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CONTENTS

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for
the braiding artisan

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The separate half-cycle tables for each essential string in the Example on pp. 756–767

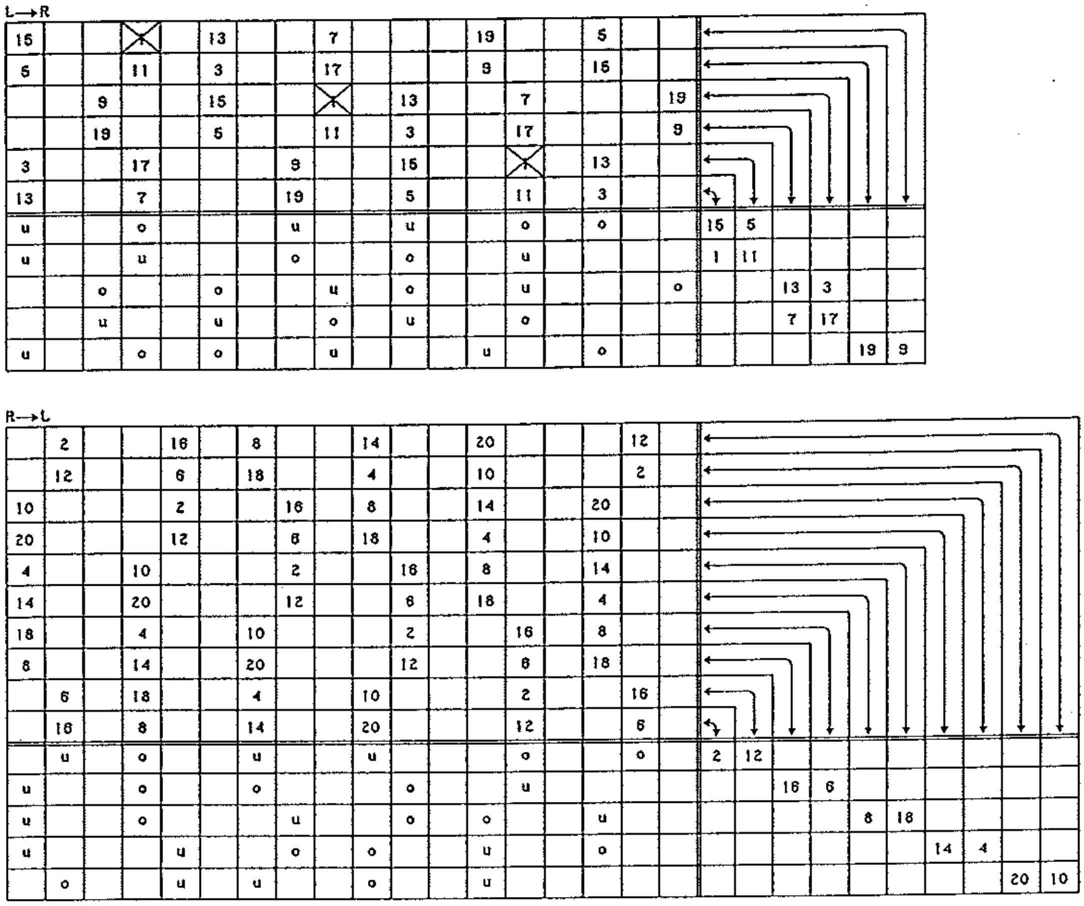


Fig. 627 — Essential string with half-cycles 1–20.

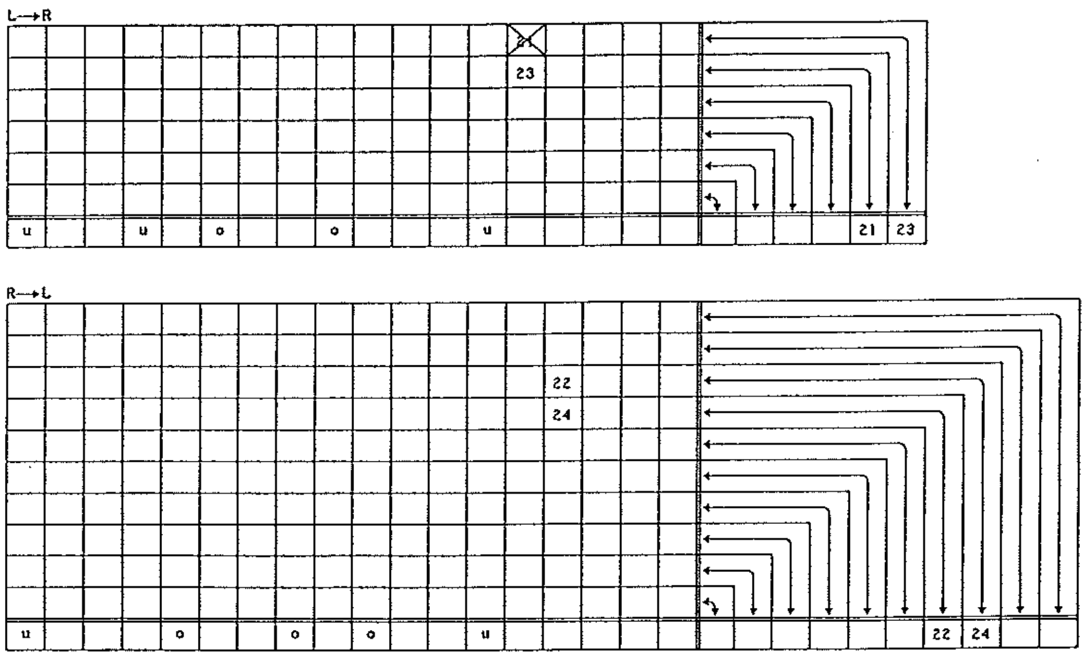


Fig. 628 — Essential string with half-cycles 21–24.

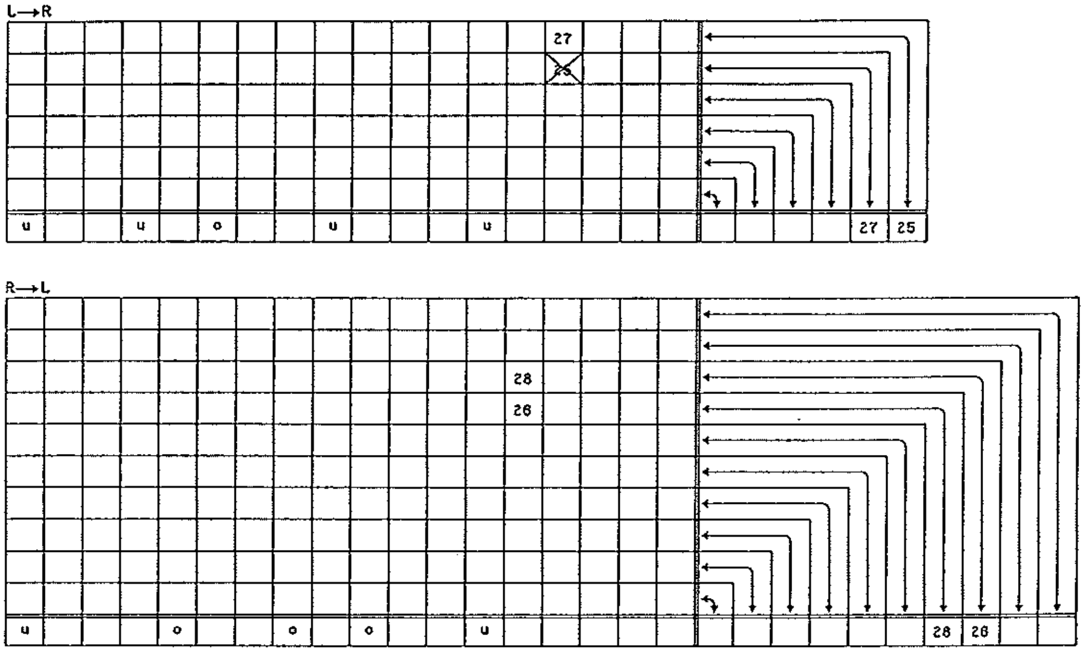


Fig. 629 — Essential string with half-cycles 25–28.

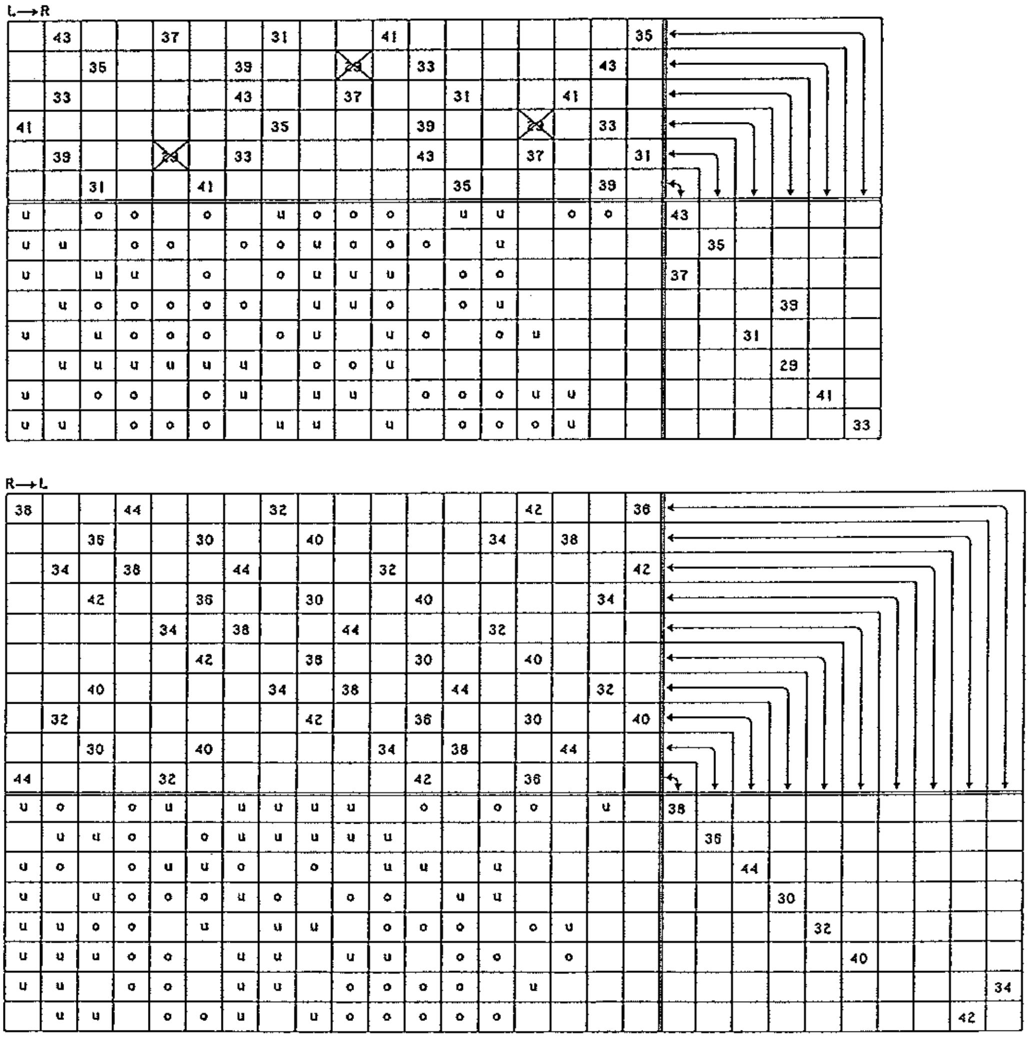


Fig. 630 — Essential string with half-cycles 29–44.

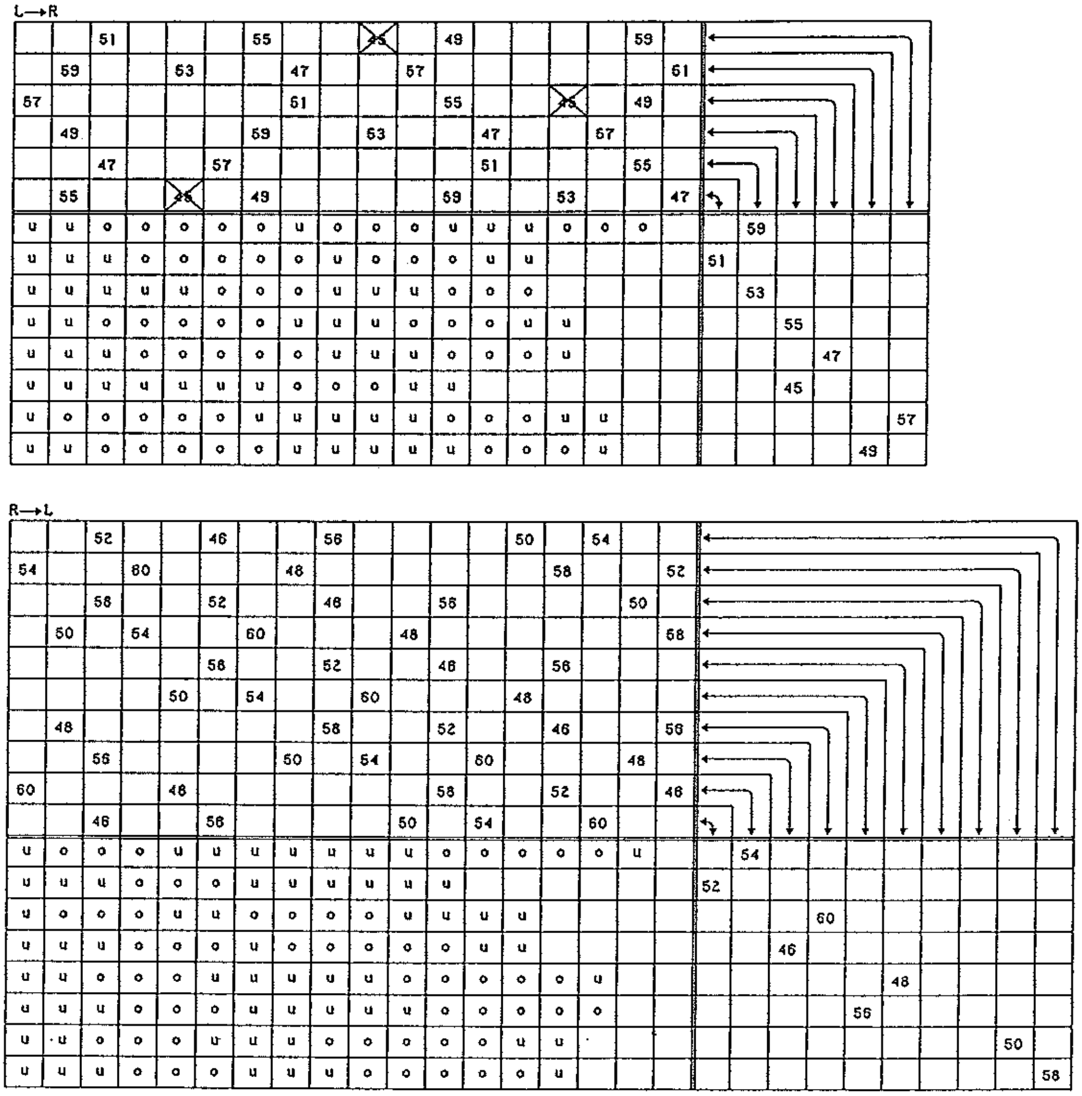


Fig. 631 — Essential string with half-cycles 45–60.

