THE BRAIDER

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A quarterly publication
for
the braiding artisan

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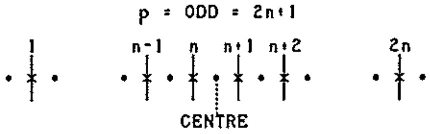
Solutions to the Questions in Issue No. 36

Question on pg.833.

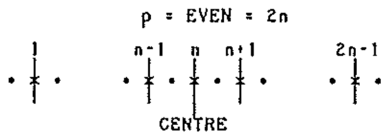
1). p is odd, hence $p = 2n+1$, where $n = 1, 2, 3, \cdots$, and $P = 2p = 4n+2$.



The number of new coding columns created by interbraiding two identical column-coded Regular Knots with p-parts is thus $(P-1)-(p-1) = (4n+1)-(2n) = 2n+1$. Hence the maximum number of different column-coded $P/B = 2p/2b$ Regular Cylindrical Braids which can be created is $2^{(2n+1)}$. This maximum number can only be created when for the identical column-coded Regular Knots p/b the condition $\{\text{coding of column } x\} = \{\text{coding of column } (2n+1-x)\}$, where $x = 1, 2, 3, \cdots, n$, does not apply for at least one x-value.

If for the column-coding of the identical column-coded Regular Knots p/b the $\{\text{coding of column } x\} = \{\text{coding of column } (2n+1-x)\}$, where $x = 1, 2, 3, \cdots, n$, for all x-values, then $2^{(n+1)}$ of the interbraided $P/B = 2p/2b$ Regular Cylindrical braids have a balanced column-coding (we have a choice out of two coding types for the columns $1, 3, 5, \cdots, (2n+1)$ of the interbraided $P/B = 2p/2b$ Regular Cylindrical Braid, then the coding of column $(2n+1+z)$ is the coding of column $(2n+1-z)$, where $z = 2, 4, 6, \cdots, 2n$). There remain thus a maximum of $\left(2^{(2n+1)} - 2^{(n+1)}\right)$ column-coded interbraided $P/B = 2p/2b$ Regular Cylindrical Braids, but only half of them are different (turn one of them through 180° and the braid obtained is already among the others of the $\left(2^{(2n+1)} - 2^{(n+1)}\right)$). Hence the number of different column-coded $P/B = 2p/2b$ interbraided Regular Cylindrical Braids we obtain is equal to $\dfrac{2^{(2n+1)} - 2^{(n+1)}}{2} + 2^{(n+1)} = 2^{2n} - 2^n + 2^{(n+1)} = 2^n(2^n - 1 + 2) = 2^n(2^n+1)$; of these $2^{(n+1)}$ have a balanced column-coding.

2). p is even, hence $p = 2n$, where $n = 1, 2, 3, \cdots$, and $P = 2p = 4n$.

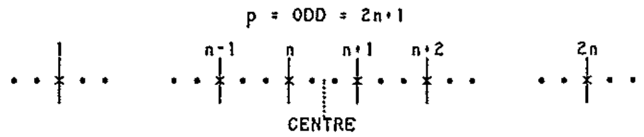


The number of new coding columns created by interbraiding two identical column-coded Regular Knots with p-parts is thus $(P-1)-(p-1) = (4n-1)-(2n-1) = 2n$. Hence the maximum number of different column-coded $P/B = 2p/2b$ Regular Cylindrical Braids which can be created is $2^{(2n)}$. This maximum number can only be created when for the identical column-coded Regular Knots p/b the condition $\{\text{coding of column } x\} = \{\text{coding of column } (2n-x)\}$, where $x = 1, 2, 3, \cdots, (n-1)$, does not apply for at least one x-value.

If for the column-coding of the identical column-coded Regular Knots p/b the $\{\text{coding of column } x\} = \{\text{coding of column } (2n-x)\}$, where $x = 1, 2, 3, \cdots, (n-1)$, for all x-values, then 2^n of the interbraided $P/B = 2p/2b$ Regular Cylindrical braids have a balanced column-coding (we have a choice out of two coding types for the columns $1, 3, 5, \cdots, (2n-1)$ of the interbraided $P/B = 2p/2b$ Regular Cylindrical Braid, then the coding of column $(2n+z)$ is the coding of column $(2n-z)$, where $z = 1, 3, 5, \cdots, (2n-1)$). There remain thus a maximum of $\left(2^{(2n)} - 2^n\right)$ column-coded interbraided $P/B = 2p/2b$ Regular Cylindrical Braids, but only half of them are different (turn one of them through 180° and the braid obtained is already among the others of the $\left(2^{(2n)} - 2^n\right)$). Hence the number of different column-coded $P/B = 2p/2b$ interbraided Regular Cylindrical Braids we obtain is equal to $\dfrac{2^{(2n)} - 2^n}{2} + 2^n = 2^{(2n-1)} - 2^{(n-1)} + 2^n = 2^{(n-1)}(2^n - 1 + 2) = 2^{(n-1)}(2^n + 1)$; there are 2^n of these which have a balanced column-coding.

Question on pg.835.

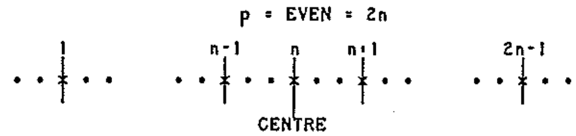
1). p is odd, hence $p = 2n+1$, where $n = 1, 2, 3, \cdots$, and $P = 3p = 6n+3$.



The number of new coding columns created by interbraiding three identical column-coded Regular Knots with p-parts is thus $(P-1) - (p-1) = (6n+2) - (2n) = 4n+2$. Hence the maximum number of different column-coded $P/B = 3p/3b$ Regular Cylindrical Braids which can be created is $2^{(4n+2)}$. This maximum number can only be created when for the identical column-coded Regular Knots p/b the condition {coding of column x} = {coding of column $(2n+1-x)$}, where $x = 1, 2, 3, \cdots, n$, does not apply for at least one x-value. When for the identical column-coded Regular Knots p/b the {coding of column x} = {opposite coding of column $(2n+1-x)$}, where $x = 1, 2, 3, \cdots, n$, for all x-values, then $2^{(2n+1)}$ of these interbraided $P/B = 3p/3b$ Regular Cylindrical braids have a balanced column-coding (we have a choice out of two coding types for the columns $1, 2, 4, 5, 7, 8, \cdots, (3n-2), (3n-1), (3n+1)$ of the interbraided $P/B = 3p/3b$ Regular Cylindrical Braid; the coding of column $(6n+3-z)$ is then the opposite to the coding of column z, where $z = 1, 2, 4, 5, 7, 8, \cdots, (3n-2), (3n-1), (3n+1)$).

If for the column-coding of the identical column-coded Regular Knots p/b the {coding of column x} = {coding of column $(2n+1-x)$}, where $x = 1, 2, 3, \cdots, n$, for all x-values, then $2^{(2n+1)}$ of the interbraided $P/B = 3p/3b$ Regular Cylindrical braids have a balanced column-coding (we have a choice out of two coding types for the columns $1, 2, 4, 5, 7, 8, \cdots, (3n-2), (3n-1), (3n+1)$ of the interbraided $P/B = 3p/3b$ Regular Cylindrical Braid; the coding of column $(6n+3-z)$ is then identical to the coding of column z, where $z = 1, 2, 4, 5, 7, 8, \cdots, (3n-2), (3n-1), (3n+1)$). There remain thus a maximum of $\left(2^{(4n+2)} - 2^{(2n+1)}\right)$ column-coded interbraided $P/B = 3p/3b$ Regular Cylindrical Braids, but only half of them are different (turn one of them through 180° and the braid obtained is already among the others of the $\left(2^{(4n+2)} - 2^{(2n+1)}\right)$). Hence the number of different column-coded $P/B = 3p/3b$ interbraided Regular Cylindrical Braids we obtain is equal to $\dfrac{2^{(4n+2)} - 2^{(2n+1)}}{2} + 2^{(2n+1)} = 2^{(4n+1)} - 2^{(2n)} + 2^{(2n+1)} = 2^{(2n)}\left(2^{(2n+1)} - 1 + 2\right) = 2^{(2n)}\left(2^{(2n+1)} + 1\right)$; of these $2^{(2n+1)}$ have a balanced column-coding.

2). p is even, hence $p = 2n$, where $n = 1, 2, 3, \cdots$, and $P = 3p = 6n$.



The number of new coding columns created by interbraiding three identical column-coded Regular Knots with p-parts is thus $(P-1)-(p-1) = (6n-1)-(2n-1) = 4n$. Hence the maximum number of different column-coded $P/B = 3p/3b$ Regular Cylindrical Braids which can be created is $2^{(4n)}$. This maximum number can only be created when for the identical column-coded Regular Knots p/b the condition $\{\text{coding of column } x\} = \{\text{coding of column } (2n-x)\}$, where $x = 1, 2, 3, \cdots, (n-1)$, does not apply for at least one x-value.

If for the column-coding of the identical column-coded Regular Knots p/b the $\{\text{coding of column } x\} = \{\text{coding of column } (2n-x)\}$, where $x = 1, 2, 3, \cdots, (n-1)$, for all x-values, then $2^{(2n)}$ of the interbraided $P/B = 3p/3b$ Regular Cylindrical braids have a balanced column-coding (we have a choice out of two coding types for the columns $1, 2, 4, 5, 7, 8, \cdots, (3n-2), (3n-1)$ of the interbraided $P/B = 3p/3b$ Regular Cylindrical Braid, then the coding of column $(6n-z)$ is identical to the coding of column z, where $z = 1, 2, 4, 5, 7, 8, \cdots, (3n-2), (3n-1)$). There remain thus a maximum of $\left(2^{(4n)} - 2^{(2n)}\right)$ column-coded interbraided $P/B = 3p/3b$ Regular Cylindrical Braids, but only half of them are different (turn one of them through 180° and the braid obtained is already among the others of the $\left(2^{(4n)} - 2^{(2n)}\right)$). Hence the number of different column-coded $P/B = 3p/3b$ interbraided Regular Cylindrical Braids is equal to $\dfrac{2^{(4n)} - 2^{(2n)}}{2} + 2^{(2n)} = 2^{(4n-1)} - 2^{(2n-1)} + 2^{(2n)} = 2^{(2n-1)}\left(2^{(2n)} - 1 + 2\right) = 2^{(2n-1)}\left(2^{(2n)} + 1\right)$; of these $2^{(2n)}$ have a balanced column-coding.

Rectangular Right Prismatic Braids

A Rectangular Right Prismatic Braid is a braid-form which can be used as a toggle (small sizes) or as a fender covering (large sizes). We find the description of some in *The Ashley Book of Knots* #2234, #2236, #2237, #2238, #2240.

Let's look at the braid $3 \times 4 \times 7$ mentioned under #2240:

In order to draw the grid-diagram of this braid, one most likely would develop it in the way a rectangular right prismatic box would be developed. The result is then as shown by the upper diagram in Fig. 672. Note that the "blank" codings are neglected in the braiding process since they only duplicate for clarity reasons some of the "black" codings (it are only the "black" codings which take part in the braiding process). As we shall see shortly, this type of braid-form requires at least two essential strings, and the braid $3 \times 4 \times 7$ mentioned under #2240 is a braid for which two strings suffice. In the lower string-run diagram of Fig. 672 the string-run of one of these two essential strings is depicted by the heavy line segments. This development procedure is, however, not the best for this type of braid-form.

Instead of folding the two 3×4 end-faces out in the plane containing the 4×7 top-face, we can fold half of each of these 3×4 end-faces out with the 4×7 top-face and the other half of each of these end-faces with the 4×7 bottom-face; this has been depicted by the upper grid-diagram of Fig. 673. The string-run can then be depicted as in the central string-run diagram of Fig. 673. Note that the heavy lined string-run in the central diagram of Fig. 673 is the heavy lined string-run in the lower diagram of Fig. 672.

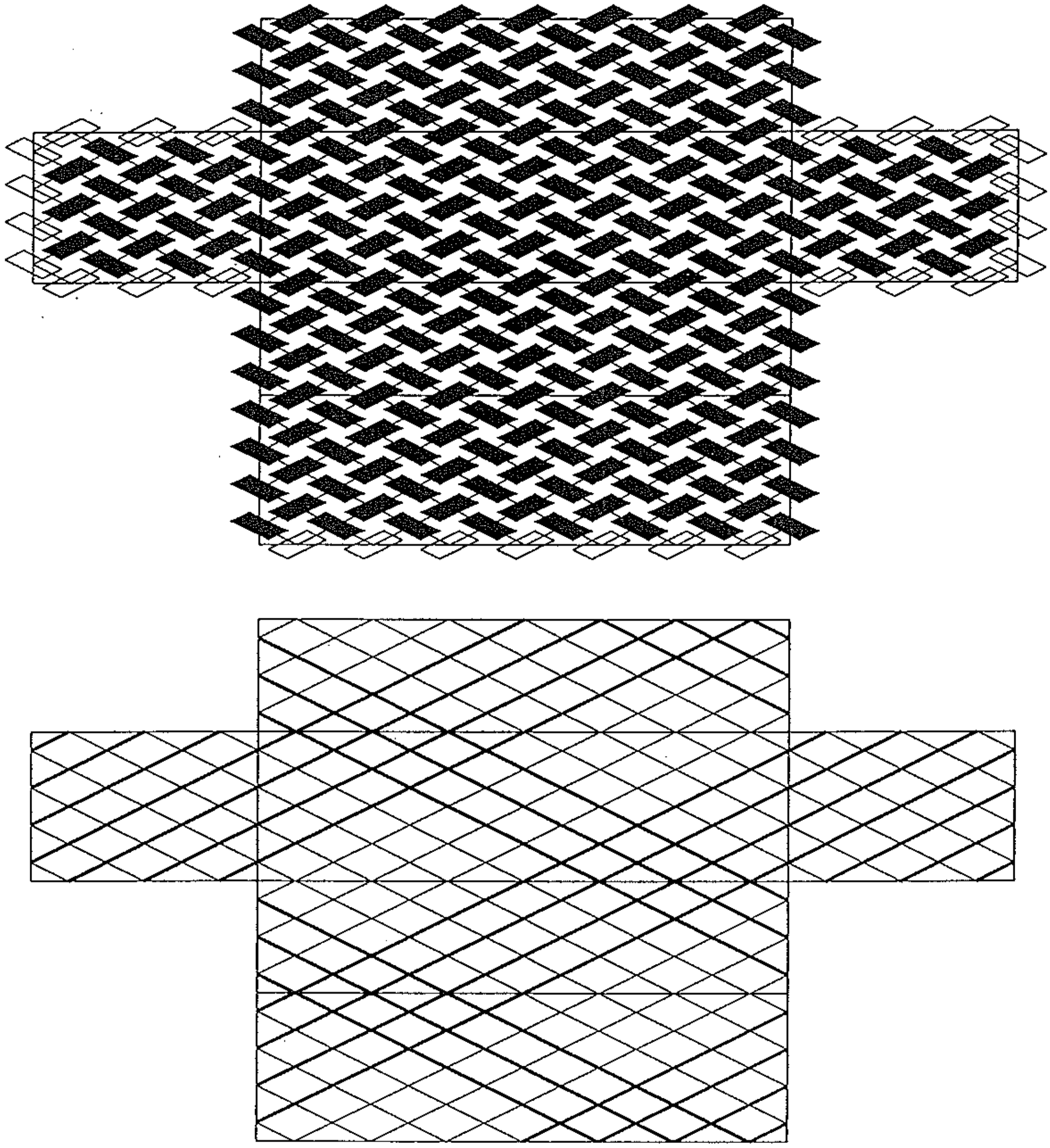


Fig. 672 — A $3 \times 4 \times 7$ over-under coded Rectangular Right Prismatic Braid.

