THE BRAIDER

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for
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Nested Cylindrical Braids

The Asymmetric Regular Nested Cylindrical Braids:

The Asymmetric Regular Nested Cylindrical Braids have the bight-boundary position specifications $222\cdots$ with $\mathcal{K}_l = A_l$ and $\mathcal{K}_r = A_r$. Their respective left and right sequence sets are:

$$1A_l(A_l-1)(A_l-2)(A_l-3)\cdots 432\,, \text{ and}$$
$$k|k+1|_{A_r}|k+2|_{A_r}|k+3|_{A_r}\cdots A_r123\cdots|k-2|_{A_r}|k-1|_{A_r}\,,$$

where $1 \leq k \leq A_r$ and $A_r \neq A_l$.

Due to these sequence sets there is no need to carry the ranking numbers, and consequently we obtain for a general left and right cycle (see Fig. 602):

$$l_{i+1} = |l_i + 2A_r + x - 2(l_i + r_i)|_{A_l}\,.$$
$$r_{i+1} = |r_i + 2A_l + x - 2(r_i + l_{i+1})|_{A_r}\,.$$

Fig. 602 — A general left and right cycle associated with the Asymmetric Regular Nested Cylindrical Braids.

The further formulae on pg. 417 modify to:

$$B_l^* = \text{number of periods at left bight-edge}\,.$$
$$B_r^* = \text{number of periods at right bight-edge}\,.$$
$$B_{total} = A_l B_l^* = A_r B_r^* = A^{**}B^{**}\,.$$
$$d = \text{g.c.d.}(A_l\,,\, A_r)\,.$$
$$A^{**} = \frac{B_{total}}{B^{**}} = \frac{A_l \cdot A_r}{d}\,.$$
$$B^{**} = \frac{B_{total}\cdot d}{A_l \cdot A_r} = \frac{B_l^* \cdot d}{A_r} = \frac{B_r^* \cdot d}{A_l}\,.$$
$$\alpha = \text{number of bights in first-return string-run}\,.$$
$$P_{component} = \frac{\alpha \cdot x + 2\alpha(A_l + A_r) - 2\sum(l_i + r_i)}{A^{**}}\,.$$
$$P_{total} = \sum P_{component} = A_l + A_r + x - 2\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs}\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{sub-components}\\ \text{in a component}\end{array}\right\} = \text{g.c.d.}(P_{component}\,, B^{**})\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components}\,.$$

Example :

Let $A_l = 5$; $A_r = 3$; $x = 13$; $B = B_{total} = 30$; $l_1 = 1$; $r_1 = k = 1$. Consequently the string-run specification of this Asymmetric Regular Nested Cylindrical Braid is (2222/13/22){15432/123}30 . Hence :

$$B_l^* = \frac{B}{A_l} = \frac{30}{5} = 6 .$$

$$B_r^* = \frac{B}{A_r} = \frac{30}{3} = 10 .$$

$$d = \text{g.c.d.}(A_l , A_r) = \text{g.c.d.}(5 , 3) = 1 .$$

$$A^{**} = \frac{A_l \cdot A_r}{d} = \frac{5 \cdot 3}{1} = 15 .$$

$$B^{**} = \frac{B_{total} \cdot d}{A_l \cdot A_r} = \frac{30 \cdot 1}{5 \cdot 3} = 2 .$$