Solution to the Question in Issue No. 33

Refer to Fig. 617, pg. 775, and to pg. 776.

For the component with the lowermost first-return string-run in Fig. 617 the number of sub-components $\lambda = \text{g.c.d.}(5, \frac{B}{2})$, hence for $B = 8$, consequently $\frac{B}{2} = 4$, we obtain $\lambda = \text{g.c.d.}(5, \frac{B}{2}) = \text{g.c.d.}(5, 4) = 1$, while for the component with the uppermost first-return string-run in Fig. 617 the number of sub-components $\lambda = \text{g.c.d.}(2, \frac{B}{2}) = \text{g.c.d.}(2, 4) = 2$. We thus require two essential strings for the interbraiding of the coloured V's. The two respective interbraids with Standing-Ends S_2 and S_3 have each one part and two bights.

The construction details for interbraiding the two $p/b = 1/2$ Regular Knots with the 2-pass Spanish Ring Knot $p/b = 5/8$, are for the four V patterns depicted in respectively Fig. 632, Fig. 633, Fig. 634 and Fig. 635.

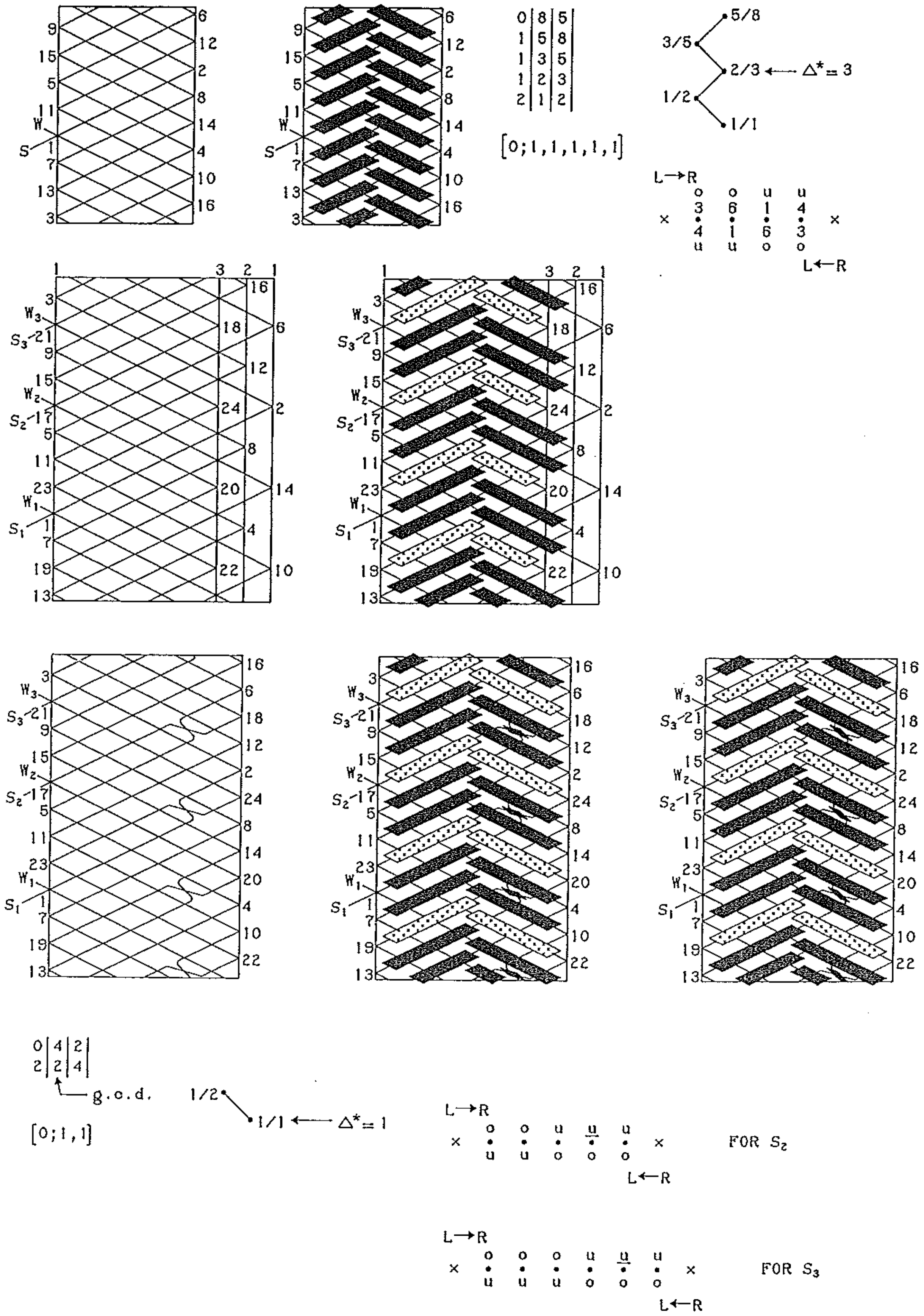


Fig. 632 — Construction details for the two interbraids $p'/b' = 1/2$ with $p/b = 5/8$.

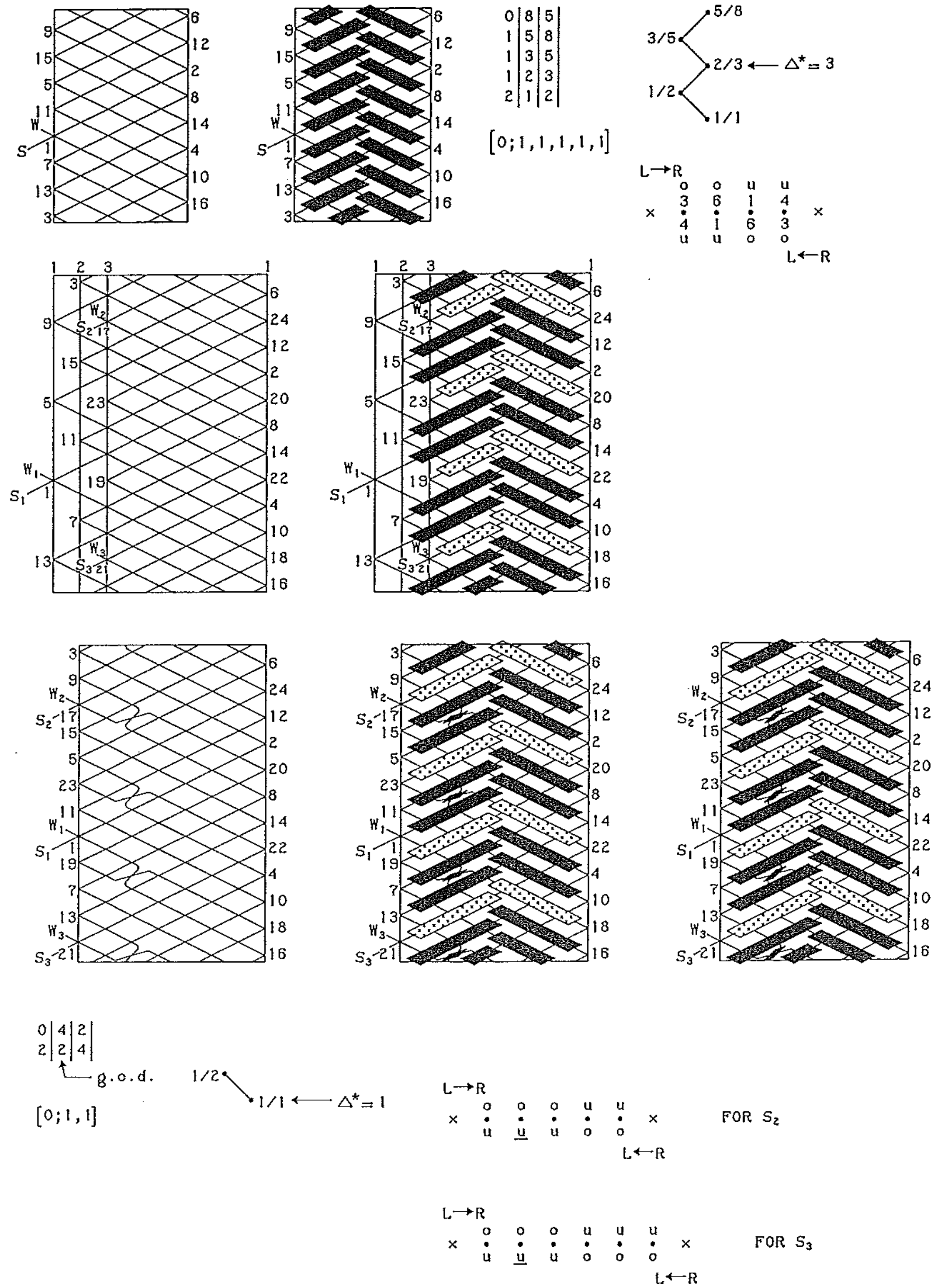


Fig. 633 — Construction details for the two interbraids $p'/b' = 1/2$ with $p/b = 5/8$.

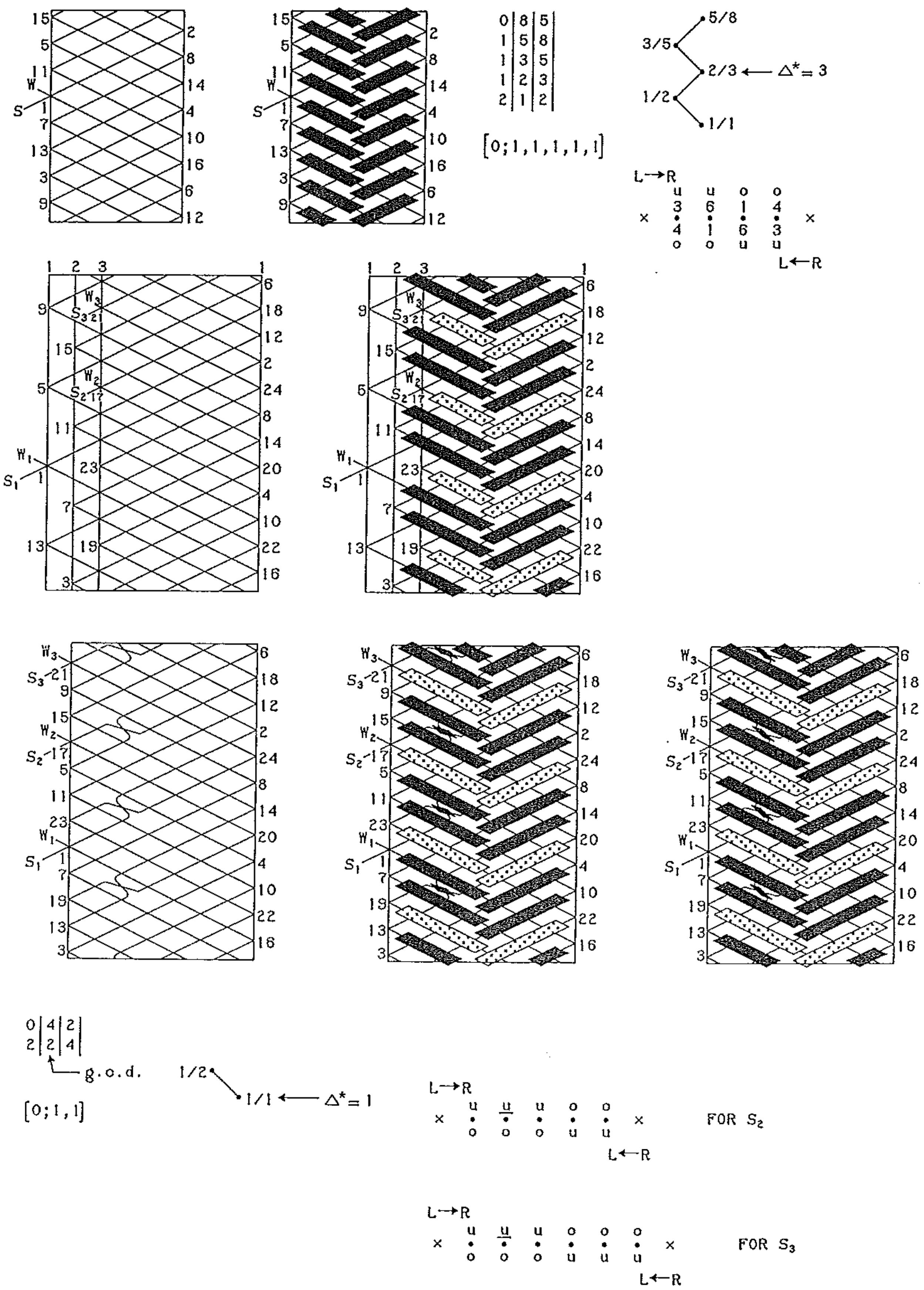


Fig. 634 — Construction details for the two interbraids $p'/b' = 1/2$ with $p/b = 5/8$.