When we now superimpose an over-under coding, we obtain the lowermost grid-diagram of this $3 \times 4 \times 7$ over-under coded Rectangular Right Prismatic Braid. The advantage of this layout (development) will immediately be obvious — fewer blank codings and a much more continuous string-run, which makes the actual braiding process much easier. To a large extent the string-run looks now like that of a Periodic Regular Nested Cylindrical Braid (there are only four points in the diagram (it are in reality only two points since the two left-hand points depict in fact only one and the same point, similarly the two right-hand points which depict in fact only one and the same point) where it deviates from a Periodic Regular Nested Cylindrical Braid).

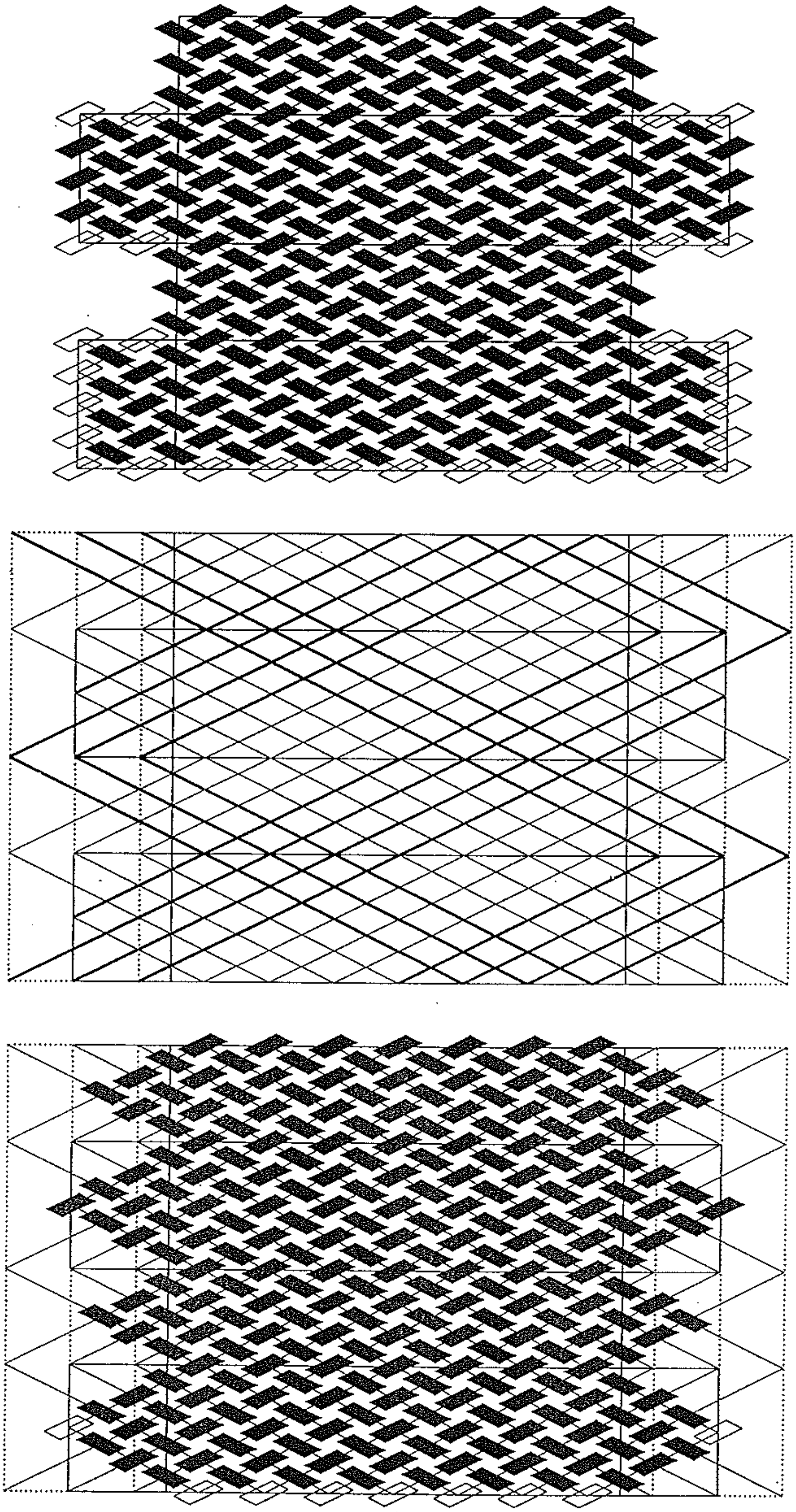


Fig. 673 — A $3 \times 4 \times 7$ over-under coded Rectangular Right Prismatic Braid.

In order to help the reader with familiarising the reading of these diagrams, we have depicted in Figs. 674 & 675 an over-under coded $2 \times 3 \times 5$ Rectangular Right Prismatic Braid, respectively developed in the same way as the over-under coded $3 \times 4 \times 7$ Rectangular Right Prismatic Braid in Figs. 672 & 673.

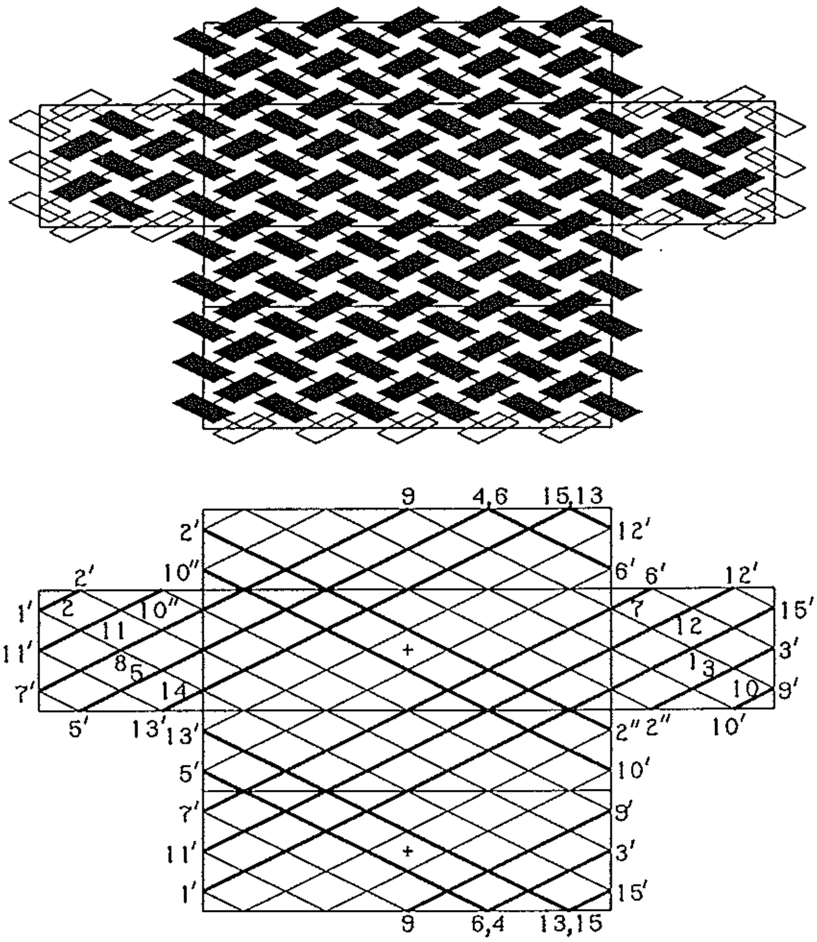


Fig. 674 — A $2 \times 3 \times 5$ over-under coded Rectangular Right Prismatic Braid.

In the lower string-run diagram of Fig. 674 the string-run of one of the two essential strings is indicated by the heavy line segments $1 - 1' - 2 - 2' - 2'' - 3 - 3' - 4 - 5 - 5' - 6 - 6' - 7 - 7' - 8 - 9 - 9' - 10 - 10' - 10'' - 11 - 11' - 12 - 12' - 13 - 13' - 14 - 15 - 15' - 1$. This string-run is depicted in Fig. 675 by the heavy line segments $1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-1$ in the central string-run diagram.

The type of string-run diagrams in Figs. 673 and 675 are generalised by the upper diagram in Fig. 676, and this generalised diagram is going to tell us that the minimum number of essential strings for these type of braids is equal to two.

There are four nests of regularly stacked bights at the right and left bight-edges, and in addition we have the boundary crossing-points $A_1 = A_3$ and $A_2 = A_4$. Let each nest of bights consist of a bights, then in total we have $8a$ bights.

Let the string-run start in A_1 in the direction of arrow 1. After leaving boundary crossing-point A_1, let the first boundary crossing-point encountered by the string-run be A_1. Hence the string-run enters A_1 in the direction of arrow 2 (see diagram 1 in Fig. 676). The string then leaves boundary crossing-point $A_3 (= A_1)$ in the direction of arrow 3. The next boundary crossing-point encountered will then be A_3 in the direction of arrow 4 and the string-run is closed. The string-run $A_1 - 1 - 2 - A_1$ and the string-run $A_3 - 3 - 4 - A_3$ can each have a maximum of $(2a - 1)$ bights, hence with the second string through the boundary crossing-points $A_2 = A_4$ the maximum number of bights through which these two strings pass is equal to $(8a - 4)$. Since the total number of bights is equal to $8a$, the braid thus requires more than two essential strings.

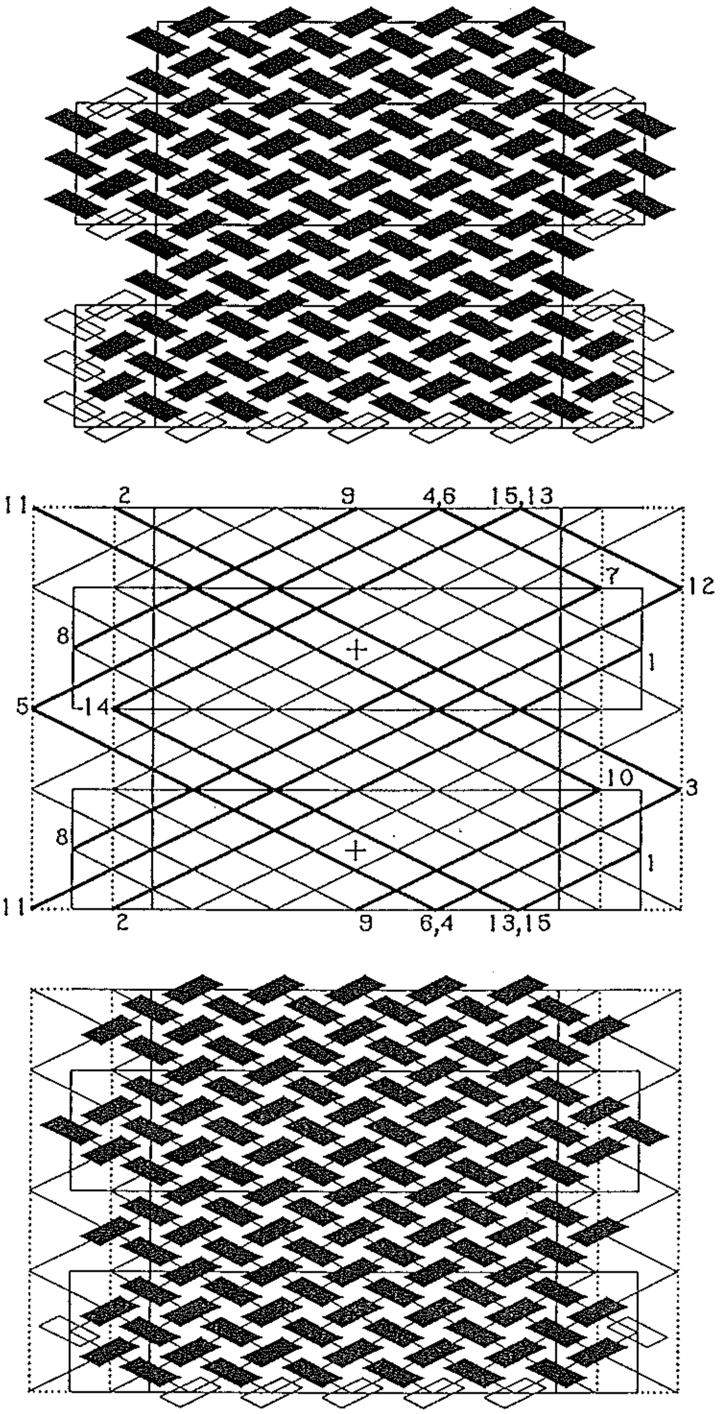


Fig. 675 — A $2 \times 3 \times 5$ over-under coded Rectangular Right Prismatic Braid.

Let the string-run start in A_1 in the direction of arrow 1. After leaving boundary crossing-point A_1 let the first boundary crossing-point encountered by the string-run be A_3. Hence the string-run enters A_3 in the direction of arrow 2 (see diagram 2 in Fig. 676). The string then leaves boundary crossing-point A_1 $(= A_3)$ in the direction of arrow 3. The next boundary crossing-point encountered will then be A_3 in the direction of arrow 4 and the string-run is closed. The string-run $A_1 - 1 - 2 - A_3$ and the string-run $A_1 - 3 - 4 - A_3$ can each have a maximum of $(2a - 1)$ bights, hence with the second string through the boundary crossing-points $A_2 = A_4$ the maximum number of bights through which these two strings pass is equal to $(8a - 4)$. Since the total number of bights is equal to $8a$, the braid thus requires more than two essential strings.

