around the back to the right, then along the front from right to left under *D*, over *F*, under *H*, over 4, under 6, over 8.

And so on.

The 12-string round braid mantle has the strings 1*, 2*, 3*, 4*, 5*, 6*, 7*, 8*, 9*, 10*, 11*, 12*. The preparation of the strings is shown in Fig. 753.

The 8-string round braid outer core has the strings *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*. The preparation of the strings is shown in the lower part of Fig. 752.

The 4-string round braid inner core, which has the flesh-side of its strings outermost, has the strings 1, 2, 3, 4. The preparation of the strings is shown in the upper part of Fig. 752:

Bring 1 around the back from right to left with the hair-side up on left. Place 1 between 2 and 4.

Bring 2 around the back from left to right with the hair-side up on right. Place 2 below 3.

Ensure that the crossings *Y* and *Z* are at the same level, and that the crossings *X* and *V* are at the same level.

Braid from the 8-string round braid outer core, hence with the strings *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, the two 4-string cores as shown in the lower part of Fig. 754. For the preparation stage, bring *A* from the left around the back to the right, then along the front from right to left under *B*, over *D*, under *F*, over *H*.

Bring *B* from the right around the back to the left, then along the front from left to right under *C*, over *E*, under *G*.

Bring *C* from the left around the back to the right, then along the front from right to left under *D*, over *F*, under *H*.

Bring *D* from the right around the back to the left, then along the front from left to right under *E*, over *G*.

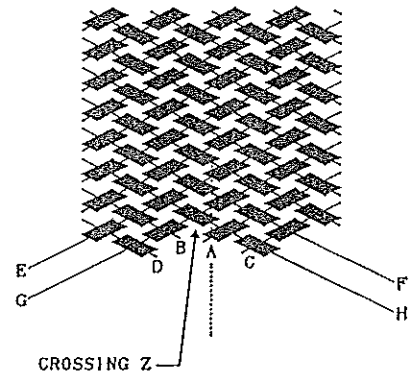
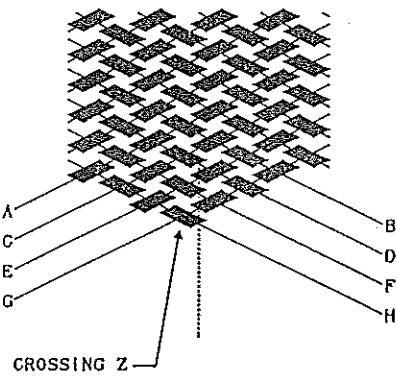
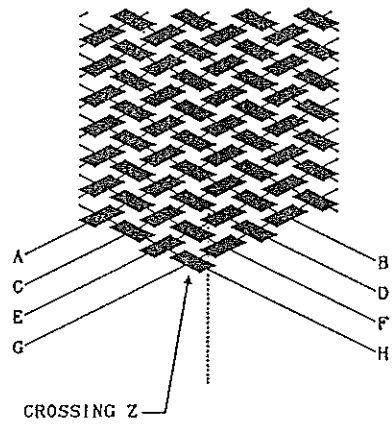
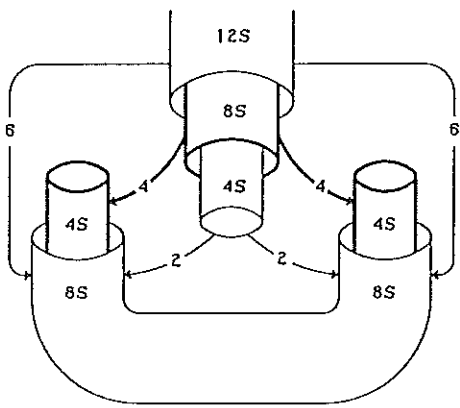
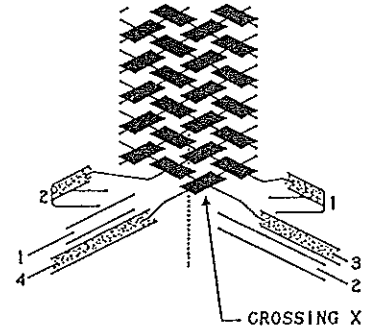
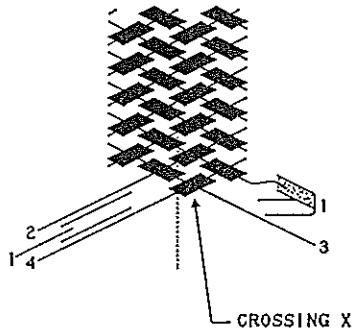
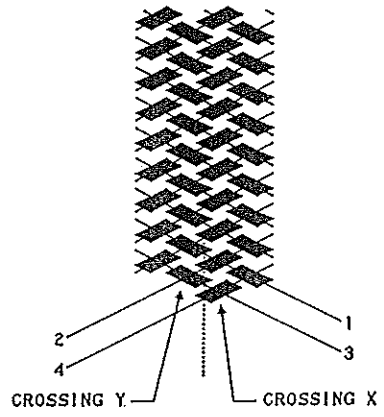
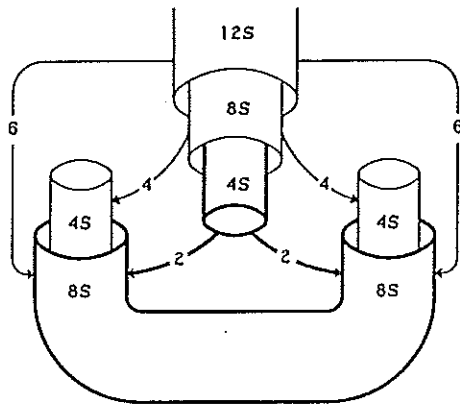


Fig. 752 — Preparation of the 4-string round braid inner core and of the 8-string round braid outer core.

