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for
the braiding artisan

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A pair of South American Hobbels

Hobbels are used to immobilise a horse by restraining either the front legs or the back legs. They generally consist of two cuffs joined by a centrepiece. In some publications it has been stated that their measurements are usually determined as follows: a man's fist should just fit through the cuff when it is buttoned closed and the distance between the cuffs should be that of the width of the hand at the height of the knuckles if the hobbels are to be used on the front legs, and about double that if the hobbels are to be used on the back legs. It may well be true that their measurements are determined that way in some quarters, but the obvious question would be: does men with small hands ride small horses and men with big hands big horses?

The pair of hobbels we are discussing here, are owned by a fellow in Wyoming who lent them to Doug Van Tassel who in turn unravelled their construction. According to the owner, the hobbels were made by an Argentinian or Chilean (he referred to the maker twice as an Argentinian and once as a Chilean) working on one of the ranches. The hobbels were braided from quite heavy rawhide; the straps and strands being all hand cut to width and not split by machine to a uniform thickness (as Doug states, this being very common for nearly all the South American equipment seen on the ranches around here), however, this did not adversely affect their appearance.

The strands in the four strand round centrepiece braid are approximately $\frac{5}{8}$ " or slightly less in width. This braid is about $\frac{3}{4}$ " in diameter and $5\frac{1}{4}$ " in length. The method of construction used for the hobbels is similar to the method described by Bruce Grant in his book *Encyclopedia of Rawhide and Leather Braiding*, Plate 141 on pg. 349, except that each strap has only two strands cut on one end so the centrepiece braid is a four strand Round Braid. A closed cuff is very slightly more than 3" in diameter. The hair-side of the rawhide is on the inside of the cuff against the horse's leg. The hair-side of the rawhide is also to the outside on the centrepiece braid.

The drawings in Plate 141 are misleading. In drawing Fig. 2 the flesh-side is up-permost. Fig. 3 would then indicate that the flesh-side also becomes outermost on the centrepiece braid. However, Fig. 7 indicates that the cuffs have been rolled closed as the centrepiece braid was being braided. This would then normally cause the flesh-side to be on the inside of the cuffs, hence against the horse's legs, while the hair-side to be on the outside of the cuffs as well as being outermost on the centrepiece braid. This is what appears to be the case for both samples in the lower photo on pg. 344 of Bruce Grant's *Encyclopedia of Rawhide and Leather Braiding*.

In order to get the hair-side on the inside of the cuffs and on the outside of the centrepiece braid, the braid can be made as shown in Fig. 658. Here, *A* represents a cuff-end which has been slit into two braiding-strands, showing the first crossing. *B* shows a cheater-strap with one end being slit into two braiding-strands. This cheater-strap can be braided from left to right together with the two braiding-strands of *A* to make a four strand round centrepiece braid. The cheater-strap is introduced into the cuff-strap as indicated by the arrow. Once the braid has been done to the desired length, the cheater-strands can be removed a little at a time and replaced with the two braiding strands of the other cuff-strap in a right to left braiding direction. The result is that the two buttons shown in Fig. 1 on pg. 349 of Bruce Grant's *Encyclopedia of Rawhide and Leather Braiding* are on the hair-side of the cuffs.

Once the cheater-strap has been replaced by the strands of the second cuff-strap, we would then have the two cuffs joined by a centrepiece four-strand Round Braid. The two

braiding-strands of either cuff will exit on the other cuff's hair-side where this other cuff was split to form its braiding-strands. At each cuff, the two emerging braiding-strands are each split into two strings in order to obtain at each cuff four knotting-strings of about $\frac{1}{4}$ " wide each. Each of these four knotting-strings are braided into a terminal knot to serve as the cuff's closure knot. In addition to these closure knots, there are two woven knots, one at each end of the centrepiece, covering the Round Braid.

So, the hobbels have overall nothing unusual in their construction.

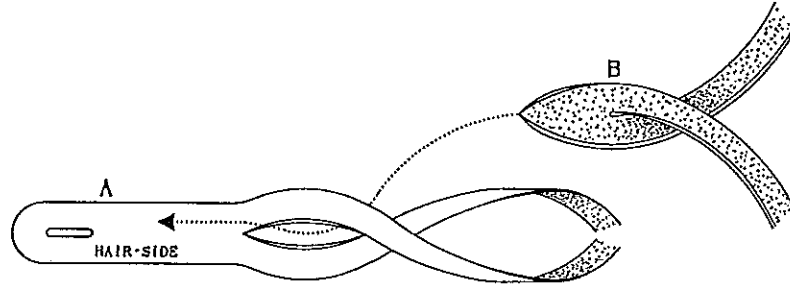


Fig. 658 — The left cuff-strap and the cheater-strap.

The two woven knots, one at each end of the centrepiece four strand Round Braid, have the general appearance of Regular Cylindrical Braids with the nominal column-coding $4u - 3o - 3u - 4o$. If this would be the actual coding of a Regular Cylindrical Braid, then the number of parts would be $\{1 + (4 + 3 + 3 + 4)\} = 15$, and since these braids appear to have each 12 bights, the number of strings required would be $\text{g.c.d.}(15, 12) = 3$. However, these woven knots are each made with two strings and the quite noticeable glitch in the braid clearly indicates that there must be an irregularity in the braid. The grid-diagram of the actual braid is depicted by the leftmost diagram in Fig. 659.

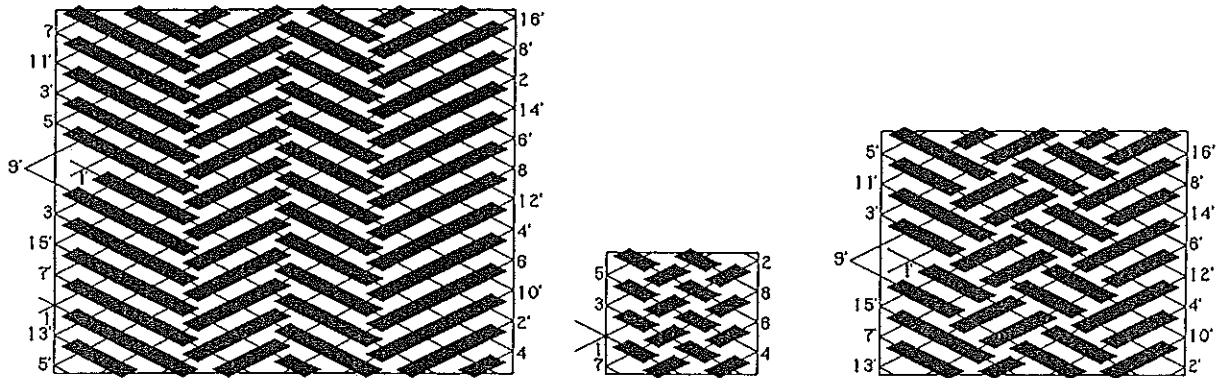


Fig. 659 — The actual woven knots at the ends of the centrepiece braid.

A Regular Cylindrical Braid with $P/B = 15/12$ would be an interbraid of three Regular Knots, each with $p/b = 5/4$. Fig. 660 depicts the three stages of braiding an interbraided $P/B = 15/12$ Regular Cylindrical Braid. It follows from the columns 3, 6, 9, 12 in the rightmost diagram of Fig. 660 that when the $P/B = 15/12$ Regular Cylindrical Braid has to be column-coded, the three $p/b = 5/4$ Regular Knots must be identical column-coded Regular Knots. In the interbraid of two of these $p/b = 5/4$ Regular Knots, the additional columns 1, 3, 5, 7, 9 have been created.

In order to obtain a column-coding for a Regular Cylindrical Braid, produced by interbraiding two identical Regular Knots, these Regular Knots must be column-coded. Let each of the two identical column-coded Regular Knots have p -parts and b -bights:

1.) $p = 2n + 1$, where $n = 1, 2, 3, \dots$.

$2^{(2n+1)}$ different column-coded $2p/2b$ Regular Cylindrical Braids can be created by interbraiding these two identical column-coded Regular Knots if for their column-coding the condition $\{\text{coding of column } x\} = \{\text{coding of column } (2n + 1 - x)\}$, where $x = 1, 2, 3, \dots, n$, does not apply for at least one x -value. If for their column-coding the condition $\{\text{coding of column } x\} = \{\text{coding of column } (2n + 1 - x)\}$, where $x = 1, 2, 3, \dots, n$, does apply, then $2^n(2^n + 1)$ different column-coded $2p/2b$ Regular Cylindrical Braids can be created by interbraiding these two identical column-coded Regular Knots, and $2^{(n+1)}$ of these interbraided $P/B = 2p/2b$ Regular Cylindrical Braids have then a balanced column-coding.

2.) $p = 2n$, where $n = 1, 2, 3, \dots$.

$2^{(2n)}$ different column-coded $2p/2b$ Regular Cylindrical Braids can be created by interbraiding these two identical column-coded Regular Knots if for their column-coding the condition $\{\text{coding of column } x\} = \{\text{coding of column } (2n - x)\}$, where $x = 1, 2, 3, \dots, (n - 1)$, does not apply for at least one x -value. If for their column-coding the condition $\{\text{coding of column } x\} = \{\text{coding of column } (2n - x)\}$, where $x = 1, 2, 3, \dots, (n - 1)$, does apply, then $2^{(n-1)}(2^n + 1)$ different column-coded $2p/2b$ Regular Cylindrical Braids can be created by interbraiding these two identical column-coded Regular Knots, and 2^n of these interbraided $P/B = 2p/2b$ Regular Cylindrical Braids have then a balanced column-coding.

Note that when the two identical p/b column-coded Regular Knots have an over-under coding, a balanced column-coding for the interbraided $P/B = 2p/2b$ Regular Cylindrical Braid can only be created when p is even.

★★ Prove the above relationships for interbraided $P/B = 2p/2b$ column-coded Regular Cylindrical Braids.

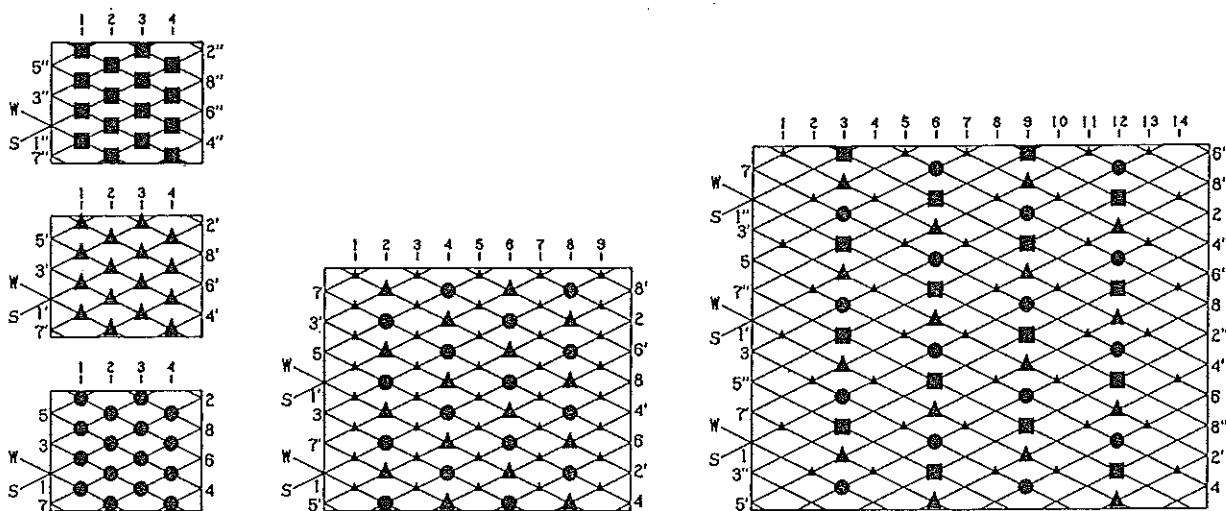


Fig. 660 — The three stages of braiding an interbraided $P/B = 15/12$ Regular Cylindrical Braid.

When a $P/B = 2p/2b = 10/8$ Regular Cylindrical Braid has to be column-coded, we can create $2^5 = 32$ different column-coded Regular Cylindrical Braids by interbraiding two identical $p/b = 5/4$ column-coded Regular Knots if for their column-coding the condition $\{\text{coding of column } x\} = \{\text{coding of column } (5 - x)\}$, where $x = 1, 2,$

does not apply for at least one x -value. If for their column-coding the condition $\{\text{coding of column } x\} = \{\text{coding of column } (5 - x)\}$, where $x = 1, 2$, does apply, then $2^2(2^2 + 1) = 20$ different column-coded $P/B = 2p/2b$ Regular Cylindrical Braids can be created by interbraiding these two identical column-coded Regular Knots, and $2^{(2+1)} = 8$ of these interbraided $P/B = 2p/2b = 10/8$ Regular Cylindrical Braids have then a balanced column-coding. Note that when the two identical $p/b = 5/4$ column-coded Regular Knots have an over-under coding, the $P/B = 10/8$ interbraided Regular Cylindrical Braids cannot have a balanced column-coding.