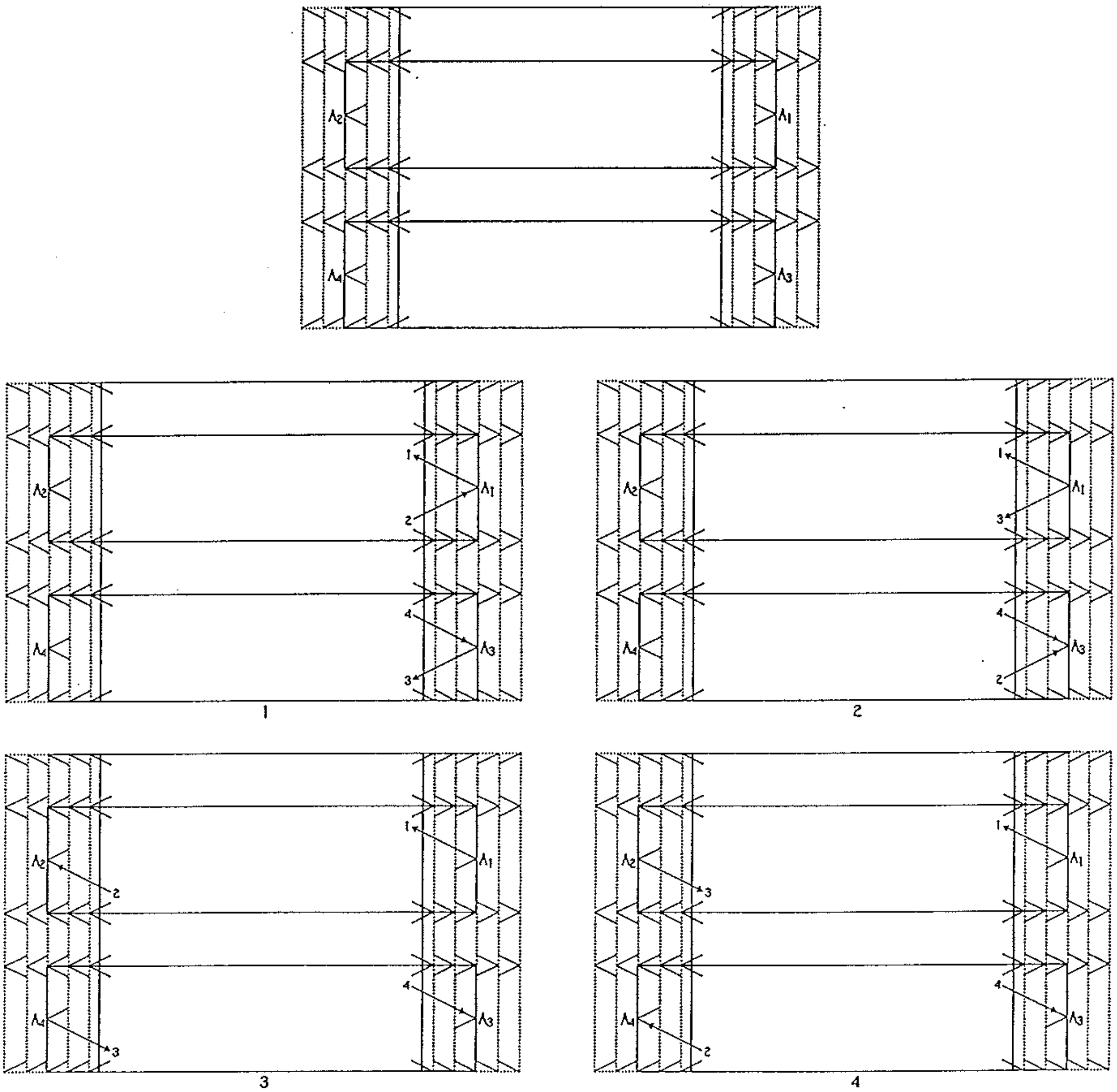


Fig. 676 — A generalised development of the string-run.

Let the string-run start in A_1 in the direction of arrow 1. After leaving boundary crossing-point A_1 let the first boundary crossing-point encountered by the string-run be A_2. Hence the string-run enters A_2 in the direction of arrow 2 (see diagram 3 in Fig. 676). The string then leaves boundary crossing-point A_4 $(= A_2)$ in the direction of arrow 3. The next boundary crossing-point encountered will then be A_3 in the direction of arrow 4 and the string-run is closed. The string-run $A_1 - 1 - 2 - A_2$ and the string-run $A_4 - 3 - 4 - A_3$ can each have a maximum of $2a$ bights, hence with the second string through the boundary crossing-points $A_1 = A_3$ and $A_2 = A_4$ the maximum number of bights through which these two strings pass is equal to $8a$. Since the total number of bights is equal to $8a$, the braid thus requires a minimum of two essential strings.

Let the string-run start in A_1 in the direction of arrow 1. After leaving boundary crossing-point A_1 let the first boundary crossing-point encountered by the string-run be A_4. Hence the string-run enters A_4 in the direction of arrow 2 (see diagram 4 in Fig. 676). The string then leaves boundary crossing-point $A_2 (= A_4)$ in the direction of arrow 3. The next boundary crossing-point encountered will then be A_3 in the direction of arrow 4 and the string-run is closed. The string-run $A_1 - 1 - 2 - A_4$ and the string-run $A_2 - 3 - 4 - A_3$ can each have a maximum of $2a$ bights, hence with the second string through the boundary crossing-points $A_1 = A_3$ and $A_4 = A_2$ the maximum number of bights through which these two strings pass is equal to $8a$. Since the total number of bights is equal to $8a$, the braid thus requires a minimum of two essential strings.

Hence the Rectangular Right Prismatic Braids with one boundary crossing-point on each the left and the right face ($A_1 = A_3$ and $A_2 = A_4$) and requiring the minimum of two essential strings, have with regards the string-run at the boundary crossing-points a string-run diagram like diagram 3 or diagram 4 in Fig. 676.

A) The Rectangular Right Prismatic Braids

i) $(2n-1) \times (2n) \times [(2m-1)(4n-1)]$,

ii) $(2n-1) \times (2n) \times [(2m-2)(4n-1)+1]$,

iii) $(2n) \times (2n+1) \times [(2m-2)(4n+1)]$,

iv) $(2n) \times (2n+1) \times [(2m-1)(4n+1)+1]$,

where n and m are independent and have the values $1, 2, 3, \cdots$, have with regards the string-run at the boundary crossing-points a string-run diagram like diagram 3 in Fig. 676.

B) The Rectangular Right Prismatic Braids

i) $(2n-1) \times (2n) \times [(2m-1)(4n-1)+1]$,

ii) $(2n-1) \times (2n) \times [(2m-2)(4n-1)]$,

iii) $(2n) \times (2n+1) \times [(2m-2)(4n+1)+1]$,

iv) $(2n) \times (2n+1) \times [(2m-1)(4n+1)]$,

where n and m are independent and have the values $1, 2, 3, \cdots$, have with regards the string-run at the boundary crossing-points a string-run diagram like diagram 4 in Fig. 676.

Example 1 :

The two development methods for an over-under coded braid of the type A)—*i)* with $n = 1$ and $m = 1$ are depicted in Figs. 677 & 678.

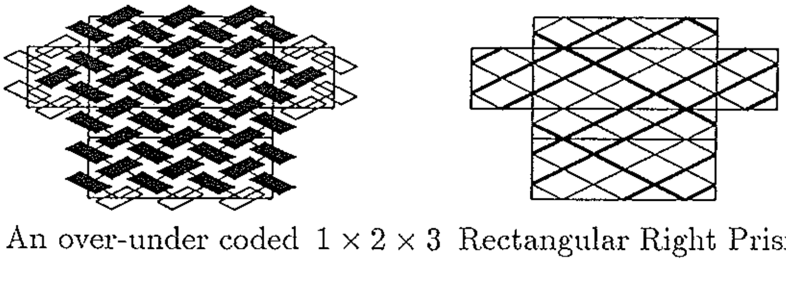


Fig. 677 — An over-under coded $1 \times 2 \times 3$ Rectangular Right Prismatic Braid.

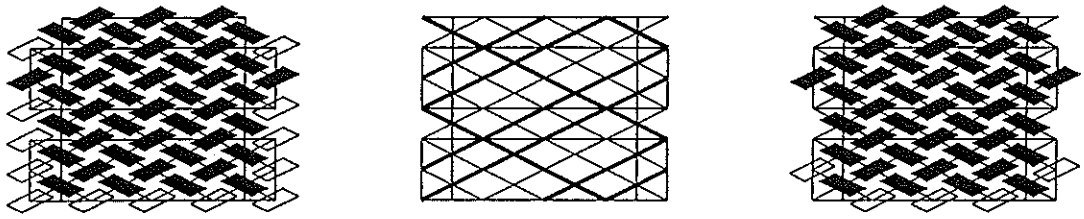


Fig. 678 — An over-under coded $1 \times 2 \times 3$ Rectangular Right Prismatic Braid.

Example 2:

The two development methods for an over-under coded braid of the type B) — *i*) with $n = 1$ and $m = 1$ are depicted in Figs. 679 & 680.

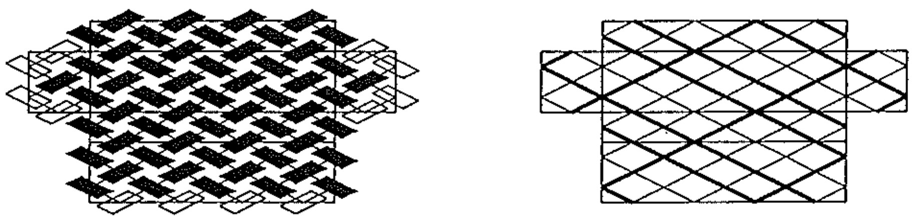


Fig. 679 — An over-under coded $1 \times 2 \times 4$ Rectangular Right Prismatic Braid.

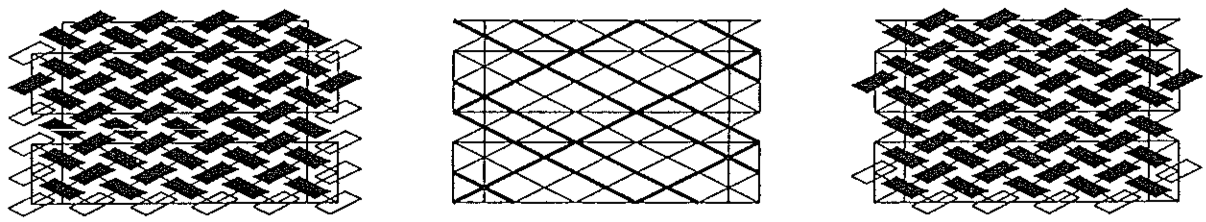


Fig. 680 — An over-under coded $1 \times 2 \times 4$ Rectangular Right Prismatic Braid.

Example 3:

The two development methods for an over-under coded braid of the type B) — *iv*) with $n = 1$ and $m = 1$ have already been depicted in Figs. 674 & 675.

Example 4:

The two development methods for an over-under coded braid of the type A) — *iv*) with $n = 1$ and $m = 1$ are depicted in Figs. 681 & 682.

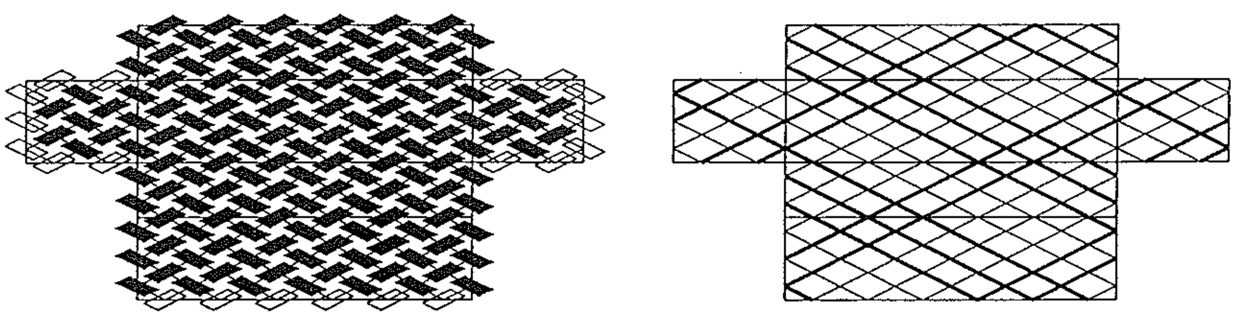


Fig. 681 — An over-under coded $2 \times 3 \times 6$ Rectangular Right Prismatic Braid.

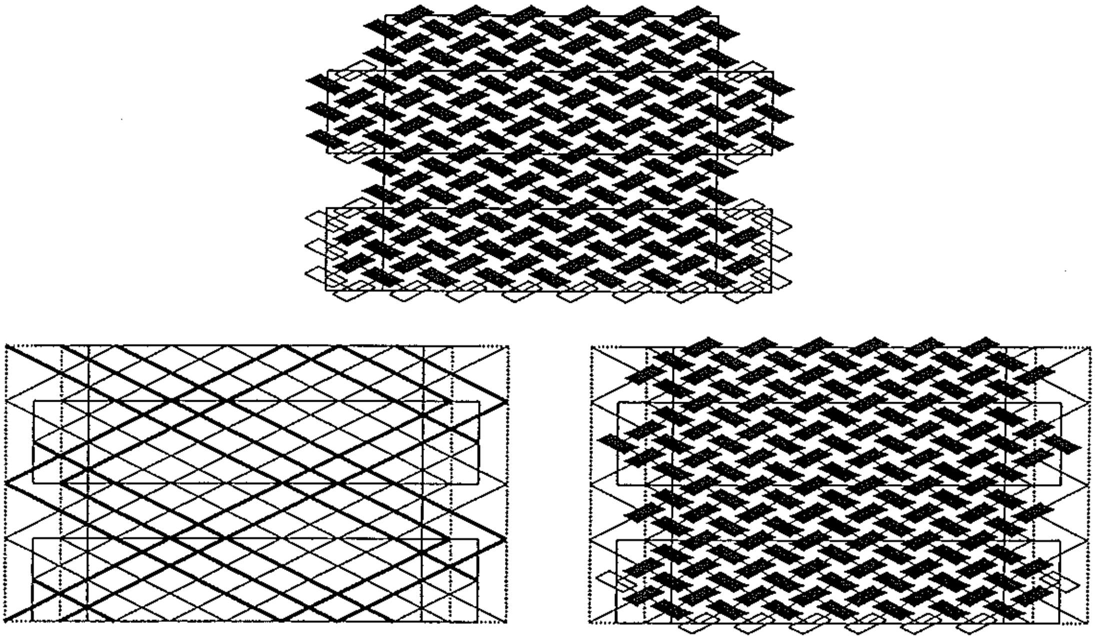


Fig. 682 — An over-under coded $2 \times 3 \times 6$ Rectangular Right Prismatic Braid.

Example 5 :

The two development methods for an over-under coded braid of the type A) — *iv*) with $n = 2$ and $m = 1$ are depicted in Figs. 683 & 684.