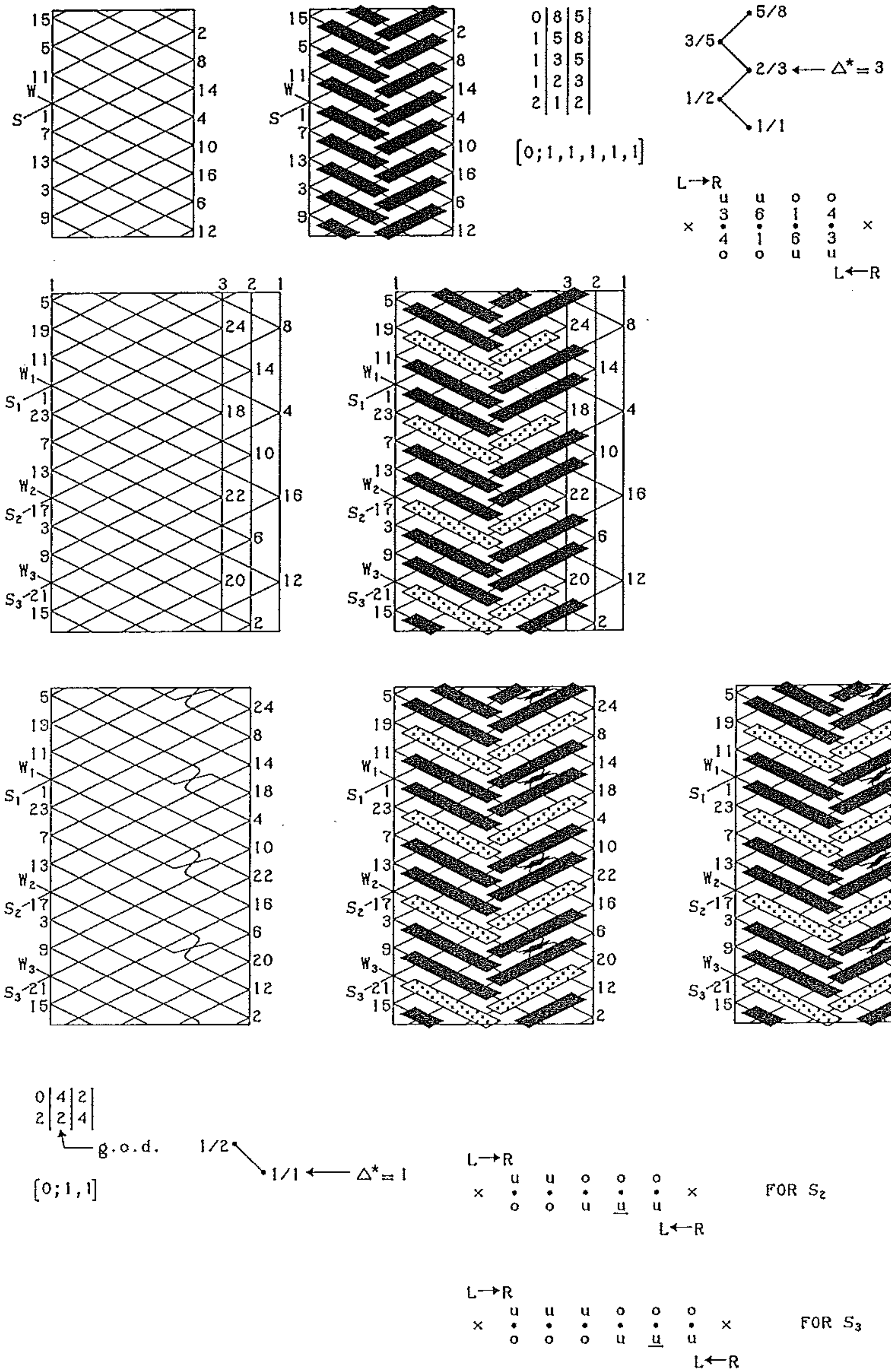


Fig. 635 — Construction details for the two interbraids $p'/b' = 1/2$ with $p/b = 5/8$.

Nested Cylindrical Braids

The Periodic Regular Nested Cylindrical Braids:

Let the left **nesting-number**(see *The Braider*, Issue No. 19, pp. 415-419) A_l being made-up of two **sub-nesting-numbers** A_{l_1} and A_{l_2}, thus $A_l = A_{l_1} + A_{l_2}$. Similarly let the right **nesting-number** A_r being made-up of two **sub-nesting-numbers** A_{r_1} and A_{r_2}, thus $A_r = A_{r_1} + A_{r_2}$. Furthermore, let $A_{l_1} > A_{l_2}$ and $A_{r_1} > A_{r_2}$.

Note that in this general case are contained the following two special cases:
1). The nesting-number on one bight-edge is made-up of two sub-nesting-numbers, and the nesting-number on the other bight-edge is made-up of only **one** sub-nesting-number.
2). The nesting-number on one bight-edge is made-up of two sub-nesting-numbers, and the nesting-number on the other bight-edge is made-up of only **one** sub-nesting-number which is equal to 1 (hence this latter bight-edge consists of one bight-boundary only).

For the left bight-edge and for the right bight-edge there are two types of bight-edges when the nesting-number of the bight-edge is made-up of two sub-nesting-numbers with the following **bight-boundary position specifications** and **bight-boundary sequence specifications**:

For the left bight-edge:

1). The two sub-nesting-numbers A_{l_1} and A_{l_2} do have the same parity.
The left bight-boundary position specification is then:

$$\underbrace{222\cdots2}_{(A_{l_1}-1)\text{ elements}}$$

While the left bight-boundary sequence specification is then:

$$1_1(z)_{(A_{l_1}+A_{l_2})}(z-1)_{(A_{l_1}+A_{l_2}-1)}(z-2)_{(A_{l_1}+A_{l_2}-2)}\cdots$$
$$(z-A_{l_2}+1)_{(A_{l_1}+1)}(A_{l_1})_{(A_{l_1})}(A_{l_1}-1)_{(A_{l_1}-1)}\cdots 2_2\,.$$

Where $z = \dfrac{A_{l_1}+A_{l_2}}{2}$.

2). The two sub-nesting-numbers A_{l_1} and A_{l_2} do not have the same parity.
The left bight-boundary position specification is then:

$$\underbrace{222\cdots2}_{\substack{z-2A_{l_2}\\ \text{elements}}}\underbrace{111\cdots1}_{\substack{2A_{l_2}\\ \text{elements}}}\underbrace{222\cdots2}_{\substack{A_l-z-1\\ \text{elements}}}$$

Note that $A_l - z - 1 = z - 2A_{l_2}$.

While the left bight-boundary sequence specification is then:

i). For $A_{l_1} = A_{l_2} + 1$:

$$1_1(z)_{(A_{l_1}+A_{l_2})}(z-2)_{(A_{l_1}+A_{l_2}-1)}(z-4)_{(A_{l_1}+A_{l_2}-2)}\cdots$$
$$2_{(A_{l_1}+1)}(z+1)_{(z+1-A_{l_2})}(z-1)_{(z-A_{l_2})}(z-3)_{(z-1-A_{l_2})}\cdots 3_2\,.$$

Where $z = \dfrac{A_{l_1}+A_{l_2}-1}{2} + A_{l_2} = 2A_{l_2}$.

ii). For $A_{l_1} \geq A_{l_2} + 3$:

$$1_1(z)_{(A_{l_1}+A_{l_2})}(z-2)_{(A_{l_1}+A_{l_2}-1)}(z-4)_{(A_{l_1}+A_{l_2}-2)}\cdots$$
$$(z+2-2A_{l_2})_{(A_{l_1}+1)}(A_{l_1}+A_{l_2})_{(A_{l_1})}(A_{l_1}+A_{l_2}-1)_{(A_{l_1}-1)}(A_{l_1}+A_{l_2}-2)_{(A_{l_1}-2)}\cdots$$
$$(z+1)_{(z+1-A_{l_2})}(z-1)_{(z-A_{l_2})}(z-3)_{(z-1-A_{l_2})}\cdots$$
$$(z+1-2A_{l_2})_{(z+1-2A_{l_2})}(z-2A_{l_2})_{(z-2A_{l_2})}(z-1-2A_{l_2})_{(z-1-2A_{l_2})}\cdots 2_2\,.$$

Where $z = \dfrac{A_{l_1}+A_{l_2}-1}{2} + A_{l_2}$.

⋆⋆ *Determine for the four left-hand bight-edges in Fig. 636 the left bight-boundary position specification and the left bight-boundary sequence set with ranking-numbers.*

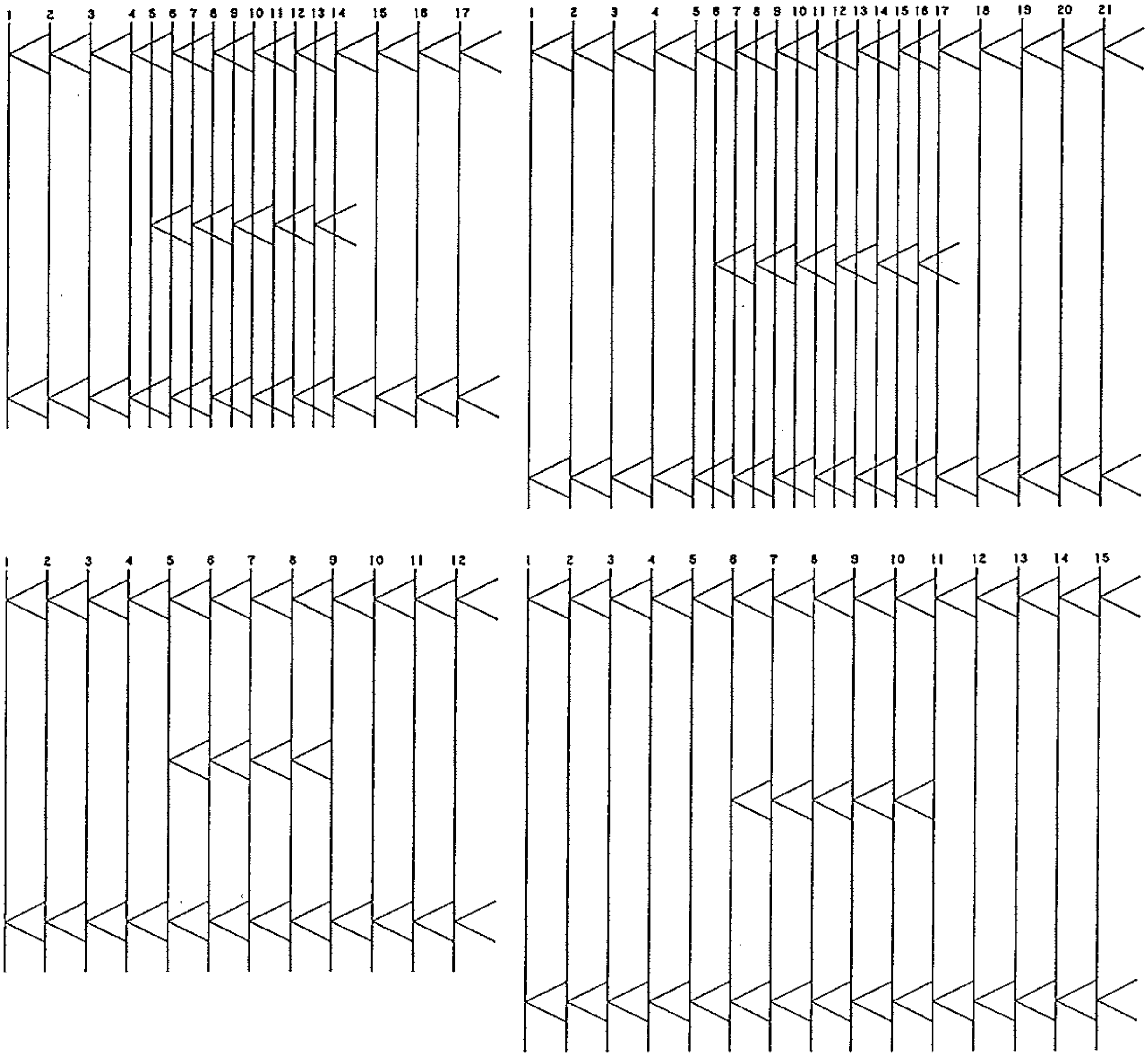


Fig. 636 — Examples of sub-nesting numbers A_{l_1} and A_{l_2}.

For the right bight-edge:

1). The two sub-nesting-numbers A_{r_1} and A_{r_2} do have the same parity:

The right bight-boundary position specification is then:

$$\underbrace{222\cdots2}_{(A_{r_1}-1)\text{ elements}}$$

While the right bight-boundary sequence specification is then:

$123\cdots(A_{r_1})(z+1-A_{r_2})(z+2-A_{r_2})(z+3-A_{r_2})\cdots z$.

Where $z = \dfrac{A_{r_1}+A_{r_2}}{2}$.

2). The two sub-nesting-numbers A_{r_1} and A_{r_2} do not have the same parity:

The right bight-boundary position specification is then:

$$\underbrace{222\cdots2}_{\substack{A_r-z-1\\ \text{elements}}}\underbrace{111\cdots1}_{\substack{2A_{r_2}\\ \text{elements}}}\underbrace{222\cdots2}_{\substack{z-2A_{r_2}\\ \text{elements}}}$$

Note that $A_r - z - 1 = z - 2A_{r_2}$.

While the right bight-boundary sequence specification is then:

i). For $A_{r_1} = A_{r_2} + 1$:

$135 \cdots (z+1)246 \cdots z$.

Where $z = \dfrac{A_{r_1} + A_{r_2} - 1}{2} + A_{r_2} = 2A_{r_2}$.

ii). For $A_{r_1} \geq A_{r_2} + 3$:

$123 \cdots (z+1-2A_{r_2})(z+3-2A_{r_2})(z+5-2A_{r_2}) \cdots (z+1)(z+2)(z+3) \cdots$
$(A_{r_1} + A_{r_2})(z+2-2A_{r_2})(z+4-2A_{r_2})(z+6-2A_{r_2}) \cdots z$.

Where $z = \dfrac{A_{r_1} + A_{r_2} - 1}{2} + A_{r_2}$.