From the rightmost diagram in Fig. 660 we see that by interbraiding three Regular Knots, each with the same p/b -values, these Regular knots must have an identical column-coding to enable the interbraided Regular Cylindrical Braid to be column-coded. We furthermore see that by interbraiding the third one of the three Regular Knots, each of the columns 1, 3, 5, 7, 9 in the central diagram splits into two columns. Hence when the interbraided Regular Cylindrical Braids depicted by the central and rightmost diagrams in Fig. 660 are both to be column-coded, then in the rightmost diagram the columns 1 and 2 must have the same coding, the columns 4 and 5 must have the same coding, the columns 7 and 8 must have the same coding, the columns 10 and 11 must have the same coding, the columns 13 and 14 must have the same coding. Consequently the same relationships with regards column-coding as the ones we formulated earlier for the interbraided $P/B = 2p/2b$ Regular Cylindrical Braids must then also apply to the interbraided $P/B = 3p/3b$ Regular Cylindrical Braids. Thus in this case when the three identical p/b Regular Knots have an over-under coding, a balanced column-coding for the interbraided $P/B = 3p/3b$ Regular Cylindrical Braid can only exist when p is *even*, hence not for $p = 5$. Consequently a balanced column-coding for the interbraided $P/B = 3p/3b$ Regular Cylindrical Braid, when the three identical p/b Regular Knots have an over-under coding with p *odd*, tells us that the interbraided $P/B = 2p/2b$ Regular Cylindrical Braid cannot be column-coded (the columns $\frac{3p-1}{2}$ and $\frac{3p+1}{2}$ in the interbraided $P/B = 3p/3b$ Regular Cylindrical Braid cannot have an identical coding, hence the column p in the interbraided $P/B = 2p/2b$ Regular Cylindrical Braid cannot be column-coded).

Let's have a look at column-coded interbraided $P/B = 3p/3b$ Regular Cylindrical Braids without worrying about the coding arrangements in the interbraided $P/B = 2p/2b$ Regular Cylindrical Braids. We have already seen that in order to obtain a column-coding for a Regular Cylindrical Braid, produced by interbraiding three identical Regular Knots, these Regular Knots must be column-coded. Let each of the three identical column-coded Regular Knots have p -parts and b -bights:

1.) $p = 2n + 1$, where $n = 1, 2, 3, \dots$

$2^{(4n+2)}$ different column-coded $3p/3b$ Regular Cylindrical Braids can be created by interbraiding these three identical column-coded Regular Knots if for their column-coding the condition $\{\text{coding of column } x\} = \{\text{coding of column } (2n + 1 - x)\}$, where $x = 1, 2, 3, \dots, n$, does not apply for at least one x -value. If for their column-coding the condition $\{\text{coding of column } x\} = \{\text{opposite coding of column } (2n + 1 - x)\}$, where $x = 1, 2, 3, \dots, n$, applies, then $2^{(2n+1)}$ of these interbraided $P/B = 3p/3b$ Regular Cylindrical Braids have a balanced column-coding.

If for their column-coding the condition $\{\text{coding of column } x\} = \{\text{coding of column } (2n + 1 - x)\}$, where $x = 1, 2, 3, \dots, n$, does apply, then $2^{2n}(2^{(2n+1)} + 1)$ different

column-coded $3p/3b$ Regular Cylindrical Braids can be created by interbraiding these three identical column-coded Regular Knots, and $2^{(2n+1)}$ of these interbraided $P/B = 3p/3b$ Regular Cylindrical Braids have then a balanced column-coding.

2.) $p = 2n$, where $n = 1, 2, 3, \dots$.

$2^{(4n)}$ different column-coded $3p/3b$ Regular Cylindrical Braids can be created by interbraiding these three identical column-coded Regular Knots if for their column-coding the condition {coding of column x } = {coding of column $(2n - x)$ }, where $x = 1, 2, 3, \dots, (n - 1)$, does not apply for at least one x -value. If for their column-coding the condition {coding of column x } = {coding of column $(2n - x)$ }, where $x = 1, 2, 3, \dots, (n - 1)$, does apply, then $2^{(2n-1)}(2^{2n} + 1)$ different column-coded $3p/3b$ Regular Cylindrical Braids can be created by interbraiding these three identical column-coded Regular Knots, and 2^{2n} of these interbraided $P/B = 3p/3b$ Regular Cylindrical Braids have then a balanced column-coding.

Note that when the three identical p/b column-coded Regular Knots have an over-under coding, a balanced column-coding for the interbraided $P/B = 3p/3b$ Regular Cylindrical Braid can be created for both p is *odd* and p is *even*. For p is *odd* we have the condition {coding of column x } = {opposite coding of column $(2n + 1 - x)$ }, where $x = 1, 2, 3, \dots, n$, and for p is *even* we have the condition {coding of column x } = {coding of column $(2n - x)$ }, where $x = 1, 2, 3, \dots, (n - 1)$.

★ ★ Prove the above relationships for interbraided $P/B = 3p/3b$ column-coded Regular Cylindrical Braids.

When a $P/B = 3p/3b = 15/12$ Regular Cylindrical Braid has to be column-coded, we can create $2^{10} = 1024$ different ones of such column-coded Regular Cylindrical Braids by interbraiding three identical $p/b = 5/4$ column-coded Regular Knots if for their column-coding the condition {coding of column x } = {coding of column $(5 - x)$ }, where $x = 1, 2$, does not apply for at least one x -value. If for their column-coding the condition {coding of column x } = {opposite coding of column $(2n + 1 - x)$ }, where $x = 1, 2, 3, \dots, n$, applies, then $2^{(4+1)} = 32$ of these interbraided $P/B = 3p/3b = 15/12$ Regular Cylindrical Braids have a balanced column-coding.

If for their column-coding the condition {coding of column x } = {coding of column $(5 - x)$ }, where $x = 1, 2$, does apply, then $2^4(2^5 + 1) = 528$ different column-coded $P/B = 3p/3b$ Regular Cylindrical Braids can be created by interbraiding these three identical column-coded Regular Knots, and $2^{(4+1)} = 32$ of these interbraided $P/B = 3p/3b = 15/12$ Regular Cylindrical Braids have then a balanced column-coding.

Note that for three identical $p/b = 5/4$ over-under coded Regular Knots the condition {coding of column x } = {coding of column $(2n + 1 - x)$ }, where $x = 1, 2, 3, \dots, n$, does not apply, but the condition {coding of column x } = {opposite coding of column $(2n + 1 - x)$ }, where $x = 1, 2, 3, \dots, n$, does apply, hence of the 1024 different column-coded $P/B = 3p/3b = 15/12$ Regular Cylindrical Braids 32 have a balanced column-coding. These 32 balanced column-coding forms are depicted in Fig. 661.

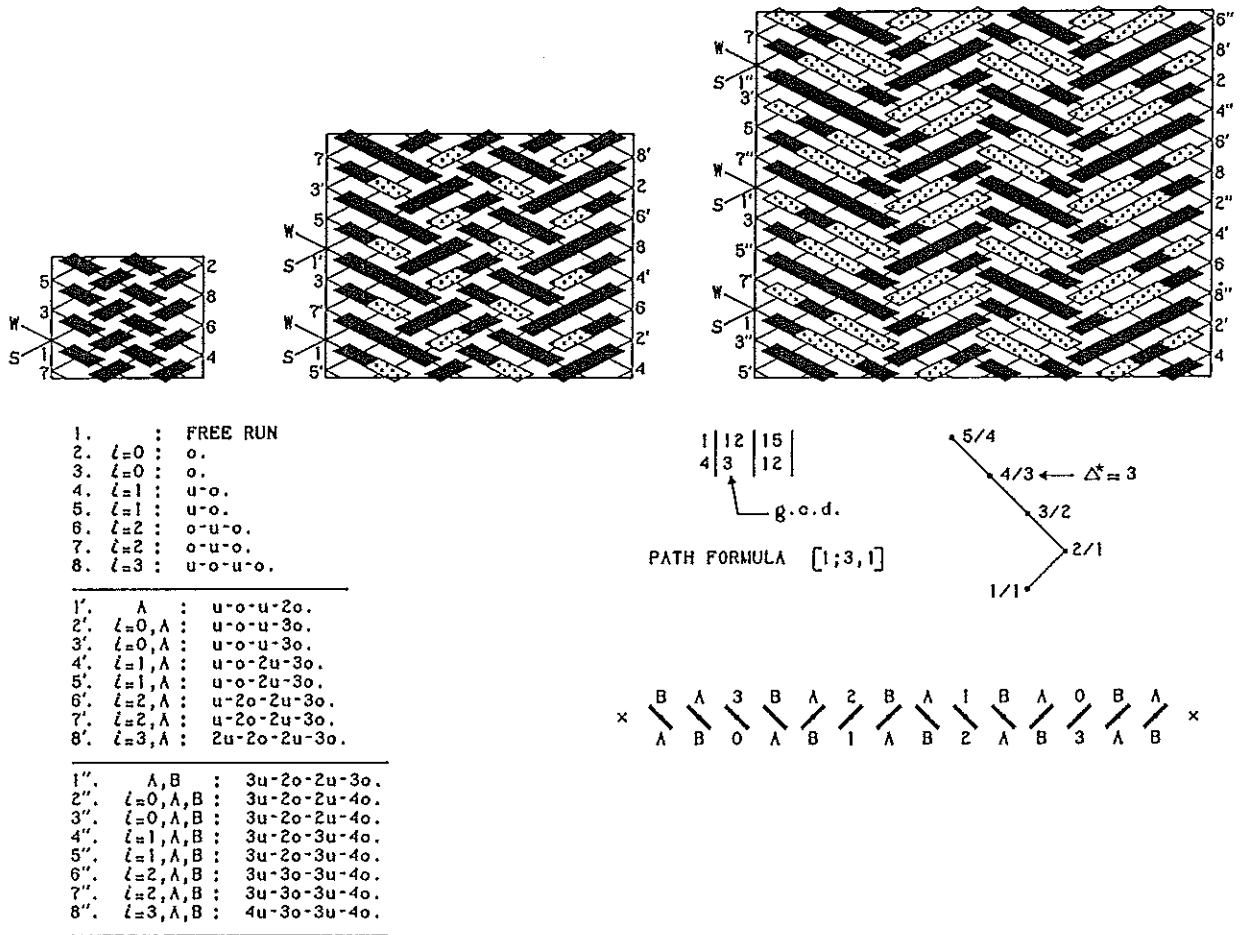
Note that the nominal coding-form used in the woven knots at each end of the centrepiece is No. 26 in Fig. 661, and that this coding-form with the coding-form of No. 30 in Fig. 661 are the nearest balanced coding-forms obtainable that superficially somewhat look like a Gaucho-coding. As we shall see shortly, one is just as easy to braid as the other one, however No. 26 resembles in appearance a Gaucho-coding a little closer and in the way the knot has been braided, the glitch created may not be quite as pronounced.

1	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
2	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
3	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
4	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
5	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
6	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
7	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
8	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
9	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
10	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
11	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
12	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
13	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
14	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
15	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
16	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
17	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
18	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
19	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
20	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
21	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
22	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
23	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
24	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
25	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
26	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
27	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
28	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
29	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
30	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙
31	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
32	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙	↙

Fig. 661 — The balanced coding-forms.

Fig. 662 shows the relevant details and construction steps of an interbraided Regular Cylindrical Braid with the coding-pattern No.26 of Fig.661. Note the consecutive half-cycle braiding algorithms for the consecutive components. Braiding half-cycle n' , where $n' = 2', 3', \dots, 8'$, can be obtained by adding braiding half-cycle n , where $n = 2, 3, \dots, 8$, to braiding half-cycle $1'$; similarly, braiding half-cycle n'' , where $n'' = 2'', 3'', \dots, 8''$, can be obtained by adding braiding half-cycle n , where $n = 2, 3, \dots, 8$, to braiding half-cycle $1''$. Observe how braiding half-cycle $1'$ relates to

braiding half-cycle 8, and how braiding half-cycle 1'' relates to braiding half-cycle 8'.