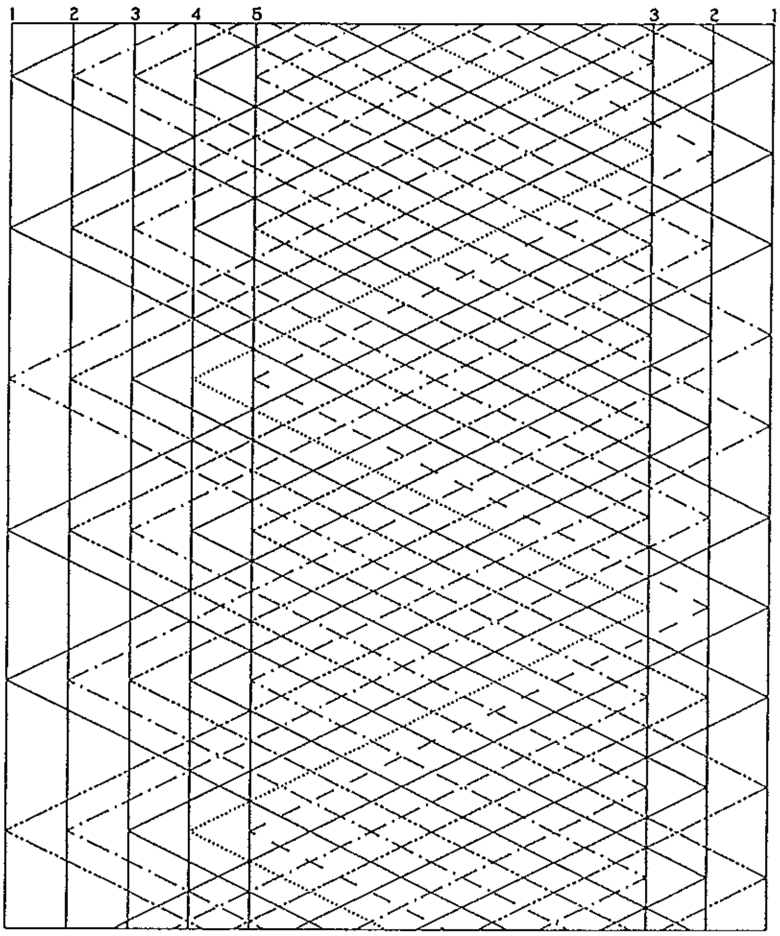


Fig. 603 — The string-run diagram of our Example.

From the given Asymmetric Regular Nested Cylindrical Braid specification we can read the lower-left to upper-right half-cycle types :

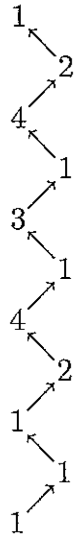
$1 \longrightarrow 1$	$1 \longrightarrow 2$	$1 \longrightarrow 3$
$2 \longrightarrow 2$	$2 \longrightarrow 1$	$2 \longrightarrow 3$
$3 \longrightarrow 1$	$3 \longrightarrow 3$	$3 \longrightarrow 2$
$4 \longrightarrow 3$	$4 \longrightarrow 2$	$4 \longrightarrow 1$
$5 \longrightarrow 2$	$5 \longrightarrow 1$	$5 \longrightarrow 3$

Anyone of these listed types may be taken as the first lower-left to upper-right half-cycle in the 1^{st} first-return string-run, but normally we take the first listed one. Every lower-left to upper-right half-cycle encountered in this 1^{st} first-return string-run gets deleted from the type-list.

Anyone of the remaining types in the type-list may be taken as the first lower-left to upper-right half-cycle in the 2^{nd} first-return string-run, and again every lower-left to upper-right half-cycle encountered in this 2^{nd} first-return string-run gets deleted from the remaining type-list.

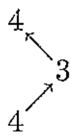
This process is carried on until all the lower-left to upper-right half-cycle types have been deleted from the type-list.

For the first-return string-runs we thus obtain:



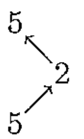
$$\begin{array}{lll}
l_1 = 1 \longrightarrow 1 = r_1 & \rightarrow & l_2 = |1+6+13-2(1+1)|_5 = 1\,. \\
l_2 = 1 \longleftarrow 1 = r_1 & \rightarrow & r_2 = |1+10+13-2(1+1)|_3 = 2\,. \\
l_2 = 1 \longrightarrow 2 = r_2 & \rightarrow & l_3 = |1+6+13-2(1+2)|_5 = 4\,. \\
l_3 = 4 \longleftarrow 2 = r_2 & \rightarrow & r_3 = |2+10+13-2(2+4)|_3 = 1\,. \\
l_3 = 4 \longrightarrow 1 = r_3 & \rightarrow & l_4 = |4+6+13-2(4+1)|_5 = 3\,. \\
l_4 = 3 \longleftarrow 1 = r_3 & \rightarrow & r_4 = |1+10+13-2(1+3)|_3 = 1\,. \\
l_4 = 3 \longrightarrow 1 = r_4 & \rightarrow & l_5 = |3+6+13-2(3+1)|_5 = 4\,. \\
l_5 = 4 \longleftarrow 1 = r_4 & \rightarrow & r_5 = |1+10+13-2(1+4)|_3 = 2\,. \\
l_5 = 4 \longrightarrow 2 = r_5 & \rightarrow & l_6 = |4+6+13-2(4+2)|_5 = 1\,. \\
l_6 = 1 \longleftarrow 2 = r_5 & \rightarrow & r_6 = |2+10+13-2(2+1)|_3 = 1\,.
\end{array}$$

$$P_c = \frac{\alpha x + 2\alpha(A_l + A_r) - 2\sum(l_i + r_i)}{A^{**}} =$$
$$\frac{5\cdot 13 + 2\cdot 5(5+3) - 2\{(1+1+4+3+4)+(1+2+1+1+2)\}}{15} = 7\,.$$
$$\text{g.c.d.}\,(P_c\,,\,B^{**}) = \text{g.c.d.}\,(7\,,\,2) = 1\,.$$