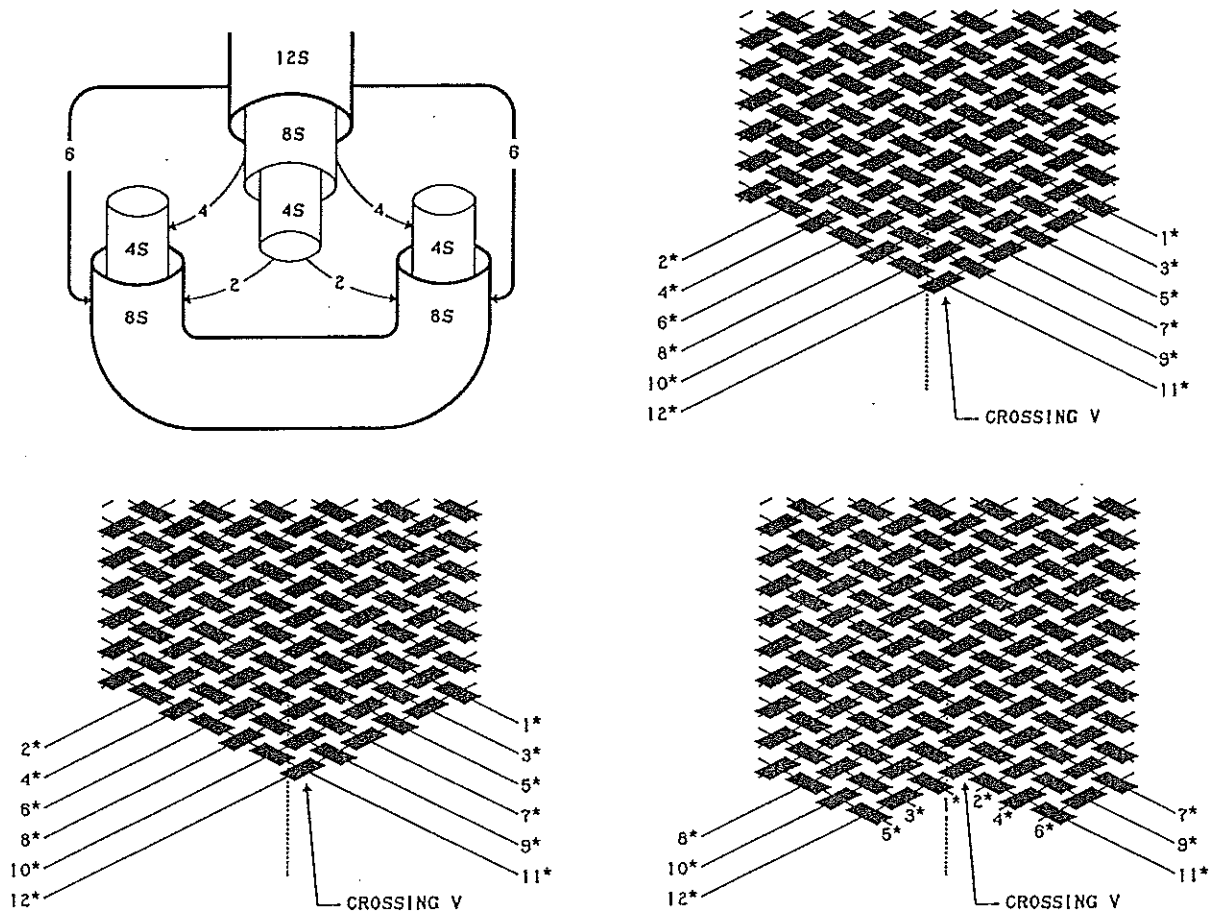


Fig. 753 — Preparation of the 12-string round braid mantle.

The left 4-string core:

Bring *E* from the left around the back to the right, then along the front from right to left under *B*, over *D*.

Bring *B* from the right around the back to the left, then along the front from left to right under *G*, over *E*.

Bring *G* from the left around the back to the right, then along the front from right to left under *D*, over *B*.

Bring *D* from the right around the back to the left, then along the front from left to right under *E*, over *G*.

And so on.

The right 4-string core:

Bring *A* from the left around the back to the right, then along the front from right to left under *F*, over *H*.

Bring *F* from the right around the back to the left, then along the front from left to right under *C*, over *A*.

Bring *C* from the left around the back to the right, then along the front from right to left under *H*, over *F*.

Bring *H* from the right around the back to the left, then along the front from left to right under *A*, over *C*.

And so on.

Braid from the strings of the 12-string round braid mantle and the strings 1, 2, 3, 4 of the inner core, the two 8-string mantles as shown in the upper part of Fig. 754.