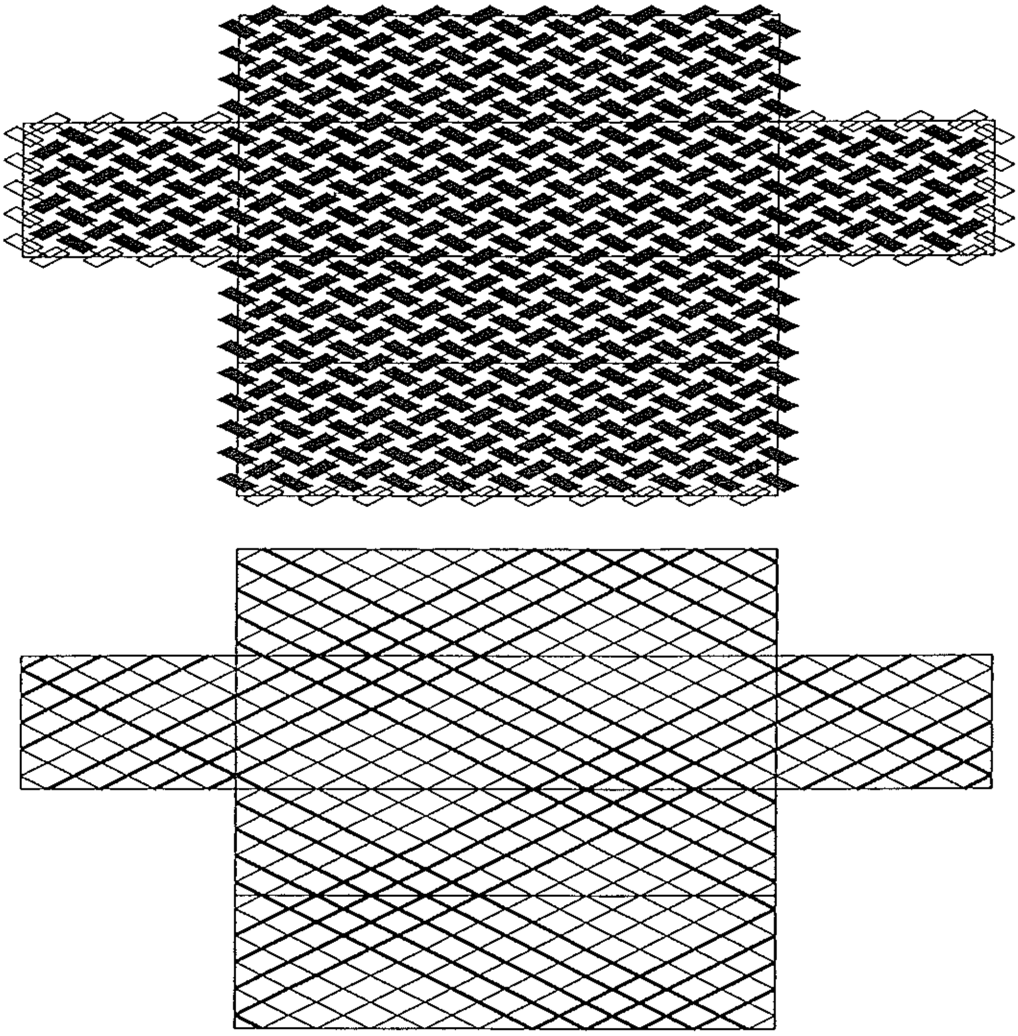


Fig. 683 — An over-under coded $4 \times 5 \times 10$ Rectangular Right Prismatic Braid.

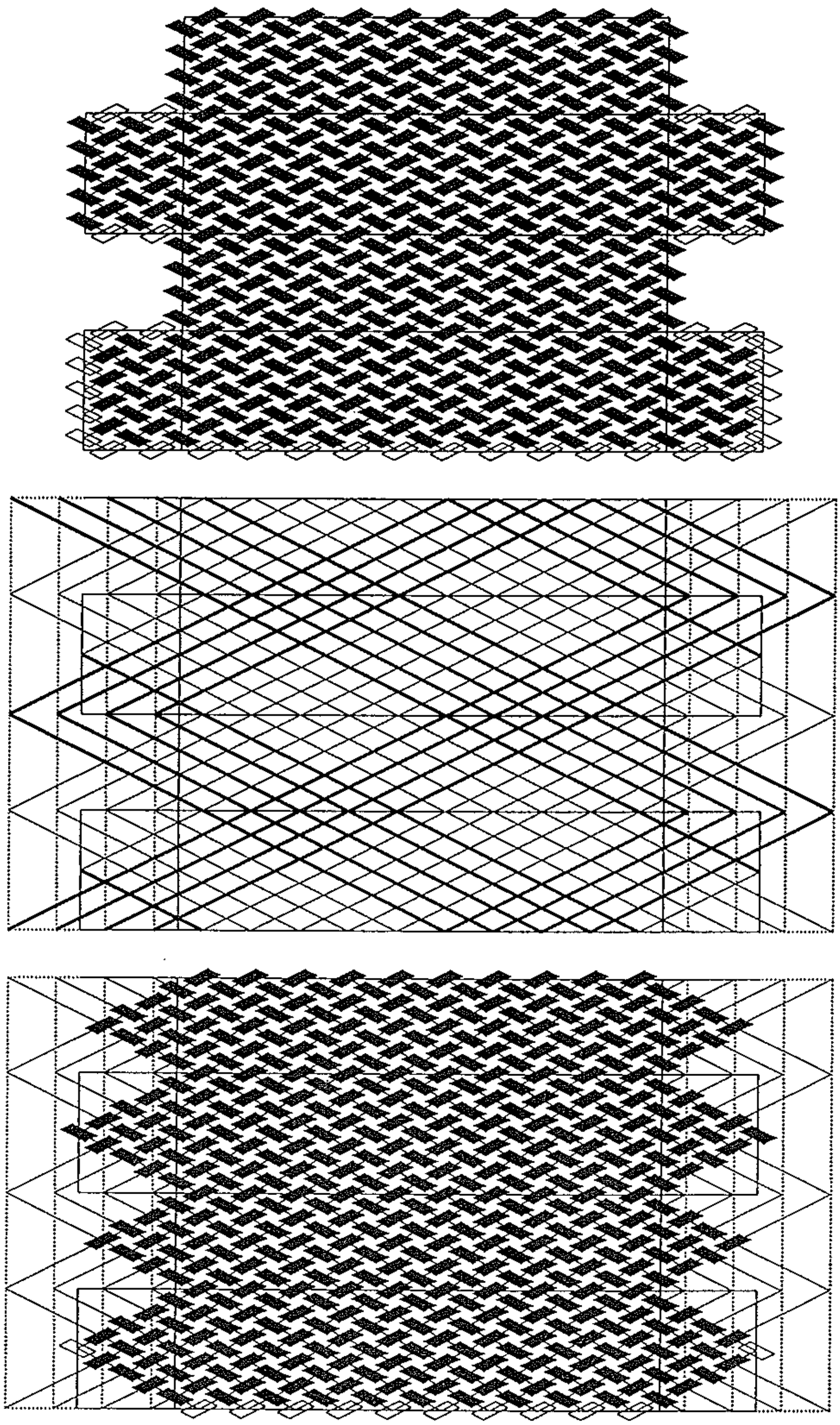


Fig. 684 — An over-under coded $4 \times 5 \times 10$ Rectangular Right Prismatic Braid.

Example 6:

The string-run of the type A)—*i*) with $n = 2$ and $m = 1$, B)—*ii*) with $n = 2$ and $m = 2$, B)—*i*) with $n = 2$ and $m = 1$, A)—*ii*) with $n = 2$ and $m = 2$, are respectively depicted in Fig. 685.

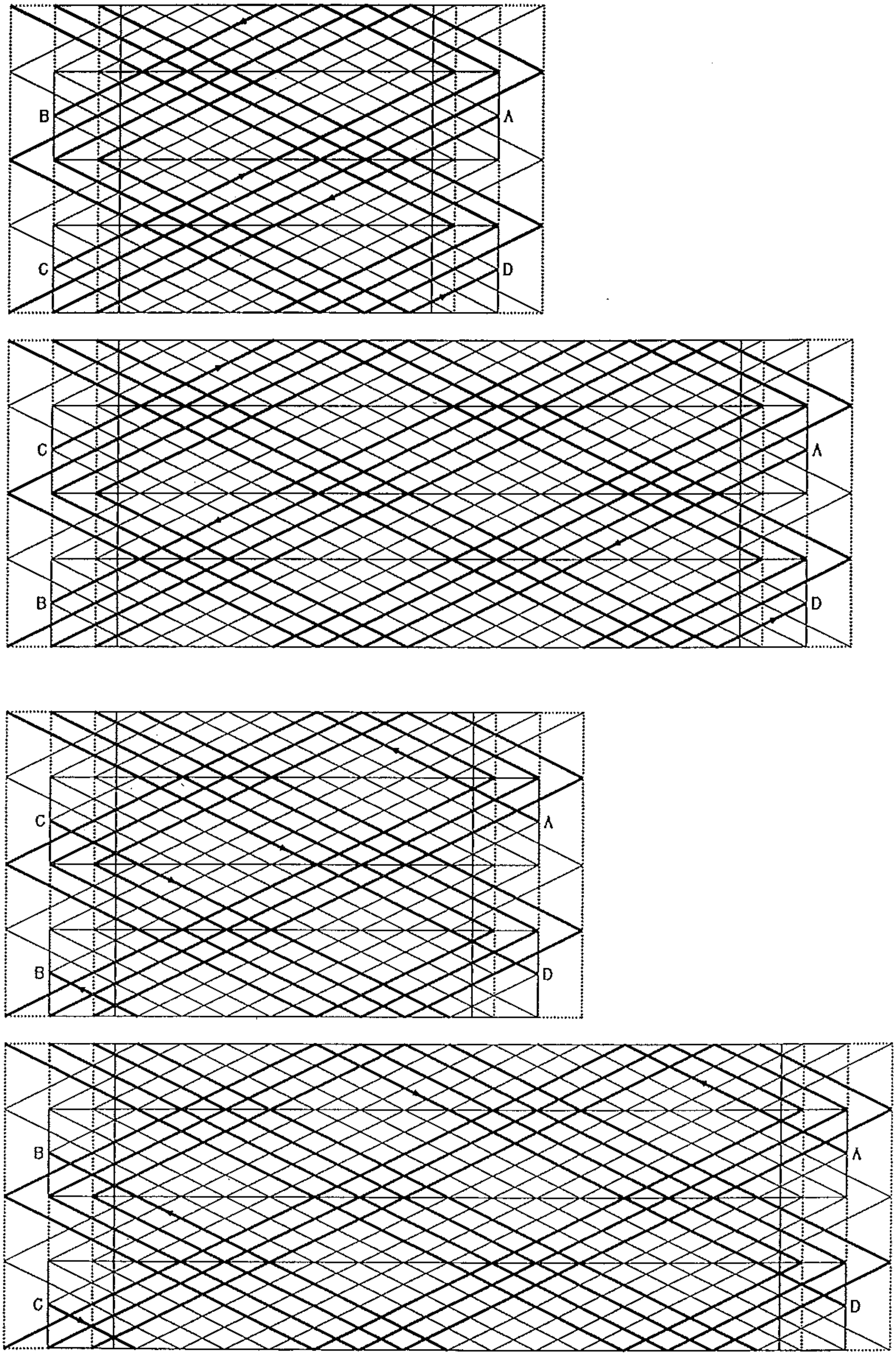


Fig. 685 — The respective string-runs of the Rectangular Right Prismatic Braids $3 \times 4 \times 7$, $3 \times 4 \times 14$, $3 \times 4 \times 8$, $3 \times 4 \times 15$.

Example 7:

The two development methods for an $1 \times 3 \times 4$ over-under coded Rectangular Right Prismatic Braid are depicted in Figs. 686 & 687.

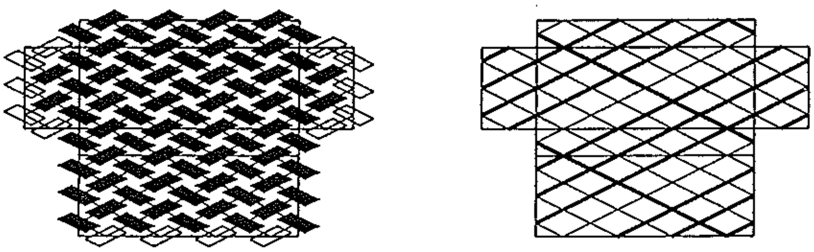


Fig. 686 — An over-under coded $1 \times 3 \times 4$ Rectangular Right Prismatic Braid.

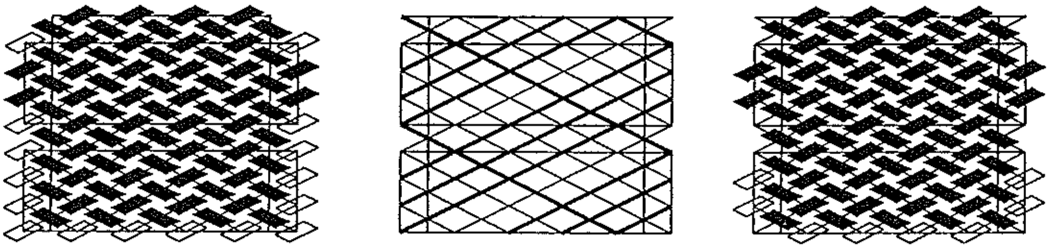


Fig. 687 — An over-under coded $1 \times 3 \times 4$ Rectangular Right Prismatic Braid.

This braid is of course identical to the identically coded $3 \times 4 \times 1$ Rectangular Right Prismatic Braid, hence its string-run is of the type A)—*ii*) with $n = 2$ and $m = 1$; its two developments are shown in Figs. 688 & 689.

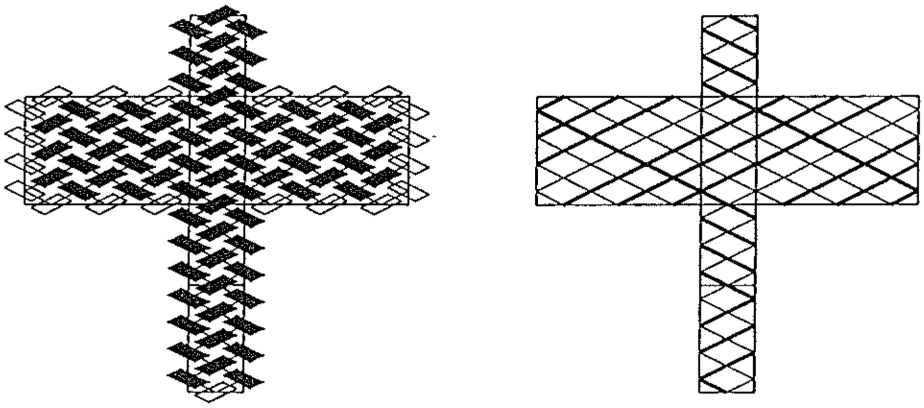


Fig. 688 — An over-under coded $3 \times 4 \times 1$ Rectangular Right Prismatic Braid.

