THE BRAIDER

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CONTENTS

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for
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Solutions to the Questions in Issue No. 34

Question on pg.791.

For determining the left bight-boundary position specification and the left bight-boundary sequence set with ranking-numbers we can use for the **Periodic Regular Nested Cylindrical Braids** the general fundamental calculation procedures as discussed in *The Braider*, Issue No. 19, pp. 415-416, or we can use their general forms as shown in *The Braider*, Issue No. 34, pg. 790.

The bight-boundary position specification and the bight-boundary sequence set with ranking-numbers for the left bight-edge depicted at the upper left in Fig. 636 is then with $A_{l_1} = 12$ and $A_{l_2} = 5$, hence $z = 13$:

$$(2221111111111222)\{1_1(13)_{17}(11)_{16}9_{15}7_{14}5_{13}(17)_{12}(16)_{11}(15)_{10}(14)_9(12)_8(10)_7 8_6 6_5 4_4 3_3 2_2\}\,.$$

The bight-boundary position specification and the bight-boundary sequence set with ranking-numbers for the left bight-edge depicted at the upper right in Fig. 636 is then with $A_{l_1} = 15$ and $A_{l_2} = 6$, hence $z = 16$:

$$(22221111111111112222)\{1_1(16)_{21}(14)_{20}(12)_{19}(10)_{18}8_{17}6_{16}(21)_{15}(20)_{14}(19)_{13}(18)_{12}(17)_{11}(15)_{10}(13)_9(11)_8 9_7 7_6 5_5 4_4 3_3 2_2\}\,.$$

The bight-boundary position specification and the bight-boundary sequence set with ranking-numbers for the left bight-edge depicted at the lower left in Fig. 636 is then with $A_{l_1} = 12$ and $A_{l_2} = 4$, hence $z = 8$:

$$(22222222222)\{1_1 8_{16}7_{15}6_{14}5_{13}(12)_{12}(11)_{11}(10)_{10}9_9 8_8 7_7 6_6 5_5 4_4 3_3 2_2\}\,.$$

The bight-boundary position specification and the bight-boundary sequence set with ranking-numbers for the left bight-edge depicted at the lower right in Fig. 636 is then with $A_{l_1} = 15$ and $A_{l_2} = 5$, hence $z = 10$:

$$(22222222222222)\{1_1(10)_{20}9_{19}8_{18}7_{17}6_{16}(15)_{15}(14)_{14}(13)_{13}(12)_{12}(11)_{11}(10)_{10}9_9 8_8 7_7 6_6 5_5 4_4 3_3 2_2\}\,.$$

Question on pg.792.

For determining the right bight-boundary position specification and the right bight-boundary sequence set with ranking-numbers in nominal cyclic form (since the RRHC[†] is not known) we can use for the **Periodic Regular Nested Cylindrical Braids** the general fundamental procedures as discussed in *The Braider*, Issue No. 19, pg. 415, or we can use their general forms as shown in *The Braider*, Issue No. 34, pp. 791-792.

The bight-boundary position specification and the bight-boundary sequence set for the right bight-edge depicted at the upper left in Fig. 637 is then with $A_{r_1} = 12$ and $A_{r_2} = 5$, hence $z = 13$:

$$(2221111111111222)\{123468(10)(12)(14)(15)(16)(17)579(11)(13)\}\,.$$

The bight-boundary position specification and the bight-boundary sequence set for the right bight-edge depicted at the upper right in Fig. 637 is then with $A_{r_1} = 15$ and $A_{r_2} = 6$, hence $z = 16$:

$$(22221111111111112222)\{1234579(11)(13)(15)(17)(18)(19)(20)(21)68(10)(12)(14)(16)\}\,.$$

The bight-boundary position specification and the bight-boundary sequence set for the right bight-edge depicted at the lower left in Fig. 637 is then with $A_{r_1} = 12$ and $A_{r_2} = 4$, hence $z = 8$:

$$(22222222222)\{123456789(10)(11)(12)5678\}\,.$$

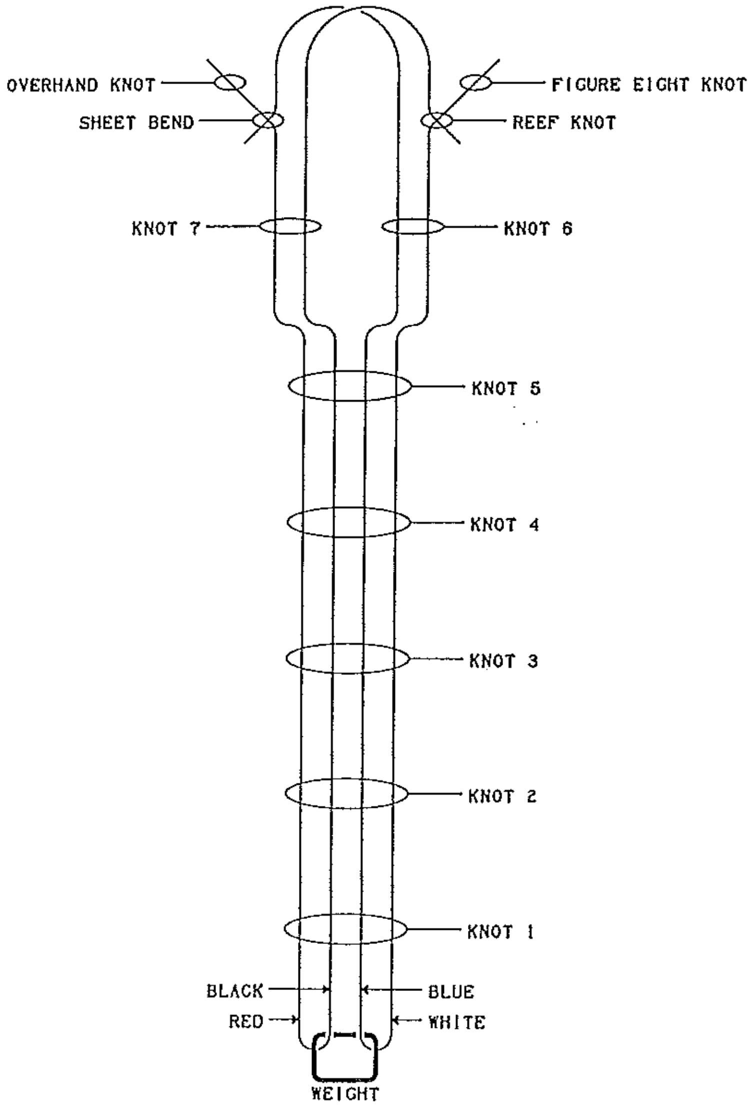
The bight-boundary position specification and the bight-boundary sequence set for the right bight-edge depicted at the lower right in Fig. 637 is then with $A_{r_1} = 15$ and $A_{r_2} = 5$, hence $z = 10$:

[†] See *The Braider*, Issue No. 19, pg. 415.

(22222222222222){123456789(10)(11)(12)(13)(14)(15)6789(10)}.

A Hanger (Lanyard) with colourful knots

In early 1999 we received from Gene Ulrich[†] a hanger decorated with a number of colourful knots. Fig. 644 shows schematically the hanger with the various knots. Two seven feet, white cotton, round braided, strands of $\frac{5}{32}$in diameter were used. Half the length of one strand was dyed blue. The other strand was dyed red for half its length, and dyed black for the other half of its length.



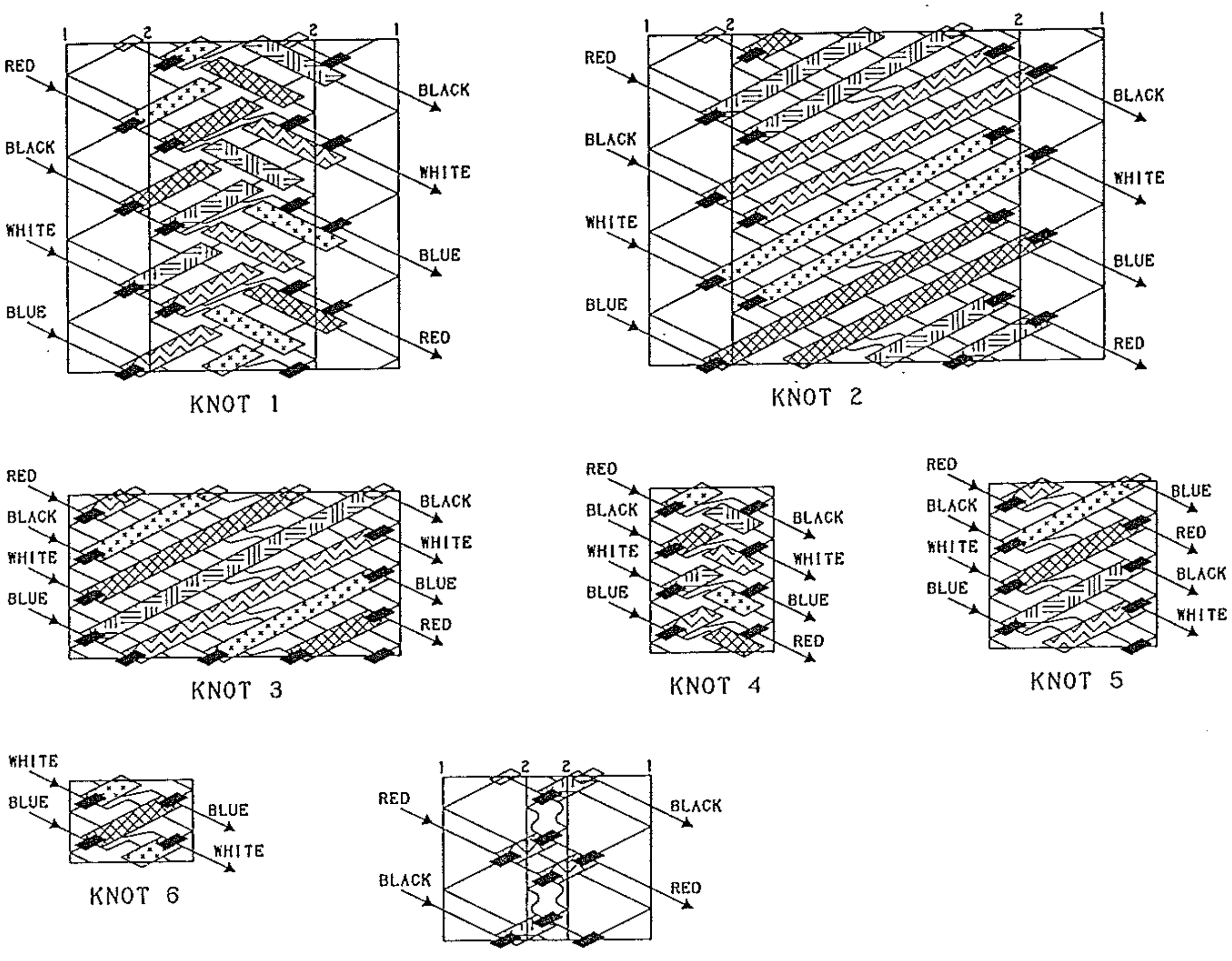
USING THIN COLOURED COTTON: PLACE A CONSTRICTOR KNOT AT THE END OF EACH STRAND.