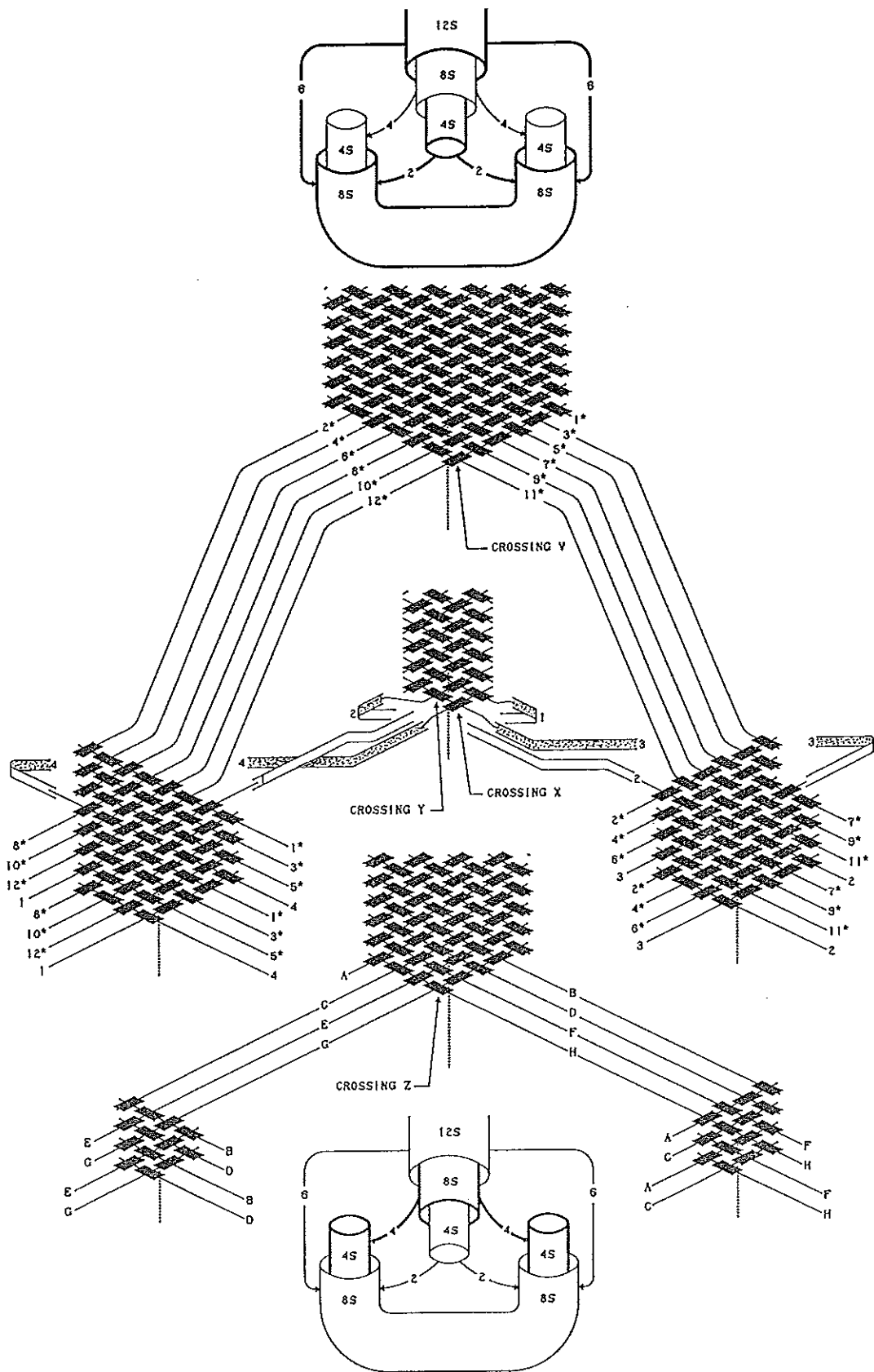


Fig. 754 — Transition from a 12-string over 8-string over 4-string round braid to two 8-string over 4-string round braids.

For the preparation stage, bring 1* from the right around the back to the left, then along the front from left to right under 2*, over 4*, under 6*, over 8*, under 10*, over 12*.

Bring 2* from the left around the back to the right, then along the front from right to left under 3*, over 5*, under 7*, over 9*, under 11*.

Bring 3* from the right around the back to the left, then along the front from left to right under 4*, over 6*, under 8*, over 10*, under 12*.

Bring 4* from the left around the back to the right, then along the front from right to left under 5*, over 7*, under 9*, over 11*.

Bring 5* from the right around the back to the left, then along the front from left to right under 6*, over 8*, under 10*, over 12*.

Bring 6* from the left around the back to the right, then along the front from right to left under 7*, over 9*, under 11*.

Bring 1 along the front to the left: over 4, over 1*, under 3*, over 5*.

Bring 4 around the back to the left, then along the front from left to right under 8*, over 10*, under 12*, over 1.

Bring 3 around the back to the right, then along the front from right to left over 7*, under 9*, under 11*.

Bring 2 along the front to the right under 2*, over 4*, under 6*, over 3.

The left 8-string mantle:

Bring 8* from the left around the back to the right, then along the front from right to left under 1*, over 3*, under 5*, over 4.

Bring 1* from the right around the back to the left, then along the front from left to right under 10*, over 12*, under 1, over 8*.

Bring 10* from the left around the back to the right, then along the front from right to left under 3*, over 5*, under 4, over 1*.

Bring 3* from the right around the back to the left, then along the front from left to right under 12*, over 1, under 8*, over 10*.

Bring 12* from the left around the back to the right, then along the front from right to left under 5*, over 4, under 1*, over 3*.

Bring 5* from the right around the back to the left, then along the front from left to right under 1, over 8*, under 10*, over 12*.

Bring 1 from the left around the back to the right, then along the front from right to left under 4, over 1*, under 3*, over 5*.

Bring 4 from the right around the back to the left, then along the front from left to right under 8*, over 10*, under 12*, over 1.

And so on.

The right 8-string mantle:

Bring 2* from the left around the back to the right, then along the front from right to left under 7*, over 9*, under 11*, over 2.

Bring 7* from the right around the back to the left, then along the front from left to right under 4*, over 6*, under 3, over 2*.

Bring 4* from the left around the back to the right, then along the front from right to left under 9*, over 11*, under 2, over 7*.

Bring 9* from the right around the back to the left, then along the front from left to right under 6*, over 3, under 2*, over 4*.

Bring 6* from the left around the back to the right, then along the front from right

to left under 11^* , over 2, under 7^* , over 9^* .

Bring 11^* from the right around the back to the left, then along the front from left to right under 3, over 2^* , under 4^* , over 6^* .

Bring 3 from the left around the back to the right, then along the front from right to left under 2, over 7^* , under 9^* , over 11^* .

Bring 2 from the right around the back to the left, then along the front from left to right under 2^* , over 4^* , under 6^* , over 3.

And so on.

THE BRAIDER'S NOTEBOOK

Most braidwork contains auxiliary knots. Their purpose, for example, may be to add weight to a section of the braidwork, to embellish the braidwork or to conceal local irregularities in the braidwork. It is important to ensure that such knots stay firmly in place at all times. A simple way to achieve this is for a knot-string to pass diametrically through the object the knot covers, and hence we ensure that the total number of bights of the knot is **even**.

When the knot is a Regular Knot, hence a Regular Cylindrical Braid requiring one essential string, then its number of parts p is **odd**, with $\text{g.c.d.}(p, b) = 1$. A string-run example of such a knot is shown in Fig. 755, where in the upper diagram the string-run has been split into two equal parts. One part forms the string-run half-cycles $1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - 11 - 12 - 13 - 14$ and the other part forms the string-run half-cycles $1' - 2' - 3' - 4' - 5' - 6' - 7' - 8' - 9' - 10' - 11' - 12' - 13' - 14'$. We can braid these string-run half-cycles of the knot in two different ways: 1. by parallel braiding or 2. by series braiding.

By parallel braiding we braid first half-cycle 1, then half-cycle $1'$, then half-cycle 2, then half-cycle $2'$, then half-cycle 3, then half-cycle $3'$, and so on.

By series braiding we braid first the half-cycles $1 - 2 - 3 - \dots - 14$, then the half-cycles $1' - 2' - 3' - \dots - 14'$.