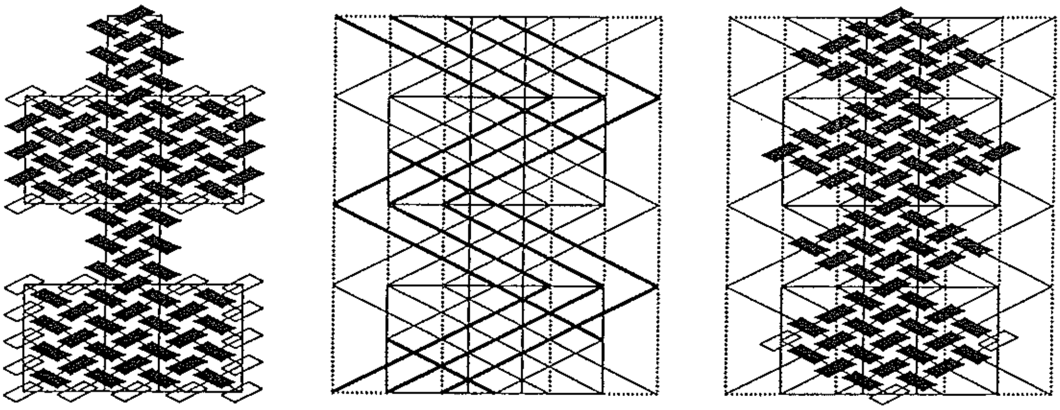


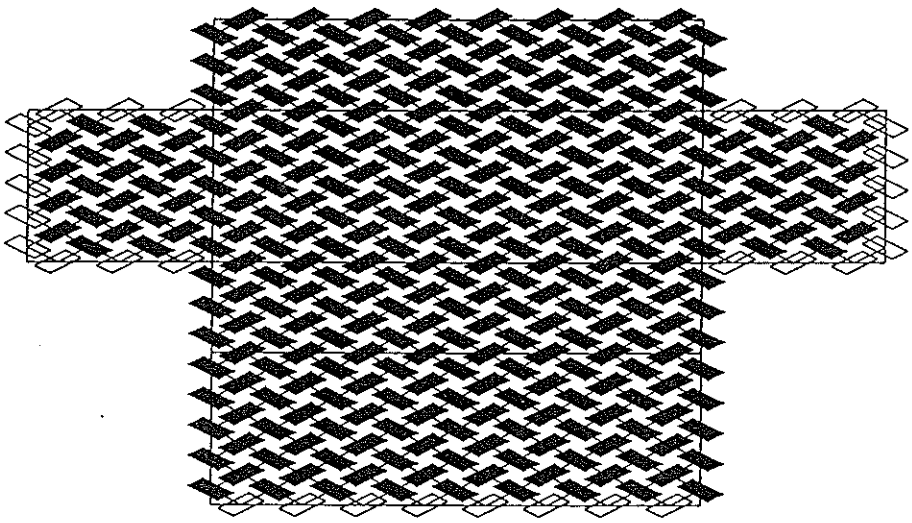
Fig. 689 — An over-under coded $3 \times 4 \times 1$ Rectangular Right Prismatic Braid.

The string-run diagrams, and hence the grid-diagrams, of the previous Examples 1-7 could be drawn with one boundary crossing-point on the left and the right bight-edge each. This will only be possible when for the Rectangular Right Prismatic Braid with sides a, b and c the difference between two of these sides is equal to 1. In the more general case, where there are no two sides which differ by 1, this will not be possible. The string-run of the over-under coded $3 \times 5 \times 8$ Rectangular Right Prismatic Braid in Example 8 requires again a minimum of two strings (the heavy lined string-run is the string-run of one of these two essential strings) but its string-run can be drawn with a minimum of two boundary crossing-points on the left and the right bight-edge each.

A rectangular box (a rectangular right prism) can be developed in six ways by the conventional development method and hence also in six ways by our special development method. For our special development method this has been shown in Example 9, Figs. 692-697, for a Rectangular Right Prismatic Braid which again requires two essential strings.

Example 8:

The two developments for an over-under coded $3 \times 5 \times 8$ Rectangular Right Prismatic Braid are depicted in Figs. 690 & 691.



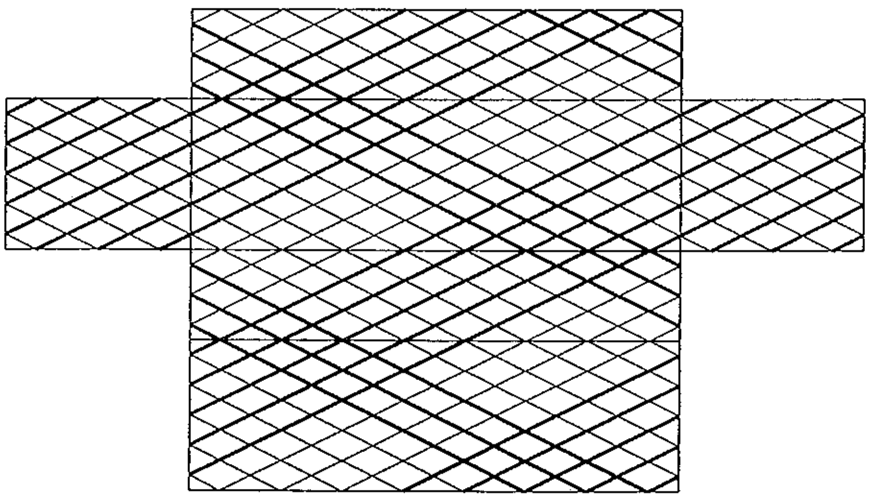


Fig. 690 — An over-under coded $3 \times 5 \times 8$ Rectangular Right Prismatic Braid.

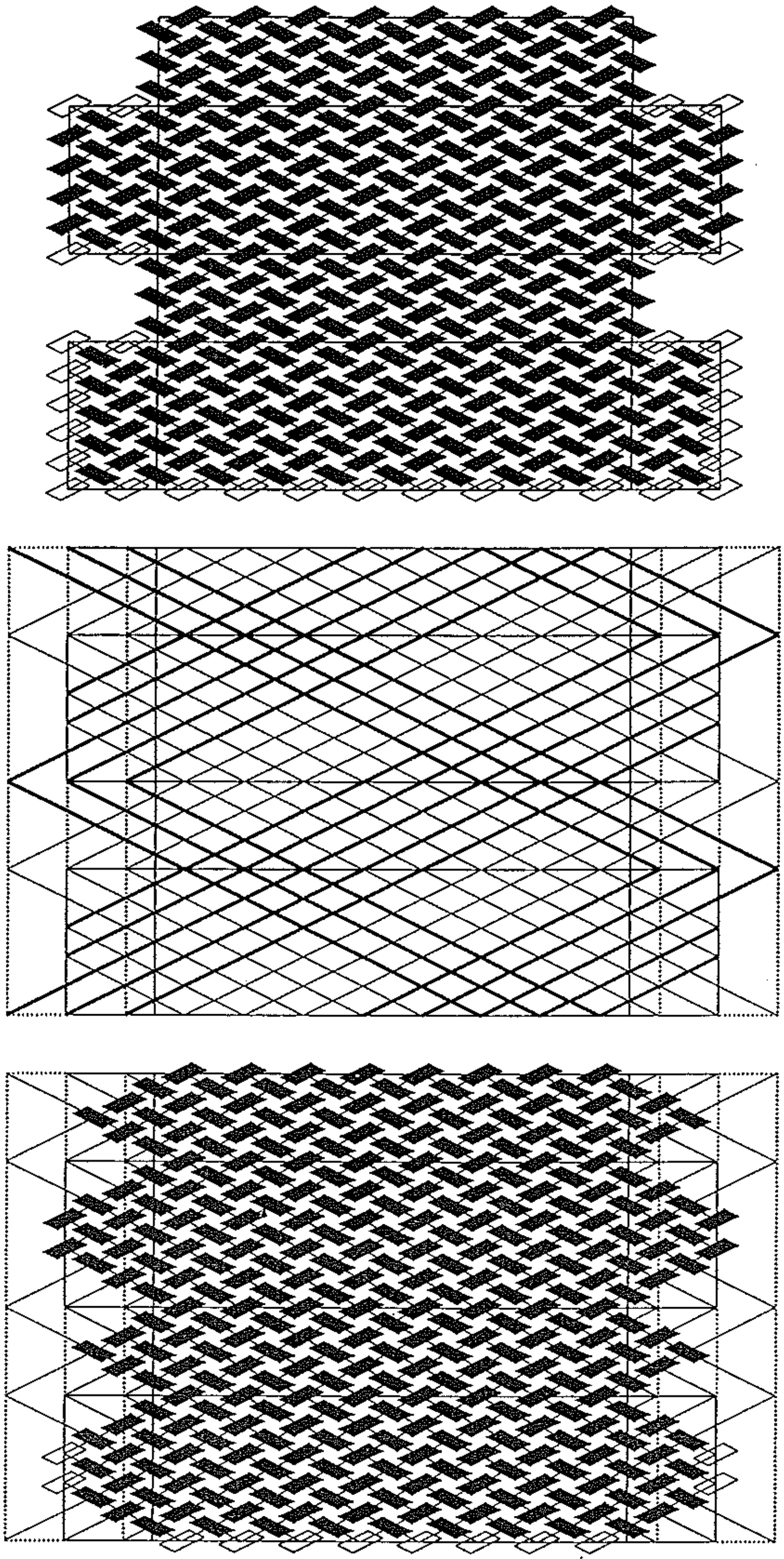


Fig. 691 — An over-under coded $3 \times 5 \times 8$ Rectangular Right Prismatic Braid.

Example 9 :

The six special developments for the string-run of a Rectangular Right Prismatic Braid with sides 3 , 8 and 11 are depicted in Figs. 692-697. Since all these six string-runs are the string-runs of the same braid, some braiders may prefer to use in the actual construction of the braid the string-run with the minimum number of boundary cossing-points, while others may prefer the string-run with the minimum number of nested bights in each nest.

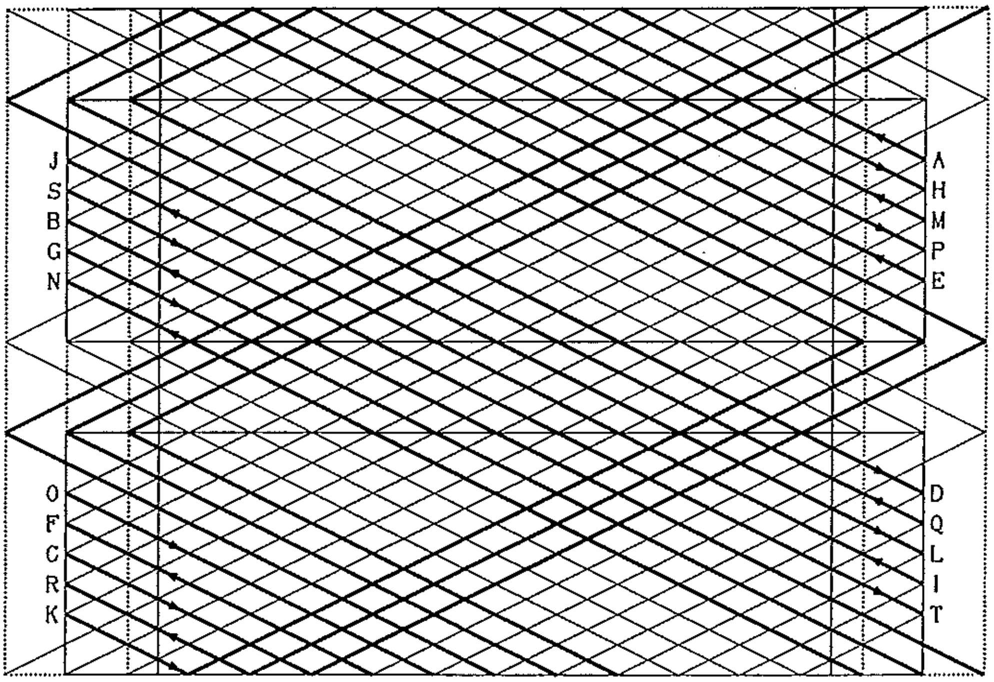


Fig. 692 — The string-run of a $3 \times 8 \times 11$ Rectangular Right Prismatic Braid.

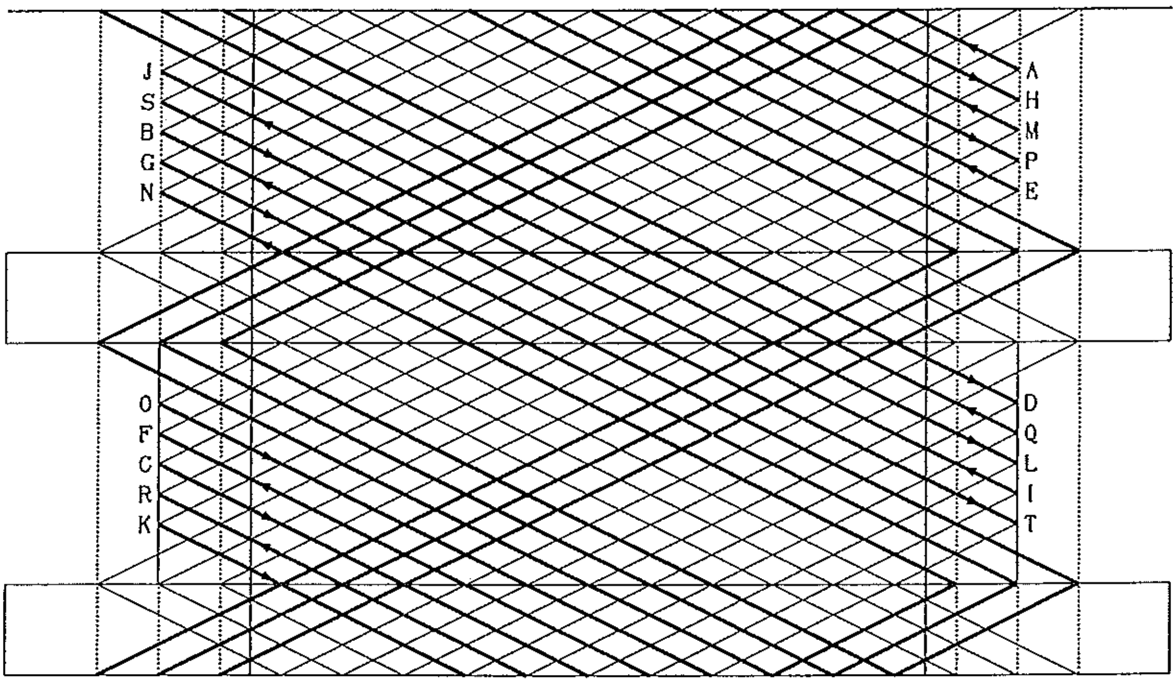


Fig. 693 — The string-run of an $8 \times 3 \times 11$ Rectangular Right Prismatic Braid.