Fig. 644 — The schematic layout of the hanger with its knots.

The grid-diagrams of the knots #1 to #7 are shown in Fig. 645. Knot #1 is a 2-ply form of a **footrope knot**[‡], which is a 4-string **Crown knot** with a 4-string **Wall knot** under it and feeding the string-ends through the crown (knot #4).

[†] See *The Braider*, Issue No. 26, pg. 599.

[‡] See *The Braider*, Issue No. 5, pg. 97, Fig. 81; pg. 99, Fig. 83; pg. 106.



KNOT 1 KNOT 2

KNOT 3 KNOT 4 KNOT 5

KNOT 6 KNOT 7

Fig. 645 — The grid-diagrams of the knots #1 to #7.

Knot #2 is a 2-ply form of a $p/b = 5/4$ (5-parts/4-bights) Matthew Walker Knot.[†]

Knot #3 is a $p/b = 8/4$ Matthew Walker Knot.

Knot #4 is a $p/b = 3/4$ Footrope Knot.

Knot #5 is a $p/b = 4/4$ Matthew Walker Knot.

Knot #6 is a $p/b = 3/2$ Matthew Walker Knot.

Knot #7 is a 2-ply form of a $p/b = 2/2$ Matthew Walker Knot (which is a $p/b = 2/2$ Wall Knot). We can draw the grid-diagram of knot #7 in Fig. 645 also as in Fig. 646.

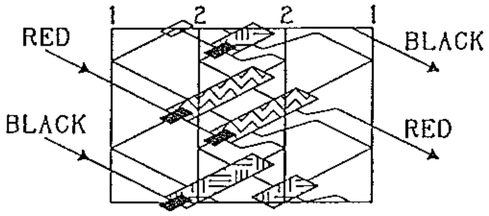


Fig. 646 — Knot #7; alternative drawing of its grid-diagram in Fig. 645.

[†] We have discussed this 2-ply form in *The Braider*, Issue No. 26, pg. 600, Fig. 488 lower right.

There are always two 2-ply forms of a Regular Cylindrical Braid; for example in Fig. 647 we have depicted the other 2-ply form of the $p/b = 5/4$ Matthew Walker Knot on which knot #2 is based. If the colour, shape and texture of all the four strings are identical, then knot #2 and the knot in Fig. 647 would look exactly the same, although they are, of course, not the same.

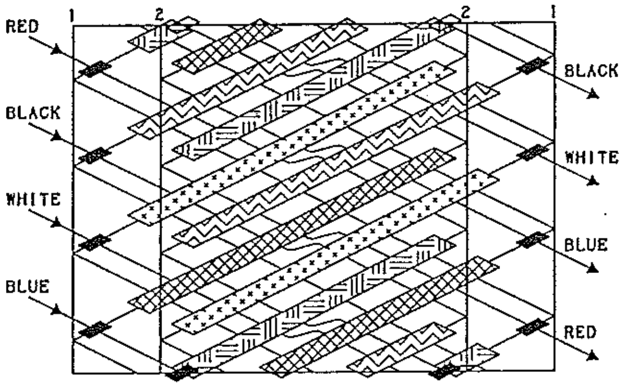


Fig. 647 — The other 2-ply form associated with the knot on which knot #2 is based.

If one would continue braiding knot #5 as the knot in Fig. 647, one would get the same colour-pattern as in knot #2, but the knot is a bit shorter of course. With the aid of grid-diagrams we can readily find the longer 2-ply Matthew Walker Knots which have the same colour-pattern.

Nested Cylindrical Braids

In *The Braider*, Issue No. 33, we discussed for the **Asymmetric Regular Nested Cylindrical Braids** the basic calculation procedures leading to their half-cycle braiding algorithms. Let's now have a further look at their first-return string-runs. We shall use as convention $A_l > A_r$.

There are two basic complementary forms associated with a first-return string-run:

1. **The basic complementary form 1 first-return string-run is the mirror-image of a first-return string-run.**

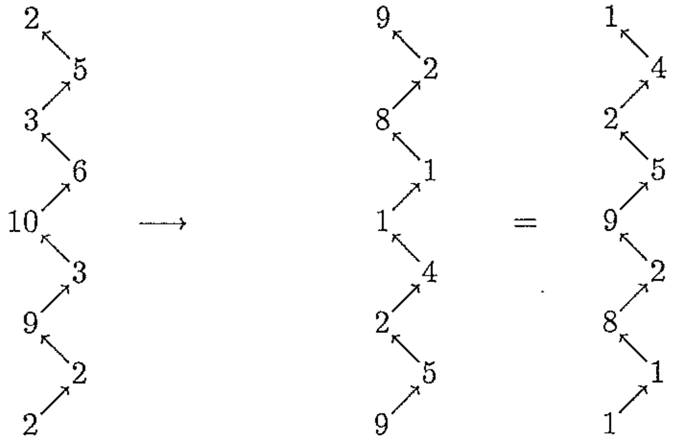
Example 1: $(A_l = 10\,;\ A_r = 6\,;\ x = 33)$

$$\begin{tikzcd}
2 & & & 2 & \\
& 5 \arrow[ul] & & & 2 \arrow[ul] \\
3 \arrow[ur] & & & 9 \arrow[ur] & \\
& 6 \arrow[ul] & & & 3 \arrow[ul] \\
10 \arrow[ur] & & \longrightarrow & 10 \arrow[ur] & \\
& 3 \arrow[ul] & & & 6 \arrow[ul] \\
9 \arrow[ur] & & & 3 \arrow[ur] & \\
& 2 \arrow[ul] & & & 5 \arrow[ul] \\
2 \arrow[ur] & & & 2 \arrow[ur] &
\end{tikzcd}$$

2. **The basic complementary form 2 first-return string-run involves the half-cycle complements with respect to A_l and A_r (the (A_l, A_r)-complement,**

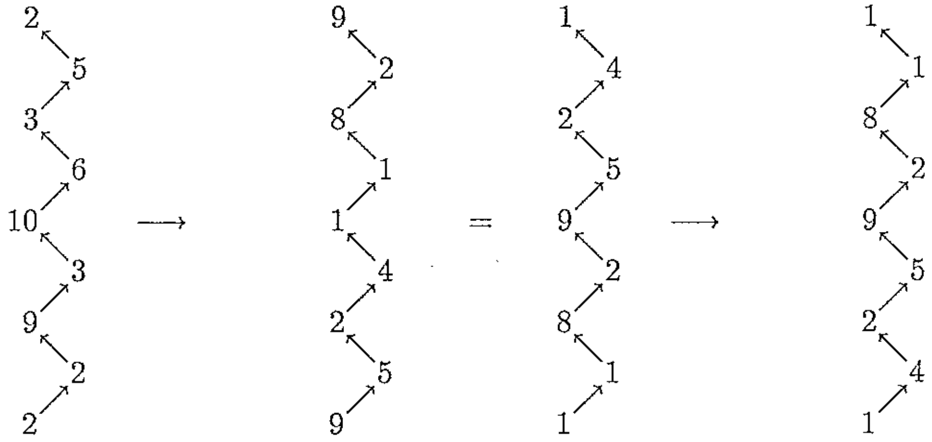
where the complement of l_i is equal to $(A_l + 1 - l_i)$ and the complement of r_i is equal to $(A_r + 1 - r_i)$).

Example 2: $(A_l = 10\,;\ A_r = 6\,;\ x = 33)$

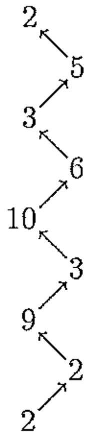


The combination of these two basic complementary first-return string-run forms results in a compound-complementary first-return string-run which is the mirror-imaged (A_l, A_r)-complement of the given first-return string-run.

Example 3: $(A_l = 10\,;\ A_r = 6\,;\ x = 33)$



It should be noted that the given first-return string-run



in the above examples is not the only first-return string-run in the Asymmetric Regular Nested Cylindrical Braid with $A_l = 10$; $A_r = 6$; $x = 33$ and a lower-left to upper-right half-cycle from left bight-boundary 2 to right bight-boundary 2.

An Asymmetric Regular Nested Cylindrical Braid has in general more than one distinct y-value associated with its first-return string-run(s). In fact, when an Asymmetric Regular Nested Cylindrical Braid has only one distinct y-value, the g.c.d. (A_l, A_r) must be A_r as we shortly shall see. When an Asymmetric Regular Nested Cylindrical Braid has more than one distinct y-value and more than one first-return string-run, then each first-return string-run is not necessarily associated with all the distinct y-values of the braid.

In Fig. 648 is depicted a section of an Asymmetric Regular Nested Cylindrical Braid. The y-values (in general not all distinct) associated with a first-return string-run are calculated with the formula

$$y_i = |2A_l + x - 2(l_i + r_i)|_{2A_r} ,$$

whereas the distinct y_m-values associated with the Asymmetric Regular Nested Cylindrical Braid are calculated from any y_n-value with the formula

$$y_m = |y_n + 2n'A_l|_{2A_r} , \quad \text{where } n' = 0\,,1\,,2\,,3\,,\cdots,(\frac{A_r}{d} - 1)\,.$$

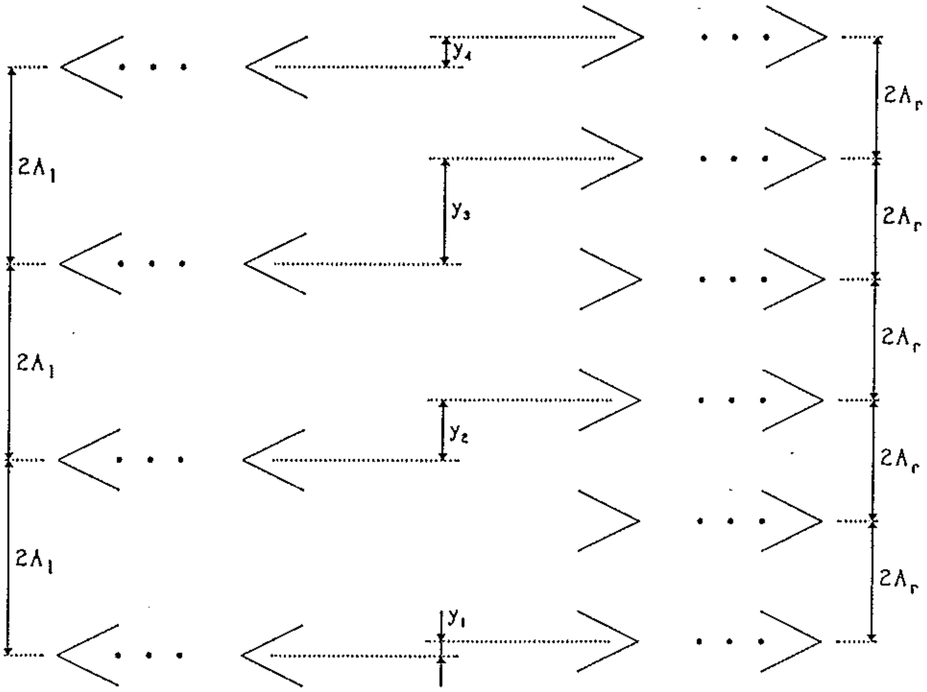


Fig. 648 — A section of an Asymmetric Regular Nested cylindrical Braid.

$$|2n'A_l|_{2A_r} = \left|2n'd\frac{A_l}{d}\right|_{2d\frac{A_r}{d}} = 2d\left|n'\frac{A_l}{d}\right|_{\frac{A_r}{d}} = 2md\,, \quad \text{where}$$

$m = 0\,,1\,,2\,,3\,,\cdots,(\frac{A_r}{d} - 1)$. Hence $y_m = |y_n + 2md|_{2A_r}$. It thus follows that $y_{\min} = |y_n|_{2d} = |y_m|_{2d}$, and $y_m = |y_n|_{2d} + 2md$.