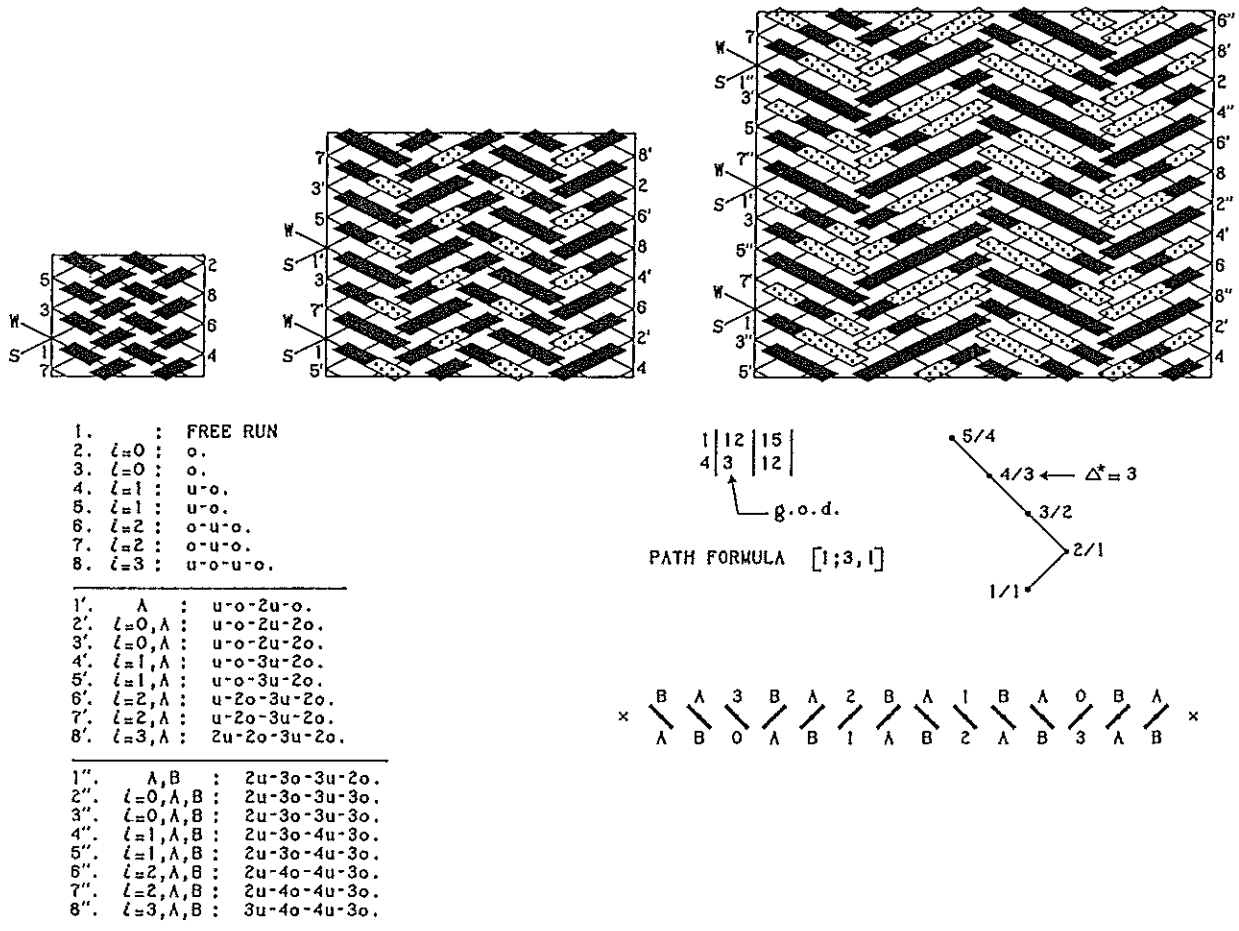


The relevant details and construction steps for Fig. 662 — an interbraided Regular Cylindrical Braid with the coding-pattern No. 26 of Fig. 661.

Fig. 663 shows the relevant details and construction steps of an interbraided Regular Cylindrical Braid with the coding-pattern No. 30 of Fig. 661. Note again the consecutive half-cycle braiding algorithms for the consecutive components. Braiding half-cycle n' , where $n' = 2', 3', \dots, 8'$, can be obtained by adding braiding half-cycle n , where $n = 2, 3, \dots, 8$, to braiding half-cycle 1'; similarly, braiding half-cycle n'' , where $n'' = 2'', 3'', \dots, 8''$, can be obtained by adding braiding half-cycle n , where $n = 2, 3, \dots, 8$, to braiding half-cycle 1''. Observe how braiding half-cycle 1' relates to braiding half-cycle 8, and how braiding half-cycle 1'' relates to braiding half-cycle 8'.

On pg. 832 we showed in Fig. 659 the actual construction of the woven knots, at the ends of the centrepiece four strand Round Braid, which have the nominal coding-pattern No. 26 in Fig. 661. In these woven knots, two components of the final interbraid in Fig. 662 have been combined into one, depicted by the rightmost grid-diagram in Fig. 659. Observe that the construction of this knot is in essence the same as the construction of the knot depicted by the uppermost second grid-diagram from the left in Fig. 456, pg. 547 in *The Braider*, Issue No. 24. Observe that the sequential half-cycle braiding algorithms for the actual woven knots at the ends of the centrepiece (see the leftmost sequential half-cycle braiding algorithms in Fig. 664) are only with the

exception of half-cycle 16' identical to the sequential half-cycle braiding algorithms for the interbraided $P/B = 15/12$ Regular Cylindrical Braid of Fig. 662.



The relevant details and construction steps for Fig. 663 — an interbraided Regular Cylindrical Braid with the coding-pattern No. 30 of Fig. 661.

The glitch in the braid of these actual interwoven knots can readily be avoided by constructing them as depicted by the lowermost left grid-diagram in Fig. 664 with the aid of the associated sequential half-cycle braiding algorithms at the lower right-hand side. This construction method, compared with the construction method actually employed, requires a change in the half-cycle braiding algorithms for half-cycles 8' and 16'.

The uppermost left grid-diagram in Fig. 665 depicts for the nominal coding-pattern No. 30 in Fig. 661 the construction of the interwoven knots by the same method as used for the actual interwoven knots at the ends of the centrepiece. The lower left-hand associated half-cycle braiding algorithms are, with the exception of only half-cycle 16', identical to the sequential half-cycle braiding algorithms for the interbraided $P/B = 15/12$ Regular Cylindrical Braid of Fig. 663. The glitch in the braid of such interwoven knots can readily be avoided by constructing them as depicted by the lowermost left grid-diagram in Fig. 665 with the aid of the associated sequential half-cycle braiding algorithms at the lower right-hand side. This construction method, compared with the construction method depicted by the uppermost right-hand grid-diagram requires a change in the half-cycle braiding algorithms for half-cycles 8' and 16'.

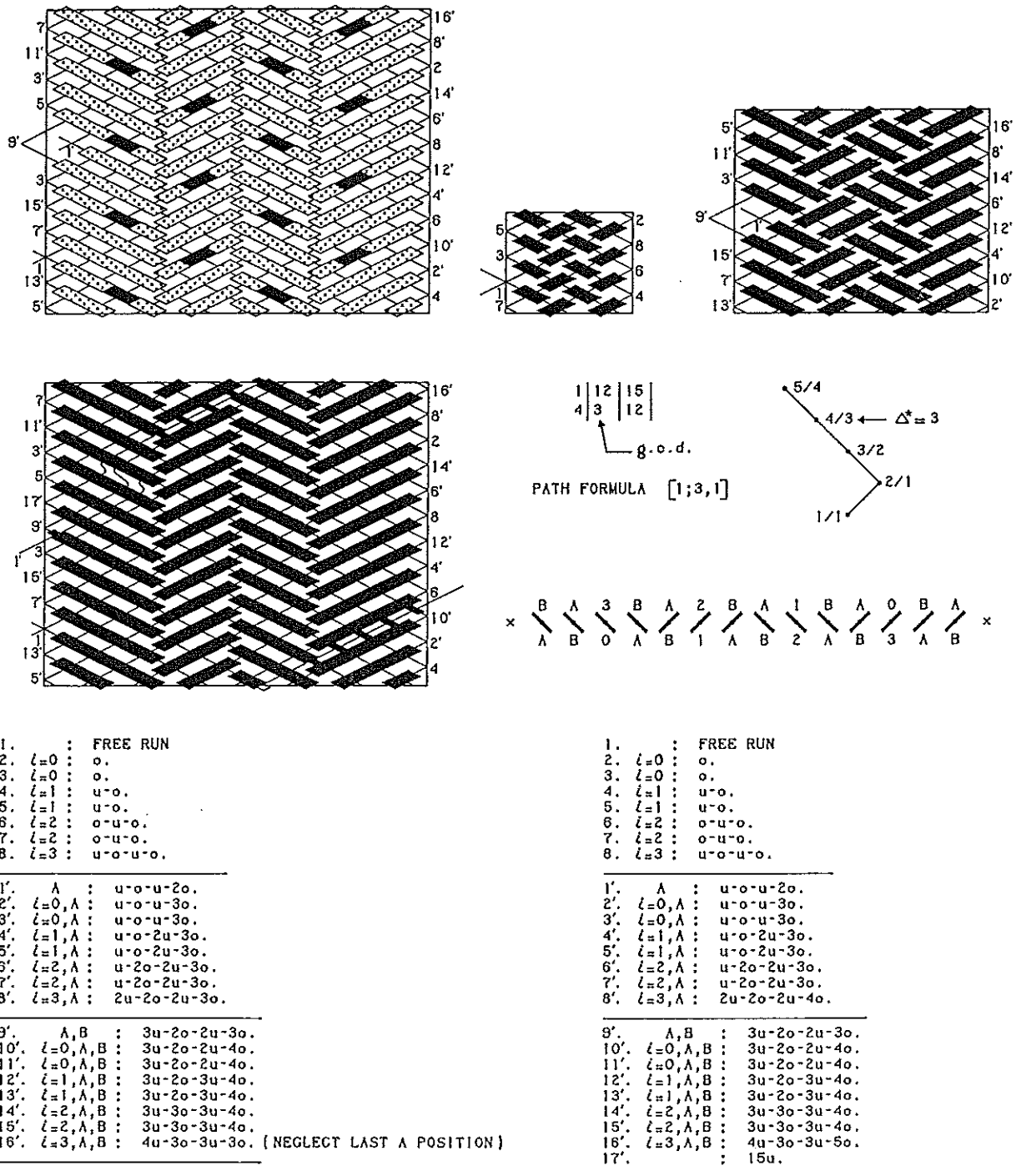
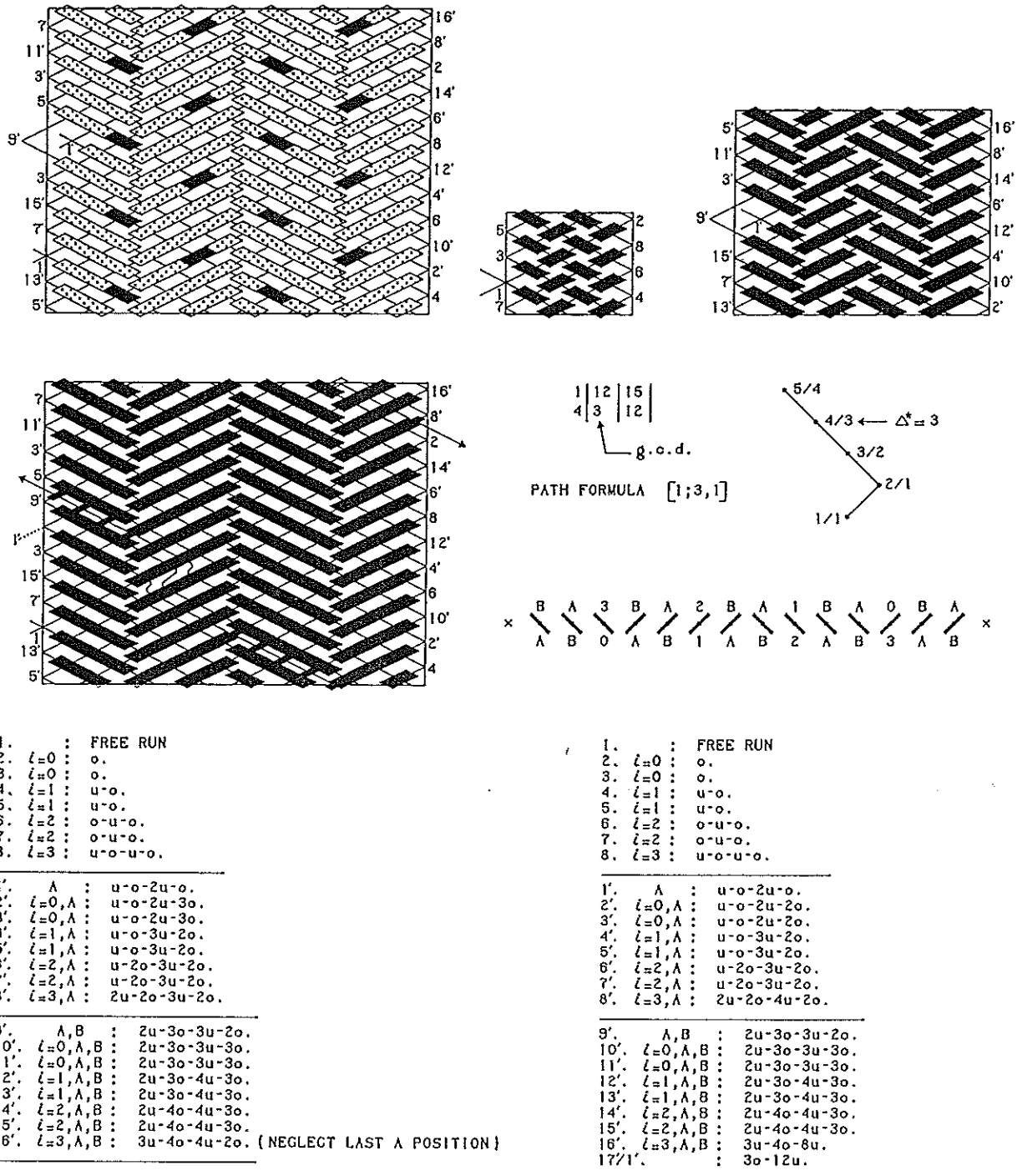


Fig. 664 — The relevant details and construction steps for an interbraided Regular Cylindrical Braid with the nominal coding-pattern No. 26 of Fig. 661.