The string-runs in the string-run diagrams of Figs. 692 and 693 have five boundary crossing-points on the left and right bight-edge each. This number of boundary crossing-points is the absolute value of the difference between the sides 3 and 8 ($|8-3| = 5$, $|3-8| = 5$).

Note that the string-run diagrams in Figs. 692 and 693 are in fact identical and only differ in the development, and hence the position, of the rectangular surfaces of the Rectangular Right Prismatic Braid. The same applies to the string-run diagrams in Figs. 694 and 695, as well as to the string-run diagrams in Figs. 696 and 697.

Note furthermore that in the string-run diagrams of Figs. 692 and 693 the sum of **bights per nest** and **boundary crossing-points per bight-edge** equals $3 + 5 = 8$.

The three boundary crossing-points on the left and right bight-edge each in the string-run diagrams of Figs. 694 and 695 are the absolute value of the difference between the sides 8 and 11 ($|11 - 8| = 3$, $|8 - 11| = 3$).

Note that in the string-run diagrams of Figs. 694 and 695 the sum of **bights per nest** and **boundary crossing-points per bight-edge** equals $8 + 3 = 11$.

Since three boundary crossing-points on the left and right bight-edge each is the minimum number for this braid, some braiders may in the actual braiding process prefer to use the string-runs depicted in Figs. 694 and 695 for this Rectangular Right Prismatic Braid.

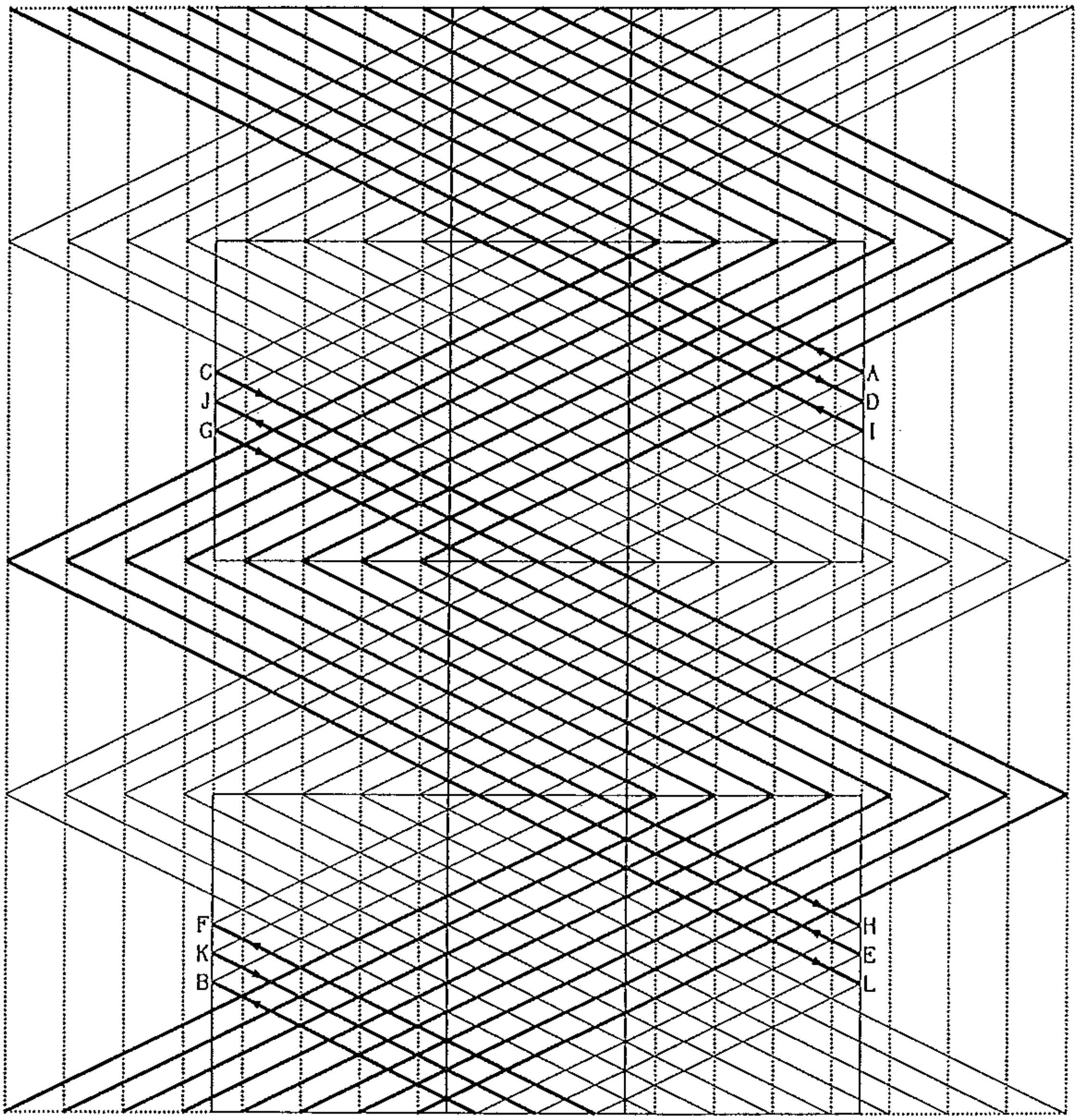


Fig. 694 — The string-run of an $8 \times 11 \times 3$ Rectangular Right Prismatic Braid.

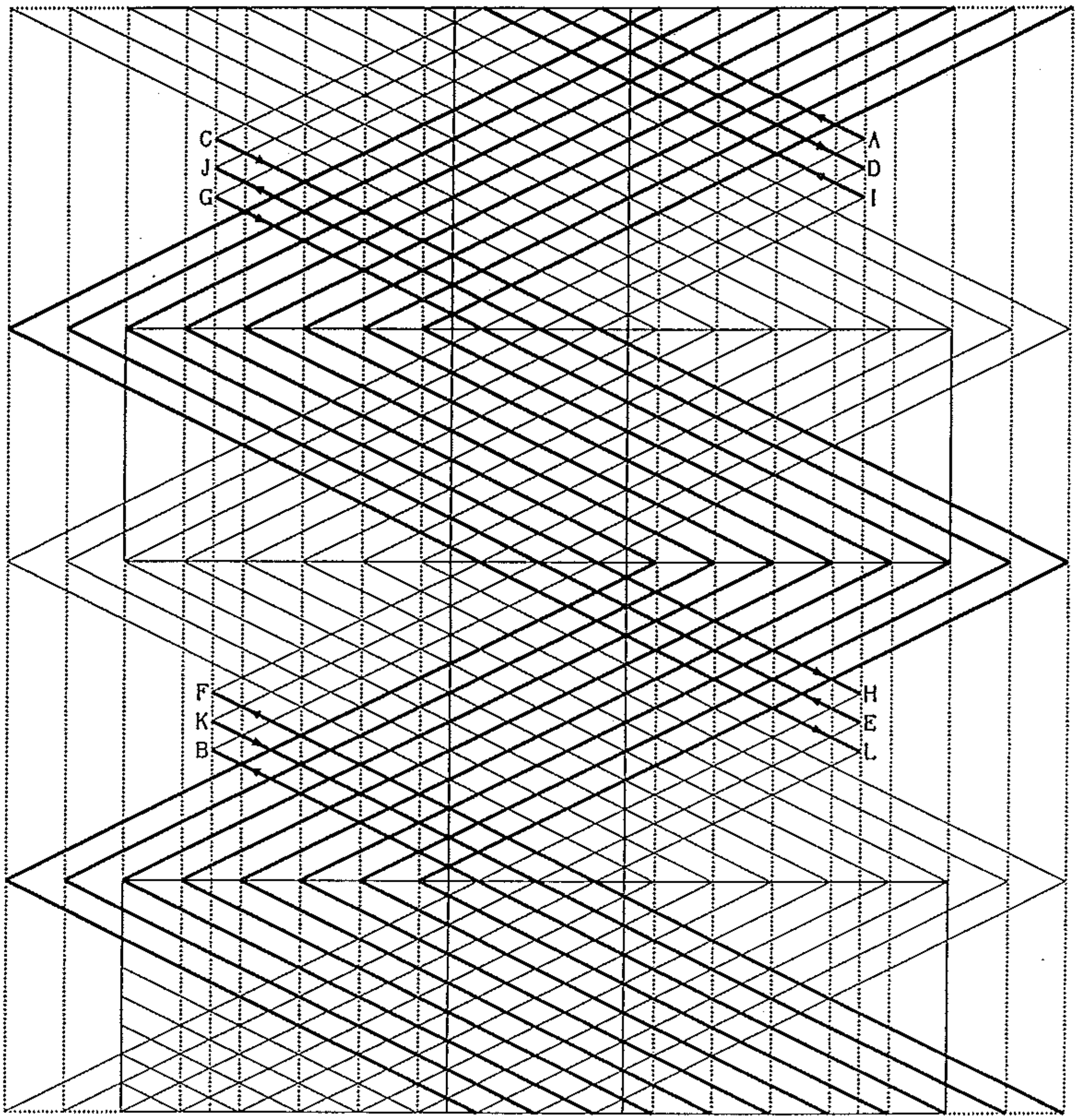


Fig. 695 — The string-run of an $11 \times 8 \times 3$ Rectangular Right Prismatic Braid.

The eight boundary crossing-points on the left and right bight-edge each in the string-run diagrams of Figs. 696 and 697 are the absolute value of the difference between the sides 3 and 11 ($|11-3| = 8$, $|3-11| = 8$).

Note that in the string-run diagrams of Figs. 696 and 697 the sum of **bights per nest** and **boundary crossing-points per bight-edge** equals $3 + 8 = 11$.

In the actual braiding process of this Rectangular Right Prismatic Braid we would not use the string-runs depicted in Figs. 696 and 697 since the number of bights per nest is the same as in the string-runs of Figs. 692 and 693, but the number of boundary crossing-points is eight per bight-edge against only five in the string-runs of Figs. 692 and 693.

The string-runs in Figs. 692 and 693 present a compromise between the minimum number of boundary crossing-points per bight-edge and the minimum number of bights per nest; their sum [(bights per nest) + (boundary crossing-points per bight-edge)] is a minimum.

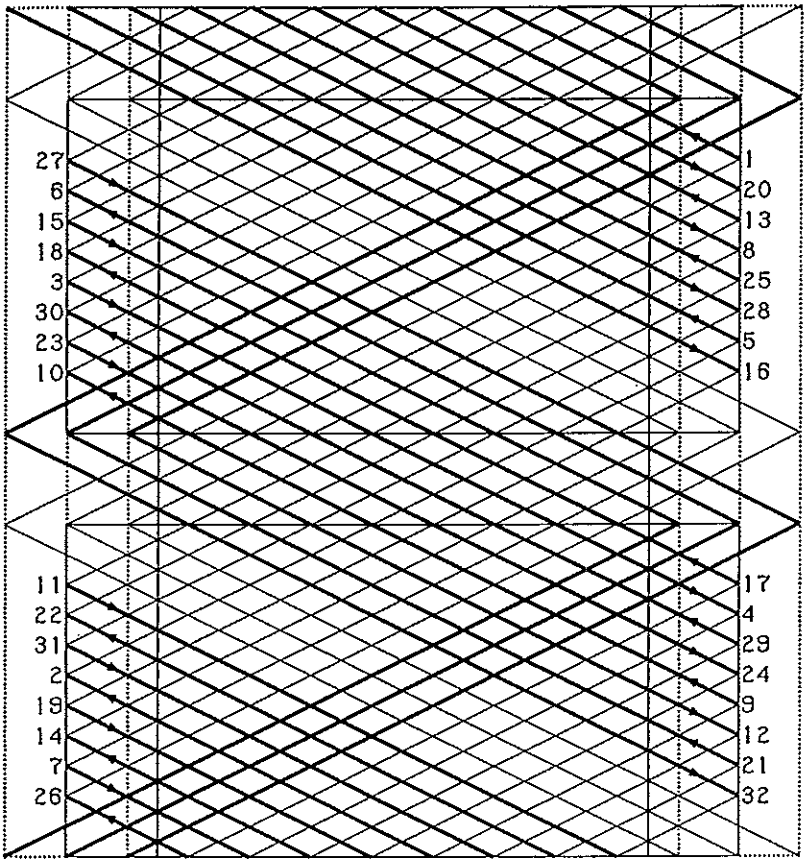


Fig. 696 — The string-run of a $3 \times 11 \times 8$ Rectangular Right Prismatic Braid.

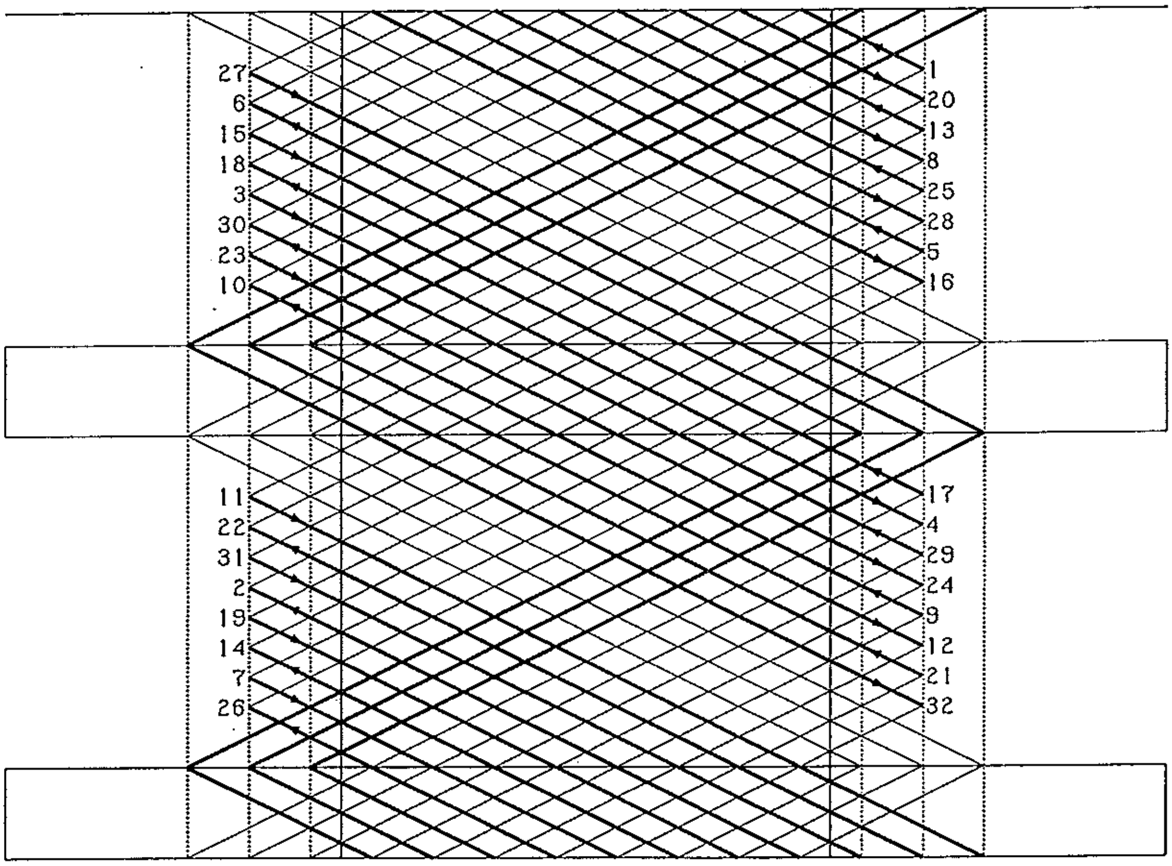


Fig. 697 — The string-run of an $11 \times 3 \times 8$ Rectangular Right Prismatic Braid.