The various distinct y_m-values associated with an Asymmetric Regular Nested Cylindrical Braid for which the g.c.d. $(A_l, A_r) = d$ are indicated by × in the left-hand diagram of Fig. 649.

When we take the mirror-image of this Asymmetric Regular Nested Cylindrical Braid (mirror-image from "top" to "bottom" in order to keep the A_l bight-edge on the left-hand side and the A_r bight-edge on the right-hand side), then the distinct y_m-values associated with this mirror-imaged Asymmetric Regular Nested Cylindrical Braid are indicated by $\star$ in the right-hand diagram of Fig. 649. Hence the lines $y = md$, where $m = 0, 1, 2, 3, \cdots, (2\frac{A_r}{d} - 1)$, are lines of symmetry for mirror-imaged Asymmetric Regular Nested Cylindrical Braids having the same x-value.

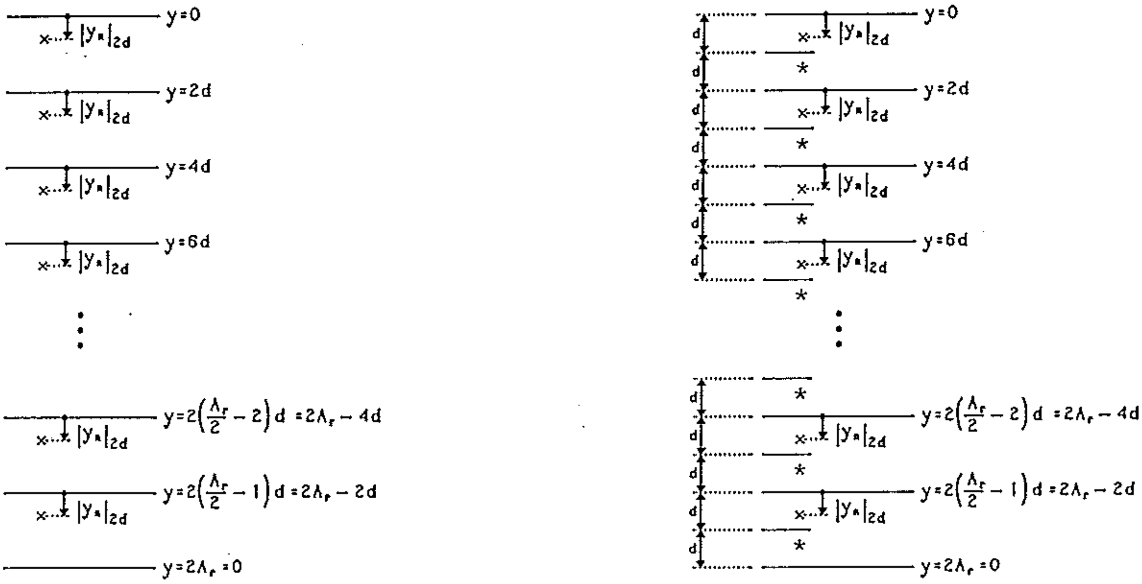


Fig. 649 — The distinct y_m-values associated with an Asymmetric Regular Nested Cylindrical Braid with g.c.d.$(A_l, A_r) = d$ and with its mirror-imaged complementary Asymmetric Regular Nested Cylindrical Braid.

Note that for $\frac{A_r}{d} =$ **even**, y_m and $|A_r + y_m|_{2A_r}$ are both distinct y-values of the same Asymmetric Regular Nested Cylindrical Braid, and that for $\frac{A_r}{d} =$ **odd**, y_m and $|A_r + y_m - d|_{2A_r}$ are both distinct y-values of the same Asymmetric Regular Nested Cylindrical Braid.

On pg. 812 we mentioned that when an Asymmetric Regular Nested Cylindrical Braid has only one distinct y_m-value, the g.c.d.(A_l, A_r) must be A_r. This follows immediately from the formula $y_m = |y_n|_{2d} + 2md$, where $m = 0, 1, 2, 3, \cdots, (\frac{A_r}{d} - 1)$, since m can then only be equal to 0, hence $(\frac{A_r}{d} - 1) = 0$. This requires $d = A_r$.

From the left-hand diagram in Fig. 649 we saw that an Asymmetric Regular Nested Cylindrical Braid having nesting-numbers A_l and A_r with g.c.d.$(A_l, A_r) = d$ has the following $\frac{A_r}{d}$ distinct y_m-values:

$$|y_n|_{2d} + 2md, \text{ where } m = 0, 1, 2, 3, \cdots, (\frac{A_r}{d} - 1).$$

We shall indicate this set of $\frac{A_r}{d}$ distinct y_m-values as the $|y_n|_{2d}$-set of distinct y_m-values.

For the Asymmetric Regular Nested Cylindrical Braids with nesting-numbers A_l and A_r, hence having g.c.d.$(A_l, A_r) = d$, we thus have $2d$ distinct $|y_n|_{2d}$-sets each of

$\dfrac{A_r}{d}$ distinct y_m-values. In the y_m-x tables we repeat the $|y_n|_{2d} = 0$-set of distinct y_m-values as the $|y_n|_{2d} = 2d$-set of distinct y_m-values. Hence in the y_m-x tables we have $(2d+1)$ rows of $|y_n|_{2d}$-sets with the row for the set $|y_n|_{2d} = d$ being the mirror-imaging row.

The minimum x-value associated with a $|y_n|_{2d}$-set of distinct y_m-values depends on the minimum y_m-value in this set, hence depends on $|y_n|_{2d}$. The minimum x-value associated with the $|y_n|_{2d}$-set is expressed by the formula:

$$x_{\min} = 2 - |y_n|_d .$$

Braid with $x = x_1$:

$$\begin{tikzcd}
l_{i+1} & \\
& r_i \arrow[ul] \\
l_i \arrow[ur] &
\end{tikzcd}$$

$$\Delta l_i = |l_{i+1} - l_i|_{A_l} .$$
$$\Delta l_i = |2A_r + x_1 - 2(l_i + r_i)|_{A_l} .$$

$$\begin{tikzcd}
& r_{j+1} \\
l_j \arrow[ur] & \\
& r_j \arrow[ul]
\end{tikzcd}$$

$$\Delta r_j = |r_{j+1} - r_j|_{A_r} .$$
$$\Delta r_j = |2A_l + x_1 - 2(r_j + l_j)|_{A_r} .$$

Complementary form (2); braid with $x = x_2$:

$$\begin{tikzcd}
{|A_l + 1 - l_{i+1}|_{A_l}} & \\
& {|A_r + 1 - r_i|_{A_r}} \arrow[ul] \\
{|A_l + 1 - l_i|_{A_l}} \arrow[ur] &
\end{tikzcd}$$

$$\Delta l_i^* = |l_i - l_{i+1}|_{A_l} .$$
$$\Delta l_i^* = |2A_r + x_2 - 2(A_l + A_r + 2 - (l_i + r_i))|_{A_l} .$$

$$\begin{tikzcd}
& {|A_r + 1 - r_{j+1}|_{A_r}} \\
{|A_l + 1 - l_j|_{A_l}} \arrow[ur] & \\
& {|A_r + 1 - r_j|_{A_r}} \arrow[ul]
\end{tikzcd}$$

$$\Delta r_j^* = |r_j - r_{j+1}|_{A_r} .$$
$$\Delta r_j^* = |2A_l + x_2 - 2(A_l + A_r + 2 - (r_j + l_j))|_{A_r} .$$

For Δl_i^* to be $|l_i - l_{i+1}|_{A_l} = |-\Delta l_i|_{A_l}$ and Δr_i^* to be $|r_i - r_{i+1}|_{A_r} = |-\Delta r_i|_{A_r}$, we must have the condition $x_2 = 2n^*A^{**} - 2(A_l + A_r - 2) - x_1$, where $|2n^*A^{**}|_{A_l} = |2n^*A^{**}|_{A_r} = 0$. Hence the x_c-lines (the lines about which (A_l, A_r) complementary/compound complementary symmetry takes place) are the lines $x = \dfrac{x_1 + x_2}{2} = n^*A^{**} + 2 - (A_l + A_r)$.

Since $|2n^*A^{**}|_{A_l} = |2n^*A^{**}|_{A_r} = 0$ (hence $2n^*\dfrac{A_r}{d}$ must be a *whole number* and $2n^*\dfrac{A_l}{d}$ must be a *whole number*), and since g.c.d.$(\dfrac{A_l}{d}, \dfrac{A_r}{d}) = 1$ (consequently $\dfrac{A_l}{d}$ and $\dfrac{A_l}{d}$ cannot both be *even*) it follows that $2n^*$ must be a *whole number*). However, since $x \geq 2 - |y_n|_d$, we therefore require n^*A^{**} to be a *natural number* such that $\{n^*A^{**} + 2 - (A_l + A_r)\} \geq \{2 - d\}$. Hence n^* is a positive integer such that $\{n^*A^{**} + 2 - (A_l + A_r)\} \geq \{2 - d\}$.

Let $|y_n|_{2d} = y_1$ be the y_m-set associated with x_1. Then:

$y_1 = |2A_l + x_1 - 2(l_i + r_i)|_{2d}$.

Thus for the Asymmetric Regular Nested Cylindrical Braid associated with x_1 the y_m-values are $y_1 + 2md$, where $m = 0, 1, 2, 3, \cdots, (\frac{A_r}{d} - 1)$. Consequently, when $\frac{A_r}{d} = even$ and y_1 is a y_m-value associated with the Asymmetric Regular Nested Cylindrical Braid, then also $y_1 + A_r$ is another y_m-value associated with the same braid.

Let $|y_n|_{2d} = y_2$ be the y_m-set associated with x_2. Then:

$$\begin{aligned} y_2 &= |2A_l + x_2 - 2(l_i^* + r_i^*)|_{2d} \\ &= |2A_l + 2n^*A^{**} - 2(A_l + A_r - 2) - x_1 - 2(A_l + A_r + 2 - (l_i + r_i))|_{2d} \\ &= |2n^*A^{**} - 2A_l + 2(l_i + r_i) - x_1|_{2d} \\ &= |2n^*A^{**} - y_1|_{2d}. \end{aligned}$$

$|2n^*A^{**}|_{2d} = 0$ for $y_2 = |-y_1|_{2d}$.