⋆⋆ *Determine for the four right-hand bight-edges in Fig. 637 the right bight-boundary position specification and in general terms the right bight-boundary sequence set.*

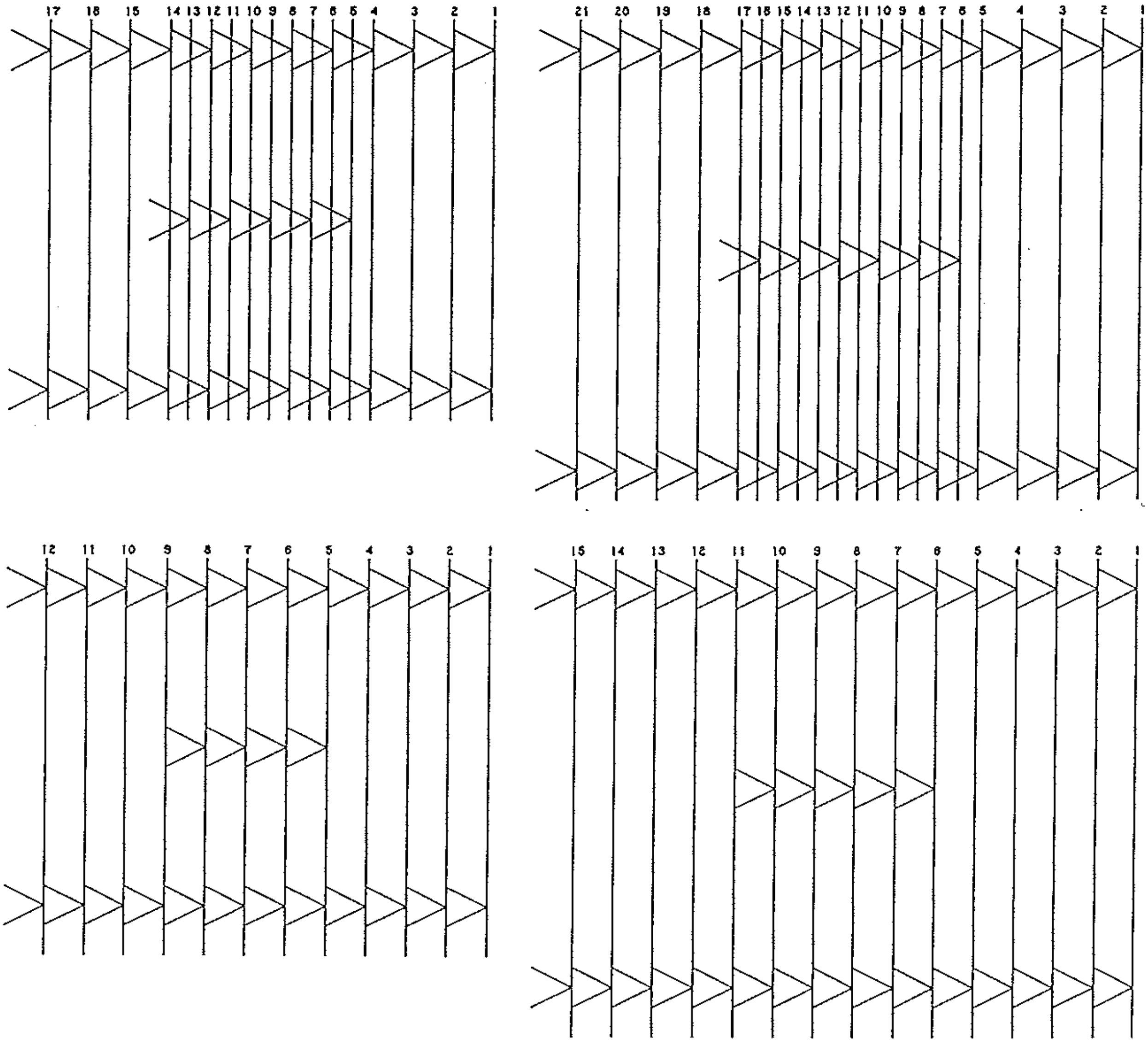


Fig. 637 — Examples of sub-nesting numbers A_{r_1} and A_{r_2} .

Note that for the left bight-edge we have used for the first lower-left to upper-right half-cycle the conventional start at left bight-boundary 1. Consequently we could fully specify in general terms the bight-boundary sequence specification (the bight-boundary sequence with their ranking-numbers). For the right bight-edge we can only specify in general terms the cyclic sequence of the bight-boundaries since the first lower-left to upper-right half-cycle can be made to end at anyone of the right bight-boundaries which then receives the ranking-number 1. The sequential order of the ranking-numbers $23\cdots(A_{r_1}+A_{r_2})$ follows then the further cyclic sequence of the bight-boundaries. For example, when $A_{r_1}=9$ and $A_{r_2}=4$, the cyclic sequence of the right bight-boundaries will be 123579(11)(12)(13)468(10). Say that the first lower-left to upper-right half-cycle ends at right bight-boundary 7, then the right bight-boundary sequence specification with its ranking-numbers becomes $7_1 9_2 11_3 12_4 13_5 4_6 6_7 8_8 10_9 1_{10} 2_{11} 3_{12} 5_{13}$.

Although there is a distinct calculation procedure associated with each of the two types of bight-edges, it is, however, more convenient to use a single, more universal, calculation procedure for both types. With the aid of the following Example we will run through this more universal calculation process.

Example 1.:

Let $A_{l_1}=5$; $A_{l_2}=2$; $A_{r_1}=4$; $A_{r_2}=2$ $x=11$.
Hence:

$$A_l = A_{l_1} + A_{l_2} = 5+2=7.$$
$$A_r = A_{r_1} + A_{r_2} = 4+2=6.$$
$$B_{total} = B_l^* \cdot A_l = B_l^* \cdot 7 = B_r^* \cdot A_r = B_r^* \cdot 6,$$
let B_{total} be equal to 42, then
$$B_l^* = 6 \text{ and } B_r^* = 7.$$
$$d = \text{g.c.d.}(A_l, A_r) = \text{g.c.d.}(7,6) = 1.$$
$$A^{**} = \frac{A_l \cdot A_r}{d} = \frac{7\cdot 6}{1} = 42.$$
$$B^{**} = \frac{B_{total}}{A^{**}} = \frac{42}{42} = 1.$$

The left bight-boundary position specification is 211112,
hence $\mathcal{K}_l = 7$.
The left bight-boundary sequence specification is $1_1 5_7 3_6 7_5 6_4 4_3 2_2$.
The right bight-boundary position specification is 222,
hence $\mathcal{K}_r = 4$.
The right bight-boundary sequence specification is $1_1 2_2 3_3 4_4 2_5 3_6$.

Hence the string-run specification of this Periodic Regular Nested Cylindrical Braid is $(211112/11/222)\{1_1 5_7 3_6 7_5 6_4 4_3 2_2 / 1_1 2_2 3_3 4_4 2_5 3_6\}42$.

We can now readily calculate the Δ_{l_i} and the Δ_{r_i} values:

$\Delta_{l_i}=8$ for $l_i=1$.	$\Delta_{r_i}=6$ for $r_i=1$.
$\Delta_{l_i}=6$ for $l_i=2$.	$\Delta_{r_i}=4$ for $r_i=2$.
$\Delta_{l_i}=5$ for $l_i=3$.	$\Delta_{r_i}=2$ for $r_i=3$.
$\Delta_{l_i}=4$ for $l_i=4$.	$\Delta_{r_i}=0$ for $r_i=4$.
$\Delta_{l_i}=3$ for $l_i=5$.	
$\Delta_{l_i}=2$ for $l_i=6$.	
$\Delta_{l_i}=0$ for $l_i=7$.	

Each lower-left to upper-right half-cycle type in a Nested Cylindrical Braid occurs only once in all the first-return string-runs of such a braid. Hence we can read the lower-left to upper-right half-cycle types from the Periodic Regular Nested Cylindrical Braid specification $(211112/11/222)\{1_1 5_7 3_6 7_5 6_4 4_3 2_2 / 1_1 2_2 3_3 4_4 2_5 3_6\}42$:

$1_1 \longrightarrow 1_1$	$1_1 \longrightarrow 3_3$	$1_1 \longrightarrow 2_5$
$5_7 \longrightarrow 2_2$	$5_7 \longrightarrow 4_4$	$5_7 \longrightarrow 3_6$
$3_6 \longrightarrow 3_3$	$3_6 \longrightarrow 2_5$	$3_6 \longrightarrow 1_1$
$7_5 \longrightarrow 4_4$	$7_5 \longrightarrow 3_6$	$7_5 \longrightarrow 2_2$
$6_4 \longrightarrow 2_5$	$6_4 \longrightarrow 1_1$	$6_4 \longrightarrow 3_3$
$4_3 \longrightarrow 3_6$	$4_3 \longrightarrow 2_2$	$4_3 \longrightarrow 4_4$
$2_2 \longrightarrow 1_1$	$2_2 \longrightarrow 3_3$	$2_2 \longrightarrow 2_5$
$1_1 \longrightarrow 2_2$	$1_1 \longrightarrow 4_4$	$1_1 \longrightarrow 3_6$
$5_7 \longrightarrow 3_3$	$5_7 \longrightarrow 2_5$	$5_7 \longrightarrow 1_1$
$3_6 \longrightarrow 4_4$	$3_6 \longrightarrow 3_6$	$3_6 \longrightarrow 2_2$
$7_5 \longrightarrow 2_5$	$7_5 \longrightarrow 1_1$	$7_5 \longrightarrow 3_3$
$6_4 \longrightarrow 3_6$	$6_4 \longrightarrow 2_2$	$6_4 \longrightarrow 4_4$
$4_3 \longrightarrow 1_1$	$4_3 \longrightarrow 3_3$	$4_3 \longrightarrow 2_5$
$2_2 \longrightarrow 2_2$	$2_2 \longrightarrow 4_4$	$2_2 \longrightarrow 3_6$

Anyone of these listed types may be taken as the first lower-left to upper-right half-cycle in the 1^{st} first-return string-run, but normally we take the first listed one. Every lower-left to upper-right half-cycle encountered in this 1^{st} first-return string-run gets deleted from the type-list.

Anyone of the remaining types in the type-list may be taken as the first lower-left to upper-right half-cycle in the 2^{nd} first-return string-run, and again every lower-left to upper-right half-cycle encountered in this 2^{nd} first-return string-run gets deleted from the remaining type-list.

This process is carried on till all the lower-left to upper-right half-cycle types have been deleted from the type-list.

For the first-return string-runs we thus obtain:

$$1_1 \nwarrow 2_2 \nearrow 4_3 \nwarrow 3_3 \nearrow 3_6 \nwarrow 3_3 \nearrow 4_3 \nwarrow 2_2 \nearrow 1_1 \nwarrow 1_1 \nearrow 7_5 \nwarrow 3_6 \nearrow 6_4$$

$$\begin{array}{llll}
l_{1_{j_l}} = 1_1 \longrightarrow 1_1 = r_{1_{j_r}} & \rightarrow \quad j'_l = |1+8+11+6|_7 = 5 & \rightarrow & l_{2_{j'_l}} = 7_5\,. \\
l_{2_{j'_l}} = 7_5 \longleftarrow 1_1 = r_{1_{j_r}} & \rightarrow \quad j'_r = |1+6+11+0|_6 = 6 & \rightarrow & r_{2_{j'_r}} = 3_6\,. \\
l_{2_{j_l}} = 7_5 \longrightarrow 3_6 = r_{2_{j'_r}} & \rightarrow \quad j'_l = |5+0+11+2|_7 = 4 & \rightarrow & l_{3_{j'_l}} = 6_4\,. \\
l_{3_{j'_l}} = 6_4 \longleftarrow 3_6 = r_{2_{j_r}} & \rightarrow \quad j'_r = |6+2+11+2|_6 = 3 & \rightarrow & r_{3_{j'_r}} = 3_3\,. \\
l_{3_{j_l}} = 6_4 \longrightarrow 3_3 = r_{3_{j'_r}} & \rightarrow \quad j'_l = |4+2+11+2|_7 = 5 & \rightarrow & l_{4_{j'_l}} = 7_5\,. \\
l_{4_{j'_l}} = 7_5 \longleftarrow 3_3 = r_{3_{j_r}} & \rightarrow \quad j'_r = |3+2+11+0|_6 = 4 & \rightarrow & r_{4_{j'_r}} = 4_4\,. \\
l_{4_{j_l}} = 7_5 \longrightarrow 4_4 = r_{4_{j'_r}} & \rightarrow \quad j'_l = |5+0+11+0|_7 = 2 & \rightarrow & l_{5_{j'_l}} = 2_2\,. \\
l_{5_{j'_l}} = 2_2 \longleftarrow 4_4 = r_{4_{j_r}} & \rightarrow \quad j'_r = |4+0+11+6|_6 = 3 & \rightarrow & r_{5_{j'_r}} = 3_3\,. \\
l_{5_{j_l}} = 2_2 \longrightarrow 3_3 = r_{5_{j'_r}} & \rightarrow \quad j'_l = |2+6+11+2|_7 = 7 & \rightarrow & l_{6_{j'_l}} = 5_7\,. \\
l_{6_{j'_l}} = 5_7 \longleftarrow 3_3 = r_{5_{j_r}} & \rightarrow \quad j'_r = |3+2+11+3|_6 = 1 & \rightarrow & r_{6_{j'_r}} = 1_1\,. \\
l_{6_{j_l}} = 5_7 \longrightarrow 1_1 = r_{6_{j'_r}} & \rightarrow \quad j'_l = |7+3+11+6|_7 = 6 & \rightarrow & l_{7_{j'_l}} = 3_6\,.
\end{array}$$

$$\nwarrow 3_3 \nearrow 7_5 \nwarrow 4_4 \nearrow 2_2 \nwarrow 3_3 \nearrow 5_7 \nwarrow 1_1 \nearrow 3_6 \nwarrow 2_5 \nearrow 7_5 \nwarrow 2_2 \nearrow 3_6 \nwarrow 4_4 \nearrow 1_1 \nwarrow 2_5 \nearrow 4_3 \nwarrow 3_6 \nearrow 3_6 \nwarrow 3_6 \nearrow 4_3 \nwarrow 2_5 \nearrow 1_1 \nwarrow 4_4 \nearrow 3_6 \nwarrow 2_2 \nearrow 7_5 \nwarrow 2_5 \nearrow 3_6 \nwarrow 1_1 \nearrow 5_7 \nwarrow 3_3 \nearrow 2_2 \nwarrow 4_4$$