We see here once again the importance of using grid-diagrams. They readily enable us to modify construction methods which may have been found by experimental ways, but contain irregularities. Sometimes an experimentally found construction method may be correct, but not be the simplest or most preferable method when it has superfluous braiding movements which are not obviously visible. We will meet a typical example

of this with the terminal knots which are intended to serve as the closure knots of the cuffs.



The relevant details and construction steps for Fig. 665 — an interbraided Regular Cylindrical Braid with the nominal coding-pattern No. 30 of Fig. 661.

The grid-diagram, and hence the construction method, of the terminal knots for closing the cuffs can be presented as depicted by the uppermost left-hand diagram in Fig. 666. This diagram, however, is equivalent to the uppermost right-hand diagram in Fig. 666, but instead of using this simplified uppermost right-hand grid-diagram, it is

much more convenient to use the lowermost diagram in Fig. 666. The convenience of using this lowermost diagram in Fig. 666 is that after the initial crowning and walling operation, the actual braiding sequence is "o-u" for each half-cycle in a set of identical half-cycles, except for the last set of identical half-cycles for which the actual sequence is all *unders* (see the solid black codings in diagrams 3, 4, 5 of Fig. 667, hence the codings as depicted under "A" below (codings actually laid down during previous half-cycles are indicated as under "B" below, and codings as indicated under "C" below are not actually laid down during these half-cycles)).

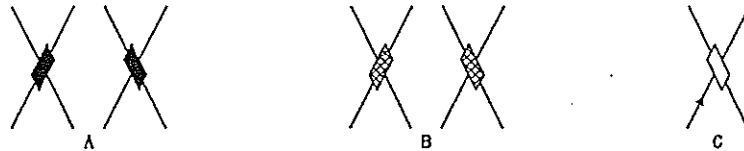


Fig. 667 depicts the construction of the lowermost knot in Fig. 666, which is the uppermost right-hand knot in Fig. 667.

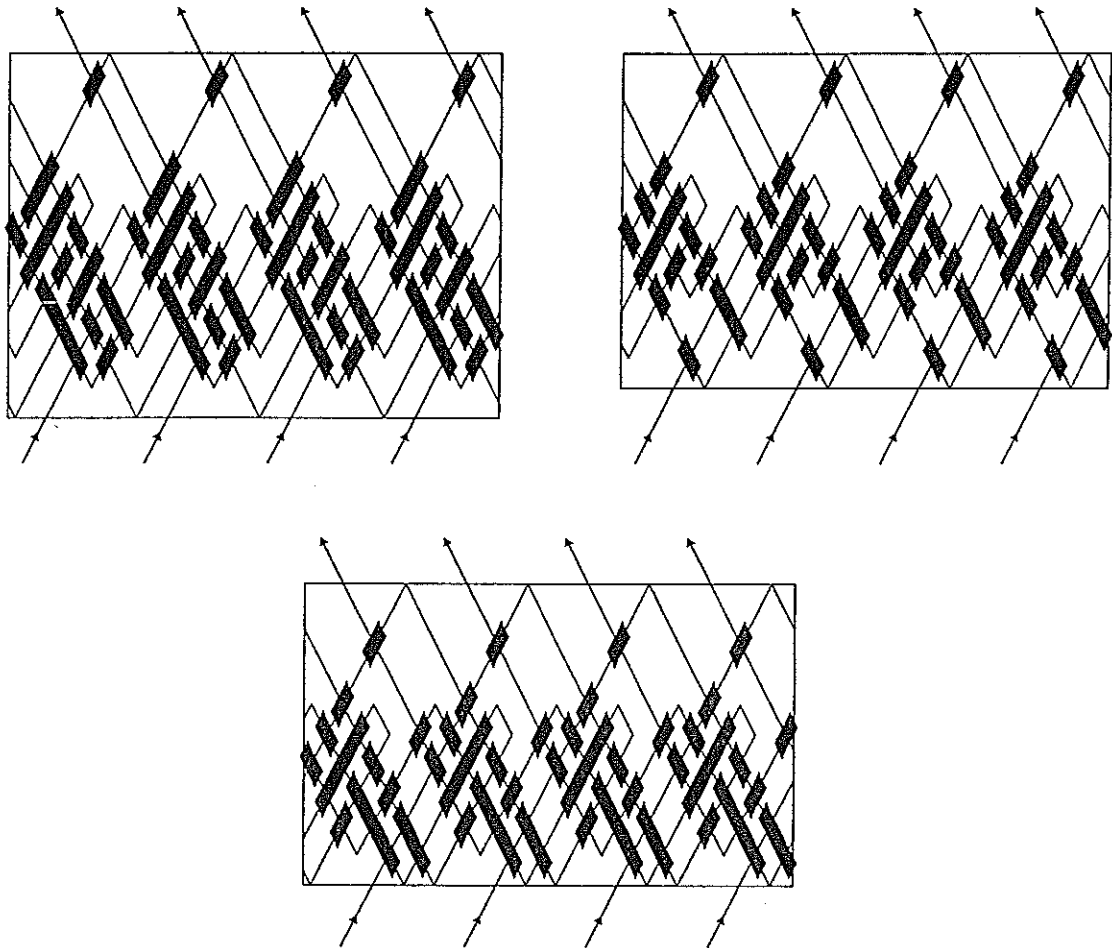


Fig. 666 — Three equivalent grid-diagrams depicting the terminal knots which are used for closing the cuffs.

It should be noted that for good braidwork the terminal knots used for closing the cuffs should be complementary. The grid-diagrams and construction steps for the complementary terminal knot are shown in Fig. 668.

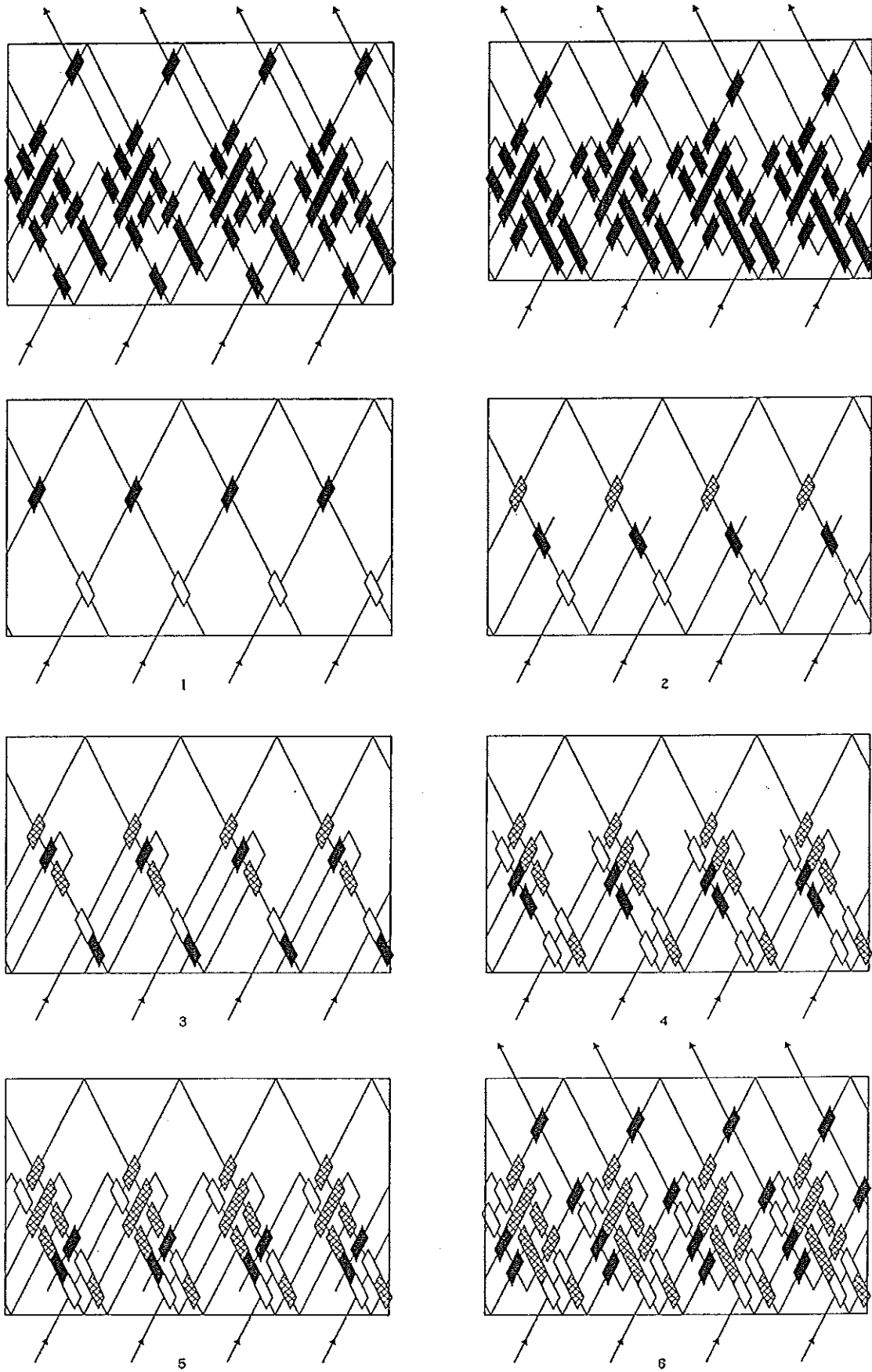


Fig. 667 — The terminal knot in Fig. 666 and its construction steps.