Hence for $|y_2|_{2d} = |-y_1|_{2d}$: (the Asymmetric Regular Nested Cylindrical braids associated with x_1 and x_2 are (A_l, A_r)-complements of each other)

When $\frac{A_l}{d} =$ **even**: (note that $A^{**} = even$ and $\frac{A_r}{d}$ is *odd*)

$2n^*$ has to be a **positive** integer such that $\{n^*A^{**} + 2 - (A_l + A_r)\} \geq \{2 - d\}$.

When $\frac{A_r}{d} =$ **even**: (note that $A^{**} = even$ and $\frac{A_l}{d}$ is *odd*)

$2n^*$ has to be a **positive** integer such that $\{n^*A^{**} + 2 - (A_l + A_r)\} \geq \{2 - d\}$.

When $\frac{A_l}{d} =$ **odd** and $\frac{A_r}{d} =$ **odd**: (note that $A^{**} = even$ or *odd*)

n^* has to be a **positive** integer such that $\{n^*A^{**} + 2 - (A_l + A_r)\} \geq \{2 - d\}$.

Since $y_2 = |-y_1|_{2d}$ for the above conditions, the row associated with the y_m-set $|y_n|_{2d} = d$ is for those conditions the row of symmetry for the (A_l, A_r)-complements associated with any of the y_m-sets. The row associated with the y_m-set $|y_n|_{2d} = d$ is also the row of symmetry for the mirror-complements associated with any y_m-set.

$|2n^*A^{**}|_{2d} = d$ for $y_2 = |d - y_1|_{2d}$.

Hence for $|y_2|_{2d} = |d - y_1|_{2d}$: (the Asymmetric Regular Nested Cylindrical braids associated with x_1 and x_2 are (A_l, A_r)-complements of each other)

$\frac{A_l}{d} =$ **odd**, $\frac{A_r}{d} =$ **odd** and $A^{**} =$ **even** (hence $d = even$):

$2n^*$ has to be a **positive odd** integer such that $\{n^*A^{**} + 2 - (A_l + A_r)\} \geq \{2 - d\}$.

Since $y_2 = |d - y_1|_{2d}$ for $\frac{A_l}{d} =$ **odd**, $\frac{A_r}{d} =$ **odd**, $A^{**} =$ **even** (hence $d = even$) and $2n^*$ a **positive odd** integer such that $\{n^*A^{**} + 2 - (A_l + A_r)\} \geq \{2 - d\}$, the row associated with the y_m-set $|y_n|_{2d} = \frac{d}{2}$ is the row of symmetry for the (A_l, A_r)-complements associated with the y_m-sets $|y_n|_{2d} \leq d$, while the row associated with the y_m-set $|y_n|_{2d} = \frac{3d}{2}$ is the row of symmetry for the (A_l, A_r)-complements associated with the y_m-sets $|y_n|_{2d} \geq d$.

The row associated with the y_m-set $|y_n|_{2d} = d$ is the row of symmetry for the mirror-complements associated with any y_m-set.

$\frac{A_l}{d} = \textbf{odd}\,,\ \frac{A_r}{d} = \textbf{odd}$ and $A^{**} = \textbf{odd}$ (hence $d = odd$):

When $2n^*$ is a **positive odd** integer then $2\{n^*A^{**} + 2 - (A_l + A_r)\}$ is **odd**, hence $\{n^*A^{**} + 2 - (A_l + A_r)\} = \left\{\frac{2n'' + 1}{2}A^{**} + 2 - (A_l + A_r)\right\}$, where n'' is a whole number such that $\left\{\frac{2n'' + 1}{2}A^{**} + 2 - (A_l + A_r)\right\} \geq \{2 - d\}$. These x_c-values are *mixed numbers*, hence are not real x-values. The lines representing them fall between the real x-values $\left\{\frac{3 + (2n'' + 1)A^{**}}{2} - (A_l + A_r)\right\}$ and $\left\{\frac{5 + (2n'' + 1)A^{**}}{2} - (A_l + A_r)\right\}$.

Since $y_2 = |d - y_1|_{2d}$ for $\frac{A_l}{d} = \textbf{odd}\,,\ \frac{A_r}{d} = \textbf{odd}\,,\ A^{**} = \textbf{odd}$ (hence $d = odd$) and $2n^*$ a **positive odd** integer such that $\{n^*A^{**} + 2 - (A_l + A_r)\} \geq \{2 - d\}$, the values of $|y_n|_{2d} = \frac{d}{2}$ and $|y_n|_{2d} = \frac{3d}{2}$ are *mixed numbers*, hence not real $|y_n|_{2d}$-values. The line representing the $\frac{d}{2}$-value is the line of symmetry for the (A_l, A_r)-complements associated with the y_m-sets $|y_n|_{2d} \leq d$, while the line representing the $\frac{3d}{2}$-value is the line of symmetry for the (A_l, A_r)-complements associated with the y_m-sets $|y_n|_{2d} \geq d$.

The row associated with the y_m-set $|y_n|_{2d} = d$ is the row of symmetry for the mirror-complements associated with any y_m-set.

Thus:

For Asymmetric Regular Nested Cylindrical Braids with $\frac{A_l}{d} = \text{odd}\,,\ \frac{A_r}{d} = \text{odd}\,,$ $A^{**} = \text{odd}$, the general y_m-x table layout is as in Fig. 650. The vertical bold x_c-lines are associated with the values $\left\{\frac{2n'' + 1}{2}A^{**} + 2 - (A_l + A_r)\right\}$, and the horizontal bold lines are associated with the values $\frac{d}{2}$ and $\frac{3d}{2}$.

For Asymmetric Regular Nested Cylindrical Braids with $\frac{A_l}{d} = \text{odd}\,,\ \frac{A_r}{d} = \text{odd}\,,$ $A^{**} = \text{even}$, the general y_m-x table layout is as in Fig. 651.

For Asymmetric Regular Nested Cylindrical Braids with $\frac{A_l}{d} = \text{even}\,,\ \frac{A_r}{d} = \text{odd}$, or with $\frac{A_l}{d} = \text{odd}\,,\ \frac{A_r}{d} = \text{even}$, the general y_m-x table layout is as in Fig. 652.

The first-return string-runs belonging to cell F_1 and cell F_2 are the mirror-image of each other.

The first-return string-runs belonging to cell F_3 and cell F_4 are the mirror-image of each other.

The first-return string-runs belonging to cell F_1 and cell F_3 are the mirror-imaged (A_l, A_r)-complements of each other.

The first-return string-runs belonging to cell F_2 and cell F_4 are the mirror-imaged (A_l, A_r)-complements of each other.

The first-return string-runs belonging to cell F_1 and cell F_4 are the (A_l, A_r)-complements of each other.

The first-return string-runs belonging to cell F_2 and cell F_3 are the (A_l, A_r)-complements of each other.

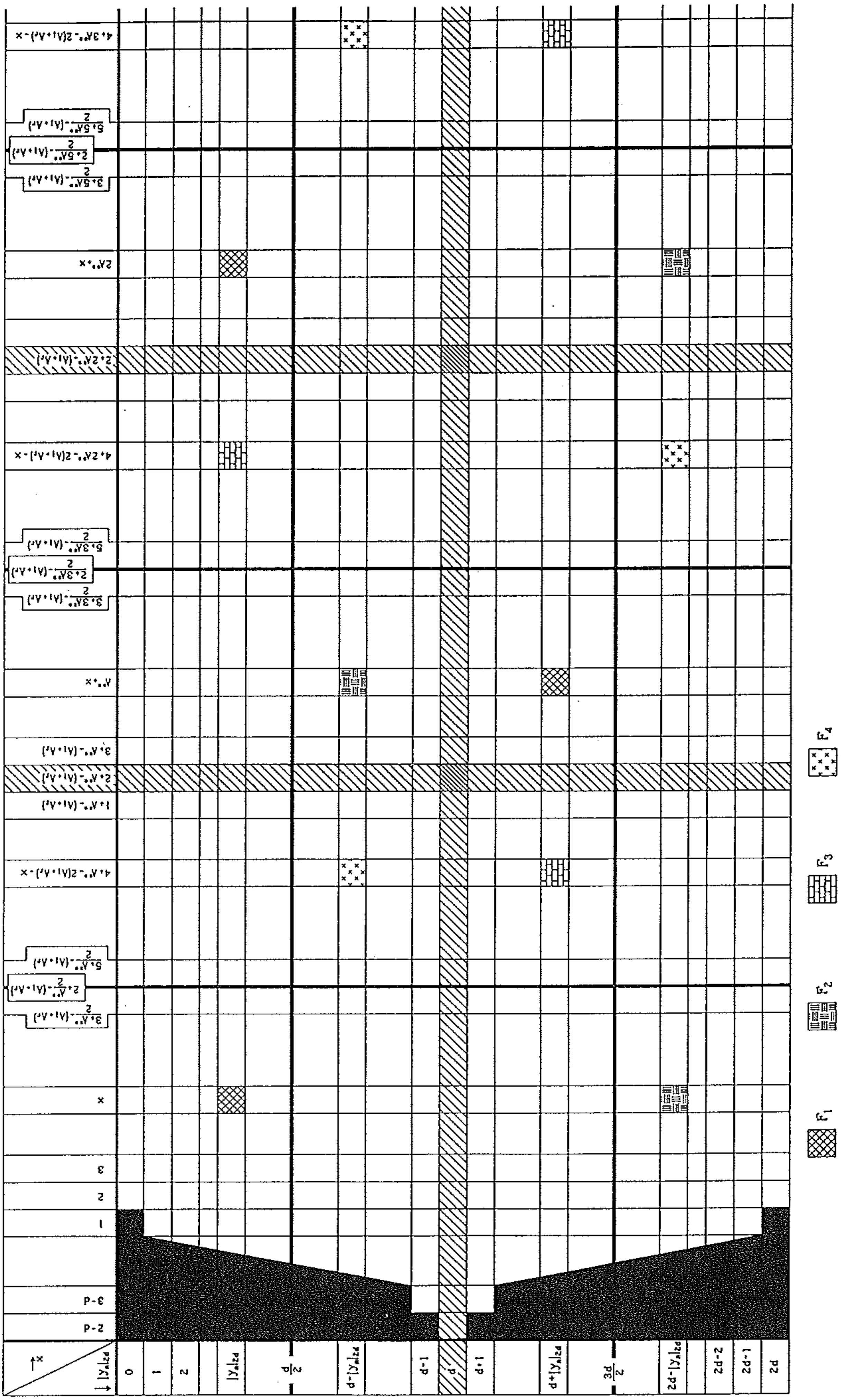


Fig. 650 — General y_m-x table layout when $\frac{A_l}{d} = \text{odd}$, $\frac{A_r}{d} = \text{odd}$, $A^{**} = \text{odd}$.