Special Rectangular Right Prismatic Braids are those which have two equal sides. They have a string-run diagram which can be drawn as $a \times a \times c$, as $a \times c \times a$ or as $c \times a \times a$; the last two are identical if we don't represent in these diagrams the development of the surfaces. We have already met earlier† the string-run diagram drawn as $a \times a \times c$ as a string-run diagram of a **Regular Nested Cylindrical Braid** with $A = a$; $x = 2c + 2$; $y = 0$; $B^* = 4$.

From the (A, x, y)-tables of these braids, it immediately follows that for $y = 0$ the only braids with a minimum of two essential strings are the braids with $A = 1$ and $x = 4n + 2$, where $n = 0, 1, 2, 3, \cdots$. Since $x = 2c + 2$, it follows that the only $a \times a \times c$ Rectangular Right Prismatic Braids with a minimum of two essential strings are the ones with $a = 1$ and $c = 2n$, where $n = 0, 1, 2, 3, \cdots$.

In order for the reader to fully understand the two types of string-run diagrams $a \times a \times c$, and $a \times c \times a$ or $c \times a \times a$, we have depicted in Figs. 698 and 699 the string-runs for respectively the $3 \times 11 \times 3$ Rectangular Right Prismatic Braid and the $3 \times 3 \times 11$ Rectangular Right Prismatic Braid. Note that these braids are of course identical and have the components 2×10 and 1×8 (which follows immediately from the $(3, x, y)$-table for $x = 2c+2 = 24$ and $y = 0$). Since g.c.d. $(10, B^*) =$ g.c.d. $(10, 4) = 2$, the component with $P_c = 10$ requires 2 essential strings (hence the two components with $P_c = 10$ require 4 essential strings), and since g.c.d. $(8, B^*) =$ g.c.d. $(8, 4) = 4$, the component with $P_c = 8$ requires 4 essential strings (hence the one component with $P_c = 8$ requires 4 essential strings); thus in total the braid requires $4 + 4 = 8$ essential strings.

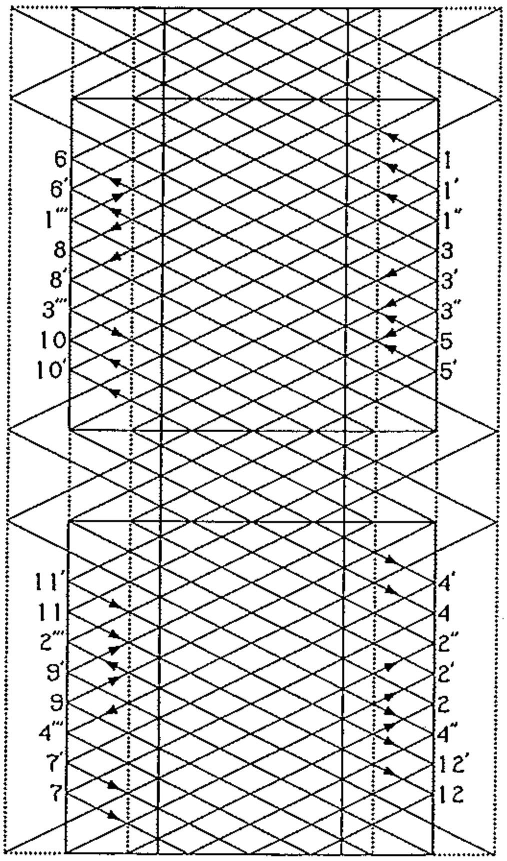


Fig. 698 — The string-run diagram of a $3 \times 11 \times 3$ Rectangular Right Prismatic Braid.

† See *The Braider*, Issue No. 25.

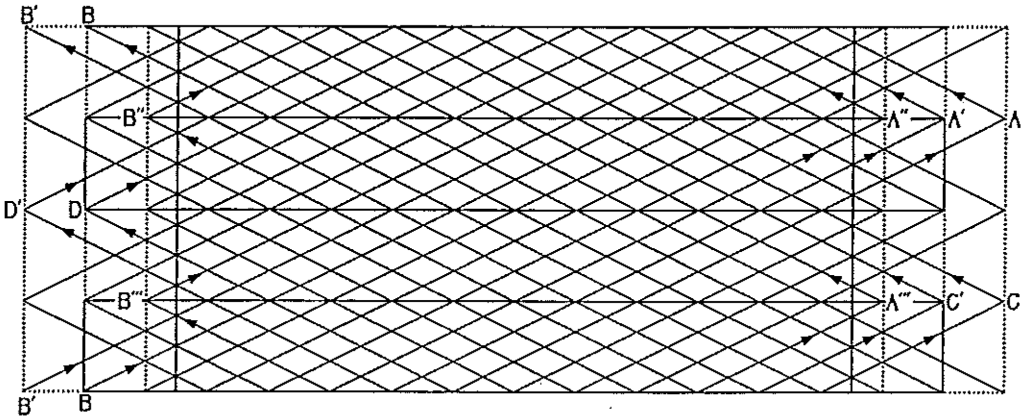


Fig. 699 — The string-run diagram of a $3 \times 3 \times 11$ Rectangular Right Prismatic Braid.

An example of the string-run diagram of a $c \times a \times a$ Rectangular Right Prismatic Braid with $a = 3$ and $c = 12$ is shown in Fig. 700. From the $(3, x, y)$-table for $x = 2c + 2 = 26$ and $y = 0$ it follows that this braid has the components 3×10 (3 components with $P_c = 10$ each), hence it requires six essential strings.

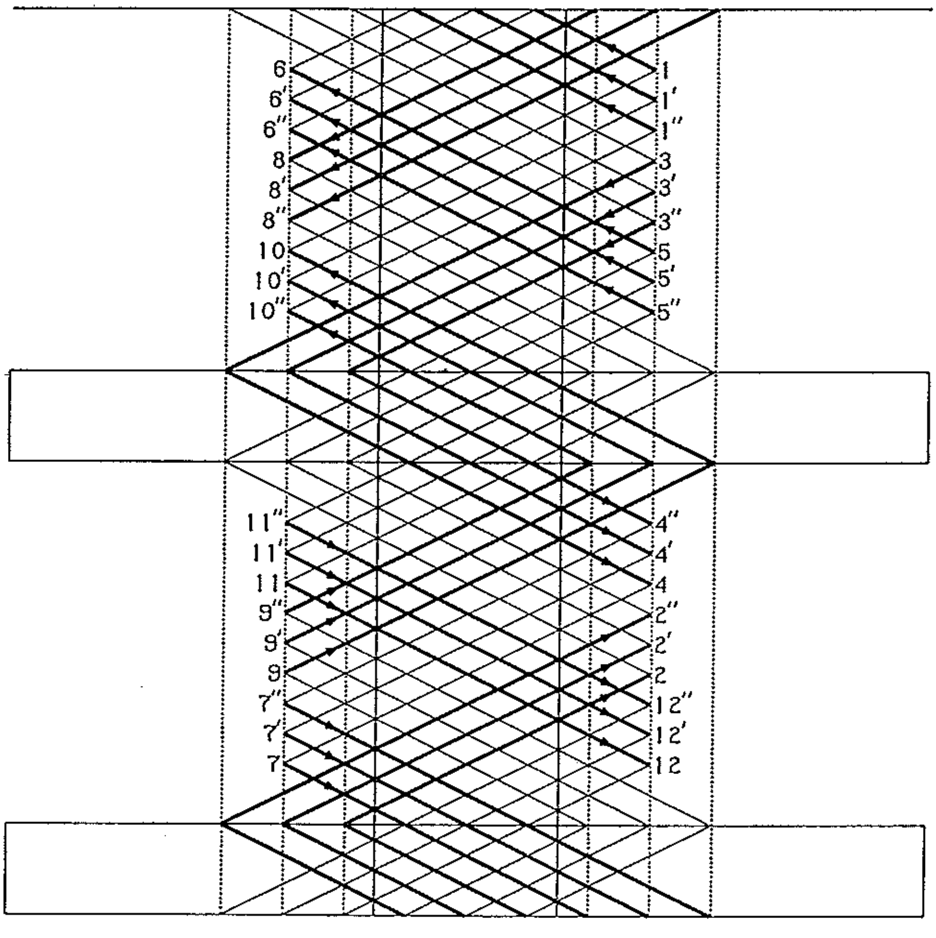


Fig. 700 — The string-run diagram of a $12 \times 3 \times 3$ Rectangular Right Prismatic Braid.

For the determination of the values a, b and c in the $a \times b \times c$ Rectangular Right Prismatic Braids, we shall restrict our further discussion here to $a \times b \times c$ Rectangular Right Prismatic Braids which require two essential strings in their construction.

From the string-run diagrams associated with our special development method it follows that if a Rectangular Right Prismatic Braid $a \times b \times c$ requires two essential strings only, then the Rectangular Right Prismatic Braid $a \times b \times \{c + n(a+b)\}$, where $n = 1, 2, 3, \cdots$, requires only two essential strings as well.

A special limit case of the $a \times b \times c$ Rectangular Right Prismatic Braid is where $c = 0$. Since the string-run of this braid is identical to the string-run of the $0 \times a \times b$ Rectangular Right Prismatic Braid, the string-run of a $0 \times a \times b$ Rectangular Right Prismatic Braid is equivalent to the string-run of an $a \times b \times 0$ Rectangular Right Prismatic Braid. The string-run in the left-hand diagram of Fig. 701 depicts an example where $a = 3$ and $b = 7$.

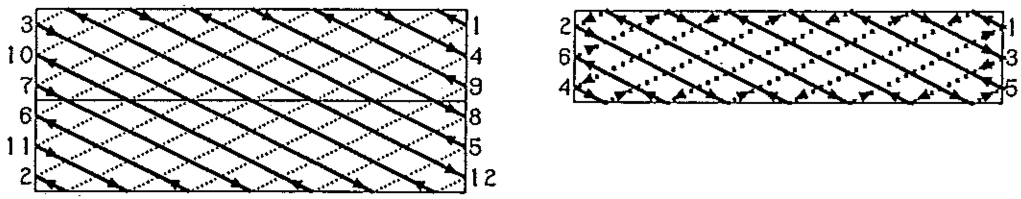


Fig. 701 — The string-run of a $0 \times 3 \times 7$ Rectangular Right Prismatic Braid.

Note that in this diagram there are no nests with bights (indicated by the lowest of the two values 0 and 3, hence 0 bights in the nests, hence no nests with bights, consequently bight-edges with no bights), and that there are $|3 - 0| = 3$ boundary crossing-points on the bight-edges with zero bights. Thus on the right-hand bight-edge point 1 is point 12, point 4 is point 5 and point 9 is point 8, while on the left-hand bight-edge point 3 is point 2, point 10 is point 11 and point 7 is point 6. In Fig. 701 crossing point 1 in the right-hand diagram is crossing-point $1 = 12$ in the left-hand diagram, crossing point 3 in the right-hand diagram is crossing-point $4 = 5$ in the left-hand diagram, crossing point 5 in the right-hand diagram is crossing-point $9 = 8$ in the left-hand diagram, crossing point 2 in the right-hand diagram is crossing-point $3 = 2$ in the left-hand diagram, crossing point 6 in the right-hand diagram is crossing-point $10 = 11$ in the left-hand diagram, crossing point 4 in the right-hand diagram is crossing-point $7 = 6$ in the left-hand diagram. Note that in Fig. 701 the heavy lines depict in the left-hand diagram half the total string-run, and that in the right-hand diagram the heavy lines together with the dotted heavy lines depict half the total string-run.

The right-hand diagram of Fig, 701 can be compared with the string-run of a **Regular Rectangular Mat.** The difference is that the number of essential strings in this right-hand diagram of Fig. 701 is twice the number of essential strings in its Regular Rectangular Mat counter part. Thus we have here a simple method for determining the number of essential stings in an $a \times b \times 0$ Rectangular Right Prismatic Braid, namely the number of essential strings in an $a \times b \times 0$ Rectangular Right Prismatic Braid is equal to twice the number of essential strings in a Regular Rectangular Mat with b bights horizontal and a bights vertical. We have seen in *The Braider*, Issue No. 10 that such a Regular Rectangular Mat can be converted into a **Regular Cylindrical Braid** with b parts and a bights. Hence for the $a \times b \times 0$ Rectangular Right Prismatic Braid to require two essential strings only, the condition g.c.d. $(a, b) = 1$ must apply; in other words: a and b must be coprime.

The string-run diagrams associated with our special development method have already disclosed that if a Rectangular Right Prismatic Braid $a \times b \times c$ requires two essential strings only, the Rectangular Right Prismatic Braid $a \times b \times \{c + n(a+b)\}$, where $n = 1, 2, 3, \cdots$, requires only two essential strings as well (see top of this page).

Furthermore, these string-run diagrams clearly show that for $|a-b|+1 \leq c \leq a+b-1$, the string-run requires more than two essential strings.