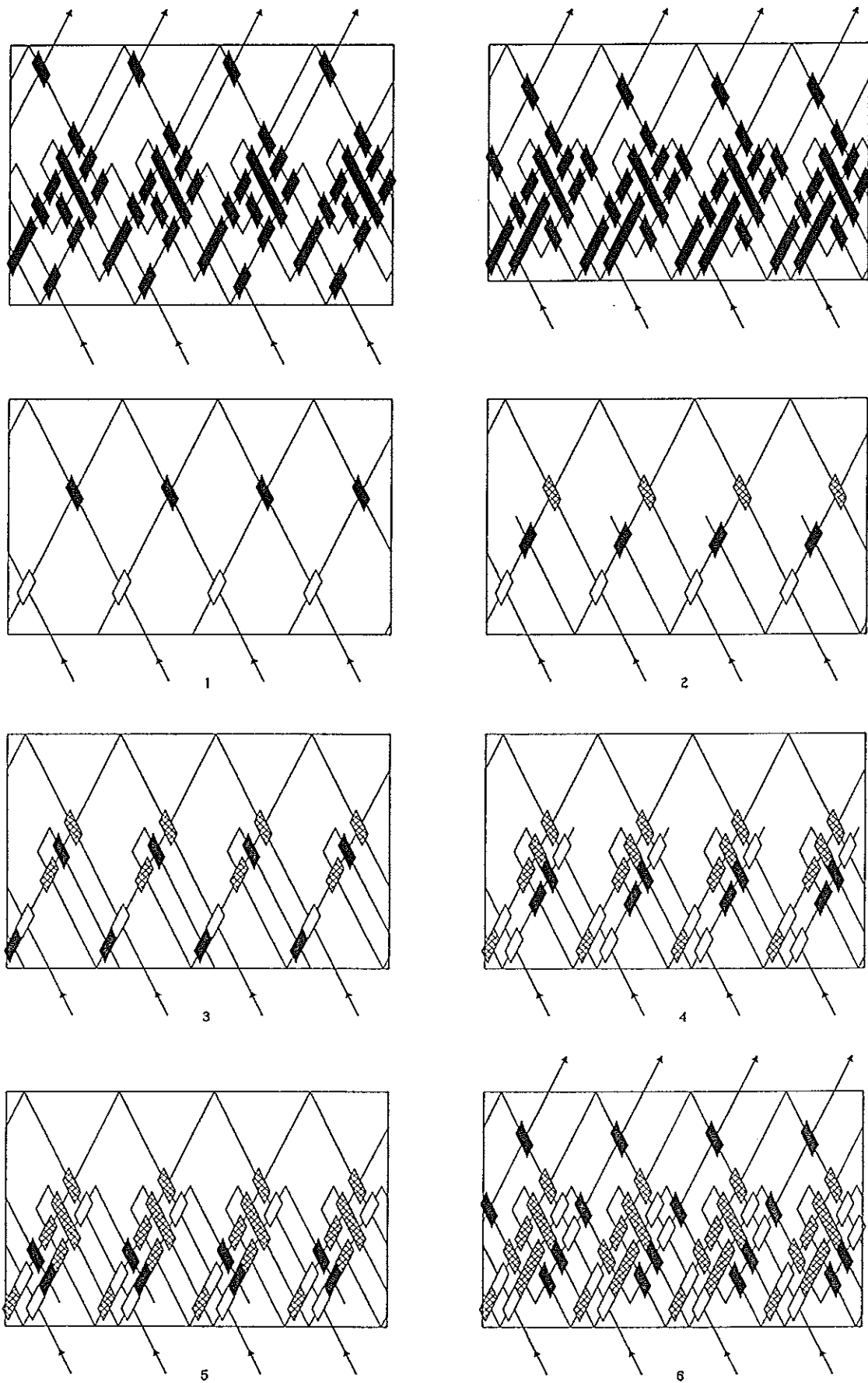
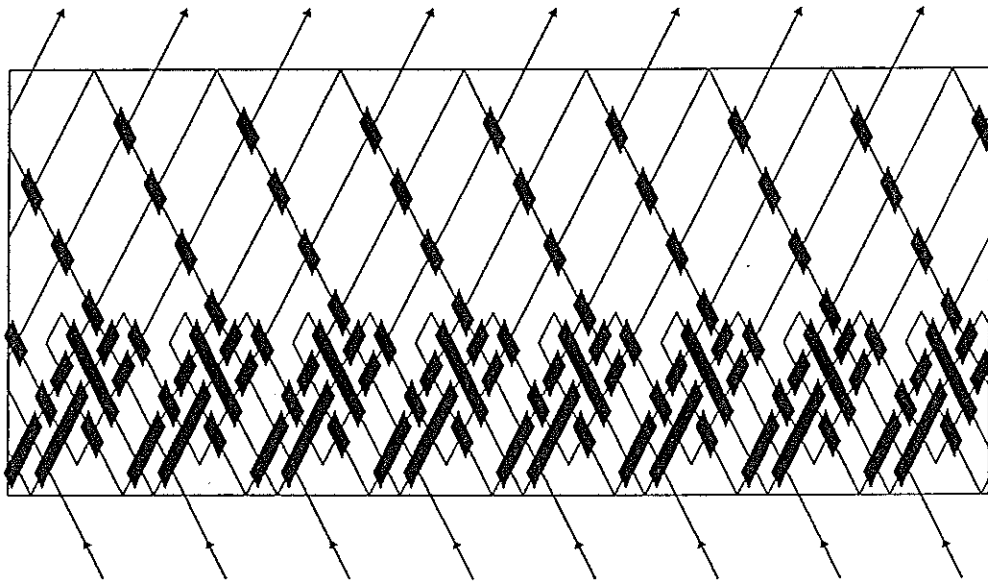


Fig. 668 — The complementary terminal knot and its construction steps.

The construction of the terminal knot in Fig. 668 is, apart from the crowning operation, the same as the construction of the "Boton de ocho tientos (redondo)" in the book *Trenzas Gauchas*, pp. 152-155 by Mario A. Lopez Osornio†.



BOTON DE OCHO TIENTOS (REDONDO)
IN *TRENZAS GAUCHAS* BY MARIO A. LOPEZ OSORNIO ; PP. 152-155

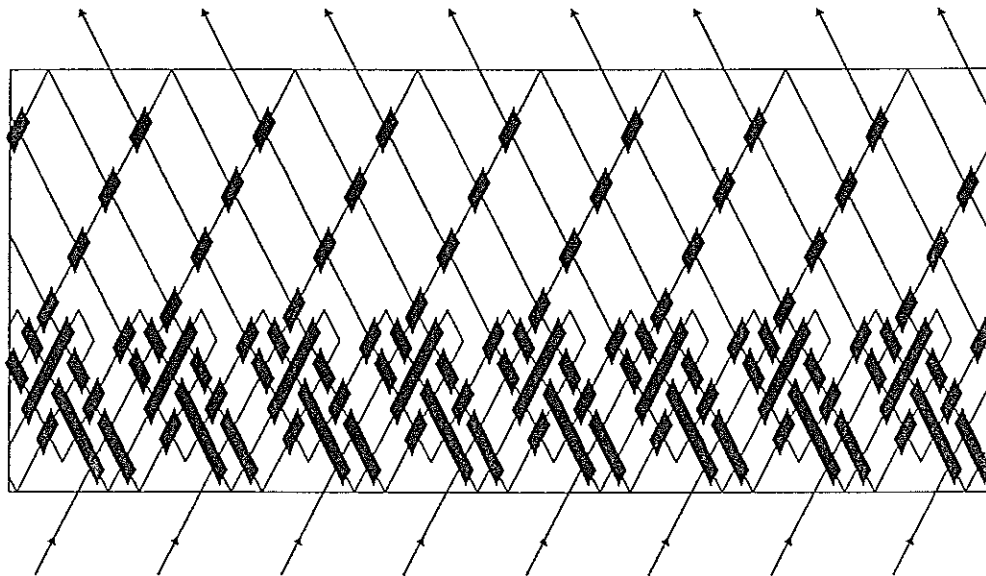


Fig. 669 — Boton de ocho tientos (redondo).

† The book *Trenzas Gauchas* by Mario A. Lopez Osornio was published in 1934 and has had very many reprints (17 up to and including 1995). It is interesting to observe that much in Bruce Grant's *Encyclopedia of Rawhide and Leather Braiding* seems to come from the book *Trenzas Gauchas*, sometimes, but not always, with the identical mistakes. Since the first edition of Grant's *Encyclopedia of Rawhide and Leather Braiding* was published in 1972, one wonders why he left some of the knots in *Trenzas Gauchas* out. The reason could have been that the errors in the drawings by Mario A. Lopez Osornio fooled him in braiding those knots.

The drawings in the book *Trenzas Gauchas* are very poor and it should be noted that he made two mistakes in the case of the “Boton de ocho tientos (redondo)”: one in Fig. N° 203 and one in Fig. N° 204. These mistakes are readily picked up by drawing the grid-diagrams. The two complementary grid-diagrams of these knots (but without the mistakes) are depicted in Fig. 669.

Nested Cylindrical Braids

In *The Braider*, Issue No. 35, we met on pg. 820 the formula $r_i = \left| A_l + \frac{x-y_i-2l_i}{2} \right|_{A_r}$. For a specified x -value and $|y_n|_{2d}$ -set of distinct y_m -values, this formula enables us to calculate the set of $\frac{A_r}{d}$ r_i -values associated with an l_i -value. Since we are dealing here with the Asymmetric Regular Nested Cylindrical Braids, the set of r_i -values associated with an l_i -value other than 1 follows directly from the set of r_i -values associated with $l_i = 1$.[†] It therefore suffices to know for the various $(|y_n|_{2d}, x)$ -values the r_i -set associated with $l_i = 1$. The general module of these r_i -sets associated with $(|y_n|_{2d}, x)$ -values, which readily can be derived from the formula $r_i = \left| A_l + \frac{x-y_i-2l_i}{2} \right|_{A_r}$, is displayed in Fig. 671. A short initial section of the $y_m - x$ table for $A_l = 10$ and $A_r = 6$ with the r_i -sets for $l_i = 1$ is shown in Fig. 670.

$\begin{matrix} \rightarrow x \\ y_n _{2d} \downarrow \end{matrix}$	0	1	2	3	4	5	6	7	8
0			1 → 2 1 → 4 1 → 6		1 → 1 1 → 3 1 → 5		1 → 2 1 → 4 1 → 6		1 → 1 1 → 3 1 → 5
1		1 → 1 1 → 3 1 → 5		1 → 2 1 → 4 1 → 6		1 → 1 1 → 3 1 → 5		1 → 2 1 → 4 1 → 6	
2	1 → 2 1 → 4 1 → 6		1 → 1 1 → 3 1 → 5		1 → 2 1 → 4 1 → 6		1 → 1 1 → 3 1 → 5		1 → 2 1 → 4 1 → 6
3		1 → 2 1 → 4 1 → 6		1 → 1 1 → 3 1 → 5		1 → 2 1 → 4 1 → 6		1 → 1 1 → 3 1 → 5	
4			1 → 2 1 → 4 1 → 6		1 → 1 1 → 3 1 → 5		1 → 2 1 → 4 1 → 6		1 → 1 1 → 3 1 → 5

Fig. 670 — The $1 \rightarrow r_i$ sets; $A_l = 10, A_r = 6$.

In order to make a $y_m - x$ table useful for practical purposes, we must in each applicable $(|y_n|_{2d}, x)$ cell enter the relevant details of its component(s) (first-return string-run(s)).

Example 1. For $A_l = 10$ and $A_r = 6$ the first-return string-run details for x -values from 0 to 16 with $|y_n|_{2d}$ -values from 0 to d are presented on pp. 847-851. In the $y_m - x$ table on pp. 852-853, ‘sbb’ stands for: sum of bight-boundary numbers; and 4×3 , for example, stands for: 4 components, each with 3 parts.

[†] $r'_i = |r_i - \Delta l_i|_{A_r}$. For example, if for $A_r = 15$ and $d = 5$ we have for $l_i = 1$ the associated r_i -set 3, 8, 13, then we have for $l_i = 4$ the associated r_i -set 5, 10, 15.

$\downarrow x$ $ y_n _{2d}$	$2n\lambda_r + 4 - 2\lambda_1 + 2md$	$2n\lambda_r + 5 - 2\lambda_1 + 2md$	$2n\lambda_r + 6 - 2\lambda_1 + 2md$	$2n\lambda_r + 7 - 2\lambda_1 + 2md$	$2n\lambda_r + 8 - 2\lambda_1 + 2md$		$2n\lambda_r + 2 - 2\lambda_1 + (2m+2)d$	$2n\lambda_r + 3 - 2\lambda_1 + (2m+2)d$
0	$1 \rightarrow 1$ $1 \rightarrow 1+d$ $1 \rightarrow 1+2d$ \vdots $1 \rightarrow 1+\lambda_r-d$		$1 \rightarrow 2$ $1 \rightarrow 2+d$ $1 \rightarrow 2+2d$ \vdots $1 \rightarrow 2+\lambda_r-d$		$1 \rightarrow 3$ $1 \rightarrow 3+d$ $1 \rightarrow 3+2d$ \vdots $1 \rightarrow 3+\lambda_r-d$		$1 \rightarrow d$ $1 \rightarrow 2d$ $1 \rightarrow 3d$ \vdots $1 \rightarrow \lambda_r$	
1		$1 \rightarrow 1$ $1 \rightarrow 1+d$ $1 \rightarrow 1+2d$ \vdots $1 \rightarrow 1+\lambda_r-d$		$1 \rightarrow 2$ $1 \rightarrow 2+d$ $1 \rightarrow 2+2d$ \vdots $1 \rightarrow 2+\lambda_r-d$				$1 \rightarrow d$ $1 \rightarrow 2d$ $1 \rightarrow 3d$ \vdots $1 \rightarrow \lambda_r$
2	$1 \rightarrow d$ $1 \rightarrow 2d$ $1 \rightarrow 3d$ \vdots $1 \rightarrow \lambda_r$		$1 \rightarrow 1$ $1 \rightarrow 1+d$ $1 \rightarrow 1+2d$ \vdots $1 \rightarrow 1+\lambda_r-d$		$1 \rightarrow 2$ $1 \rightarrow 2+d$ $1 \rightarrow 2+2d$ \vdots $1 \rightarrow 2+\lambda_r-d$		$1 \rightarrow d-1$ $1 \rightarrow 2d-1$ $1 \rightarrow 3d-1$ \vdots $1 \rightarrow \lambda_r-1$	
3		$1 \rightarrow d$ $1 \rightarrow 2d$ $1 \rightarrow 3d$ \vdots $1 \rightarrow \lambda_r$		$1 \rightarrow 1$ $1 \rightarrow 1+d$ $1 \rightarrow 1+2d$ \vdots $1 \rightarrow 1+\lambda_r-d$				$1 \rightarrow d-1$ $1 \rightarrow 2d-1$ $1 \rightarrow 3d-1$ \vdots $1 \rightarrow \lambda_r-1$
4	$1 \rightarrow d-1$ $1 \rightarrow 2d-1$ $1 \rightarrow 3d-1$ \vdots $1 \rightarrow \lambda_r-1$		$1 \rightarrow d$ $1 \rightarrow 2d$ $1 \rightarrow 3d$ \vdots $1 \rightarrow \lambda_r$		$1 \rightarrow 1$ $1 \rightarrow 1+d$ $1 \rightarrow 1+2d$ \vdots $1 \rightarrow 1+\lambda_r-d$		$1 \rightarrow d-2$ $1 \rightarrow 2d-2$ $1 \rightarrow 3d-2$ \vdots $1 \rightarrow \lambda_r-2$	
$2d-2$	$1 \rightarrow 2$ $1 \rightarrow 2+d$ $1 \rightarrow 2+2d$ \vdots $1 \rightarrow 2+\lambda_r-d$		$1 \rightarrow 3$ $1 \rightarrow 3+d$ $1 \rightarrow 3+2d$ \vdots $1 \rightarrow 3+\lambda_r-d$		$1 \rightarrow 4$ $1 \rightarrow 4+d$ $1 \rightarrow 4+2d$ \vdots $1 \rightarrow 4+\lambda_r-d$		$1 \rightarrow 1$ $1 \rightarrow 1+d$ $1 \rightarrow 1+2d$ \vdots $1 \rightarrow 1+\lambda_r-d$	
$2d-1$		$1 \rightarrow 2$ $1 \rightarrow 2+d$ $1 \rightarrow 2+2d$ \vdots $1 \rightarrow 2+\lambda_r-d$		$1 \rightarrow 3$ $1 \rightarrow 3+d$ $1 \rightarrow 3+2d$ \vdots $1 \rightarrow 3+\lambda_r-d$				$1 \rightarrow 1$ $1 \rightarrow 1+d$ $1 \rightarrow 1+2d$ \vdots $1 \rightarrow 1+\lambda_r-d$
$2d$	$1 \rightarrow 1$ $1 \rightarrow 1+d$ $1 \rightarrow 1+2d$ \vdots $1 \rightarrow 1+\lambda_r-d$		$1 \rightarrow 2$ $1 \rightarrow 2+d$ $1 \rightarrow 2+2d$ \vdots $1 \rightarrow 2+\lambda_r-d$		$1 \rightarrow 3$ $1 \rightarrow 3+d$ $1 \rightarrow 3+2d$ \vdots $1 \rightarrow 3+\lambda_r-d$		$1 \rightarrow d$ $1 \rightarrow 2d$ $1 \rightarrow 3d$ \vdots $1 \rightarrow \lambda_r$	