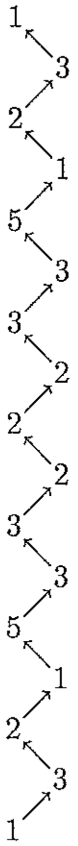
$$\begin{array}{lll}
l_1 = 4 \longrightarrow 3 = r_1 & \rightarrow & l_2 = |4+6+13-2(4+3)|_5 = 4\,. \\
l_2 = 4 \longleftarrow 3 = r_1 & \rightarrow & r_2 = |3+10+13-2(3+4)|_3 = 3\,.
\end{array}$$

$$P_c = \frac{\alpha x + 2\alpha(A_l + A_r) - 2\sum(l_i + r_i)}{A^{**}} = \frac{1\cdot 13 + 2\cdot 1(5+3) - 2\{(4)+(3)\}}{15} = 1\,.$$
$$\text{g.c.d.}\,(P_c\,,\,B^{**}) = \text{g.c.d.}\,(1\,,\,2) = 1\,.$$



$$\begin{array}{lll}
l_1 = 5 \longrightarrow 2 = r_1 & \rightarrow & l_2 = |5+6+13-2(5+2)|_5 = 5\,. \\
l_2 = 5 \longleftarrow 2 = r_1 & \rightarrow & r_2 = |2+10+13-2(2+5)|_3 = 3\,.
\end{array}$$

$$P_c = \frac{\alpha x + 2\alpha(A_l + A_r) - 2\sum(l_i + r_i)}{A^{**}} = \frac{1\cdot 13 + 2\cdot 1(5+3) - 2\{(5)+(2)\}}{15} = 1\,.$$
$$\text{g.c.d.}\,(P_c\,,\,B^{**}) = \text{g.c.d.}\,(1\,,\,2) = 1\,.$$



$$\begin{array}{lcl}
l_1 = 1 \longrightarrow 3 = r_1 & \to & l_2 = |1+6+13-2(1+3)|_5 = 2\,. \\
l_2 = 2 \longleftarrow 3 = r_1 & \to & r_2 = |3+10+13-2(3+2)|_3 = 1\,. \\
l_2 = 2 \longrightarrow 1 = r_2 & \to & l_3 = |2+6+13-2(2+1)|_5 = 5\,. \\
l_3 = 5 \longleftarrow 1 = r_2 & \to & r_3 = |1+10+13-2(1+5)|_3 = 3\,. \\
l_3 = 5 \longrightarrow 3 = r_3 & \to & l_4 = |5+6+13-2(5+3)|_5 = 3\,. \\
l_4 = 3 \longleftarrow 3 = r_3 & \to & r_4 = |3+10+13-2(3+3)|_3 = 2\,. \\
l_4 = 3 \longrightarrow 2 = r_4 & \to & l_5 = |3+6+13-2(3+2)|_5 = 2\,. \\
l_5 = 2 \longleftarrow 2 = r_4 & \to & r_5 = |2+10+13-2(2+2)|_3 = 2\,. \\
l_5 = 2 \longrightarrow 2 = r_5 & \to & l_6 = |2+6+13-2(2+2)|_5 = 3\,. \\
l_6 = 3 \longleftarrow 2 = r_5 & \to & r_6 = |2+10+13-2(2+3)|_3 = 3\,. \\
l_6 = 3 \longrightarrow 3 = r_6 & \to & l_7 = |3+6+13-2(3+3)|_5 = 5\,. \\
l_7 = 5 \longleftarrow 3 = r_6 & \to & r_7 = |3+10+13-2(3+5)|_3 = 1\,. \\
l_7 = 5 \longrightarrow 1 = r_7 & \to & l_8 = |5+6+13-2(5+1)|_5 = 2\,. \\
l_8 = 2 \longleftarrow 1 = r_7 & \to & r_8 = |1+10+13-2(1+2)|_3 = 3\,. \\
l_8 = 2 \longrightarrow 3 = r_8 & \to & l_9 = |2+6+13-2(2+3)|_5 = 1\,. \\
l_9 = 1 \longleftarrow 3 = r_8 & \to & r_9 = |3+10+13-2(3+1)|_3 = 3\,.
\end{array}$$