⋆⋆ *Show how this last statement is derived from these string-run diagrams.*

In order to determine whether or not an $a \times b \times c$ Rectangular Right Prismatic Braid requires only two essential strings, we can employ the following method:

Let the values for a and b be given. We have then to determine which values for c will give us a Rectangular Right Prismatic Braid which requires two essential strings only. Since we can use the relationship:

if an $a \times b \times c$ Rectangular Right Prismatic Braid requires two essential strings only, then an $a \times b \times \{c + n(a+b)\}$ Rectangular Right Prismatic Braid, where $n = 1, 2, 3, \cdots$, also requires two essential strings only,

we can restrict our basic c-value determination to $0 \leq c \leq |a-b|$. Recall that for $|a-b|+1 \leq c \leq a+b-1$, the two essential strings only condition is not being fulfilled.

Example 10:

Does the $3 \times 11 \times 16$ Rectangular Right Prismatic Braid require two essential strings only? What are the basic values for c which would give a two essential string $3 \times 11 \times c$ Rectangular Right Prismatic Braid?

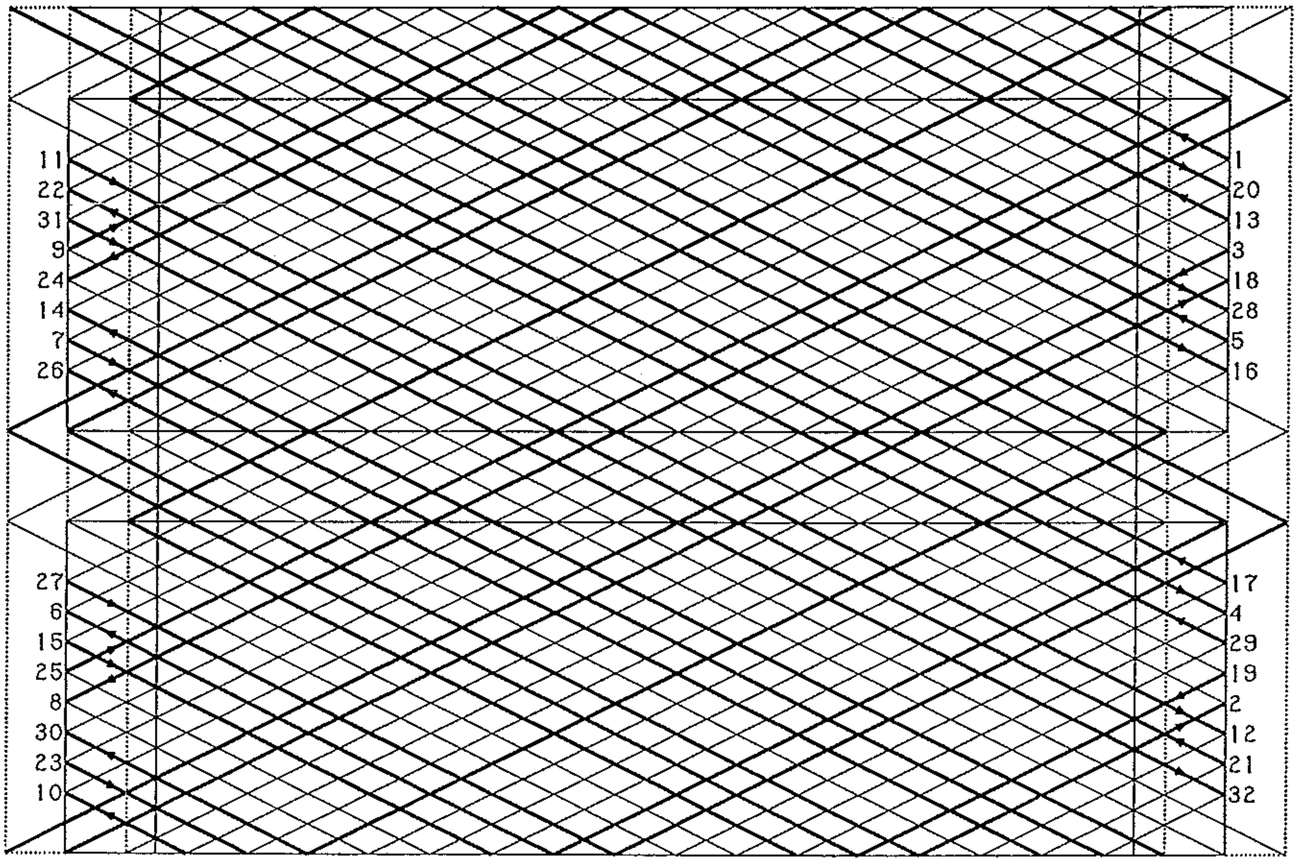


Fig. 702 — The string-run of a $3 \times 11 \times 16$ Rectangular Right Prismatic Braid.

Although we have already shown in Fig. 702 the string-run of a $3 \times 11 \times 16$ Rectangular Right Prismatic Braid, and from this string-run we can determine that it indeed requires two essential strings only, we would of course first have ensured via the calculations below, in which we can restrict the basic c-values to $0 \leq c \leq |3-11|$, hence to $0 \leq c \leq 8$, that this was the case.

3 and 11 are coprime, hence $3\times11\times0$ fulfills the two essential strings only condition.

$3 \times 11 \times 1$ is identical to $1 \times 3 \times 11$, which reduces to $1 \times 3 \times 3$ $(c - 2(a + b) = 11 - 2(1+3) = 3)$. Since this braid contains two identical sides other than 1 with a third side other than 0, the braid requires more than two essential strings. Consequently, the braid $3 \times 11 \times 1$ does not fulfil the two essential strings only condition.

$3 \times 11 \times 2$ is identical to $2 \times 3 \times 11$, which reduces to $2 \times 3 \times 1$ $(c - 2(a + b) = 11 - 2(2 + 3) = 1)$. $2 \times 3 \times 1$ is identical to $1 \times 2 \times 3$, which reduces to $1 \times 2 \times 0$ $(c - 1(a + b) = 3 - 1(1 + 2) = 0)$, and since 1 and 2 are coprime, the $1 \times 2 \times 0$ fulfills the two essential strings only condition. Consequently, the braid $3 \times 11 \times 2$ fulfills the two essential strings only condition.

$3 \times 11 \times 3$ contains two identical sides other than 1 with a third side other than 0, hence the braid requires more than two essential strings. Hence the braid $3 \times 11 \times 1$ does not fulfil the two essential strings only condition.

$3 \times 11 \times 4$ is identical to $3 \times 4 \times 11$, which reduces to $3 \times 4 \times 4$ $(c - 1(a + b) = 11 - 1(3 + 4) = 4)$. $3 \times 4 \times 4$ contains two identical sides other than 1 with a third side other than 0, hence the braid requires more than two essential strings. Hence the braid $3 \times 11 \times 4$ does not fulfil the two essential strings only condition.

$3 \times 11 \times 5$ is identical to $3 \times 5 \times 11$, which reduces to $3 \times 5 \times 3$ $(c - 1(a + b) = 11 - 1(3 + 5) = 3)$. $3 \times 11 \times 3$ contains two identical sides other than 1 with a third side other than 0, hence the braid requires more than two essential strings. Hence the braid $3 \times 11 \times 5$ does not fulfil the two essential strings only condition.

$3 \times 11 \times 6$ is identical to $3 \times 6 \times 11$, which reduces to $3 \times 6 \times 2$ $(c - 1(a + b) = 11 - 1(3 + 6) = 2)$. $3 \times 6 \times 2$ is identical to $2 \times 3 \times 6$, which reduces to $2 \times 3 \times 1$ $(c - 1(a + b) = 6 - 1(2 + 3) = 1)$. $2 \times 3 \times 1$ is identical to $1 \times 2 \times 3$, which reduces to $1 \times 2 \times 0$ $(c - 1(a + b) = 3 - 1(1 + 2) = 0)$, and since 1 and 2 are coprime, the $1 \times 2 \times 0$ fulfills the two essential strings only condition. Consequently, the braid $3 \times 11 \times 6$ fulfills the two essential strings only condition.

$3 \times 11 \times 7$ is identical to $3 \times 7 \times 11$, which reduces to $3 \times 7 \times 1$ $(c - 1(a + b) = 11 - 1(3 + 7) = 1)$. $3 \times 7 \times 1$ is identical to $1 \times 3 \times 7$, which reduces to $1 \times 3 \times 3$ $(c - 1(a + b) = 7 - 1(1 + 3) = 3)$. $1 \times 3 \times 3$ contains two identical sides other than 1 with a third side other than 0, hence the braid requires more than two essential strings. Hence the braid $3 \times 11 \times 7$ does not fulfil the two essential strings only condition.

$3 \times 11 \times 8$: since 3 and 8 are coprime, $3 \times 8 \times 0$ fulfills the two essential strings only condition and consequently so does $3 \times 8 \times 11$ $(c + n(a + b) = 0 + 1(3 + 8) = 11)$. $3 \times 8 \times 11$ is identical to $3 \times 11 \times 8$, hence $3 \times 11 \times 8$ fulfills the two essential strings only condition.

Since $3 \times 11 \times 2$ fulfills the two essential strings only condition, so does $3 \times 11 \times 16$ $(c + n(a + b) = 2 + 1(3 + 11) = 16)$ fulfil the two essential strings only condition.

The basic values for c which would give a two essential string $3 \times 11 \times c$ Rectangular Right Prismatic Braid are 0, 2, 6 and 8.

Example 11:

Does the $7 \times 10 \times 2$ Rectangular Right Prismatic Braid require two essential strings only? What are the basic values for c which would give a two essential string $7 \times 10 \times c$ Rectangular Right Prismatic Braid?

We can restrict the basic values for c to $0 \leq c \leq |7 - 10|$, hence to $0 \leq c \leq 3$.

7 and 10 are coprime, hence $7\times10\times0$ fulfills the two essential strings only condition.

$7 \times 10 \times 1$ is identical to $1 \times 7 \times 10$, which reduces to $1 \times 7 \times 2$ $(c - 1(a+b) = 10 - 1(1+7) = 2)$. $1 \times 7 \times 2$ is identical to $1 \times 2 \times 7$, which reduces to $1 \times 2 \times 1$ $(c - 2(a+b) = 7 - 2(1+2) = 1)$. $1 \times 2 \times 1$ is identical to $1 \times 1 \times 2$, which reduces to $1 \times 1 \times 0$ $(c - 1(a+b) = 2 - 1(1+1) = 0)$. Since this braid contains two identical sides of 1 with a third side of 0, the braid requires two essential strings only. Consequently, the braid $7 \times 10 \times 1$ fulfills the two essential strings only condition.

$7 \times 10 \times 2$ is identical to $2 \times 7 \times 10$, which reduces to $2 \times 7 \times 1$ $(c - 1(a+b) = 10 - 1(2+7) = 1)$. $2 \times 7 \times 1$ is identical to $1 \times 2 \times 7$, which reduces to $1 \times 2 \times 1$ $(c - 2(a+b) = 7 - 2(1+2) = 1)$. $1 \times 2 \times 1$ is identical to $1 \times 1 \times 2$, which reduces to $1 \times 1 \times 0$ $(c - 1(a+b) = 2 - 1(1+1) = 0)$. Since this braid contains two identical sides of 1 with a third side of 0, the braid requires two essential strings only. Consequently, the braid $7 \times 10 \times 2$ fulfills the two essential strings only condition.

$7 \times 10 \times 3$: since 7 and 3 are coprime, $7 \times 3 \times 0$ fulfills the two essential strings only condition and consequently so does $7 \times 3 \times 10$ $(c + n(a+b) = 0 + 1(7+3) = 10)$. $7 \times 3 \times 10$ is identical to $7 \times 10 \times 3$, hence $7 \times 10 \times 3$ fulfills the two essential strings only condition.

The basic values for c which would give a two essential string $7 \times 10 \times c$ Rectangular Right Prismatic Braid are 0, 1, 2 and 3.

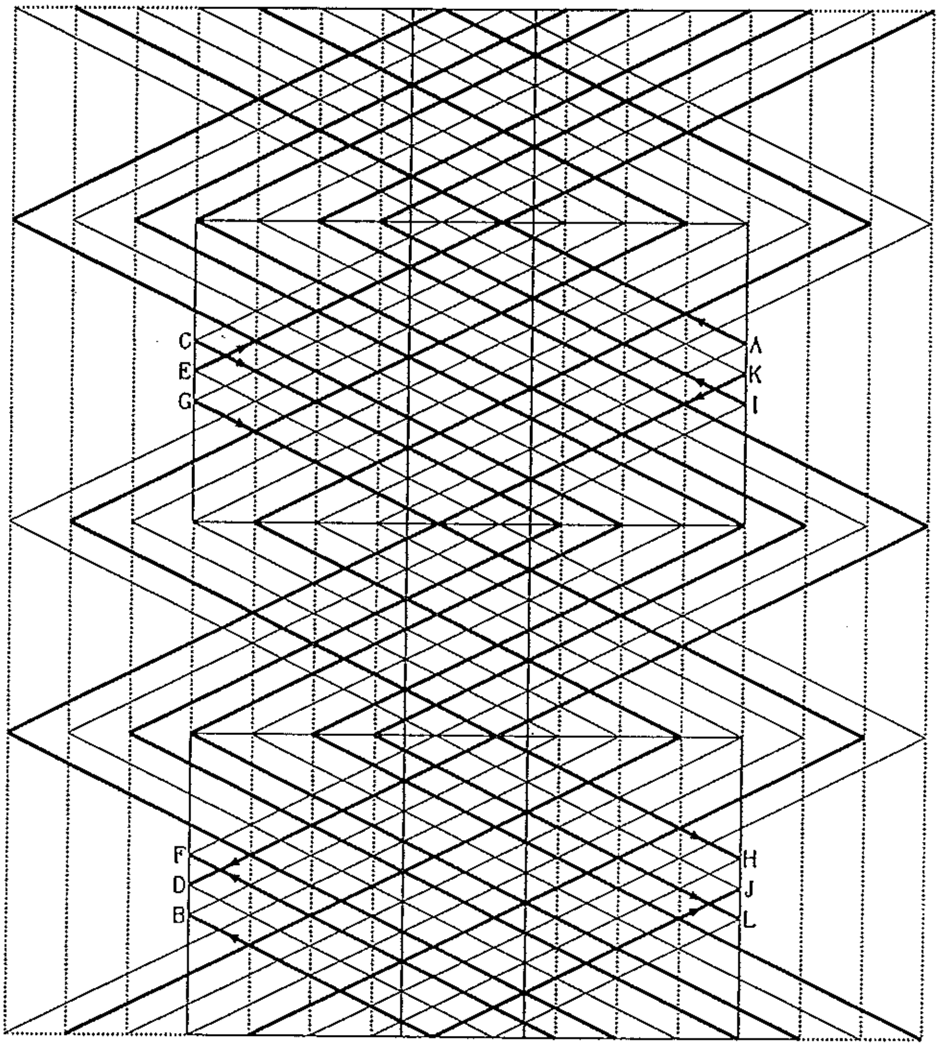


Fig. 703 — The string-run of a $7 \times 10 \times 2$ Rectangular Right Prismatic Braid.