Fig. 671 — The general $(|y_n|_{2d}, x)$ module of r_i -sets.

$$\Lambda_1=10; \Lambda_r=6; d=2; x=0; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 \cdot \Lambda_r) \cdot x=32; |y_n|_{zd}=d=2; y=2,6,10; l_{r,i}=|12-(l_i \cdot 2r_i)|_{10}; r_{r,i}=|20-(r_i \cdot 2l_i)|_6.$$

$2\sum(l_i \cdot r_i)=70$	$2\sum(l_i \cdot r_i)=66$	$2\sum(l_i \cdot r_i)=70$	$2\sum(l_i \cdot r_i)=66$	$2\sum(l_i \cdot r_i)=68$	$2\sum(l_i \cdot r_i)=68$	$2\sum(l_i \cdot r_i)=66$	$2\sum(l_i \cdot r_i)=66$
$\alpha=5$	$\alpha=3$	$\alpha=5$	$\alpha=3$	$\alpha=4$	$\alpha=4$	$\alpha=3$	$\alpha=3$
$P_o = \frac{5 \cdot 32 - 70}{30} = 3$	$P_o = \frac{3 \cdot 32 - 66}{30} = 1$	$P_o = \frac{5 \cdot 32 - 70}{30} = 3$	$P_o = \frac{3 \cdot 32 - 66}{30} = 1$	$P_o = \frac{4 \cdot 32 - 68}{30} = 2$	$P_o = \frac{4 \cdot 32 - 68}{30} = 2$	$P_o = \frac{3 \cdot 32 - 66}{30} = 1$	$P_o = \frac{3 \cdot 32 - 66}{30} = 1$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=1; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 \cdot \Lambda_r) \cdot x=33; |y_n|_{zd}=1; y=1,3,5; l_{r,i}=|13-(l_i \cdot 2r_i)|_{10}; r_{r,i}=|21-(r_i \cdot 2l_i)|_6.$$

$2\sum(l_i \cdot r_i)=36$	$2\sum(l_i \cdot r_i)=108$	$2\sum(l_i \cdot r_i)=72$	$2\sum(l_i \cdot r_i)=108$	$2\sum(l_i \cdot r_i)=72$	$2\sum(l_i \cdot r_i)=108$	$2\sum(l_i \cdot r_i)=36$
$\alpha=2$	$\alpha=6$	$\alpha=4$	$\alpha=6$	$\alpha=4$	$\alpha=6$	$\alpha=2$
$P_o = \frac{2 \cdot 33 - 36}{30} = 1$	$P_o = \frac{6 \cdot 33 - 108}{30} = 3$	$P_o = \frac{4 \cdot 33 - 72}{30} = 2$	$P_o = \frac{6 \cdot 33 - 108}{30} = 3$	$P_o = \frac{4 \cdot 33 - 72}{30} = 2$	$P_o = \frac{6 \cdot 33 - 108}{30} = 3$	$P_o = \frac{2 \cdot 33 - 36}{30} = 1$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=2; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 \cdot \Lambda_r) \cdot x=34; |y_n|_{zd}=0; y=0,4,8,12=0; l_{r,i}=|14-(l_i \cdot 2r_i)|_{10}; r_{r,i}=|22-(r_i \cdot 2l_i)|_6.$$

$2\sum(l_i \cdot r_i)=42$	$2\sum(l_i \cdot r_i)=42$	$2\sum(l_i \cdot r_i)=42$	$2\sum(l_i \cdot r_i)=42$	$2\sum(l_i \cdot r_i)=76$	$2\sum(l_i \cdot r_i)=110$	$2\sum(l_i \cdot r_i)=76$	$2\sum(l_i \cdot r_i)=110$
$\alpha=3$	$\alpha=3$	$\alpha=3$	$\alpha=3$	$\alpha=4$	$\alpha=5$	$\alpha=4$	$\alpha=5$
$P_o = \frac{3 \cdot 34 - 42}{30} = 2$	$P_o = \frac{3 \cdot 34 - 42}{30} = 2$	$P_o = \frac{3 \cdot 34 - 42}{30} = 2$	$P_o = \frac{3 \cdot 34 - 42}{30} = 2$	$P_o = \frac{4 \cdot 34 - 76}{30} = 2$	$P_o = \frac{5 \cdot 34 - 110}{30} = 2$	$P_o = \frac{4 \cdot 34 - 76}{30} = 2$	$P_o = \frac{5 \cdot 34 - 110}{30} = 2$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=2; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 \cdot \Lambda_r) \cdot x=34; |y_n|_{zd}=2; y=2,6,10; l_{r,i}=|14-(l_i \cdot 2r_i)|_{10}; r_{r,i}=|22-(r_i \cdot 2l_i)|_6.$$

$2\sum(l_i \cdot r_i)=4$	$2\sum(l_i \cdot r_i)=80$	$2\sum(l_i \cdot r_i)=80$	$2\sum(l_i \cdot r_i)=72$	$2\sum(l_i \cdot r_i)=80$	$2\sum(l_i \cdot r_i)=76$	$2\sum(l_i \cdot r_i)=76$	$2\sum(l_i \cdot r_i)=72$
$\alpha=1$	$\alpha=5$	$\alpha=5$	$\alpha=3$	$\alpha=5$	$\alpha=4$	$\alpha=4$	$\alpha=3$
$P_o = \frac{1 \cdot 34 - 4}{30} = 1$	$P_o = \frac{5 \cdot 34 - 80}{30} = 3$	$P_o = \frac{5 \cdot 34 - 80}{30} = 3$	$P_o = \frac{3 \cdot 34 - 72}{30} = 1$	$P_o = \frac{5 \cdot 34 - 80}{30} = 3$	$P_o = \frac{4 \cdot 34 - 76}{30} = 2$	$P_o = \frac{4 \cdot 34 - 76}{30} = 2$	$P_o = \frac{3 \cdot 34 - 72}{30} = 1$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=3; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 \cdot \Lambda_r) \cdot x=35; |y_n|_{zd}=1; y=1,5,9; l_{r,i}=|15-(l_i \cdot 2r_i)|_{10}; r_{r,i}=|23-(r_i \cdot 2l_i)|_6.$$

$2\sum(l_i \cdot r_i)=180$	$2\sum(l_i \cdot r_i)=120$	$2\sum(l_i \cdot r_i)=80$	$2\sum(l_i \cdot r_i)=120$	$2\sum(l_i \cdot r_i)=40$
$\alpha=12$	$\alpha=6$	$\alpha=4$	$\alpha=6$	$\alpha=2$
$P_o = \frac{12 \cdot 35 - 180}{30} = 8$	$P_o = \frac{6 \cdot 35 - 120}{30} = 3$	$P_o = \frac{4 \cdot 35 - 80}{30} = 2$	$P_o = \frac{6 \cdot 35 - 120}{30} = 3$	$P_o = \frac{2 \cdot 35 - 40}{30} = 1$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=4; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 \Lambda_r) \cdot x=36; |y_n|_{2d}=0; y=0,4,8,12=0; l_{t,1}=\lfloor 16-(l_t+2r_t) \rfloor_{10}; r_{t,1}=\lfloor 24-(r_t+2l_t) \rfloor_6.$$

$2\sum(l_t+r_t)=60$	$2\sum(l_t+r_t)=48$	$2\sum(l_t+r_t)=48$	$2\sum(l_t+r_t)=48$	$2\sum(l_t+r_t)=48$	$2\sum(l_t+r_t)=84$	$2\sum(l_t+r_t)=120$	$2\sum(l_t+r_t)=84$
$\alpha=5$	$\alpha=3$	$\alpha=3$	$\alpha=3$	$\alpha=3$	$\alpha=4$	$\alpha=5$	$\alpha=4$
$P_o = \frac{5 \cdot 36 - 60}{30} = 4$	$P_o = \frac{3 \cdot 36 - 48}{30} = 2$	$P_o = \frac{3 \cdot 36 - 48}{30} = 2$	$P_o = \frac{3 \cdot 36 - 48}{30} = 2$	$P_o = \frac{3 \cdot 36 - 48}{30} = 2$	$P_o = \frac{4 \cdot 36 - 84}{30} = 2$	$P_o = \frac{5 \cdot 36 - 120}{30} = 2$	$P_o = \frac{4 \cdot 36 - 84}{30} = 2$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=4; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 \Lambda_r) \cdot x=36; |y_n|_{2d}=2; y=2,6,10; l_{t,1}=\lfloor 16-(l_t+2r_t) \rfloor_{10}; r_{t,1}=\lfloor 24-(r_t+2l_t) \rfloor_6.$$

$2\sum(l_t+r_t)=6$	$2\sum(l_t+r_t)=90$	$2\sum(l_t+r_t)=6$	$2\sum(l_t+r_t)=90$	$2\sum(l_t+r_t)=90$	$2\sum(l_t+r_t)=90$	$2\sum(l_t+r_t)=84$	$2\sum(l_t+r_t)=84$
$\alpha=1$	$\alpha=5$	$\alpha=1$	$\alpha=5$	$\alpha=5$	$\alpha=5$	$\alpha=4$	$\alpha=4$
$P_o = \frac{1 \cdot 36 - 6}{30} = 1$	$P_o = \frac{5 \cdot 36 - 90}{30} = 3$	$P_o = \frac{1 \cdot 36 - 6}{30} = 1$	$P_o = \frac{5 \cdot 36 - 90}{30} = 3$	$P_o = \frac{5 \cdot 36 - 90}{30} = 3$	$P_o = \frac{5 \cdot 36 - 90}{30} = 3$	$P_o = \frac{4 \cdot 36 - 84}{30} = 2$	$P_o = \frac{4 \cdot 36 - 84}{30} = 2$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=5; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 \Lambda_r) \cdot x=37; |y_n|_{2d}=1; y=1,5,9; l_{t,1}=\lfloor 17-(l_t+2r_t) \rfloor_{10}; r_{t,1}=\lfloor 25-(r_t+2l_t) \rfloor_6.$$