$$\begin{array}{llll}
l_{7_{j_l'}} = 3_6 \longleftarrow 1_1 = r_{6_{j_r}} & \to & j_r' = |1+6+11+5|_6 = 5 & \to \; r_{7_{j_r'}} = 2_5 . \\
l_{7_{j_l}} = 3_6 \longrightarrow 2_5 = r_{7_{j_r'}} & \to & j_l' = |6+5+11+4|_7 = 5 & \to \; l_{8_{j_l'}} = 7_5 . \\
l_{8_{j_l'}} = 7_5 \longleftarrow 2_5 = r_{7_{j_r}} & \to & j_r' = |5+4+11+0|_6 = 2 & \to \; r_{8_{j_r'}} = 2_2 . \\
l_{8_{j_l}} = 7_5 \longrightarrow 2_2 = r_{8_{j_r'}} & \to & j_l' = |5+0+11+4|_7 = 6 & \to \; l_{9_{j_l'}} = 3_6 . \\
l_{9_{j_l'}} = 3_6 \longleftarrow 2_2 = r_{8_{j_r}} & \to & j_r' = |2+4+11+5|_6 = 4 & \to \; r_{9_{j_r'}} = 4_4 . \\
l_{9_{j_l}} = 3_6 \longrightarrow 4_4 = r_{9_{j_r'}} & \to & j_l' = |6+5+11+0|_7 = 1 & \to \; l_{10_{j_l'}} = 1_1 . \\
l_{10_{j_l'}} = 1_1 \longleftarrow 4_4 = r_{9_{j_r}} & \to & j_r' = |4+0+11+8|_6 = 5 & \to \; r_{10_{j_r'}} = 2_5 . \\
l_{10_{j_l}} = 1_1 \longrightarrow 2_5 = r_{10_{j_r'}} & \to & j_l' = |1+8+11+4|_7 = 3 & \to \; l_{11_{j_l'}} = 4_3 . \\
l_{11_{j_l'}} = 4_3 \longleftarrow 2_5 = r_{10_{j_r}} & \to & j_r' = |5+4+11+4|_6 = 6 & \to \; r_{11_{j_r'}} = 3_6 . \\
l_{11_{j_l}} = 4_3 \longrightarrow 3_6 = r_{11_{j_r'}} & \to & j_l' = |3+4+11+2|_7 = 6 & \to \; l_{12_{j_l'}} = 3_6 . \\
l_{12_{j_l'}} = 3_6 \longleftarrow 3_6 = r_{11_{j_r}} & \to & j_r' = |6+2+11+5|_6 = 6 & \to \; r_{12_{j_r'}} = 3_6 . \\
l_{12_{j_l}} = 3_6 \longrightarrow 3_6 = r_{12_{j_r'}} & \to & j_l' = |6+5+11+2|_7 = 3 & \to \; l_{13_{j_l'}} = 4_3 . \\
l_{13_{j_l'}} = 4_3 \longleftarrow 3_6 = r_{12_{j_r}} & \to & j_r' = |6+2+11+4|_6 = 5 & \to \; r_{13_{j_r'}} = 2_5 . \\
l_{13_{j_l}} = 4_3 \longrightarrow 2_5 = r_{13_{j_r'}} & \to & j_l' = |3+4+11+4|_7 = 1 & \to \; l_{14_{j_l'}} = 1_1 . \\
l_{14_{j_l'}} = 1_1 \longleftarrow 2_5 = r_{13_{j_r}} & \to & j_r' = |5+4+11+8|_6 = 4 & \to \; r_{14_{j_r'}} = 4_4 . \\
l_{14_{j_l}} = 1_1 \longrightarrow 4_4 = r_{14_{j_r'}} & \to & j_l' = |1+8+11+0|_7 = 6 & \to \; l_{15_{j_l'}} = 3_6 . \\
l_{15_{j_l'}} = 3_6 \longleftarrow 4_4 = r_{14_{j_r}} & \to & j_r' = |4+0+11+5|_6 = 2 & \to \; r_{15_{j_r'}} = 2_2 . \\
l_{15_{j_l}} = 3_6 \longrightarrow 2_2 = r_{15_{j_r'}} & \to & j_l' = |6+5+11+4|_7 = 5 & \to \; l_{16_{j_l'}} = 7_5 . \\
l_{16_{j_l'}} = 7_5 \longleftarrow 2_2 = r_{15_{j_r}} & \to & j_r' = |2+4+11+0|_6 = 5 & \to \; r_{16_{j_r'}} = 2_5 . \\
l_{16_{j_l}} = 7_5 \longrightarrow 2_5 = r_{16_{j_r'}} & \to & j_l' = |5+0+11+4|_7 = 6 & \to \; l_{17_{j_l'}} = 3_6 . \\
l_{17_{j_l'}} = 3_6 \longleftarrow 2_5 = r_{16_{j_r}} & \to & j_r' = |5+4+11+5|_6 = 1 & \to \; r_{17_{j_r'}} = 1_1 . \\
l_{17_{j_l}} = 3_6 \longrightarrow 1_1 = r_{17_{j_r'}} & \to & j_l' = |6+5+11+6|_7 = 7 & \to \; l_{18_{j_l'}} = 5_7 . \\
l_{18_{j_l'}} = 5_7 \longleftarrow 1_1 = r_{17_{j_r}} & \to & j_r' = |1+6+11+3|_6 = 3 & \to \; r_{18_{j_r'}} = 3_3 . \\
l_{18_{j_l}} = 5_7 \longrightarrow 3_3 = r_{18_{j_r'}} & \to & j_l' = |7+3+11+2|_7 = 2 & \to \; l_{19_{j_l'}} = 2_2 . \\
l_{19_{j_l'}} = 2_2 \longleftarrow 3_3 = r_{18_{j_r}} & \to & j_r' = |3+2+11+6|_6 = 4 & \to \; r_{19_{j_r'}} = 4_4 . \\
l_{19_{j_l}} = 2_2 \longrightarrow 4_4 = r_{19_{j_r'}} & \to & j_l' = |2+6+11+0|_7 = 5 & \to \; l_{20_{j_l'}} = 7_5 . \\
l_{20_{j_l'}} = 7_5 \longleftarrow 4_4 = r_{19_{j_r}} & \to & j_r' = |4+0+11+0|_6 = 3 & \to \; r_{20_{j_r'}} = 3_3 . \\
l_{20_{j_l}} = 7_5 \longrightarrow 3_3 = r_{20_{j_r'}} & \to & j_l' = |5+0+11+2|_7 = 4 & \to \; l_{21_{j_l'}} = 6_4 . \\
l_{21_{j_l'}} = 6_4 \longleftarrow 3_3 = r_{20_{j_r}} & \to & j_r' = |3+2+11+2|_6 = 6 & \to \; r_{21_{j_r'}} = 3_6 . \\
l_{21_{j_l}} = 6_4 \longrightarrow 3_6 = r_{21_{j_r'}} & \to & j_l' = |4+2+11+2|_7 = 5 & \to \; l_{22_{j_l'}} = 7_5 . \\
l_{22_{j_l'}} = 7_5 \longleftarrow 3_6 = r_{21_{j_r}} & \to & j_r' = |6+2+11+0|_6 = 1 & \to \; r_{22_{j_r'}} = 1_1 . \\
l_{22_{j_l}} = 7_5 \longrightarrow 1_1 = r_{22_{j_r'}} & \to & j_l' = |5+0+11+6|_7 = 1 & \to \; l_{23_{j_l'}} = 1_1 . \\
l_{23_{j_l'}} = 1_1 \longleftarrow 1_1 = r_{22_{j_r}} & \to & j_r' = |1+6+11+8|_6 = 2 & \to \; r_{23_{j_r'}} = 2_2 . \\
l_{23_{j_l}} = 1_1 \longrightarrow 2_2 = r_{23_{j_r'}} & \to & j_l' = |1+8+11+4|_7 = 3 & \to \; l_{24_{j_l'}} = 4_3 . \\
l_{24_{j_l'}} = 4_3 \longleftarrow 2_2 = r_{23_{j_r}} & \to & j_r' = |2+4+11+4|_6 = 3 & \to \; r_{24_{j_r'}} = 3_3 . \\
l_{24_{j_l}} = 4_3 \longrightarrow 3_3 = r_{24_{j_r'}} & \to & j_l' = |3+4+11+2|_7 = 6 & \to \; l_{25_{j_l'}} = 3_6 . \\
l_{25_{j_l'}} = 3_6 \longleftarrow 3_3 = r_{24_{j_r}} & \to & j_r' = |3+2+11+5|_6 = 3 & \to \; r_{25_{j_r'}} = 3_3 .
\end{array}$$

$\nearrow$
$7_5 \nwarrow$
3_3
$6_4 \nwarrow \nearrow$
3_6
$7_5 \nwarrow \nearrow$
1_1
$1_1 \nearrow$

$$\begin{array}{llll}
l_{25_{j_l}} = 3_6 \longrightarrow 3_3 = r_{25_{j'_r}} & \to\ j'_l = |6+5+11+2|_7 = 3 & \to & l_{26_{j'_l}} = 4_3\,. \\
l_{26_{j'_l}} = 4_3 \longleftarrow 3_3 = r_{25_{j_r}} & \to\ j'_r = |3+2+11+4|_6 = 2 & \to & r_{26_{j'_r}} = 2_2\,. \\
l_{26_{j_l}} = 4_3 \longrightarrow 2_2 = r_{26_{j'_r}} & \to\ j'_l = |3+4+11+4|_7 = 1 & \to & l_{27_{j'_l}} = 1_1\,. \\
l_{27_{j'_l}} = 1_1 \longleftarrow 2_2 = r_{26_{j_r}} & \to\ j'_r = |2+4+11+8|_6 = 1 & \to & r_{27_{j'_r}} = 1_1\,.
\end{array}$$

$P_c = \frac{\alpha\cdot x+\sum(\Delta_{l_i}+\Delta_{r_i})}{A^{**}} = \frac{26\cdot 11+(4\cdot 8+2\cdot 6+6\cdot 5+4\cdot 4+2\cdot 3+2\cdot 2+6\cdot 0)+(4\cdot 6+8\cdot 4+10\cdot 2+4\cdot 0)}{42} = 11\,.$
g.c.d.$(P_c\,,\,B^{**})$ = g.c.d.$(11\,,\,1) = 1\,.$

$1_1 \nwarrow$
3_6
$1_1 \nearrow \nwarrow$
3_3
$1_1 \nearrow$

$$\begin{array}{llll}
l_{1_{j_l}} = 1_1 \longrightarrow 3_3 = r_{1_{j_r}} & \to\ j'_l = |1+8+11+2|_7 = 1 & \to & l_{2_{j'_l}} = 1_1\,. \\
l_{2_{j'_l}} = 1_1 \longleftarrow 3_3 = r_{1_{j_r}} & \to\ j'_r = |3+2+11+8|_6 = 6 & \to & r_{2_{j'_r}} = 3_6\,. \\
l_{2_{j_l}} = 1_1 \longrightarrow 3_6 = r_{2_{j'_r}} & \to\ j'_l = |1+8+11+2|_7 = 1 & \to & l_{3_{j'_l}} = 1_1\,. \\
l_{3_{j'_l}} = 1_1 \longleftarrow 3_6 = r_{2_{j_r}} & \to\ j'_r = |6+2+11+8|_6 = 3 & \to & r_{3_{j'_r}} = 3_3\,.
\end{array}$$

$P_c = \frac{\alpha\cdot x+\sum(\Delta_{l_i}+\Delta_{r_i})}{A^{**}} = \frac{2\cdot 11+(2\cdot 8)+(2\cdot 2)}{42} = 1\,.$
g.c.d.$(P_c\,,\,B^{**})$ = g.c.d.$(1\,,\,1) = 1\,.$

$2_2 \nwarrow$
2_2
$2_2 \nearrow \nwarrow$
2_5
$2_2 \nearrow$

$$\begin{array}{llll}
l_{1_{j_l}} = 2_2 \longrightarrow 2_5 = r_{1_{j_r}} & \to\ j'_l = |2+6+11+4|_7 = 2 & \to & l_{2_{j'_l}} = 2_2\,. \\
l_{2_{j'_l}} = 2_2 \longleftarrow 2_5 = r_{1_{j_r}} & \to\ j'_r = |5+4+11+6|_6 = 2 & \to & r_{2_{j'_r}} = 2_2\,. \\
l_{2_{j_l}} = 2_2 \longrightarrow 2_2 = r_{2_{j'_r}} & \to\ j'_l = |2+6+11+4|_7 = 2 & \to & l_{3_{j'_l}} = 2_2\,. \\
l_{3_{j'_l}} = 2_2 \longleftarrow 2_2 = r_{2_{j_r}} & \to\ j'_r = |2+4+11+6|_6 = 5 & \to & r_{3_{j'_r}} = 2_5\,.
\end{array}$$

$P_c = \frac{\alpha\cdot x+\sum(\Delta_{l_i}+\Delta_{r_i})}{A^{**}} = \frac{2\cdot 11+(2\cdot 6)+(2\cdot 4)}{42} = 1\,.$
g.c.d.$(P_c\,,\,B^{**})$ = g.c.d.$(1\,,\,1) = 1\,.$