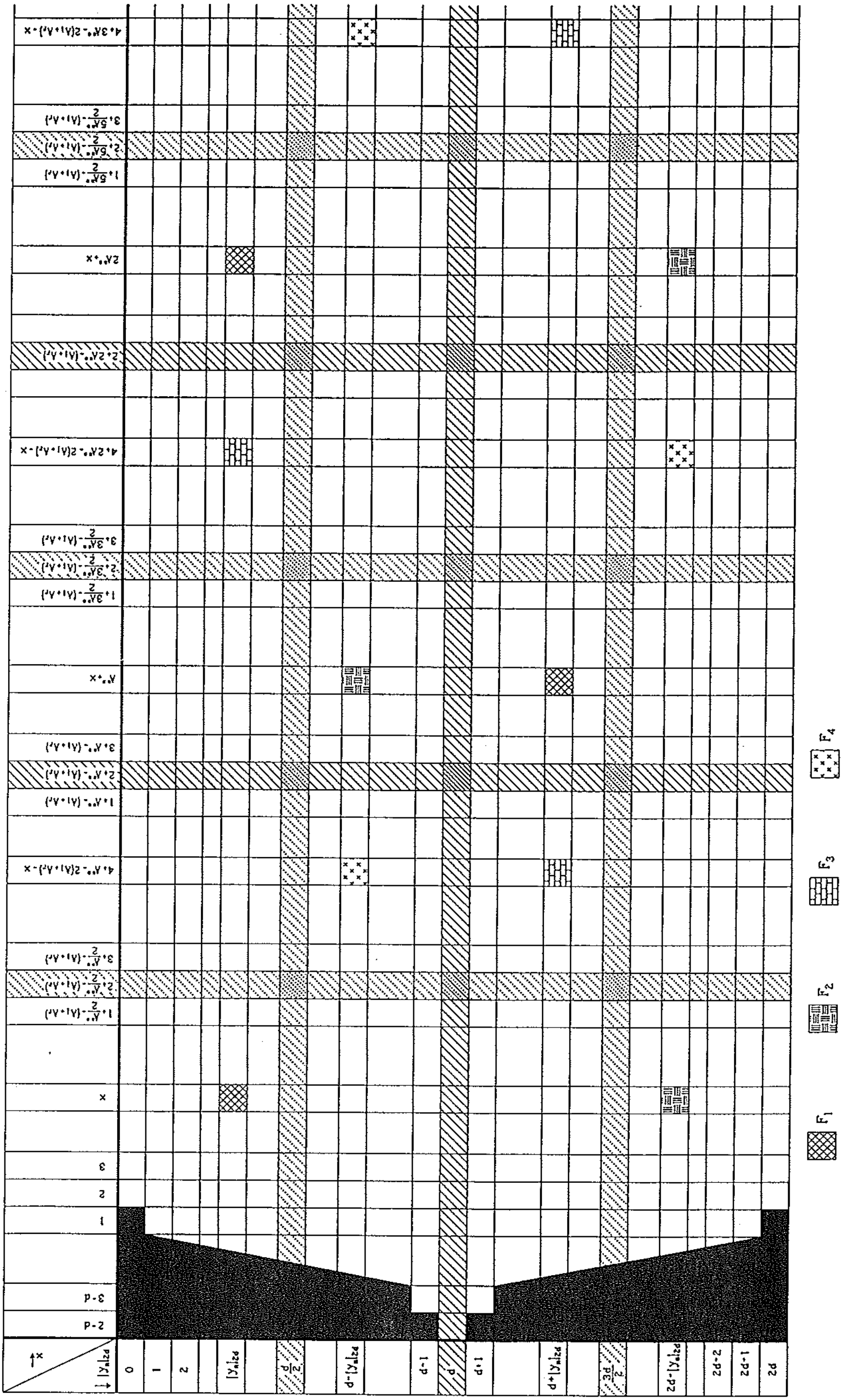


Fig. 651 — General y_m-x table layout when $\frac{A_l}{d}$ = odd, $\frac{A_r}{d}$ = odd, A^{**} = even.

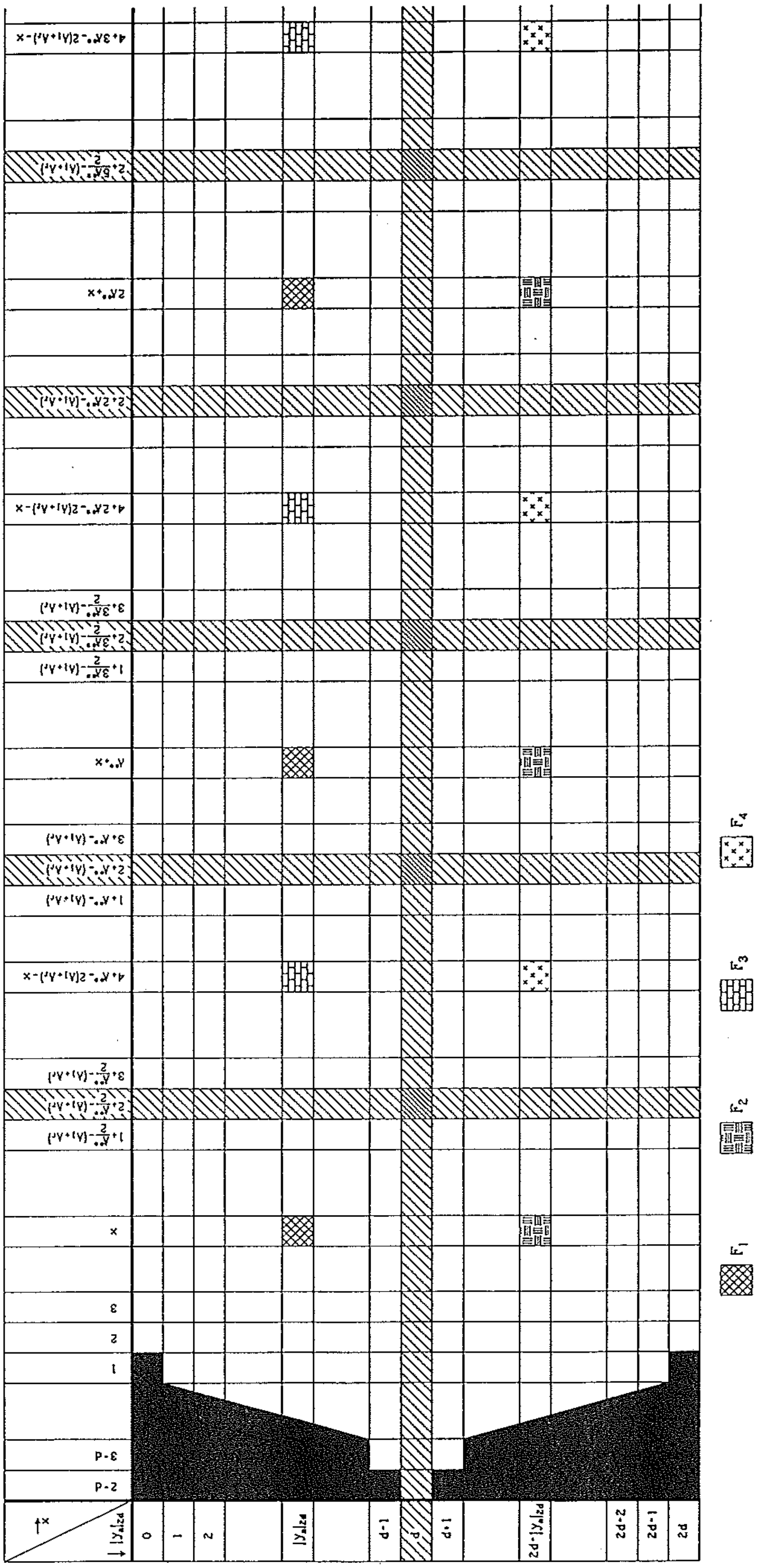


Fig. 652 — General y_m-x table layout when $\frac{A_l}{d} =$ even, or $\frac{A_r}{d} =$ even.

Hence:

In the x/y-tables for $\frac{A_l}{d} = odd$, $\frac{A_r}{d} = odd$, and $A^{**} = odd$, cells in the same $|y_n|_{2d}$-row (hence belonging to the same $|y_n|_{2d}$-set) at equiperpendicular distance from a compound complementary x_c-column equal to $n^*A^{**} + 2 - (A_l + A_r)$, where n^* is a positive integer such that $n^*A^{**} + 2 - (A_l + A_r) \geq 2 - d$, where $d = \text{g.c.d.}(A_l, A_r)$, belong to Asymmetric Regular Nested Cylindrical Braids whose first-return string-runs are compound complementary. See Fig. 650.

In the x/y-tables for $\frac{A_l}{d} = odd$, $\frac{A_r}{d} = odd$, and $A^{**} = even$, cells in the same $|y_n|_{2d}$-row (hence belonging to the same $|y_n|_{2d}$-set) at equiperpendicular distance from a compound complementary x_c-column equal to $n^*A^{**} + 2 - (A_l + A_r)$, where n^* is a positive integer such that $n^*A^{**} + 2 - (A_l + A_r) \geq 2 - d$, where $d = \text{g.c.d.}(A_l, A_r)$) belong to Asymmetric Regular Nested Cylindrical Braids whose first-return string-runs are compound complementary. See Fig. 651.

In the x/y-tables for $\frac{A_l}{d} = even$ or $\frac{A_r}{d} = even$, cells in the same $|y_n|_{2d}$-row (hence belonging to the same $|y_n|_{2d}$-set) at equiperpendicular distance from a compound complementary x_c-column equal to $n^*A^{**} + 2 - (A_l + A_r)$, where $2n^*$ is a positive integer such that $n^*A^{**} + 2 - (A_l + A_r) \geq 2 - d$, where $d = \text{g.c.d.}(A_l, A_r)$) belong to Asymmetric Regular Nested Cylindrical Braids whose first-return string-runs are compound complementary. See Fig. 652.

Example 4:

$A_l = 15$, $A_r = 9$. Hence $\text{g.c.d.}(A_l, A_r) = d = 3$ and $A^{**} = 45$. Take for braid 1: $x_1 = 13$, $y_1 = 1$, $l_1 = 1$. The set of y_m-values is then 1, $1 + 2d = 7$, $1 + 4d = 13$.