$2\sum(l_t+r_t)=364$	$2\sum(l_t+r_t)=132$	$2\sum(l_t+r_t)=44$	
$\alpha=22$	$\alpha=6$	$\alpha=2$	
$P_o = \frac{22 \cdot 37 - 364}{30} = 15$	$P_o = \frac{6 \cdot 37 - 132}{30} = 3$	$P_o = \frac{2 \cdot 37 - 44}{30} = 1$	

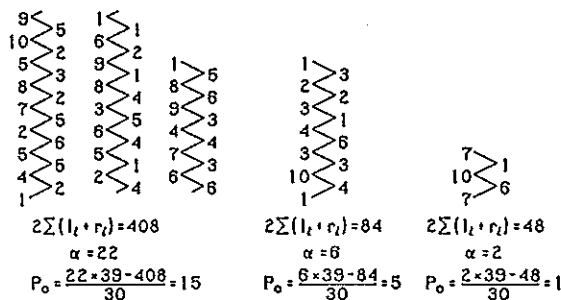
$$\Lambda_1=10; \Lambda_r=6; d=2; x=6; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 \Lambda_r) \cdot x=38; |y_n|_{2d}=0; y=0,4,8,12=0; l_{t,1}=\lfloor 18-(l_t+2r_t) \rfloor_{10}; r_{t,1}=\lfloor 26-(r_t+2l_t) \rfloor_6.$$

$2\sum(l_t+r_t)=70$	$2\sum(l_t+r_t)=54$	$2\sum(l_t+r_t)=70$	$2\sum(l_t+r_t)=54$	$2\sum(l_t+r_t)=54$	$2\sum(l_t+r_t)=54$	$2\sum(l_t+r_t)=92$	$2\sum(l_t+r_t)=92$
$\alpha=5$	$\alpha=3$	$\alpha=5$	$\alpha=3$	$\alpha=3$	$\alpha=3$	$\alpha=4$	$\alpha=4$
$P_o = \frac{5 \cdot 38 - 70}{30} = 4$	$P_o = \frac{3 \cdot 38 - 54}{30} = 2$	$P_o = \frac{5 \cdot 38 - 70}{30} = 4$	$P_o = \frac{3 \cdot 38 - 54}{30} = 2$	$P_o = \frac{3 \cdot 38 - 54}{30} = 2$	$P_o = \frac{3 \cdot 38 - 54}{30} = 2$	$P_o = \frac{4 \cdot 38 - 92}{30} = 2$	$P_o = \frac{4 \cdot 38 - 92}{30} = 2$

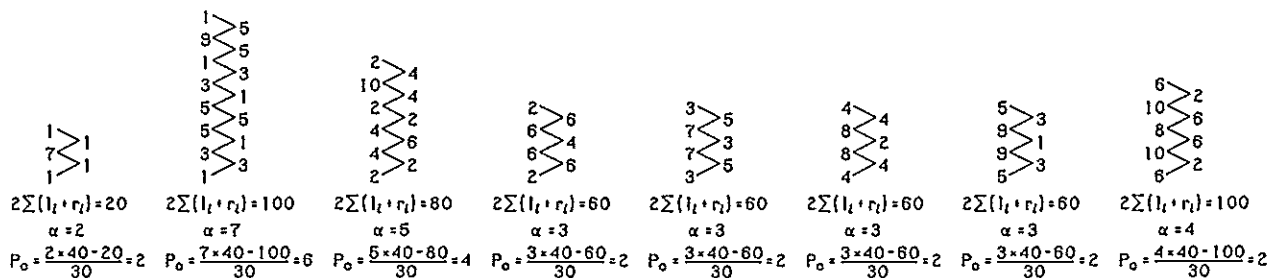
$$\Lambda_1=10; \Lambda_r=6; d=2; x=6; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 \Lambda_r) \cdot x=38; |y_n|_{2d}=2; y=2,6,10; l_{t,1}=\lfloor 18-(l_t+2r_t) \rfloor_{10}; r_{t,1}=\lfloor 26-(r_t+2l_t) \rfloor_6.$$

$2\sum(l_t+r_t)=124$	$2\sum(l_t+r_t)=8$	$2\sum(l_t+r_t)=8$	$2\sum(l_t+r_t)=100$	$2\sum(l_t+r_t)=8$	$2\sum(l_t+r_t)=100$	$2\sum(l_t+r_t)=100$	$2\sum(l_t+r_t)=92$
$\alpha=8$	$\alpha=1$	$\alpha=1$	$\alpha=5$	$\alpha=1$	$\alpha=5$	$\alpha=5$	$\alpha=4$
$P_o = \frac{8 \cdot 38 - 124}{30} = 6$	$P_o = \frac{1 \cdot 38 - 8}{30} = 1$	$P_o = \frac{1 \cdot 38 - 8}{30} = 1$	$P_o = \frac{5 \cdot 38 - 100}{30} = 3$	$P_o = \frac{1 \cdot 38 - 8}{30} = 1$	$P_o = \frac{5 \cdot 38 - 100}{30} = 3$	$P_o = \frac{5 \cdot 38 - 100}{30} = 3$	$P_o = \frac{4 \cdot 38 - 92}{30} = 2$

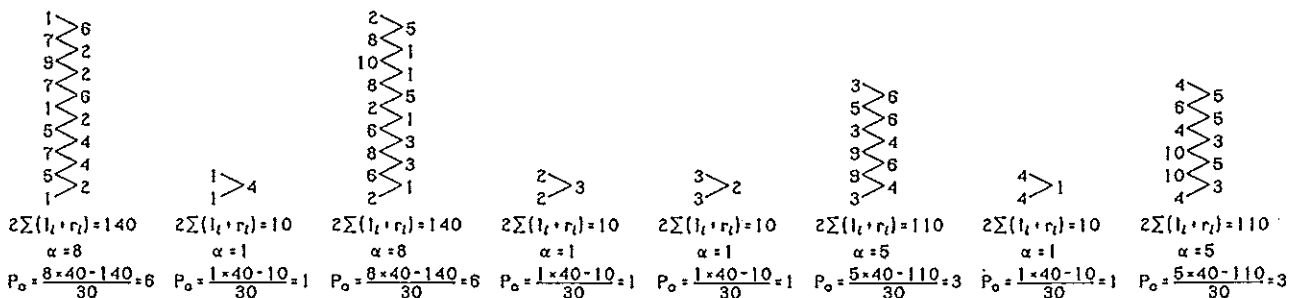
$\Lambda_1=10; \Lambda_r=6; d=2; x=7; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 + \Lambda_r) \cdot x=39; |y_n|_{2d}=1; y=1,5,9; l_{t,1}=[19-(l_t+2r_t)]_{10}; r_{t,1}=[27-(r_t+2l_t)]_6.$



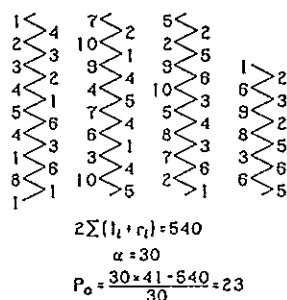
$\Lambda_1=10; \Lambda_r=6; d=2; x=8; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 + \Lambda_r) \cdot x=40; |y_n|_{2d}=0; y=0,4,8,12=0; l_{t,1}=[20-(l_t+2r_t)]_{10}; r_{t,1}=[28-(r_t+2l_t)]_6.$



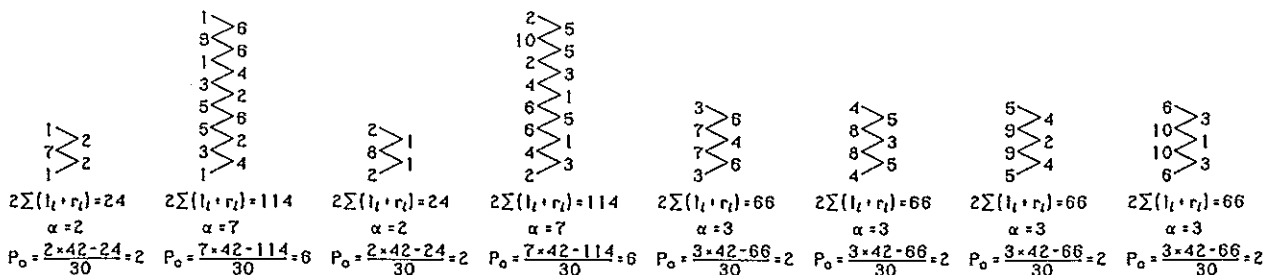
$\Lambda_1=10; \Lambda_r=6; d=2; x=8; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 + \Lambda_r) \cdot x=40; |y_n|_{2d}=2; y=2,6,10; l_{t,1}=[20-(l_t+2r_t)]_{10}; r_{t,1}=[28-(r_t+2l_t)]_6.$



$\Lambda_1=10; \Lambda_r=6; d=2; x=9; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 + \Lambda_r) \cdot x=41; |y_n|_{2d}=1; y=1,5,9; l_{t,1}=[21-(l_t+2r_t)]_{10}; r_{t,1}=[28-(r_t+2l_t)]_6.$



$\Lambda_1=10; \Lambda_r=6; d=2; x=10; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1 + \Lambda_r) \cdot x=42; |y_n|_{2d}=0; y=0,4,8,12=0; l_{t,1}=[22-(l_t+2r_t)]_{10}; r_{t,1}=[30-(r_t+2l_t)]_6.$



$$\Lambda_l=10; \Lambda_r=6; d=2; x=10; \frac{\Lambda_l \Lambda_r}{d}=30; 2(\Lambda_l + \Lambda_r) \cdot x=42; |y_n|_{2d}=2; y=2,6,10; l_{r,1}=[22-(l_r+2r_l)]_{10}; r_{r,1}=[30-(r_r+2l_r)]_6.$$

$2\sum(l_i+r_i)=204$ $\alpha=12$ $P_0=\frac{12 \times 42 - 204}{30}=10$	$2\sum(l_i+r_i)=12$ $\alpha=1$ $P_0=\frac{1 \times 42 - 12}{30}=1$	$2\sum(l_i+r_i)=156$ $\alpha=8$ $P_0=\frac{8 \times 42 - 156}{30}=6$	$2\sum(l_i+r_i)=12$ $\alpha=1$ $P_0=\frac{1 \times 42 - 12}{30}=1$	$2\sum(l_i+r_i)=12$ $\alpha=1$ $P_0=\frac{1 \times 42 - 12}{30}=1$	$2\sum(l_i+r_i)=12$ $\alpha=1$ $P_0=\frac{1 \times 42 - 12}{30}=1$	$2\sum(l_i+r_i)=120$ $\alpha=5$ $P_0=\frac{5 \times 42 - 120}{30}=3$	$2\sum(l_i+r_i)=12$ $\alpha=1$ $P_0=\frac{1 \times 42 - 12}{30}=1$

$$\Lambda_l=10; \Lambda_r=6; d=2; x=11; \frac{\Lambda_l \Lambda_r}{d}=30; 2(\Lambda_l + \Lambda_r) \cdot x=43; |y_n|_{2d}=1; y=1,5,9; l_{r,1}=[23-(l_r+2r_l)]_{10}; r_{r,1}=[31-(r_r+2l_r)]_6.$$

$2\sum(l_i+r_i)=108$ $\alpha=6$ $P_0=\frac{6 \times 43 - 108}{30}=5$	$2\sum(l_i+r_i)=108$ $\alpha=6$ $P_0=\frac{6 \times 43 - 108}{30}=5$	$2\sum(l_i+r_i)=324$ $\alpha=18$ $P_0=\frac{18 \times 43 - 324}{30}=15$

$$\Lambda_l=10; \Lambda_r=6; d=2; x=12; \frac{\Lambda_l \Lambda_r}{d}=30; 2(\Lambda_l + \Lambda_r) \cdot x=44; |y_n|_{2d}=0; y=0,4,8,12=0; l_{r,1}=[24-(l_r+2r_l)]_{10}; r_{r,1}=[32-(r_r+2l_r)]_6.$$

$2\sum(l_i+r_i)=112$ $\alpha=8$ $P_0=\frac{8 \times 44 - 112}{30}=8$	$2\sum(l_i+r_i)=28$ $\alpha=2$ $P_0=\frac{2 \times 44 - 28}{30}=2$	$2\sum(l_i+r_i)=28$ $\alpha=2$ $P_0=\frac{2 \times 44 - 28}{30}=2$	$2\sum(l_i+r_i)=128$ $\alpha=7$ $P_0=\frac{7 \times 44 - 128}{30}=6$	$2\sum(l_i+r_i)=28$ $\alpha=2$ $P_0=\frac{2 \times 44 - 28}{30}=2$	$2\sum(l_i+r_i)=72$ $\alpha=3$ $P_0=\frac{3 \times 44 - 72}{30}=2$	$2\sum(l_i+r_i)=72$ $\alpha=3$ $P_0=\frac{3 \times 44 - 72}{30}=2$	$2\sum(l_i+r_i)=72$ $\alpha=3$ $P_0=\frac{3 \times 44 - 72}{30}=2$

$$\Lambda_l=10; \Lambda_r=6; d=2; x=12; \frac{\Lambda_l \Lambda_r}{d}=30; 2(\Lambda_l + \Lambda_r) \cdot x=44; |y_n|_{2d}=2; y=2,6,10; l_{r,1}=[24-(l_r+2r_l)]_{10}; r_{r,1}=[32-(r_r+2l_r)]_6.$$