$6_4 \nwarrow$
4_4
$4_3 \nearrow \nwarrow$
1_1
$4_3 \nearrow \nwarrow$
4_4

$$\begin{array}{llll}
l_{1_{j_l}} = 6_4 \longrightarrow 2_5 = r_{1_{j_r}} & \to\ j'_l = |4+2+11+4|_7 = 7 & \to & l_{2_{j'_l}} = 5_7\,. \\
l_{2_{j'_l}} = 5_7 \longleftarrow 2_5 = r_{1_{j_r}} & \to\ j'_r = |5+4+11+3|_6 = 5 & \to & r_{2_{j'_r}} = 2_5\,. \\
l_{2_{j_l}} = 5_7 \longrightarrow 2_5 = r_{2_{j'_r}} & \to\ j'_l = |7+3+11+4|_7 = 4 & \to & l_{3_{j'_l}} = 6_4\,. \\
l_{3_{j'_l}} = 6_4 \longleftarrow 2_5 = r_{2_{j_r}} & \to\ j'_r = |5+4+11+2|_6 = 4 & \to & r_{3_{j'_r}} = 4_4\,. \\
l_{3_{j_l}} = 6_4 \longrightarrow 4_4 = r_{3_{j'_r}} & \to\ j'_l = |4+2+11+0|_7 = 3 & \to & l_{4_{j'_l}} = 4_3\,.
\end{array}$$

$$\begin{array}{l} 6_4 \nearrow \\ 6_4 \nwarrow \\ \quad 2_5 \\ 5_7 \nearrow \\ 5_7 \nwarrow \\ \quad 2_5 \\ 6_4 \nearrow \end{array}$$

$$\begin{array}{llll}
l_{4_{j_l'}} = 4_3 \longleftarrow 4_4 = r_{3_{j_r}} & \rightarrow \; j_r' = |4+0+11+4|_6 = 1 & \rightarrow & r_{4_{j_r'}} = 1_1\,. \\
l_{4_{j_l}} = 4_3 \longrightarrow 1_1 = r_{4_{j_r'}} & \rightarrow \; j_l' = |3+4+11+6|_7 = 3 & \rightarrow & l_{5_{j_l'}} = 4_3\,. \\
l_{5_{j_l'}} = 4_3 \longleftarrow 1_1 = r_{4_{j_r}} & \rightarrow \; j_r' = |1+6+11+4|_6 = 4 & \rightarrow & r_{5_{j_r'}} = 4_4\,. \\
l_{5_{j_l}} = 4_3 \longrightarrow 4_4 = r_{5_{j_r'}} & \rightarrow \; j_l' = |3+4+11+0|_7 = 4 & \rightarrow & l_{6_{j_l'}} = 6_4\,. \\
l_{6_{j_l'}} = 6_4 \longleftarrow 4_4 = r_{5_{j_r}} & \rightarrow \; j_r' = |4+0+11+2|_6 = 5 & \rightarrow & r_{6_{j_r'}} = 2_5\,.
\end{array}$$

$P_c = \frac{\alpha\cdot x + \sum(\Delta_{l_i}+\Delta_{r_i})}{A^{**}} = \frac{5\cdot 11+(2\cdot 2+1\cdot 3+2\cdot 4)+(2\cdot 4+2\cdot 0+1\cdot 6)}{42} = 2\,.$
g.c.d. $(P_c\,,\, B^{**})$ = g.c.d. $(2\,,\,1) = 1\,.$

$$\begin{array}{l} 6_4 \nwarrow \\ \quad 1_1 \\ 2_2 \nearrow \\ 2_2 \nwarrow \\ \quad 3_6 \\ 5_7 \nearrow \\ 5_7 \nwarrow \\ \quad 4_4 \\ 5_7 \nearrow \\ 5_7 \nwarrow \\ \quad 3_6 \\ 2_2 \nearrow \\ 2_2 \nwarrow \\ \quad 1_1 \\ 6_4 \nearrow \\ 6_4 \nwarrow \\ \quad 2_2 \\ 5_7 \nearrow \\ 5_7 \nwarrow \\ \quad 2_2 \\ 6_4 \nearrow \end{array}$$

$$\begin{array}{llll}
l_{1_{j_l}} = 6_4 \longrightarrow 2_2 = r_{1_{j_r}} & \rightarrow \; j_l' = |4+2+11+4|_7 = 7 & \rightarrow & l_{2_{j_l'}} = 5_7\,. \\
l_{2_{j_l'}} = 5_7 \longleftarrow 2_2 = r_{1_{j_r}} & \rightarrow \; j_r' = |2+4+11+3|_6 = 2 & \rightarrow & r_{2_{j_r'}} = 2_2\,. \\
l_{2_{j_l}} = 5_7 \longrightarrow 2_2 = r_{2_{j_r'}} & \rightarrow \; j_l' = |7+3+11+4|_7 = 4 & \rightarrow & l_{3_{j_l'}} = 6_4\,. \\
l_{3_{j_l'}} = 6_4 \longleftarrow 2_2 = r_{2_{j_r}} & \rightarrow \; j_r' = |2+4+11+2|_6 = 1 & \rightarrow & r_{3_{j_r'}} = 1_1\,. \\
l_{3_{j_l}} = 6_4 \longrightarrow 1_1 = r_{3_{j_r'}} & \rightarrow \; j_l' = |4+2+11+6|_7 = 2 & \rightarrow & l_{4_{j_l'}} = 2_2\,. \\
l_{4_{j_l'}} = 2_2 \longleftarrow 1_1 = r_{3_{j_r}} & \rightarrow \; j_r' = |1+6+11+6|_6 = 6 & \rightarrow & r_{4_{j_r'}} = 3_6\,. \\
l_{4_{j_l}} = 2_2 \longrightarrow 3_6 = r_{4_{j_r'}} & \rightarrow \; j_l' = |2+6+11+2|_7 = 7 & \rightarrow & l_{5_{j_l'}} = 5_7\,. \\
l_{5_{j_l'}} = 5_7 \longleftarrow 3_6 = r_{4_{j_r}} & \rightarrow \; j_r' = |6+2+11+3|_6 = 4 & \rightarrow & r_{5_{j_r'}} = 4_4\,. \\
l_{5_{j_l}} = 5_7 \longrightarrow 4_4 = r_{5_{j_r'}} & \rightarrow \; j_l' = |7+3+11+0|_7 = 7 & \rightarrow & l_{6_{j_l'}} = 5_7\,. \\
l_{6_{j_l'}} = 5_7 \longleftarrow 4_4 = r_{5_{j_r}} & \rightarrow \; j_r' = |4+0+11+3|_6 = 6 & \rightarrow & r_{6_{j_r'}} = 3_6\,. \\
l_{6_{j_l}} = 5_7 \longrightarrow 3_6 = r_{6_{j_r'}} & \rightarrow \; j_l' = |7+3+11+2|_7 = 2 & \rightarrow & l_{7_{j_l'}} = 2_2\,. \\
l_{7_{j_l'}} = 2_2 \longleftarrow 3_6 = r_{6_{j_r}} & \rightarrow \; j_r' = |6+2+11+6|_6 = 1 & \rightarrow & r_{7_{j_r'}} = 1_1\,. \\
l_{7_{j_l}} = 2_2 \longrightarrow 1_1 = r_{7_{j_r'}} & \rightarrow \; j_l' = |2+6+11+6|_7 = 4 & \rightarrow & l_{8_{j_l'}} = 6_4\,. \\
l_{8_{j_l'}} = 6_4 \longleftarrow 1_1 = r_{7_{j_r}} & \rightarrow \; j_r' = |1+6+11+2|_6 = 2 & \rightarrow & r_{8_{j_r'}} = 2_2\,.
\end{array}$$

$P_c = \frac{\alpha\cdot x + \sum(\Delta_{l_i}+\Delta_{r_i})}{A^{**}} = \frac{7\cdot 11+(2\cdot 2+3\cdot 3+2\cdot 6)+(2\cdot 4+2\cdot 6+2\cdot 2)+1\cdot 0}{42} = 3\,.$
g.c.d. $(P_c\,,\, B^{**})$ = g.c.d. $(3\,,\,1) = 1\,.$

Although the calculation process is simple, it is nevertheless a bit tedious, and hence a suitable computer program will be advantageous.

In order to obtain an overview of the actual positions of the bights on the left-hand and right-hand bight-edges we attach to the bight-boundary number with ranking-number of a bight the nest-index number of the nest to which the bight belongs. We can obtain the nest-index number which belongs to a bight-boundary number with ranking-number from the list of lower-left to upper-right half-cycle types (see pg. 794) after we have indicated in this list the nest-index numbers associated with the left-hand and right-hand bight-points, or we can obtain the nest-index number which belongs to a bight-boundary number with ranking-number by means of a calculation. The first method is the quickest and simplest, and hence will be one we shall use.

For our Example the list of lower-left to upper-right half-cycle types with the nest-index numbers will be as shown in Fig. 638.

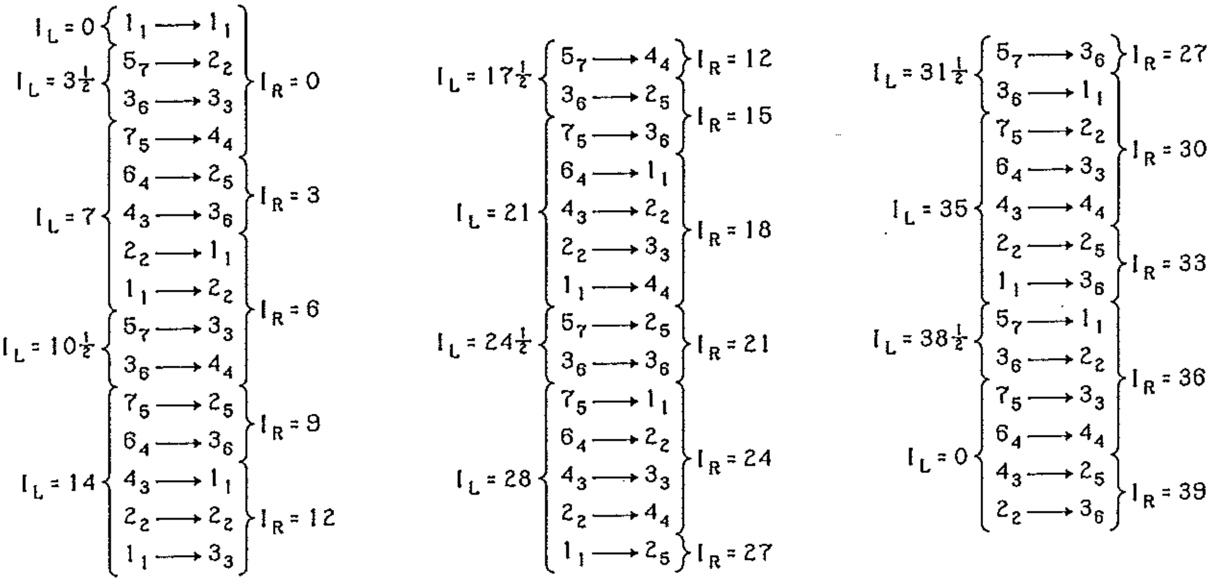


Fig. 638 — The lower-left to upper-right half-cycle types.

The nest-index number interval between adjacent sets of half-cycle types is for the left-hand bight-edge equal to $\frac{A_{l_1}+A_{l_2}}{2} = \frac{A_l}{2}$, and is for the right-hand bight-edge equal to $\frac{A_{r_1}+A_{r_2}}{2} = \frac{A_r}{2}$.

The first-return string-runs in which the bight-boundary numbers with ranking-numbers are provided with their associated nest-index numbers are then as follows:

$0/1_1$
$\nwarrow$
$2_2/18$
$\nearrow$
$21/4_3$
$\nwarrow$
$3_3/0$
$\nearrow$
$3\frac{1}{2}/3_6$
$\nwarrow$
$3_3/24$
$\nearrow$
$28/4_3$
$\nwarrow$
$2_2/6$
$\nearrow$
$7/1_1$
$\nwarrow$
$1_1/24$
$\nearrow$
$28/7_5$
$\nwarrow$
$3_6/9$
$\nearrow$
$14/6_4$
$\nwarrow$
$3_3/36$

$\nearrow$
$0/7_5$
$\nwarrow$
$4_4/24$
$\nearrow$
$28/2_2$
$\nwarrow$
$3_3/6$
$\nearrow$
$10\frac{1}{2}/5_7$
$\nwarrow$
$1_1/30$
$\nearrow$
$31\frac{1}{2}/3_6$
$\nwarrow$
$2_5/9$
$\nearrow$
$14/7_5$
$\nwarrow$
$2_2/36$
$\nearrow$
$38\frac{1}{2}/3_6$
$\nwarrow$
$4_4/18$
$\nearrow$
$21/1_1$
$\nwarrow$
$2_5/39$
$\nearrow$
$0/4_3$
$\nwarrow$
$3_6/21$
$\nearrow$
$24\frac{1}{2}/3_6$
$\nwarrow$
$3_6/3$
$\nearrow$
$7/4_3$
$\nwarrow$
$2_5/27$
$\nearrow$
$28/1_1$
$\nwarrow$
$4_4/6$
$\nearrow$
$10\frac{1}{2}/3_6$
$\nwarrow$
$2_2/30$
$\nearrow$
$35/7_5$
$\nwarrow$
$2_5/15$
$\nearrow$
$17\frac{1}{2}/3_6$
$\nwarrow$
$1_1/36$
$\nearrow$
$38\frac{1}{2}/5_7$
$\nwarrow$
$3_3/18$
$\nearrow$
$21/2_2$

$7/6_4$
$\nwarrow$
$4_4/30$
$\nearrow$
$35/4_3$

$28/6_4$
$\nwarrow$
$1_1/6$
$\nearrow$
$7/2_2$
$\nwarrow$
$3_6/27$
$\nearrow$
$31\frac{1}{2}/5_7$
$\nwarrow$
$4_4/12$
$\nearrow$
$17\frac{1}{2}/5_7$

$4_4/0$			$1_1/12$	$3_6/39$
$7/7_5$			$14/4_3$	$0/2_2$
$3_3/30$			$4_4/36$	$1_1/18$
$35/6_4$	$14/1_1$	$35/2_2$	$0/6_4$	$21/6_4$
$3_6/15$	$3_6/33$	$2_2/12$	$2_5/21$	$2_2/0$
$21/7_5$	$35/1_1$	$14/2_2$	$24\frac{1}{2}/5_7$	$3\frac{1}{2}/5_7$
$1_1/0$	$3_3/12$	$2_5/33$	$2_5/3$	$2_2/24$
$0/1_1$	$14/1_1$	$35/2_2$	$7/6_4$	$28/6_4$

We can now construct the half-cycle pattern of the first-return string-runs (see Fig. 639).

1	2	3	4	5	6	7	
		29/16		11/16			— $38\frac{1}{2}$
55/16	57/16		69/12		5/12	15/12	— 35
		33/17		81/15			— $31\frac{1}{2}$
19/17	37/13		47/13		71/13	43/13	— 28
		23/16		63/16			— $24\frac{1}{2}$
27/14	9/14		51/14		75/14	3/12	— 21
		13/17		79/13			— $17\frac{1}{2}$
53/15	59/15		67/15		41/13	31/13	— 14
		17/14		35/14			— $10\frac{1}{2}$
45/16	83/16		21/14		61/14	7/10	— 7
		49/15		73/15			— $3\frac{1}{2}$
1/17	77/15		25/15		65/11	39/11	— 0

	4	3	2	1
39 —		78/15	26/17	
36 —	66/12	40/12	30/12	12/17
33 —		56/16	58/16	
30 —	70/11	6/11	16/16	34/16
27 —		82/15	20/15	
24 —	38/10	48/15	72/15	44/17
21 —		24/14	64/14	
18 —	28/14	10/14	52/16	76/16
15 —		4/13	14/13	
12 —	80/13	54/15	60/15	68/15
9 —		42/12	32/17	
6 —	18/14	36/14	46/14	84/14
3 —		22/16	62/16	
0 —	8/13	50/13	74/13	2/13

Fig. 639 — The half-cycle pattern of the first-return string-runs.