Parallel braiding:

For the string-run half-cycles $1 - 2 - 3 - \dots - 14$ or the string-run half-cycles $1' - 2' - 3' - \dots - 14'$, the algorithm diagram with $\Delta^* = 3$ is constructed in the usual way, resulting in the two inner rows of i -values. The outer two rows of i -values belong respectively to the string-run half-cycles $1' - 2' - 3' - \dots - 14'$ or to the string-run half-cycles $1 - 2 - 3 - \dots - 14$ and are derived from the inner rows of i -values by adding or subtracting in modular fashion $\frac{b}{2}$. Thus for the half-cycles $1 - 2 - 3 - \dots - 14$, the inner row of i -values are associated with their intersections with the half-cycles $1 - 2 - 3 - \dots - 14$, and the outer row of i -values are associated with their intersections with the half-cycles $1' - 2' - 3' - \dots - 14'$. For the half-cycles $1' - 2' - 3' - \dots - 14'$, the inner row of i -values are associated with their intersections with the half-cycles $1' - 2' - 3' - \dots - 14'$, and the outer row of i -values are associated with their intersections with the half-cycles $1 - 2 - 3 - \dots - 14$. From the inner row $i = 3$ we thus obtain the outer row $i = |3 + 7|_{14} = 10$ (or $i = |3 - 7|_{14} = 10$), from the inner row $i = 6$ we thus obtain the outer row $i = |6 + 7|_{14} = 13$ (or $i = |6 - 7|_{14} = 13$), from the inner row $i = 9$ we thus obtain the outer row $i = |9 + 7|_{14} = 2$ (or $i = |9 - 7|_{14} = 2$), etc. Hence for half-cycle 7 we obtain its half-cycle braiding algorithm by reading the sequential

codings associated with $i = 2, 1, 0$. For half-cycle 8 we obtain its half-cycle braiding algorithm by reading the sequential codings associated with $i = 3, 2, 1, 0, 3$. For half-cycle 10' we obtain its half-cycle braiding algorithm by reading the sequential codings associated with $i = 3, 2, 1, 4, 0, 3$. For half-cycle 13' we obtain its half-cycle braiding algorithm by reading the sequential codings associated with $i = 3, 2, 5, 1, 4, 0, 3$.

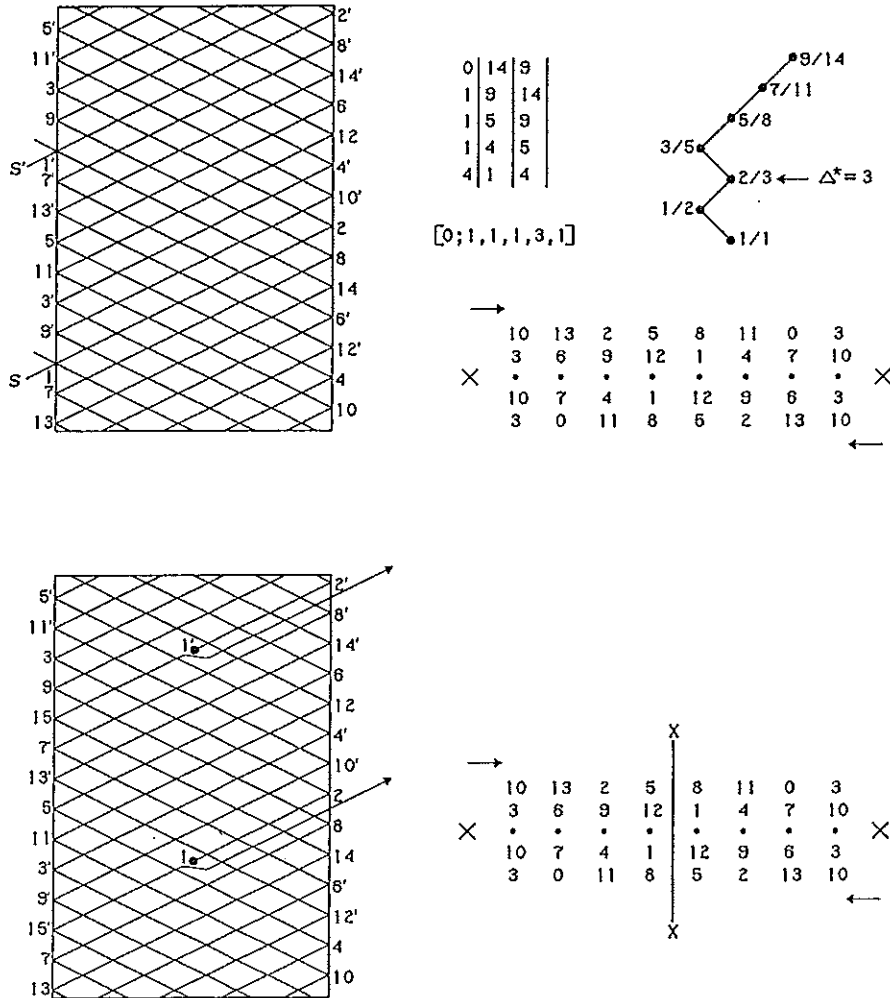


Fig. 755 — A Regular Knot with $p/b = 9/14$.

In the lower diagram of Fig. 755 the Standing Ends of the two strings have diametrically been joined. Hence the half-cycles 1 and 1' run from the X—X line to the right, while the half-cycles 15 and 15' are the last ones to be laid down; their half-cycle braiding algorithms are the sequential codings associated with $i = 3, 6, 2, 5$ followed by four unders. For half-cycle 7 we thus obtain its half-cycle braiding algorithm by reading the sequential codings associated with $i = 2, 1, 0$. For half-cycle 8 we obtain its half-cycle braiding algorithm by reading the sequential codings associated with $i = 3, 2, 1, 0$ (note that $i = 3$ in the outer lower row represents the intersection of half-cycle 8 with half-cycle 15 (the extended Standing End half-cycle 1' left of the X—X line) and hence has to be neglected). For half-cycle 10' we obtain its half-cycle braiding algorithm by reading the sequential codings associated with $i = 3, 2, 1, 0, 3$ (note that $i = 4$ in the inner lower row represents the intersection of half-cycle 10' with half-cycle 15 (the extended Standing End half-cycle 1' left of the X—X line) and hence has to be neglected). For half-cycle 13' we obtain its half-cycle braiding algorithm by reading the sequential codings associated with $i = 3, 2, 5, 1, 4, 0, 3$.

Let the superimposed coding on this string-run be as shown in Fig. 756.