$2\sum(l_i+r_i)=228$ $\alpha=12$ $P_0=\frac{12 \times 44 - 228}{30}=10$	$2\sum(l_i+r_i)=14$ $\alpha=1$ $P_0=\frac{1 \times 44 - 14}{30}=1$	$2\sum(l_i+r_i)=228$ $\alpha=12$ $P_0=\frac{12 \times 44 - 228}{30}=10$	$2\sum(l_i+r_i)=14$ $\alpha=1$ $P_0=\frac{1 \times 44 - 14}{30}=1$	$2\sum(l_i+r_i)=14$ $\alpha=1$ $P_0=\frac{1 \times 44 - 14}{30}=1$	$2\sum(l_i+r_i)=14$ $\alpha=1$ $P_0=\frac{1 \times 44 - 14}{30}=1$	$2\sum(l_i+r_i)=14$ $\alpha=1$ $P_0=\frac{1 \times 44 - 14}{30}=1$	$2\sum(l_i+r_i)=14$ $\alpha=1$ $P_0=\frac{1 \times 44 - 14}{30}=1$

$$\Lambda_l=10; \Lambda_r=6; d=2; x=13; \frac{\Lambda_l \Lambda_r}{d}=30; 2(\Lambda_l + \Lambda_r) \cdot x=45; |y_n|_{2d}=1; y=1,5,9; l_{r,1}=[25-(l_r+2r_l)]_{10}; r_{r,1}=[33-(r_r+2l_r)]_6.$$

$2\sum(l_i+r_i)=300$ $\alpha=18$ $P_0=\frac{18 \times 45 - 300}{30}=17$	$2\sum(l_i+r_i)=120$ $\alpha=6$ $P_0=\frac{6 \times 45 - 120}{30}=5$	$2\sum(l_i+r_i)=120$ $\alpha=6$ $P_0=\frac{6 \times 45 - 120}{30}=5$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=14; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1+\Lambda_r) \cdot x=46; |y_n|_{2d}=0; y=0,4,8,12=0; l_{r,1}=[26-(l_r+2r_l)]_{10}; r_{r,1}=[34-(r_r+2l_r)]_6.$$

$2\sum(l_i+r_i)=128$	$2\sum(l_i+r_i)=32$	$2\sum(l_i+r_i)=128$	$2\sum(l_i+r_i)=32$	$2\sum(l_i+r_i)=32$	$2\sum(l_i+r_i)=32$	$2\sum(l_i+r_i)=78$	$2\sum(l_i+r_i)=78$
$\alpha=8$	$\alpha=2$	$\alpha=8$	$\alpha=2$	$\alpha=2$	$\alpha=2$	$\alpha=3$	$\alpha=3$
$P_o = \frac{8 \times 46 - 128}{30} = 8$	$P_o = \frac{2 \times 46 - 32}{30} = 2$	$P_o = \frac{8 \times 46 - 128}{30} = 8$	$P_o = \frac{2 \times 46 - 32}{30} = 2$	$P_o = \frac{2 \times 46 - 32}{30} = 2$	$P_o = \frac{2 \times 46 - 32}{30} = 2$	$P_o = \frac{3 \times 46 - 78}{30} = 2$	$P_o = \frac{3 \times 46 - 78}{30} = 2$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=14; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1+\Lambda_r) \cdot x=46; |y_n|_{2d}=2; y=2,6,10; l_{r,1}=[26-(l_r+2r_l)]_{10}; r_{r,1}=[34-(r_r+2l_r)]_6.$$

$2\sum(l_i+r_i)=96$	$2\sum(l_i+r_i)=48$	$2\sum(l_i+r_i)=48$	$2\sum(l_i+r_i)=252$	$2\sum(l_i+r_i)=16$	$2\sum(l_i+r_i)=16$	$2\sum(l_i+r_i)=16$	$2\sum(l_i+r_i)=16$
$\alpha=6$	$\alpha=3$	$\alpha=3$	$\alpha=12$	$\alpha=1$	$\alpha=1$	$\alpha=1$	$\alpha=1$
$P_o = \frac{6 \times 46 - 96}{30} = 6$	$P_o = \frac{3 \times 46 - 48}{30} = 3$	$P_o = \frac{3 \times 46 - 48}{30} = 3$	$P_o = \frac{12 \times 46 - 252}{30} = 10$	$P_o = \frac{1 \times 46 - 16}{30} = 1$	$P_o = \frac{1 \times 46 - 16}{30} = 1$	$P_o = \frac{1 \times 46 - 16}{30} = 1$	$P_o = \frac{1 \times 46 - 16}{30} = 1$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=15; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1+\Lambda_r) \cdot x=47; |y_n|_{2d}=1; y=1,5,9; l_{r,1}=[27-(l_r+2r_l)]_{10}; r_{r,1}=[35-(r_r+2l_r)]_6.$$

$2\sum(l_i+r_i)=540$				
$\alpha=30$				
$P_o = \frac{30 \times 47 - 540}{30} = 29$				

$$\Lambda_1=10; \Lambda_r=6; d=2; x=16; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1+\Lambda_r) \cdot x=48; |y_n|_{2d}=0; y=0,4,8,12=0; l_{r,1}=[28-(l_r+2r_l)]_{10}; r_{r,1}=[36-(r_r+2l_r)]_6.$$

$2\sum(l_i+r_i)=24$	$2\sum(l_i+r_i)=36$	$2\sum(l_i+r_i)=144$	$2\sum(l_i+r_i)=36$	$2\sum(l_i+r_i)=144$	$2\sum(l_i+r_i)=36$	$2\sum(l_i+r_i)=36$	$2\sum(l_i+r_i)=84$
$\alpha=3$	$\alpha=2$	$\alpha=8$	$\alpha=2$	$\alpha=8$	$\alpha=2$	$\alpha=2$	$\alpha=3$
$P_o = \frac{3 \times 48 - 24}{30} = 4$	$P_o = \frac{2 \times 48 - 36}{30} = 2$	$P_o = \frac{8 \times 48 - 144}{30} = 8$	$P_o = \frac{2 \times 48 - 36}{30} = 2$	$P_o = \frac{8 \times 48 - 144}{30} = 8$	$P_o = \frac{2 \times 48 - 36}{30} = 2$	$P_o = \frac{2 \times 48 - 36}{30} = 2$	$P_o = \frac{3 \times 48 - 84}{30} = 2$

$$\Lambda_1=10; \Lambda_r=6; d=2; x=16; \frac{\Lambda_1 \Lambda_r}{d}=30; 2(\Lambda_1+\Lambda_r) \cdot x=48; |y_n|_{2d}=2; y=2,6,10; l_{r,1}=[28-(l_r+2r_l)]_{10}; r_{r,1}=[36-(r_r+2l_r)]_6.$$

$2\sum(l_i+r_i)=108$	$2\sum(l_i+r_i)=54$	$2\sum(l_i+r_i)=54$	$2\sum(l_i+r_i)=108$	$2\sum(l_i+r_i)=54$	$2\sum(l_i+r_i)=54$	$2\sum(l_i+r_i)=18$	$2\sum(l_i+r_i)=18$
$\alpha=6$	$\alpha=3$	$\alpha=3$	$\alpha=6$	$\alpha=3$	$\alpha=3$	$\alpha=1$	$\alpha=1$
$P_o = \frac{6 \times 48 - 108}{30} = 6$	$P_o = \frac{3 \times 48 - 54}{30} = 3$	$P_o = \frac{3 \times 48 - 54}{30} = 3$	$P_o = \frac{6 \times 48 - 108}{30} = 6$	$P_o = \frac{3 \times 48 - 54}{30} = 3$	$P_o = \frac{3 \times 48 - 54}{30} = 3$	$P_o = \frac{1 \times 48 - 18}{30} = 1$	$P_o = \frac{1 \times 48 - 18}{30} = 1$

The first-return string-run details for x ranging from 1 to 16 and $|y_n|_{2d}$ ranging from 0 to d supply us with all the necessary information for the first-return string-runs of all Asymmetric Regular Nested Cylindrical Braids with $A_l = 10$ and $A_r = 6$.

Let the cells $(|y_n|_{2d}, x)$ and $(|y_n|'_{2d}, x')$ be (A_l, A_r) -complementary.

Let $\Delta x = x' - x$.

Let P_c be the number of parts associated with a first-return string-run belonging to cell $(|y_n|_{2d}, x)$ and let P'_c be the number of parts associated with its (A_l, A_r) -complementary first-return string-run belonging to cell $(|y_n|'_{2d}, x')$. Let $\Delta P_c = P'_c - P_c$.

Let $2 \times \text{sbb} = 2 \sum (l_i + r_i)$ for a first-return string-run belonging to cell $(|y_n|_{2d}, x)$ and let $2 \times \text{sbb}' = 2 \sum (l'_i + r'_i)$ for its (A_l, A_r) -complementary first-return string-run belonging to cell $(|y_n|'_{2d}, x')$. Then:

$$\Delta P_c = \frac{\alpha \cdot \Delta x + 2(2 \times \text{sbb}) - 2\alpha(A_l + A_r + 2)}{A^{**}}, \quad \text{where } A^{**} = \frac{A_l \cdot A_r}{d}.$$

$$2 \times \text{sbb}' = 2\alpha(A_l + A_r + 2) - 2 \times \text{sbb}.$$

Example 1(i):

Cell $(|y_n|_{2d}, x) = (2, 0)$ is (A_l, A_r) -complementary to cell $(|y_n|'_{2d}, x') = (0, 2)$.

components for cell (2, 0)	→	for cell (0, 2)
$4 \times 2 \quad ; \quad \alpha = 3 \quad ; \quad 2 \times \text{sbb} = 42$		$\Delta P_c = \frac{3(-2) + 2 \cdot 42 - 6 \cdot 18}{30} = -1;$ $2 \times \text{sbb}' = 6 \cdot 18 - 42 = 66.$
$2 \times 2 \quad ; \quad \alpha = 4 \quad ; \quad 2 \times \text{sbb} = 76$		$\Delta P_c = \frac{4(-2) + 2 \cdot 76 - 8 \cdot 18}{30} = 0;$ $2 \times \text{sbb}' = 8 \cdot 18 - 76 = 68.$
$2 \times 2 \quad ; \quad \alpha = 5 \quad ; \quad 2 \times \text{sbb} = 110$		$\Delta P_c = \frac{5(-2) + 2 \cdot 110 - 10 \cdot 18}{30} = +1;$ $2 \times \text{sbb}' = 10 \cdot 18 - 110 = 70.$

Example 1(ii):

Cell $(|y_n|'_{2d}, x') = (1, 19)$ is (A_l, A_r) -complementary to cell $(|y_n|_{2d}, x) = (3, 13)$ which in turn is the mirror image of cell $(|y_n|_{2d}, x) = (1, 13)$. Hence cell $(1, 19)$ is the mirror-imaged (A_l, A_r) -complement to cell $(1, 13)$.

components for cell (1, 13)	→	for cell (1, 19)
$2 \times 5 \quad ; \quad \alpha = 6 \quad ; \quad 2 \times \text{sbb} = 120$		$\Delta P_c = \frac{6 \cdot 6 + 2 \cdot 120 - 12 \cdot 18}{30} = +2;$ $2 \times \text{sbb}' = 12 \cdot 18 - 120 = 96.$
$1 \times 17 \quad ; \quad \alpha = 18 \quad ; \quad 2 \times \text{sbb} = 300$		$\Delta P_c = \frac{18 \cdot 6 + 2 \cdot 300 - 36 \cdot 18}{30} = +2;$ $2 \times \text{sbb}' = 36 \cdot 18 - 300 = 348.$

Example 1(iii):

Cell $(|y_n|'_{2d}, x') = (1, 49)$ is (A_l, A_r) -complementary to cell $(|y_n|_{2d}, x) = (3, 43)$ which is the (A_l, A_r) -complement to cell $(3, 19)$ which in turn is the mirror image of cell $(|y_n|_{2d}, x) = (1, 19)$. Hence the first-return string-runs of cell $(1, 49)$ are identical to the first-return string-run(s) of cell $(3, 19)$ (apart from the y_i -values). Mirroring of a first-return string-run has no effect on its associated P_c -value and sbb-value, consequently the ΔP_c -values between the cells $(1, 19)$ and $(1, 49)$ are calculated with the formula:

$$\Delta P_c = \frac{\alpha \cdot \Delta x}{A^{**}}.$$