The number of crossings on a half-cycle are again indicated below the half-cycle number (see *The Braider*, Issue No. 33, pg. 763). The number of crossings on a half-cycle are calculated as the sum of three sets of crossings:

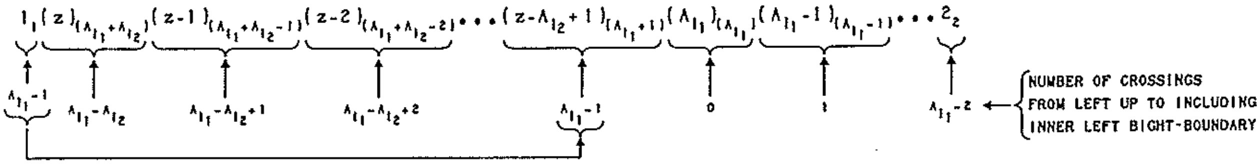
i). The number of crossings between the left-hand bight-boundary 1 up to and including the innermost left-hand bight-boundary.

ii). The number of crossings between the innermost left-hand bight-boundary and the innermost right-hand bight-boundary; hence equal to $(x-1)$.

iii). The number of crossings between the right-hand bight-boundary 1 up to and including the innermost right-hand bight-boundary.

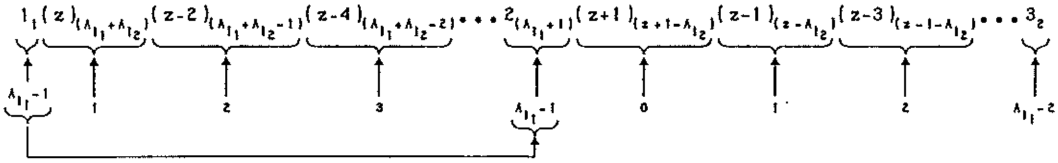
The general scheme for the number of crossings under **i)** is as follows:

a). The two sub-nesting numbers A_{l_1} and A_{l_2} do have the same parity.

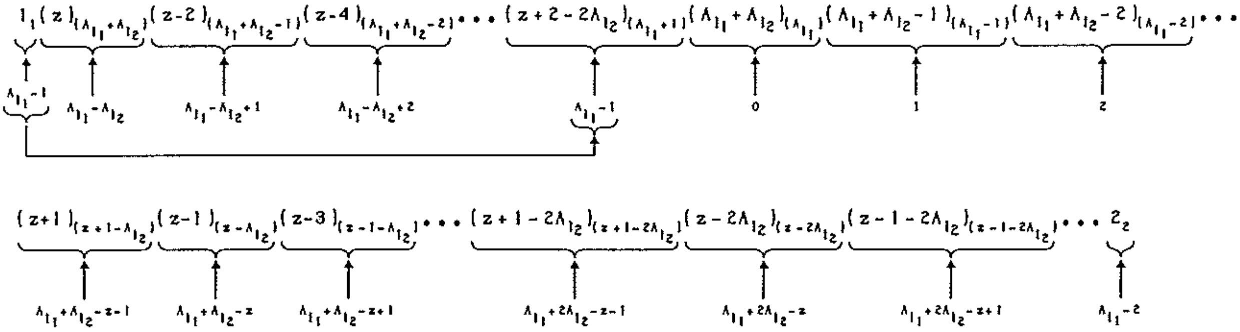


b). The two sub-nesting numbers A_{l_1} and A_{l_2} do not have the same parity.

1). For $A_{l_1} = A_{l_2} + 1$:

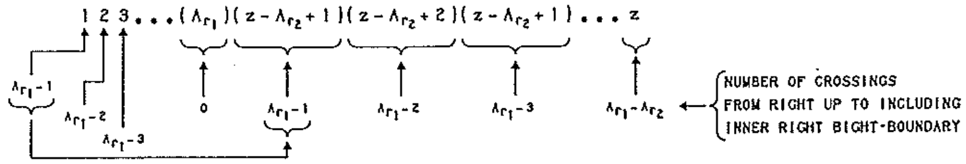


2). For $A_{l_1} \geq A_{l_2} + 3$:



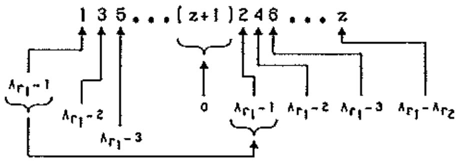
The general scheme for the number of crossings under **iii)** is as follows:

a). The two sub-nesting numbers A_{r_1} and A_{r_2} do have the same parity.

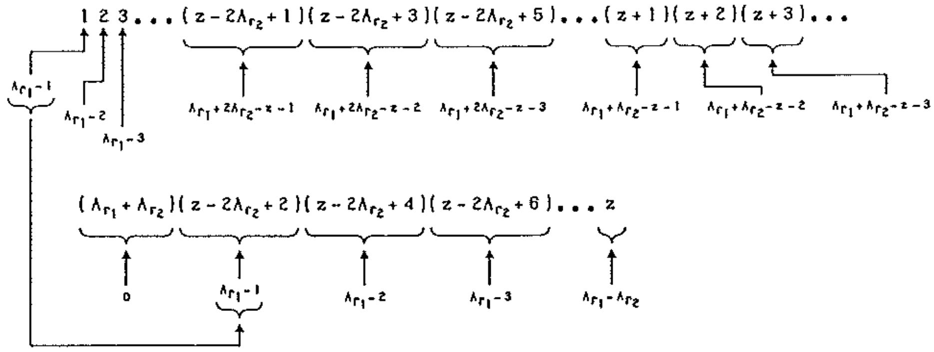


b). The two sub-nesting numbers A_{r_1} and A_{r_2} do not have the same parity.

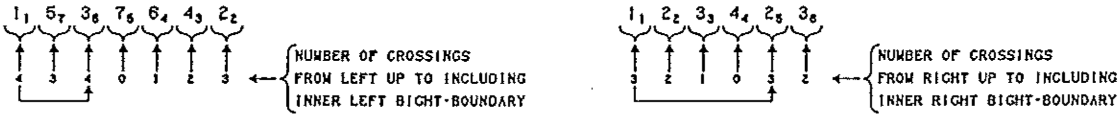
1). For $A_{r_1} = A_{r_2} + 1$:



2). For $A_{r_1} \geq A_{r_2} + 3$:



Thus for our Example we obtain:



Since in our Example $B^{**} = 1$ we know that the string-run of each component consists of only one first-return string-run of that component, hence in the half-cycle pattern of Fig. 639 all bight-points are occupied.

Say we superimpose on the string-run of our Example an under–over coding, then we can now assemble the half-cycle tables for the components; Fig. 640 presents the table for the odd-numbered half-cycles (the half-cycles from lower-left to upper-right), and Fig. 641 presents the even-numbered half-cycles (the half-cycles from lower-right to upper left). Note that in the table of the odd-numbered half-cycles (see Fig. 640) some of the half-cycle numbers have one or more stars attached to them. When a half-cycle number in the rightmost section of this table has one or more star(s), then in its associated row of half-cycles in the uppermost section of this table only those half-cycle numbers equal to or smaller than the half-cycle number in the rightmost table-section with an identical number of stars are neglected†.

1*	77	25	65	39	49	73	45	83	21	61	7	17	35	53	59	67												
29	11	1*	77	25	65	39	49	73	45	83	21	61	7	17	35	53												
55	57	69	5	15	29	11	1*	77	25	65	39	49	73	45	83	21												
33	81	55	57	69	5	15	29	11	1*	77	25	65	39	49	73	45												
19	37	47	71	43	33	81	55	57	69	5	15	29	11	1*	77	25												
23	63	19	37	47	71***	43	33	81	55	57	69	5	15	29	11	1												
27	9	51	75	3	23	63	19	37	47	71***	43	33	81	55	57	69												
13	79	27	9	51	75	3	23	63	19	37	47	71	43	33	81	55												
53	59	67	41	31	13	79	27	9	51	75	3	23	63	19	37	47												
17	35	53	59	67	41	31	13	79	27	9	51	75	3	23	63	19												
45	83	21	61	7	17	35	53	59	67	41	31	13	79	27	9	51												
49	73	45	83	21	61**	7	17	35	53	59	67	41	31	13	79	27												
u	o	u	o	u	o	u	o	u	o	u	o	u	o	u	o	u	1					13			19*	33*		
u	o	u	o	u	o	u	o	u	o	u	o	u	o	u	o	X			45					23*			55	29*
o	u	o	u	o	u	o	u	o	u	o	u	o	u	o	u	X			83					63			57	11*
u	o	u	o	u	o	u	o	u	o	u	o	u	o	u	X	X	25	49			53 67							
o	u	o	u	o	u	o	u	o	u	o	u	o	u	o	X	X	77	73			59					81		
u	o	u	o	u	o	u	o	u	o	u	o	u	o	X	X	X			21	17			27 51					
o	u	o	u	o	u	o	u	o	u	o	u	o	u	X	X	X			61	35			9 75***					
u	o	u	o	u	o	u	o	u	o	u	o	u	X	X	X	X					31				47* 43*			
o	u	o	u	o	u	o	u	o	u	o	u	o	X	X	X	X					41	79***			37* 71			
u	o	u	o	u	o	u	o	u	o	u	o	X	X	X	X	X							3				15* 69	
o	u	o	u	o	u	o	u	o	u	o	u	X	X	X	X	X											5*	
u	o	u	o	u	o	u	o	u	o	u	X	X	X	X	X	X	39											
o	u	o	u	o	u	o	u	o	u	o	X	X	X	X	X	X	65**											
u	o	u	o	u	o	u	o	u	o	X	X	X	X	X	X	X			7									

Fig. 640 — The half-cycle table for the lower-left to upper-right half-cycles.

The half-cycle braiding algorithms are subsequently being read from these half-cycle tables.

† See *The Braider*, Issue No. 33, pp. 764-765.

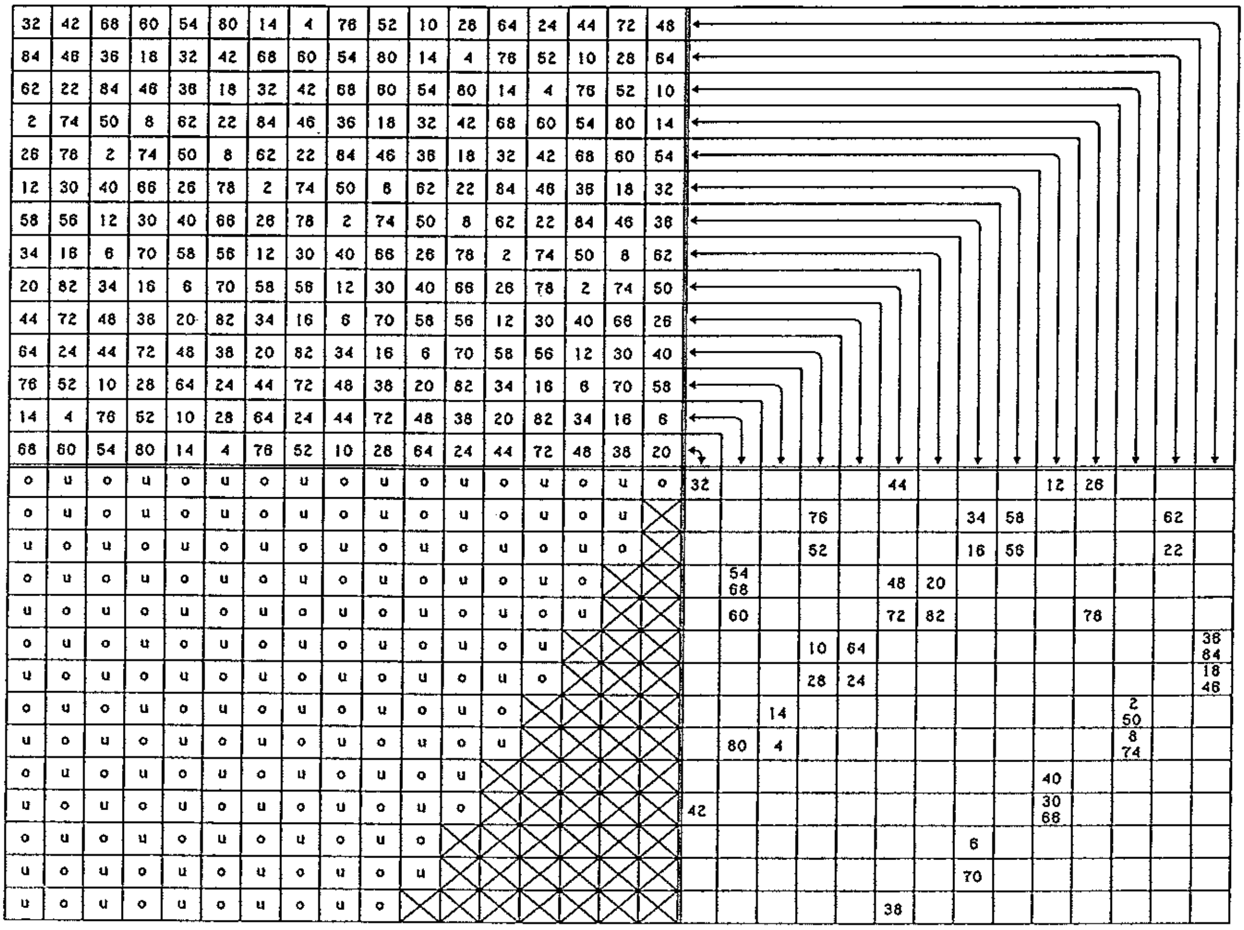


Fig. 641 — The half-cycle table for the lower-right to upper-left half-cycles.