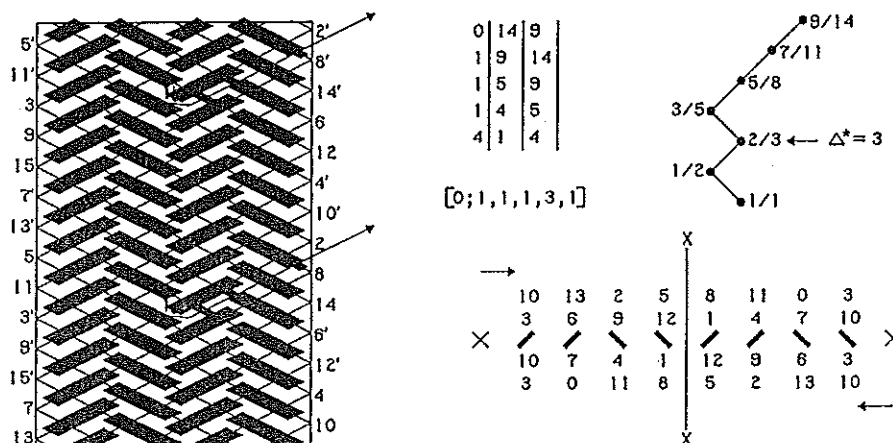


Fig. 756 — The superimposed coding on the string-run of Fig. 755.

The half-cycle braiding algorithms are then as follows:

- half-cycles 1 & 1' : : $L \rightarrow R$: Free run.
- half-cycles 2 & 2' : $i = 0$; $i \neq 0$ left of X-X : $L \leftarrow R$: Free run.
- half-cycles 3 & 3' : $i = 0$; : $L \rightarrow R$: u .
- half-cycles 4 & 4' : $i \leq 1$; $i \neq 1$ left of X-X : $L \leftarrow R$: u .
- half-cycles 5 & 5' : $i \leq 1$; : $L \rightarrow R$: $o - u$.
- half-cycles 6 & 6' : $i \leq 2$; $i \neq 2$ left of X-X : $L \leftarrow R$: $u - o - u$.
- half-cycles 7 & 7' : $i \leq 2$; : $L \rightarrow R$: $u - o - u$.
- half-cycles 8 & 8' : $i \leq 3$; $i \neq 3$ left of X-X : $L \leftarrow R$: $o - u - o - u$.
- half-cycles 9 & 9' : $i \leq 3$; : $L \rightarrow R$: $o - u - o - 2u$.
- half-cycles 10 & 10' : $i \leq 4$; $i \neq 4$ left of X-X : $L \leftarrow R$: $o - u - o - 2u$.
- half-cycles 11 & 11' : $i \leq 4$; : $L \rightarrow R$: $o - u - 2o - 2u$.
- half-cycles 12 & 12' : $i \leq 5$; $i \neq 5$ left of X-X : $L \leftarrow R$: $o - 2u - 2o - 2u$.
- half-cycles 13 & 13' : $i \leq 5$; : $L \rightarrow R$: $o - 2u - 2o - 2u$.
- half-cycles 14 & 14' : $i \leq 6$; $i \neq 6$ left of X-X : $L \leftarrow R$: $2o - 2u - 2o - 2u$.
- half-cycles 15 & 15' : $i \leq 6$; unders right of X-X : $L \rightarrow R$: $2o - 6u$.

Series braiding :

By series braiding we braid first the half-cycles 1 - 2 - 3 - ... - 14 - 15, then the half-cycles 1' - 2' - 3' - ... - 14' - 15'. By series braiding it is more convenient to number these two sets of half-cycles sequentially as in Fig. 757; thus we braid first half-cycles 1 - 2 - 3 - ... - 14 - 15, then the half-cycles 15* - 16 - 17 - ... - 28 - 29. The i -value $\frac{h_e - 2}{2}$ to the left of the line X-X (hence in the bottom row of the algorithm diagram), associated with the string-run 1 - 2 - 3 - ... - 14 - 15; 15* - 16 - 17 - ... - 28 - 29, must be neglected for the even numbered half-cycle h_e (intersection of half-cycle h_e with half-cycle 1 (the extended Standing End half-cycle 1 left of the X-X line). The i -values $\frac{h_e - 3}{2}$ to the right of the line X-X (hence in the top row of the algorithm diagram), associated with half-cycle 15, correspond to unders. The i -values $\frac{h_e - 3}{2}$ to the left of the line X-X (hence in the top row of the algorithm diagram), associated with half-cycle 15*, must be neglected. The i -value $|\frac{h_e - 2}{2} - \frac{b}{2}|_b$ to the right of the line X-X (hence in the bottom row of the algorithm diagram), associated with the even numbered half-cycle h_e of the string-run 15* - 16 - 17 - ... - 28 - 29, corresponds to an $o - *$, where the $*$ stands for the coding that is associated with $i = |\frac{h_e - 2}{2} - \frac{b}{2}|_b$. Note

that $\left| \frac{h_e-2}{2} - \frac{b}{2} \right|_b$ is identical to $\left| \frac{h_e-2}{2} + \frac{b}{2} \right|_b$. The i -values $\frac{h_e-3}{2}$ to the right of the line X—X (hence in the top row of the algorithm diagram), associated with half-cycle 29, correspond to unders.

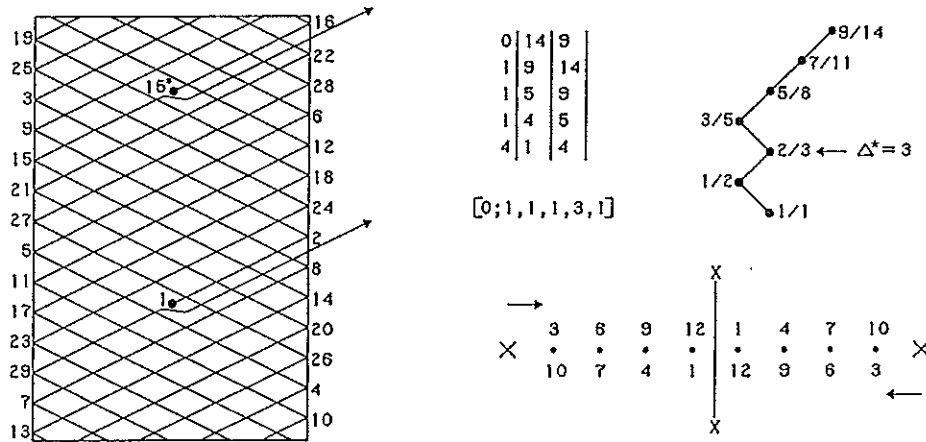


Fig. 757 — Half-cycle numbering for series braiding.