$$P_c = \frac{\alpha x + 2\alpha(A_l + A_r) - 2\sum(l_i + r_i)}{A^{**}} =$$

$$\frac{8\cdot 13 + 2\cdot 8(5+3) - 2\{(1+2+5+3+2+3+5+2)+(3+1+3+2+2+3+1+3)\}}{15} = 10\,.$$

g.c.d. $(P_c\,, B^{**})$ = g.c.d. $(10\,, 2) = 2$.

Hence:

$$P_{total} = \sum P_{component} = 7 + 1 + 1 + 10 = 19\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 4\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1+1+1+2 = 5\,.$$

Note that we can also calculate P_{total} by means of the formula $P_{total} = A_l + A_r + x - 2$.

The above calculations are the very basic ones for the string-run and are carried out without using the string-run diagram. The string-run diagram can of course readily be drawn up by using the string-run specification, which is in case of our Example: (2222/13/22){15432/123}30. From the general layout, or the actual layout, of the string-run we can also readily determine the first-return string-runs and the essential string-number conditions of the components by using the general law of the common divisor†. In order to show here the use of the general law of the common divisor, we

† Refer to *The Braider*, Issues No. 29 & 30, where this law has been discussed.

have drawn in Fig. 604 the actual layout of the string-run (a page-length would be too short for a general layout in which none of the half-cycles of any first-return string-run cross each other). The string-run of each component has been drawn in a separate diagram and is depicted by solidly drawn half-cycles; in each diagram the half-cycles in bold depict the first-return string-run of the component.