For calculating the first-return string-run(s) we use the general formulae:

$$r_i = \left| A_l + \frac{x - y_i - 2l_i}{2} \right|_{A_r} .$$

$$l_{i+1} = |2A_r + x - (l_i + 2r_i)|_{A_l} .$$

$$r_{j+1} = |2A_l + x - (r_j + 2l_j)|_{A_r} .$$

$$y_i = |2A_l + x - 2(l_i + r_i)|_{A_r} .$$

Hence for $A_l = 15$, $A_r = 9$, $x = 13$, $y_1 = 1$, $l_1 = 1$ we obtain:

$$r_1 = \left| 15 + \frac{13 - 1 - 2}{2} \right|_9 = 2 .$$

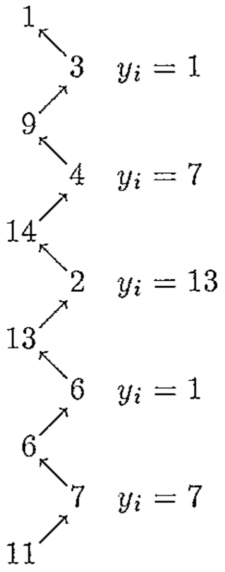
$$y_i = |30 + 13 - 2(l_i + r_i)|_{18} = |43 - 2(l_i + r_i)|_{18} .$$

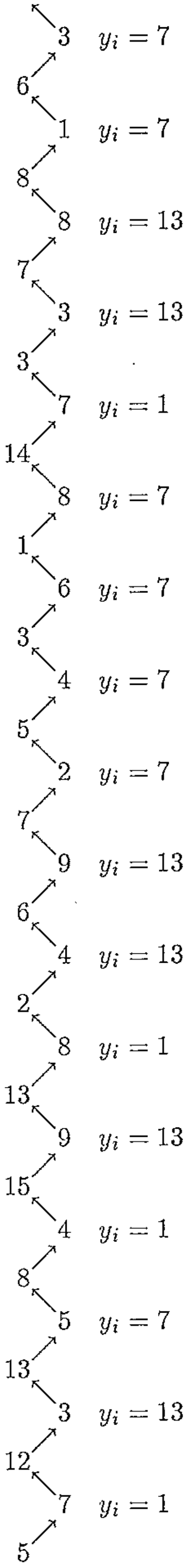
$$l_{i+1} = |18 + 13 - (l_i + 2r_i)|_{15} = |31 - (l_i + 2r_i)|_{15} .$$

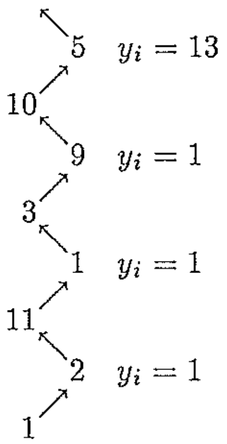
$$r_{j+1} = |30 + 13 - (r_j + 2l_j)|_9 = |43 - (r_j + 2l_j)|_9 .$$

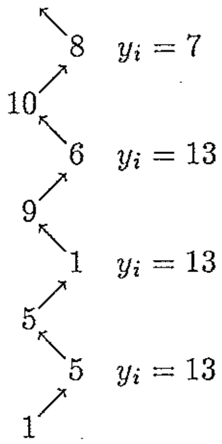
As we shall see below, the first-return string-run of which the first half-cycle runs from left bight-boundary 1 to right bight-boundary 2 consists of 18 half-cycles. There are thus another $2A^{**} - 18 = 72$ further half-cycles which are contained in one or more further first-return string-runs. Since we have the half-cycle from left bight-boundary 1 to right bight-boundary 2, we also have the half-cycle from left bight-boundary 1 to right bight boundary $2 + d = 5$ and the half-cycle from left bight-boundary 1 to right bight boundary $2 + 2d = 8$. The first-return string-run which consists of 18 half-cycle contains the half-cycle from left bight-boundary 1 to right bight-boundary 2 only, hence the second first-return string-run can start with the half-cycle from left bight-boundary 1 to right bight-boundary 5. As we shall see below, the second first-return string-run consists of 72 half-cycles. There are thus no further first-return string-runs.

$$\begin{tikzcd}
1 & & \\
& 9 \arrow[ul] & y_i = 1 \\
12 \arrow[ur] & & \\
& 1 \arrow[ul] & y_i = 1 \\
2 \arrow[ur] & & \\
& 2 \arrow[ul] & y_i = 1 \\
10 \arrow[ur] & & \\
& 3 \arrow[ul] & y_i = 7 \\
15 \arrow[ur] & & \\
& 1 \arrow[ul] & y_i = 13 \\
14 \arrow[ur] & & \\
& 5 \arrow[ul] & y_i = 1 \\
7 \arrow[ur] & & \\
& 6 \arrow[ul] & y_i = 7 \\
12 \arrow[ur] & & \\
& 4 \arrow[ul] & y_i = 13 \\
11 \arrow[ur] & & \\
& 8 \arrow[ul] & y_i = 1 \\
4 \arrow[ur] & & \\
& 9 \arrow[ul] & y_i = 7 \\
9 \arrow[ur] & & \\
& 7 \arrow[ul] & y_i = 13 \\
8 \arrow[ur] & & \\
& 2 \arrow[ul] & y_i = 13 \\
4 \arrow[ur] & & \\
& 6 \arrow[ul] & y_i = 1 \\
15 \arrow[ur] & & \\
& 7 \arrow[ul] & y_i = 7 \\
2 \arrow[ur] & & \\
& 5 \arrow[ul] & y_i = 7 \\
4 \arrow[ur] & &
\end{tikzcd}$$

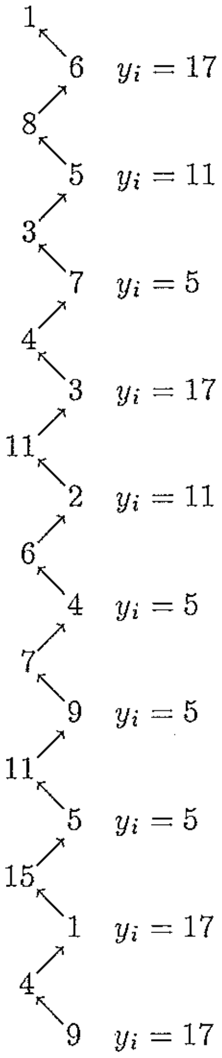

1
3 $y_i = 1$
9
4 $y_i = 7$
14
2 $y_i = 13$
13
6 $y_i = 1$
6
7 $y_i = 7$
11


3 $y_i = 7$
6
1 $y_i = 7$
8
8 $y_i = 13$
7
3 $y_i = 13$
3
7 $y_i = 1$
14
8 $y_i = 7$
1
6 $y_i = 7$
3
4 $y_i = 7$
5
2 $y_i = 7$
7
9 $y_i = 13$
6
4 $y_i = 13$
2
8 $y_i = 1$
13
9 $y_i = 13$
15
4 $y_i = 1$
8
5 $y_i = 7$
13
3 $y_i = 13$
12
7 $y_i = 1$
5

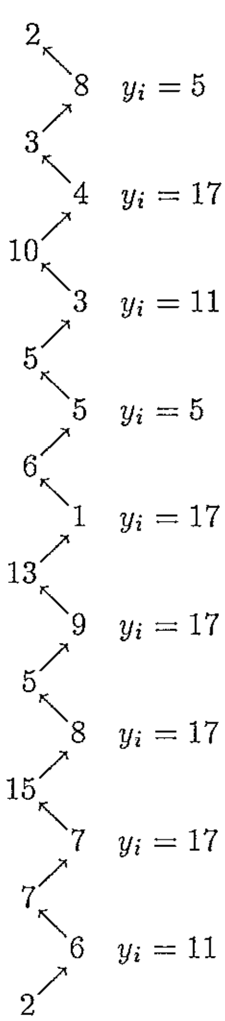


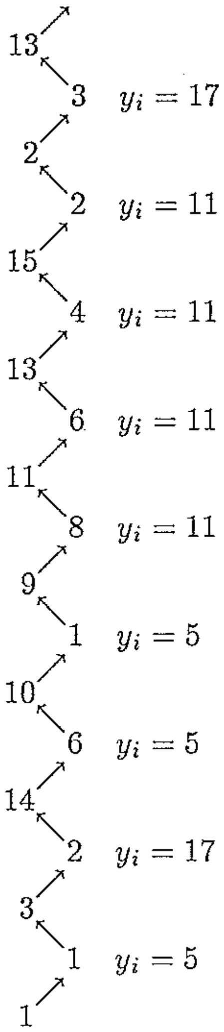


The (A_l, A_r)-complements of these two first-return string-runs can now readily be derived from them, and for convenience we start with their respective half-cycles from left bight-boundary 14 to right bight-boundary 4 and from left bight-boundary 15 to right bight-boundary 9. Hence the (A_l, A_r)-complementary first-return string-runs start respectively with the half-cycles from left bight-boundary 2 to right bight-boundary 6 and from left bight-boundary 1 to right bight-boundary 1. The indicated y_i-values associated with these (A_l, A_r)-complementary first-return string-runs have been calculated with the general formulae on pg. 825.



$$\begin{tikzcd}
 & {} & \\
14 \arrow[ur] & & \\
 & 8 \arrow[ul] & y_i = 17 \\
6 \arrow[ur] & & \\
 & 7 \arrow[ul] & y_i = 11 \\
1 \arrow[ur] & & \\
 & 9 \arrow[ul] & y_i = 5 \\
2 \arrow[ur] & & \\
 & 5 \arrow[ul] & y_i = 17 \\
9 \arrow[ur] & & \\
 & 4 \arrow[ul] & y_i = 11 \\
4 \arrow[ur] & & \\
 & 6 \arrow[ul] & y_i = 5 \\
5 \arrow[ur] & & \\
 & 2 \arrow[ul] & y_i = 17 \\
12 \arrow[ur] & & \\
 & 1 \arrow[ul] & y_i = 11 \\
7 \arrow[ur] & & \\
 & 3 \arrow[ul] & y_i = 5 \\
8 \arrow[ur] & & \\
 & 8 \arrow[ul] & y_i = 5 \\
12 \arrow[ur] & & \\
 & 4 \arrow[ul] & y_i = 17 \\
1 \arrow[ur] & & \\
 & 3 \arrow[ul] & y_i = 11 \\
14 \arrow[ur] & & \\
 & 5 \arrow[ul] & y_i = 11 \\
12 \arrow[ur] & & \\
 & 7 \arrow[ul] & y_i = 11 \\
10 \arrow[ur] & & \\
 & 9 \arrow[ul] & y_i = 11 \\
8 \arrow[ur] & & \\
 & 2 \arrow[ul] & y_i = 5 \\
9 \arrow[ur] & & \\
 & 7 \arrow[ul] & y_i = 5
\end{tikzcd}$$





From the general formulae on pg. 820 we readily derive the following general formulae:

$$
\begin{aligned}
n_l A_l &= n_r A_r + (\Delta l_i + \Delta r_i) - 2(A_l - A_r)\,. \\
x &= n_l A_l + l_i + l_{i+1} + 2r_i - 2A_r\,. \\
&= n_r A_r + r_j + r_{j+1} + 2l_j - 2A_l\,. \\
y_i &= |(n_l + 2)A_l + \Delta l_i|_{2A_r}\,. \\
|y_i|_{2d} &= |\Delta l_i + n_l A_l|_{2d}\,.
\end{aligned}
$$

Hence for the above (A_l, A_r)-complementary first-return string-runs we obtain with $l_i = 1$, $r_i = 1$, $l_{i+1} = 3$, $r_{i+1} = 2$:

$$
\begin{aligned}
n_l A_l &= n_r A_r + (\Delta l_i + \Delta r_i) - 2(A_l - A_r) \\
15 n_l &= 9 n_r + (2 + 1) - 2(15 - 9) \\
5 n_l &= 3 n_r - 3\,.
\end{aligned}
$$

$n_r = 6 \quad \to \quad n_l = 3 \quad \to \quad x = 33 \quad \to \quad |y_n|_{2d} = 5\,.$

$n_r = 11 \quad \to \quad n_l = 6 \quad \to \quad x = 78 \quad \to \quad |y_n|_{2d} = 2\,.$

$n_r = 16 \quad \to \quad n_l = 9 \quad \to \quad x = 123 \quad \to \quad |y_n|_{2d} = 5\,.$

$n_r = 21 \quad \to \quad n_l = 12 \quad \to \quad x = 168 \quad \to \quad |y_n|_{2d} = 2\,.$

$n_r = 26 \quad \to \quad n_l = 15 \quad \to \quad x = 213 \quad \to \quad |y_n|_{2d} = 5\,.$