Thus for half-cycle 10 we obtain its half-cycle braiding algorithm by reading the sequential codings associated with $i = 3, 1$ (bottom row, right to left). Note that $i = 4$ in the bottom row represents the intersection of half-cycle 10 with half-cycle 1 (the extended Standing End half-cycle 1 left of the X—X line) and hence has to be neglected. For half-cycle 15 we obtain its half-cycle braiding algorithm by reading the sequential codings associated with $i = 3, 6$ (top row, left to right) followed by two unders. For half-cycle 15* we obtain its half-cycle braiding algorithm by reading the sequential codings associated with $i = 1, 4$ (top row, left to right). Note that $i = 3, 6$ in the top row are to the left of the X—X line and hence have to be neglected. For half-cycle 18 we obtain its half-cycle braiding algorithms by reading the sequential codings associated with $i = 3, 6, 1, 4, 7$ (bottom row, right to left). For half-cycle 22 we obtain its half-cycle braiding algorithms by reading the sequential codings o and those associated with $i = 3, 6, 9, 1, 4, 7$ (bottom row, right to left). Note that $i = \left| \frac{22-2}{2} - \frac{14}{2} \right|_{14} = 3$ in the bottom row, situated at the right of the X—X line, gives us the o — $*$ codings, where the $*$ -coding is associated with $i = 3$; furthermore that $i = \frac{22-2}{2} = 10$ in the bottom row represents the intersection of half-cycle 22 with half-cycle 1 (the extended Standing End half-cycle 1 left of the X—X line) and hence has to be neglected.

Let the superimposed coding on this string-run be as shown in Fig. 758.

The half-cycle braiding algorithms are then as follows:

- half-cycle 1 : $i = 0$; $i \neq 0$ left of X-X : $L \rightarrow R$: Free run.
- half-cycle 2 : $i = 0$; $i \neq 0$ left of X-X : $L \leftarrow R$: Free run.
- half-cycle 3 : $i = 0$; $i \neq 0$ left of X-X : $L \rightarrow R$: Free run.
- half-cycle 4 : $i \leq 1$; $i \neq 1$ left of X-X : $L \leftarrow R$: Free run.
- half-cycle 5 : $i \leq 1$; $i \neq 1$ left of X-X : $L \rightarrow R$: o .
- half-cycle 6 : $i \leq 2$; $i \neq 2$ left of X-X : $L \leftarrow R$: o .
- half-cycle 7 : $i \leq 2$; $i \neq 2$ left of X-X : $L \rightarrow R$: o .
- half-cycle 8 : $i \leq 3$; $i \neq 3$ left of X-X : $L \leftarrow R$: $2o$.
- half-cycle 9 : $i \leq 3$; $i \neq 3$ left of X-X : $L \rightarrow R$: $2o$.
- half-cycle 10 : $i \leq 4$; $i \neq 4$ left of X-X : $L \leftarrow R$: $2o$.
- half-cycle 11 : $i \leq 4$; $i \neq 4$ left of X-X : $L \rightarrow R$: $3o$.
- half-cycle 12 : $i \leq 5$; $i \neq 5$ left of X-X : $L \leftarrow R$: $3o$.

- half-cycle 13 : $i \leq 5$; : $L \rightarrow R$: $3o$.
- half-cycle 14 : $i \leq 6$; $i \neq 6$ left of X-X : $L \leftarrow R$: $4o$.
- half-cycle 15 : $i \leq 6$; unders right of X-X : $L \rightarrow R$: $2o - 2u$.
- half-cycle 15* : $i \leq 6$; : $L \rightarrow R$: $2o$.
- half-cycle 16 : $i \leq 7$; $i \neq 7$ left of X-X ; coding $o - \star = \text{none}$ for
 $i = |7 - 7|_{14} = 0$ right of X-X : $L \leftarrow R$: $4o$.
- half-cycle 17 : $i \leq 7$; : $L \rightarrow R$: $4o - u$.
- half-cycle 18 : $i \leq 8$; $i \neq 8$ left of X-X ; coding $o - \star = \text{none}$ for
 $i = |8 - 7|_{14} = 1$ right of X-X : $L \leftarrow R$: $4o - u$.
- half-cycle 19 : $i \leq 8$; : $L \rightarrow R$: $4o - u$.
- half-cycle 20 : $i \leq 9$; $i \neq 9$ left of X-X ; coding $o - \star = \text{none}$ for
 $i = |9 - 7|_{14} = 2$ right of X-X : $L \leftarrow R$: $2o - u - 2o - u$.
- half-cycle 21 : $i \leq 9$; : $L \rightarrow R$: $2o - u - 2o - u$.
- half-cycle 22 : $i \leq 10$; $i \neq 10$ left of X-X ; coding $o - \star = 2o$ for
 $i = |10 - 7|_{14} = 3$ right of X-X : $L \leftarrow R$: $3o - u - 2o - u$.
- half-cycle 23 : $i \leq 10$; : $L \rightarrow R$: $2o - u - 2o - 2u$.
- half-cycle 24 : $i \leq 11$; $i \neq 11$ left of X-X ; coding $o - \star = \text{none}$ for
 $i = |11 - 7|_{14} = 4$ right of X-X : $L \leftarrow R$: $2o - u - 2o - 2u$.
- half-cycle 25 : $i \leq 11$; : $L \rightarrow R$: $2o - u - 2o - 2u$.
- half-cycle 26 : $i \leq 12$; $i \neq 12$ left of X-X ; coding $o - \star = \text{none}$ for
 $i = |12 - 7|_{14} = 5$ right of X-X : $L \leftarrow R$: $2o - 2u - 2o - 2u$.
- half-cycle 27 : $i \leq 12$; : $L \rightarrow R$: $2o - 2u - 2o - 2u$.
- half-cycle 28 : $i \leq 13$; $i \neq 13$ left of X-X ; coding $o - \star = 2o$ for
 $i = |13 - 7|_{14} = 6$ right of X-X : $L \leftarrow R$: $3o - 2u - 2o - 2u$.
- half-cycle 29 : $i \leq 13$; unders right of X-X : $L \rightarrow R$: $2o - 6u$.