The braiding half-cycle algorithms are as follows:

1. $1_1 \longrightarrow 1_1$: Free run.
2. $7_5 \longleftarrow 1_1$: Free run.
3. $7_5 \longrightarrow 3_6$: Free run.
4. $6_4 \longleftarrow 3_6$: Free run.
5. $6_4 \longrightarrow 3_3$: Free run.
6. $7_5 \longleftarrow 3_3$: o
7. $7_5 \longrightarrow 4_4$: Free run.
8. $2_2 \longleftarrow 4_4$: Free run.
9. $2_2 \longrightarrow 3_3$: o.
10. $5_7 \longleftarrow 3_3$: o.
11. $5_7 \longrightarrow 1_1$: u.
12. $3_6 \longleftarrow 1_1$: $o - u$.
13. $3_6 \longrightarrow 2_5$: $o - u$.
14. $7_5 \longleftarrow 2_5$: o.
15. $7_5 \longrightarrow 2_2$: o.
16. $3_6 \longleftarrow 2_2$: $2u - o$.
17. $3_6 \longrightarrow 4_4$: $o - u - o$.
18. $1_1 \longleftarrow 4_4$: $u - o - u$.
19. $1_1 \longrightarrow 2_5$: $o - 2u$.
20. $4_3 \longleftarrow 2_5$: $u - 3o$.

21. $4_3 \longrightarrow 3_6$: $u-o-u-o$.
22. $3_6 \longleftarrow 3_6$: $o-u-o-u$.
23. $3_6 \longrightarrow 3_6$: $2u-2o$.
24. $4_3 \longleftarrow 3_6$: $u-o-2u$.
25. $4_3 \longrightarrow 2_5$: $2u-o-u$.
26. $1_1 \longleftarrow 2_5$: $o-3u-o$.
27. $1_1 \longrightarrow 4_4$: $3u-o$.
28. $3_6 \longleftarrow 4_4$: $o-u-o-u$.
29. $3_6 \longrightarrow 2_2$: $u-2o-u$.
30. $7_5 \longleftarrow 2_2$: $2u-3o$.
31. $7_5 \longrightarrow 2_5$: $2u-o-u-2o$.
32. $3_6 \longleftarrow 2_5$: $o-u-o-2u-o$.
33. $3_6 \longrightarrow 1_1$: $o-u-o-u-o-u$.
34. $5_7 \longleftarrow 1_1$: $o-u-2o-2u$.
35. $5_7 \longrightarrow 3_3$: $o-2u-o-u-o$.
36. $2_2 \longleftarrow 3_3$: $2o-u-o-2u$.
37. $2_2 \longrightarrow 4_4$: $o-u-o-u-o-u$.
38. $7_5 \longleftarrow 4_4$: $2u-o-2u-o$.
39. $7_5 \longrightarrow 3_3$: $2u-o-u$.
40. $6_4 \longleftarrow 3_3$: $2o-2u-o-u$.
41. $6_4 \longrightarrow 3_6$: $2o-u-o-2u-o$.
42. $7_5 \longleftarrow 3_6$: $u-o-u-2o$.
43. $7_5 \longrightarrow 1_1$: $u-o-u-o-u-o$.
44. $1_1 \longleftarrow 1_1$: $2o-u-2o-u-3o$.
45. $1_1 \longrightarrow 2_2$: $u-2o-u-2o-u-o-u$.
46. $4_3 \longleftarrow 2_2$: $u-o-u-o-u-2o$.
47. $4_3 \longrightarrow 3_3$: $u-o-u-o-u-o$.
48. $3_6 \longleftarrow 3_3$: $2o-u-2o-u-3o$.
49. $3_6 \longrightarrow 3_3$: $3u-o-2u-o-2u$.
50. $4_3 \longleftarrow 3_3$: $2u-o-u-o-u-o$.
51. $4_3 \longrightarrow 2_2$: $2u-o-2u-o-u-o$.
52. $1_1 \longleftarrow 2_2$: $o-2u-o-2u-o-2u-o$.

---

53. $1_1 \longrightarrow 3_3$: $2u-o-2u-2o-u-2o-u$.
54. $1_1 \longleftarrow 3_3$: $o-2u-o-2u-2o-u-2o$.
55. $1_1 \longrightarrow 3_6$: $u-o-3u-4o-u-o$.
56. $1_1 \longleftarrow 3_6$: $u-o-4u-3o-u-o$.

---

57. $2_2 \longrightarrow 2_5$: $o-u-3o-4u-o-u$.
58. $2_2 \longleftarrow 2_5$: $o-u-4o-3u-o-u$.
59. $2_2 \longrightarrow 2_2$: $2o-u-2o-2u-o-2u-o$.
60. $2_2 \longleftarrow 2_2$: $u-2o-u-2o-2u-o-2u$.

---

61. $6_4 \longrightarrow 2_5$: $o-u-o-2u-o-2u-o-2u$.
62. $5_7 \longleftarrow 2_5$: $u-o-u-o-2u-2o-2u-o-u$.
63. $5_7 \longrightarrow 2_5$: $o-u-2o-2u-2o-u-o-u-o$.
64. $6_4 \longleftarrow 2_5$: $2o-u-2o-u-2o-u-o-u$.
65. $6_4 \longrightarrow 4_4$: $4o-u-o-u-o$.

66. $4_3 \longleftarrow 4_4$: $3u - o - u - 2o - u - o$.
67. $4_3 \longrightarrow 1_1$: $2u - o - 2u - o - u - o - u - 2o - u$.
68. $4_3 \longleftarrow 1_1$: $o - 2u - o - u - o - u - 2o - u - 2o$.
69. $4_3 \longrightarrow 4_4$: $u - o - 2u - o - u - 3o$.
70. $6_4 \longleftarrow 4_4$: $u - o - u - o - u - o - 3u$.

---

71. $6_4 \longrightarrow 2_2$: $2o - u - o - u - o - u - o - 2u - o$.
72. $5_7 \longleftarrow 2_2$: $2u - o - u - o - u - o - u - o - u - o - 2u$.
73. $5_7 \longrightarrow 2_2$: $2o - u - o - u - o - u - o - u - o - u - 2o$.
74. $6_4 \longleftarrow 2_2$: $u - 2o - u - o - u - o - u - o - 2u$.
75. $6_4 \longrightarrow 1_1$: $o - u - o - u - 2o - 2u - o - u - o - u$.
76. $2_2 \longleftarrow 1_1$: $o - u - o - u - o - u - 2o - u - o - u - o - u - o - u$.
77. $2_2 \longrightarrow 3_6$: $o - u - 2o - u - o - u - o - u - o - u - o - u - o$.
78. $5_7 \longleftarrow 3_6$: $u - o - u - o - u - 2o - u - o - u - o - u - o - u$.
79. $5_7 \longrightarrow 4_4$: $o - u - o - u - o - u - o - u - o - 2u - o$.
80. $5_7 \longleftarrow 4_4$: $u - o - u - o - u - o - u - o - u - o - u - o - u$.
81. $5_7 \longrightarrow 3_6$: $o - u - o - u - o - u - o - u - o - u - o - u - o - u - o$.
82. $2_2 \longleftarrow 3_6$: $u - o - u - o - u - o - u - o - u - o - u - o - u - o - u$.
83. $2_2 \longrightarrow 1_1$: $o - u - o - u - o - u - o - u - o - u - o - u - o - u - o - u$.
84. $6_4 \longleftarrow 1_1$: $o - u - o - u - o - u - o - u - o - u - o - u - o - u$.

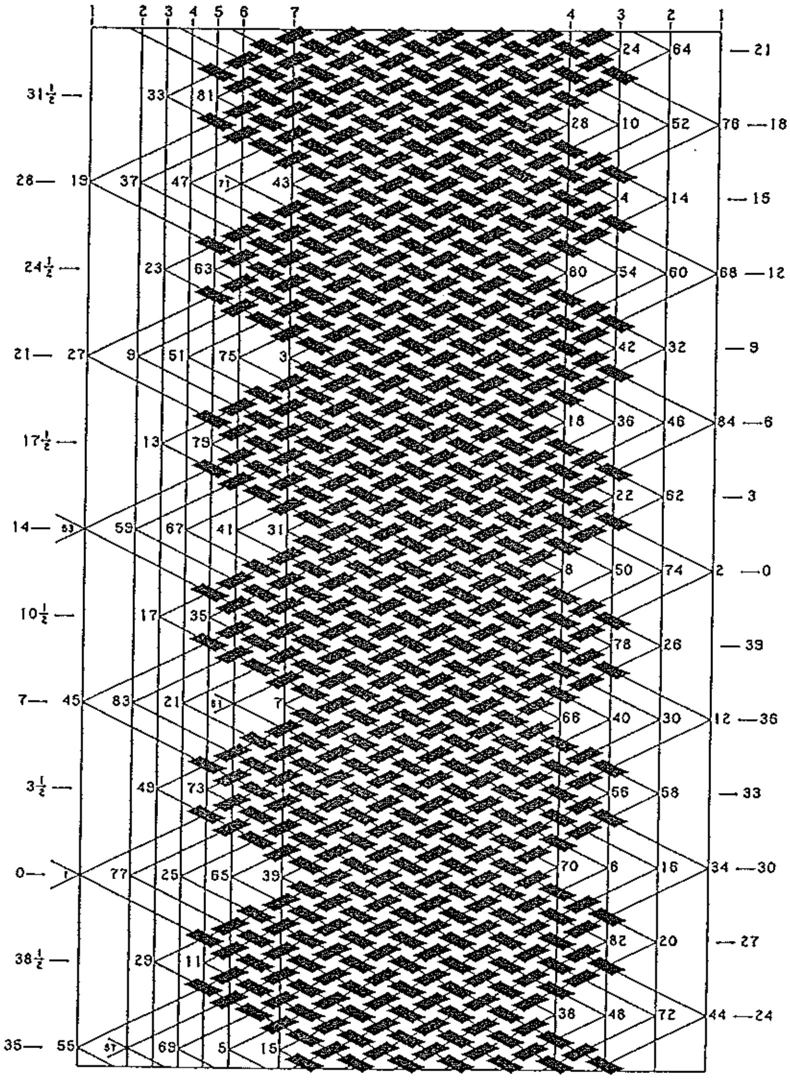


Fig. 642 — The grid-diagram of Example 1.

It is generally more convenient to read the half-cycle braiding algorithms for each component (and in case a component consists of more than one sub-component, for each sub-component) from its two own specific half-cycle tables (one for the odd-numbered half-cycles and one for the even-numbered half-cycles). Although the calculation procedures are simple, there are many to be performed and consequently mistakes are easily made. Hence in practice it will be advantageous to make use of a suitable software program which will enable a computer not only to do these calculations, but also will enable it to produce all the various associated lists, tables, and of course the grid-diagram.

Toggle Knots at the end of a 4-string Round Braid

Fig. 643 depicts two toggle knots; each toggle knot forms an integral part with a 4-string round braid. The end of the 4-string round braid is indicated by A, C, G, I.

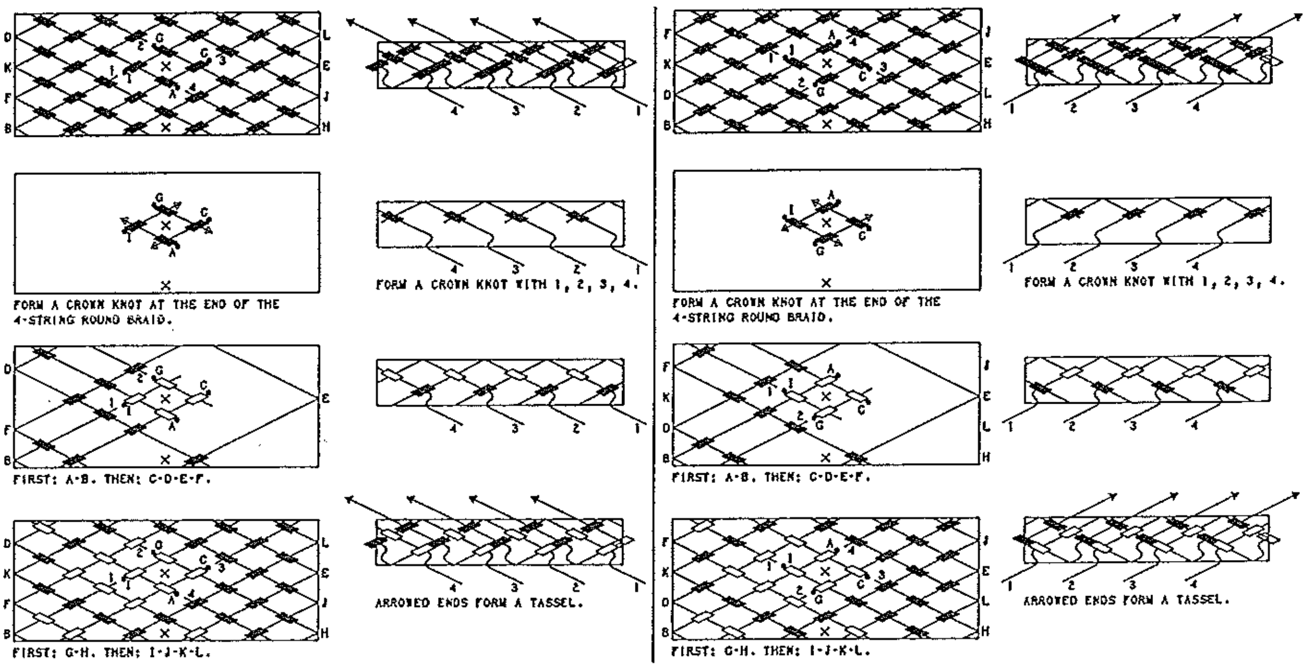


Fig. 643 — Toggle Knots at the end of a 4-string Round Braid.

Braiding procedure:

Form a Crown Knot with the string-ends A, C, G and I by crowning as indicated.

Lay down the rest of A : Free run.
B : Free run. Ends at 1.
Lay down the rest of C : u.
D : $o - u$.
E : $o - u - o$.
F : $o - 2u$. Ends at 2.

Lay down the rest of G : u.
H : o. Ends at 3.
Lay down the rest of I : $2u$.
J : $2o - u - 2o - u - o$.
K : $u - o - u - o - 2u - o - u$.
L : $o - u - o - u$. Ends at 4.

Form a Crown Knot with the string-ends 1, 2, 3 and 4 by crowning as indicated. Then finish off as further depicted in the subsequent diagrams.