$n_r = 31 \quad \to \quad n_l = 18 \quad \to \quad x = 258 \quad \to \quad |y_n|_{2d} = 2\,.$

$n_r = 36 \quad \to \quad n_l = 21 \quad \to \quad x = 303 \quad \to \quad |y_n|_{2d} = 5\,.$

For the original first-return string-runs (the string-runs on pg. 823 → 822 → 821) we obtain with $l_i = 1$, $r_i = 2$, $l_{i+1} = 11$, $r_{i+1} = 1$:

$$\begin{aligned} n_l A_l &= n_r A_r + (\Delta l_i + \Delta r_i) - 2(A_l - A_r) \\ 15 n_l &= 9 n_r + (10 - 1) - 2(15 - 9) \\ 5 n_l &= 3 n_r - 1 . \end{aligned}$$

$$\begin{array}{lllllll} n_r = 2 & \rightarrow & n_l = 1 & \rightarrow & x = 13 & \rightarrow & |y_n|_{2d} = 1 . \\ n_r = 7 & \rightarrow & n_l = 4 & \rightarrow & x = 58 & \rightarrow & |y_n|_{2d} = 4 . \\ n_r = 12 & \rightarrow & n_l = 7 & \rightarrow & x = 103 & \rightarrow & |y_n|_{2d} = 1 . \\ n_r = 17 & \rightarrow & n_l = 10 & \rightarrow & x = 148 & \rightarrow & |y_n|_{2d} = 4 . \\ n_r = 22 & \rightarrow & n_l = 13 & \rightarrow & x = 193 & \rightarrow & |y_n|_{2d} = 1 . \\ n_r = 27 & \rightarrow & n_l = 16 & \rightarrow & x = 238 & \rightarrow & |y_n|_{2d} = 4 . \\ n_r = 32 & \rightarrow & n_l = 19 & \rightarrow & x = 283 & \rightarrow & |y_n|_{2d} = 1 . \end{array}$$

The positions in the y_m-x table of these and their (A_l, A_r)-complementary Asymmetric Regular Nested Cylindrical Braids are depicted in Fig. 653.

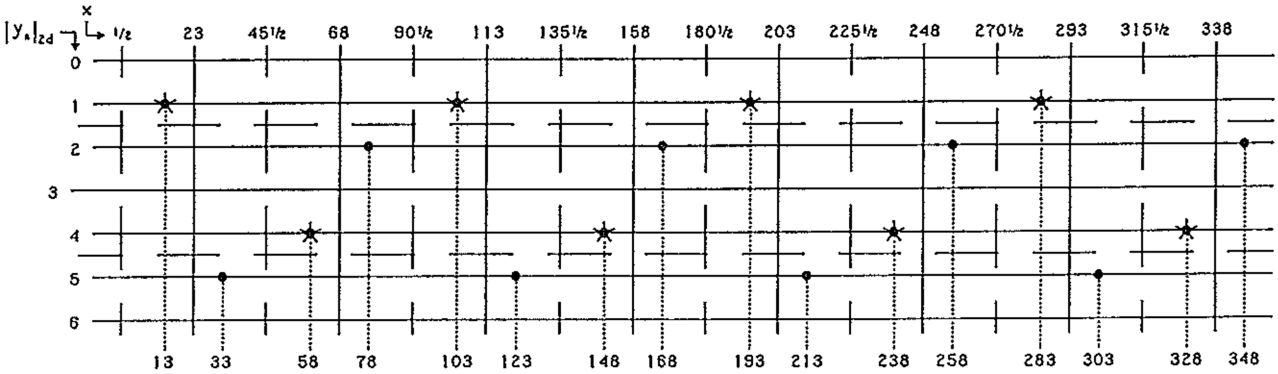


Fig. 653 — The y_m-x table associated with Example 4.

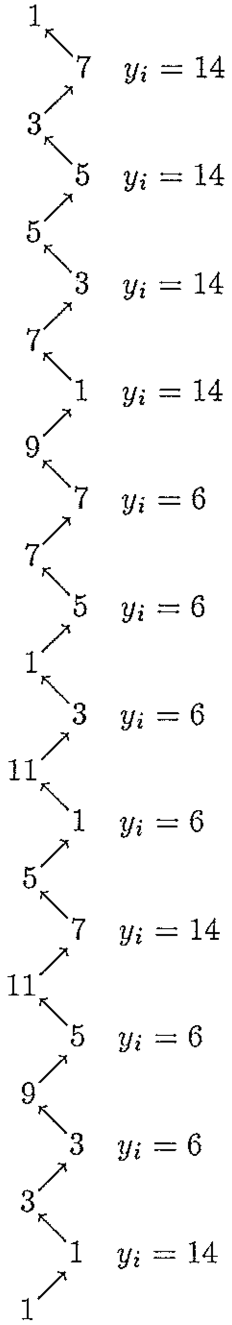
Example 5:

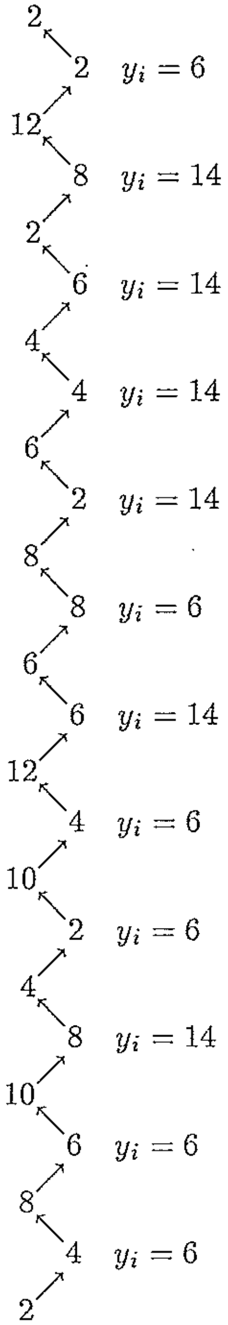
$A_l = 12$, $A_r = 8$. Hence g.c.d.$(A_l, A_r) = d = 4$ and $A^{**} = 24$. Take for braid 1: $x_1 = 26$, $y_1 = 6$, $l_1 = 1$. The set of y_m-values is then 6, $1 + 2d = 14$.

For calculating the first-return string-run(s) we use the general formulae on pg. 820: Hence for $A_l = 12$, $A_r = 8$, $x = 26$, $y_1 = 6$, $l_1 = 1$ we obtain:

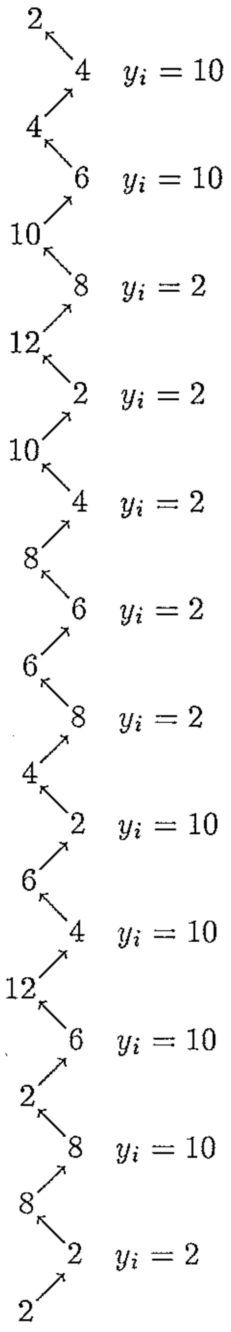
$$\begin{aligned} r_1 &= \left| 12 + \frac{26 - 6 - 2}{2} \right|_8 = 5 . \\ y_i &= |24 + 26 - 2(l_i + r_i)|_{16} = |50 - 2(l_i + r_i)|_{16} . \\ l_{i+1} &= |16 + 26 - (l_i + 2r_i)|_{12} = |42 - (l_i + 2r_i)|_{12} . \\ r_{j+1} &= |24 + 26 - (r_j + 2l_j)|_8 = |50 - (r_j + 2l_j)|_8 . \end{aligned}$$

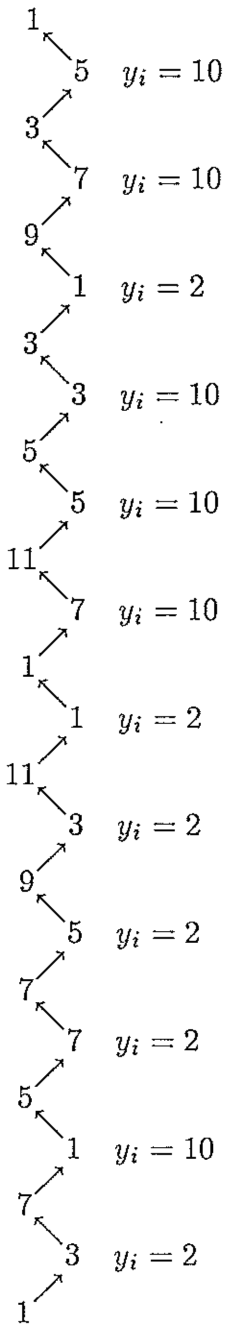
When there is a half-cycle from left bight-boundary 1 to right bight-boundary 5, then there is a half-cycle from left bight-boundary 1 to right bight-boundary $|5 + d|_{A_r} = |5 + 4|_8 = 1$. As we shall see, there are two first-return string-runs. Say we start the first one of these with the half-cycle from left bight-boundary 1 to right bight-boundary 1. As we shall see, there is in this first-return string-run no half-cycle from left bight-boundary 2 to a right bight-boundary, hence we can start the second first-return string-run with such a half-cycle. Since there is a half-cycle from left bight-boundary 1 to right bight-boundary 1 (in the first first-return string-run), there must exist a half-cycle from left bight-boundary 2 to right bight-boundary $A_r = 8$, hence there must be a half-cycle from left bight-boundary 2 to right bight-boundary $|8+d|_{A_r} = |8+4|_8 = 4$. Say we start the second first-return string-run with the half-cycle from left bight-boundary 2 to right bight-boundary 4. The following two first-return string-runs are then obtained:





The (A_l, A_r)-complements of these two first-return string-runs can again readily be derived from them, and for convenience we start with their respective half-cycles from left bight-boundary 11 to right bight-boundary 7 and from left bight-boundary 12 to right bight-boundary 6. Hence the (A_l, A_r)-complementary first-return string-runs start respectively with the half-cycles from left bight-boundary 2 to right bight-boundary 2 and from left bight-boundary 1 to right bight-boundary 3. The indicated y_i-values associated with these (A_l, A_r)-complementary first-return string-runs have again been calculated with the general formulae on pg. 825.