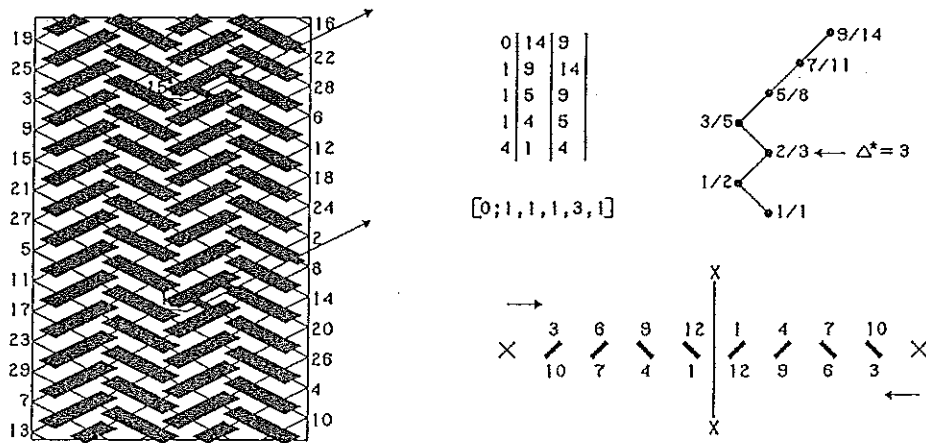
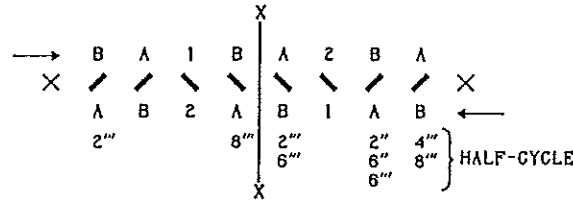


Fig. 758 — The superimposed coding on the string-run of Fig. 757.

When the knot is a Semi-Regular Knot, hence a Regular Cylindrical Braid with $\text{g.c.d.}(p, b) = \lambda$, where λ is the number of essential strings required, then we ensure that $\frac{b}{\lambda} = \text{even}$ and $\frac{p}{\lambda} = \text{odd}$. We furthermore ensure that the string-run of one of the interbraided Regular Knots passes diametrically through the object the Semi-Regular Knot covers. An example of such a Semi-Regular Knot is shown in Fig. 759 where the string-run of the component Regular Knot, which passes diametrically through the object the Semi-Regular Knot covers, is $1 - 2 - 3 - 4 - 5$; $1' - 2' - 3' - 4' - 5'$. One part of this component forms the string-run half-cycles $1 - 2 - 3 - 4 - 5$ and the other part forms the string-run half-cycles $1' - 2' - 3' - 4' - 5'$. We can again braid these string-run

Next we interbraid the Regular Knot $1'' - 2'' - 3'' - \dots - 8'' - 9''$, and finally we interbraid the Regular Knot $1''' - 2''' - 3''' - \dots - 8''' - 9'''$. We position the starting-points of the half-cycles $5'$, $1''$ and $1'''$ regularly around the left-hand bight-boundary. Let half-cycle $5'$ start at bight-index number $I_0 = 0$, then half-cycle 5 starts at bight-index number $I'_0 = \frac{b}{2} = 6$, half-cycle $1''$ starts at bight-index number $I''_0 = \frac{b}{3} = 4$, and half-cycle $1'''$ starts at bight-index number $I'''_0 = 2 \times \frac{b}{3} = 8$. Thus from each of the positions 1 and $1'$ we have two parallel strings to the right-hand bight-boundary (hence to the right of the X—X line), ending at this bight-boundary respectively at bight-index number $I_0 = 0$ and bight-index number $I'_0 = 6$. Furthermore, we have two parallel strings from each of the left bight-boundary bight-index numbers $I''_0 = 4$ and $I'''_0 = 8$ to respectively the right bight-boundary bight-index numbers $I''_0 = 4$ and $I'''_0 = 8$. An even numbered half-cycle to be laid down which will intersect such an already laid down parallel pair of strings will do so in the sequence $o - \star$ when the intersection is to the right of the X—X line and in the sequence $\star - o$ when the intersection is to the left of the X—X line. Hence for the even numbered half-cycles of the string-run sequence $1'' - 2'' - 3'' - \dots - 8'' - 9''$, such intersections to the right of the X—X line are associated with the right bight-boundary bight-index numbers $I_0 = 0$ and $I'_0 = 6$; there are no such intersections to the left of the X—X line. For the even numbered half-cycles of the string-run sequence $1''' - 2''' - 3''' - \dots - 8''' - 9'''$, such intersections to the right of the X—X line are associated with the right bight-boundary bight-index numbers $I_0 = 0$, $I'_0 = 6$, and $I''_0 = 4$, while such intersections to the left of the X—X line are associated with the right bight-boundary bight-index number $I''_0 = 4$. Thus for half-cycle $2''$, an $o - \star$ intersection occurs on the second intersection-column from the right ($|0 - 4|_b + nb = |0 - 4|_{12} + 12n = 8 + 12n$, and $|6 - 4|_b + nb = |6 - 4|_{12} + 12n = 2 + 12n = 2 + 12n$ (or $8 - 6 + nb = 2 + 12n$)). For half-cycle $4''$ there is no such an intersection ($|0 - (4 + p)|_b + nb = |0 - (4 + 9)|_{12} + 12n = 11 + 12n$, and $|6 - (4 + p)|_b + nb = |6 - (4 + 9)|_{12} + 12n = 5 + 12n$ (or $11 - 6 + nb = 5 + 12n$)). For half-cycle $6''$, an $o - \star$ intersection occurs on the second intersection-column from the right ($|0 - (4 + 2p)|_b + nb = |0 - (4 + 18)|_{12} + 12n = 2 + 12n$, and $|6 - (4 + 2p)|_b + nb = |6 - (4 + 18)|_{12} + 12n = 8 + 12n$ (or $2 + 6 + nb = 8 + 12n$)). For half-cycle $8''$ there is no such an intersection ($|0 - (4 + 3p)|_b + nb = |0 - (4 + 27)|_{12} + 12n = 5 + 12n$, and $|6 - (4 + 3p)|_b + nb = |6 - (4 + 27)|_{12} + 12n = 11 + 12n$ (or $5 + 6 + nb = 11 + 12n$)). For half-cycle $2'''$, an $o - \star$ intersection occurs on the fourth intersection-column from the right and a $\star - o$ intersection occurs on the eighth intersection-column from the right ($|0 - 8|_b + nb = |0 - 8|_{12} + 12n = 4 + 12n$, $|6 - 8|_b + nb = |6 - 8|_{12} + 12n = 10 + 12n$ (or $4 + 6 + nb = 10 + 12n$), and $|4 - 8|_b + nb = |4 - 8|_{12} + 12n = 8 + 12n$). For half-cycle $4'''$, an $o - \star$ intersection occurs on the first intersection-column from the right ($|0 - (8 + p)|_b + nb = |0 - (8 + 9)|_{12} + 12n = 7 + 12n$, $|6 - (8 + p)|_b + nb = |6 - (8 + 9)|_{12} + 12n = 1 + 12n$ (or $7 - 6 + nb = 1 + 12n$), and $|4 - (8 + p)|_b + nb = |4 - (8 + 9)|_{12} + 12n = 11 + 12n$). For half-cycle $6'''$, an $o - \star$ intersection occurs on the second and the fourth intersection-column from the right ($|0 - (8 + 2p)|_b + nb = |0 - (8 + 18)|_{12} + 12n = 10 + 12n$, $|6 - (8 + 2p)|_b + nb = |6 - (8 + 18)|_{12} + 12n = 4 + 12n$ (or $10 - 6 + nb = 4 + 12n$), and $|4 - (8 + 2p)|_b + nb = |4 - (8 + 18)|_{12} + 12n = 2 + 12n$). For half-cycle $8'''$, an $o - \star$ intersection occurs on the first intersection-column from the right and a $\star - o$ intersection occurs on the fifth intersection-column from the right ($|0 - (8 + 3p)|_b + nb = |0 - (8 + 27)|_{12} + 12n = 1 + 12n$, $|6 - (8 + 3p)|_b + nb = |6 - (8 + 27)|_{12} + 12n = 7 + 12n$ (or $1 + 6 + nb = 7 + 12n$), and $|4 - (8 + 3p)|_b + nb = |4 - (8 + 27)|_{12} + 12n = 5 + 12n$).

For the string-runs $1'' - 2'' - 3'' - \dots - 8'' - 9''$ and $1''' - 2''' - 3''' - \dots - 8''' - 9'''$ we thus use the following algorithm diagram:



The intersections to the left of the X—X line on the first half-cycle of the string-runs $1'' - 2'' - 3'' - \dots - 8'' - 9''$ and $1''' - 2''' - 3''' - \dots - 8''' - 9'''$ are **unders**. The intersections to the right of the X—X line on the last half-cycle of the string-runs $1'' - 2'' - 3'' - \dots - 8'' - 9''$ and $1''' - 2''' - 3''' - \dots - 8''' - 9'''$ are **unders**.

For the string-run $1'' - 2'' - 3'' - \dots - 8'' - 9''$ the half-cycle braiding algorithms are:

- half-cycle $1''$: $i = A : L \rightarrow R : 2u - o.$
- half-cycle $2''$: $i = A \ \& \ i = 0 : L \leftarrow R : o - u - o - u.$
- half-cycle $3''$: $i = A \ \& \ i = 0 : L \rightarrow R : o - u - o.$
- half-cycle $4''$: $i = A \ \& \ i \leq 1 : L \leftarrow R : u - 2o - u.$
- half-cycle $5''$: $i = A \ \& \ i \leq 1 : L \rightarrow R : o - 2u - o.$
- half-cycle $6''$: $i = A \ \& \ i \leq 2 : L \leftarrow R : o - u - 3o - u.$
- half-cycle $7''$: $i = A \ \& \ i \leq 2 : L \rightarrow R : o - 3u - o.$
- half-cycle $8''$: $i = A \ \& \ i \leq 3 : L \leftarrow R : u - 3o - u.$
- half-cycle $9''$: $i = A \ \& \ i \leq 3 : L \rightarrow R : o - 4u.$

For the string-run $1''' - 2''' - 3''' - \dots - 8''' - 9'''$ the half-cycle braiding algorithms are:

- half-cycle $1'''$: $i = A, B : L \rightarrow R : 4u - 2o.$
- half-cycle $2'''$: $i = A, B \ \& \ i = 0 : L \leftarrow R : 2u - 3o - 2u - o.$
- half-cycle $3'''$: $i = A, B \ \& \ i = 0 : L \rightarrow R : 2o - 2u - 2o.$
- half-cycle $4'''$: $i = A, B \ \& \ i \leq 1 : L \leftarrow R : o - 2u - 3o - 2u.$
- half-cycle $5'''$: $i = A, B \ \& \ i \leq 1 : L \rightarrow R : 2o - 3u - 2o.$
- half-cycle $6'''$: $i = A, B \ \& \ i \leq 2 : L \leftarrow R : u - o - u - 5o - 2u.$
- half-cycle $7'''$: $i = A, B \ \& \ i \leq 2 : L \rightarrow R : 2o - 4u - 2o.$
- half-cycle $8'''$: $i = A, B \ \& \ i \leq 3 : L \leftarrow R : o - 2u - 5o - 2u.$
- half-cycle $9'''$: $i = A, B \ \& \ i \leq 3 : L \rightarrow R : 2o - 6u.$

Series braiding:

Series braiding affects only the braiding of the sequence $1 - 2 - 3 - 4 - 5; 1' - 2' - 3' - 4' - 5'$ which is then braided in the sequence $1 - 2 - 3 - 4 - 5; 5^* - 6 - 7 - 8 - 9.$

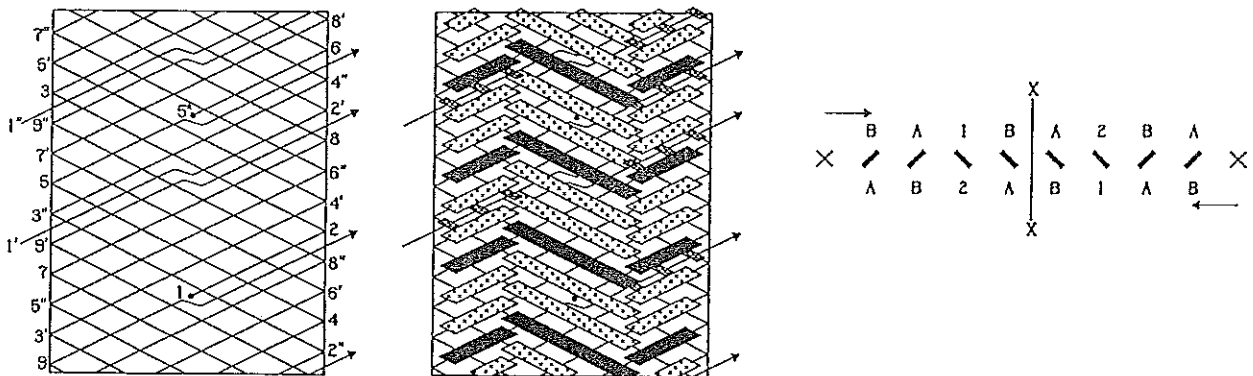


Fig. 760 — Half-cycle numbering for series braiding.