With the general formulae on pg. 825 we obtain for these (A_l, A_r)-complementary first-return string-runs with $l_i = 2$, $r_i = 2$, $l_{i+1} = 8$, $r_{i+1} = 8$:

$$\begin{aligned} n_l A_l &= n_r A_r + (\Delta l_i + \Delta r_i) - 2(A_l - A_r) \\ 12 n_l &= 8 n_r + (6+6) - 2(12-8) \\ 3 n_l &= 2 n_r + 1 \, . \end{aligned}$$

$$\begin{array}{lllllll} n_r = 1 & \longrightarrow & n_l = 1 & \longrightarrow & x = 10 & \longrightarrow & |y_n|_{2d} = 2 \, . \\ n_r = 4 & \longrightarrow & n_l = 3 & \longrightarrow & x = 34 & \longrightarrow & |y_n|_{2d} = 2 \, . \\ n_r = 7 & \longrightarrow & n_l = 5 & \longrightarrow & x = 58 & \longrightarrow & |y_n|_{2d} = 2 \, . \\ n_r = 10 & \longrightarrow & n_l = 7 & \longrightarrow & x = 82 & \longrightarrow & |y_n|_{2d} = 2 \, . \\ n_r = 13 & \longrightarrow & n_l = 9 & \longrightarrow & x = 106 & \longrightarrow & |y_n|_{2d} = 2 \, . \\ n_r = 16 & \longrightarrow & n_l = 11 & \longrightarrow & x = 130 & \longrightarrow & |y_n|_{2d} = 2 \, . \\ n_r = 19 & \longrightarrow & n_l = 13 & \longrightarrow & x = 154 & \longrightarrow & |y_n|_{2d} = 2 \, . \end{array}$$

For the original first-return string-runs (the string-runs on pg. 827) we obtain with $l_i = 1$, $r_i = 1$, $l_{i+1} = 3$, $r_{i+1} = 3$:

$$\begin{aligned} n_l A_l &= n_r A_r + (\Delta l_i + \Delta r_i) - 2(A_l - A_r) \\ 12 n_l &= 8 n_r + (2+2) - 2(12-8) \\ 3 n_l &= 2 n_r - 1 \,. \end{aligned}$$

$$\begin{array}{lllllll} n_r = 2 & \to & n_l = 1 & \to & x = 2 & \to & |y_n|_{2d} = 6 \,. \\ n_r = 5 & \to & n_l = 3 & \to & x = 26 & \to & |y_n|_{2d} = 6 \,. \\ n_r = 8 & \to & n_l = 5 & \to & x = 50 & \to & |y_n|_{2d} = 6 \,. \\ n_r = 11 & \to & n_l = 7 & \to & x = 74 & \to & |y_n|_{2d} = 6 \,. \\ n_r = 14 & \to & n_l = 9 & \to & x = 98 & \to & |y_n|_{2d} = 6 \,. \\ n_r = 17 & \to & n_l = 11 & \to & x = 122 & \to & |y_n|_{2d} = 6 \,. \\ n_r = 20 & \to & n_l = 13 & \to & x = 146 & \to & |y_n|_{2d} = 6 \,. \end{array}$$

The positions in the y_m-x table of these and their (A_l, A_r)-complementary Asymmetric Regular Nested Cylindrical Braids are depicted in Fig. 654.

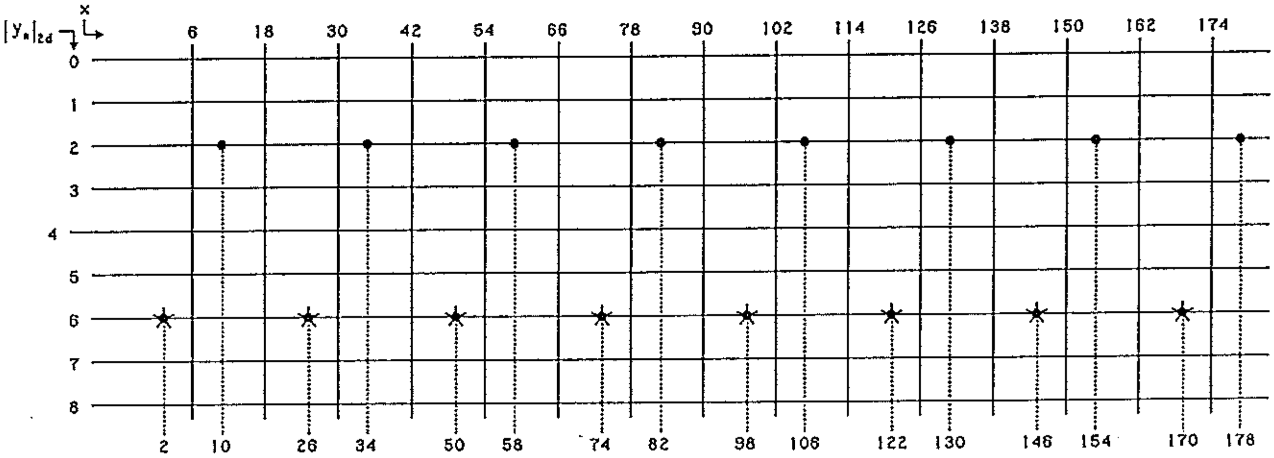
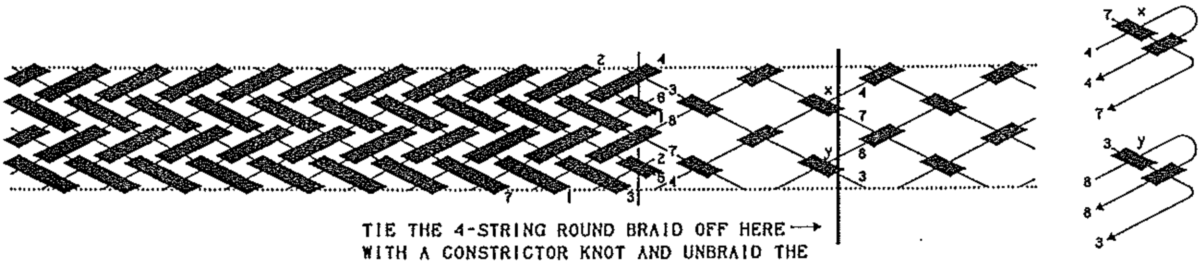


Fig. 654 — The y_m-x table associated with Example 5.

An Integrated Pineapple Knot for an 8-string 2u-2o Round Braid.



TIE THE 4-STRING ROUND BRAID OFF HERE →
WITH A CONSTRICTOR KNOT AND UNBRAID THE
RIGHT-HAND PART OF THE 4-THONG ROUND BRAID

Fig. 655 — Preparing the Round Braid for the integrated Pineapple Knot.

The consecutive braiding steps for the integrated Pineapple Knot are depicted in Fig. 656.

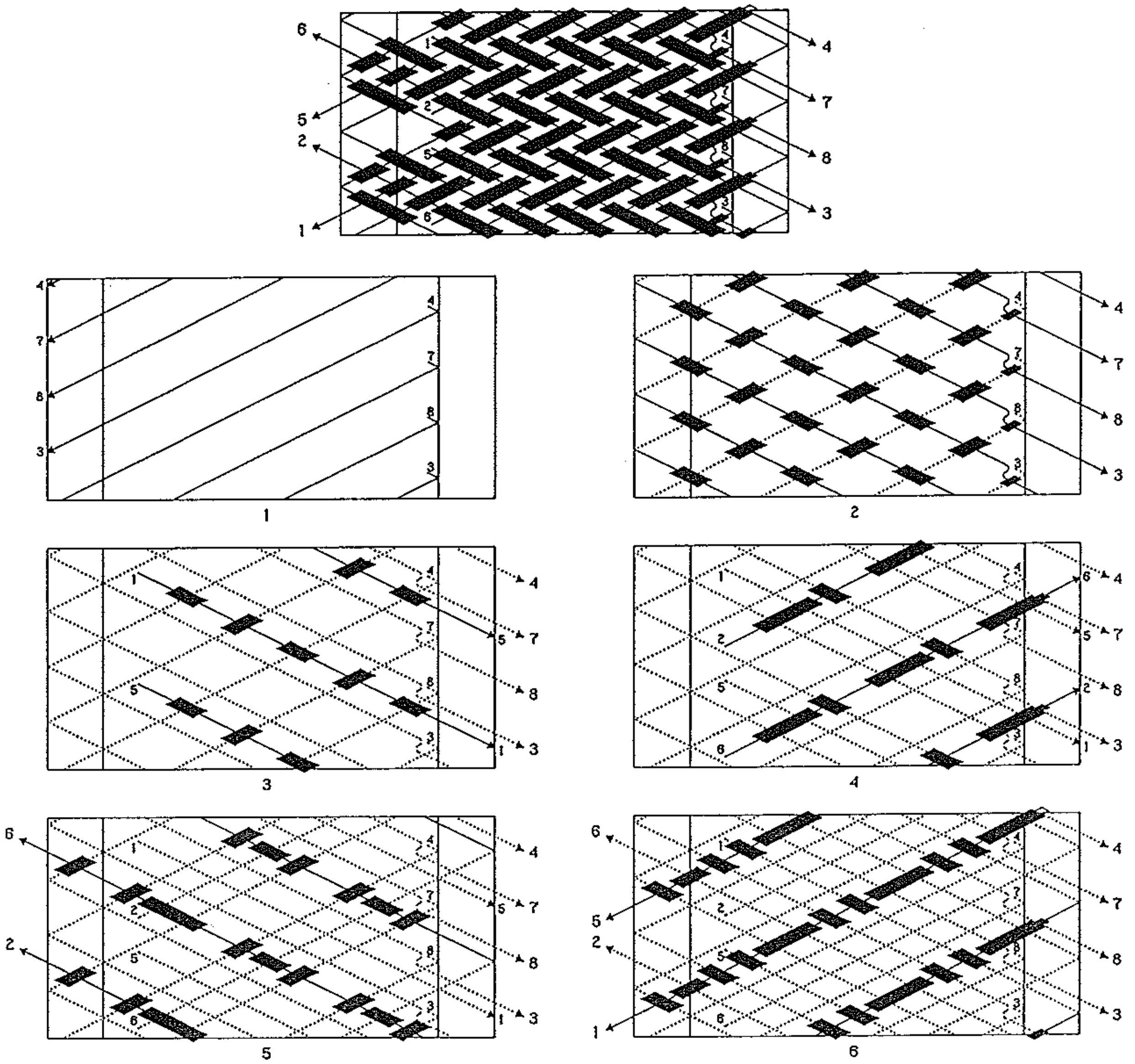


Fig. 656 — The consecutive braiding steps for the integrated Pineapple Knot.

This integrated Pineapple Knot can be closed completely on the right-hand side.

As Fig. 657 shows, a good colour pattern is obtained when in the 8-string round braid the strings 1, 2, 5, 6 all have the same colour and the strings 3, 4, 7, 8 all have the same but a different colour. In this case the colour pattern of the 8-string round braid is as depicted by the upper-right grid-diagrams on pp. 219 and 221 in *The Braider* No. 10.

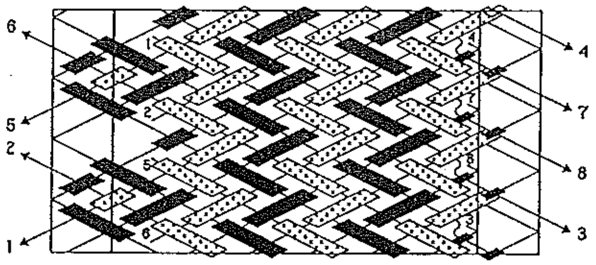


Fig. 657 — A good colour pattern for the integrated Pineapple Knot.