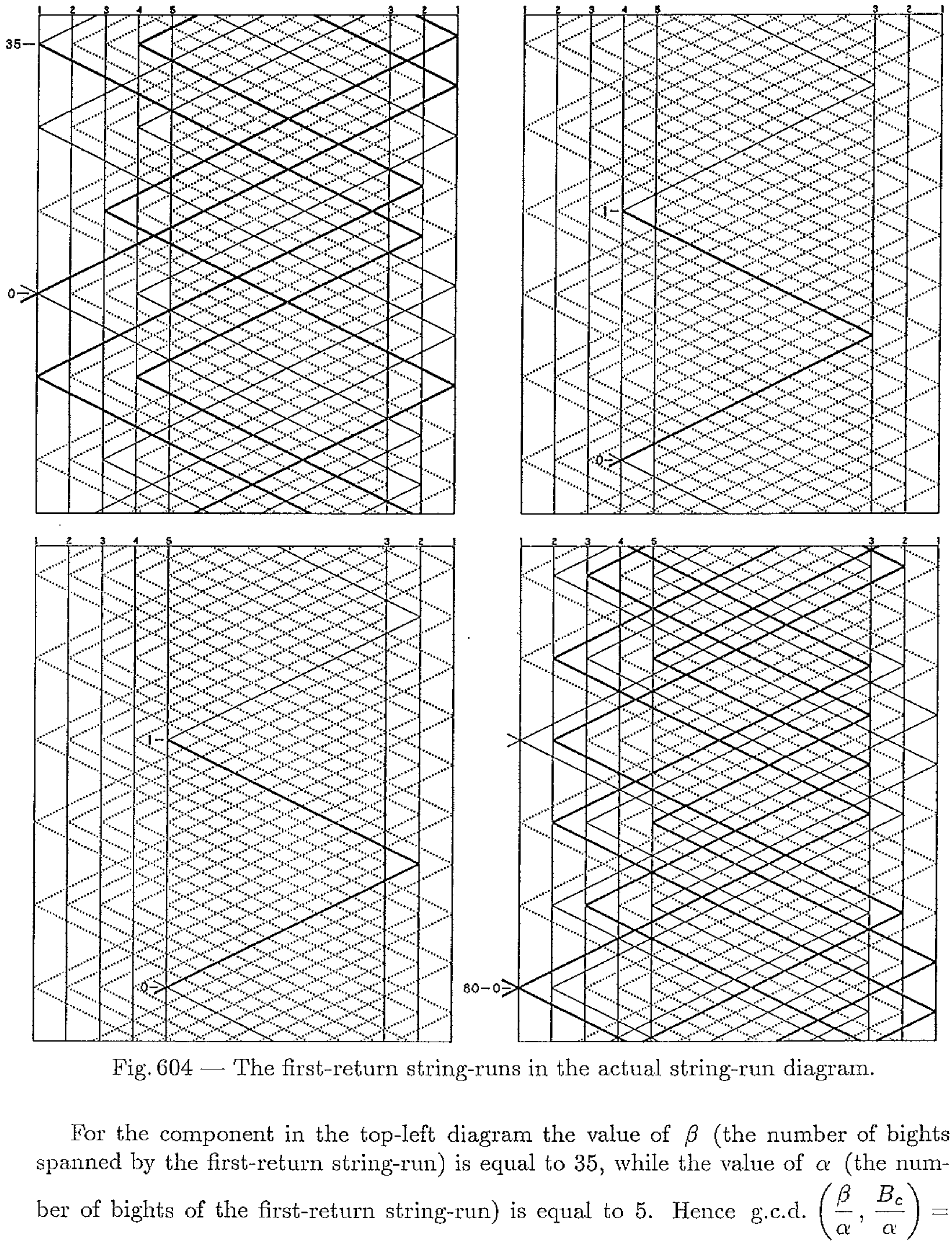


Fig. 604 — The first-return string-runs in the actual string-run diagram.

For the component in the top-left diagram the value of β (the number of bights spanned by the first-return string-run) is equal to 35, while the value of α (the number of bights of the first-return string-run) is equal to 5. Hence g.c.d. $\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) =$

g.c.d. $\left(\frac{35}{5}, \frac{10}{5}\right)$ = g.c.d. $(7, 2) = 1 = \lambda$, where λ is the number of essential strings in the component.

For the component in the top-right diagram the value of β (the number of bights spanned by the first-return string-run) is equal to 1, while the value of α (the number of bights of the first-return string-run) is also equal to 1. Hence g.c.d. $\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right)$ = g.c.d. $\left(\frac{1}{1}, \frac{2}{1}\right)$ = g.c.d. $(1, 2) = 1 = \lambda$, where λ is the number of essential strings in the component.

For the component in the bottom-left diagram the value of β (the number of bights spanned by the first-return string-run) is equal to 1, while the value of α (the number of bights of the first-return string-run) is also equal to 1. Hence g.c.d. $\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right)$ = g.c.d. $\left(\frac{1}{1}, \frac{2}{1}\right)$ = g.c.d. $(1, 2) = 1 = \lambda$, where λ is the number of essential strings in the component.

For the component in the bottom-right diagram the value of β (the number of bights spanned by the first-return string-run) is equal to 80, while the value of α (the number of bights of the first-return string-run) is equal to 8. Hence g.c.d. $\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right)$ = g.c.d. $\left(\frac{80}{8}, \frac{16}{8}\right)$ = g.c.d. $(10, 2) = 2 = \lambda$, where λ is the number of essential strings in the component.

Note that $\frac{\beta}{\alpha} = P_c$, and that $\frac{B_c}{\alpha} = B^{**}$.

Similarly as with the Regular Nested Cylindrical Braids, we attach to the bight-boundary number of a bight of an Asymmetric Regular Nested Cylindrical Braid the **nest-index number** of the nest to which the bight belongs†.

Let the initial Standing End half-cycle, running from lower-left to upper-right, start at the left-hand bight-boundary l_1 and end at the right-hand bight-boundary r_1. The nest, to which its left-hand bight-point belongs, receives the nest-index number $I_L = I_{L_1} = 0$, and the nest, to which its right-hand bight-point belongs, receives the nest-index number $I_R = I_{R_1} = 0$. The left-hand bight-point of this Standing End half-cycle is indicated by $I_L/l_1 = I_{L_1}/l_1 = 0/l_1$, and the right-hand bight-point of this Standing End half-cycle is indicated by $r_1/I_R = r_1/I_{R_1} = r_1/0$.

The next half-cycle (the second half-cycle), running from lower-right to upper-left, starts at the right-hand bight-point $r_1/0$ and ends at the left-hand bight-boundary l_2. The nest, to which its bight-point on the left-hand bight-boundary l_2 belongs, receives the nest-index number $I_L = I_{L_2} = |I_{L_1} + 2(A_l + A_r) + x - (l_1 + l_2 + 2r_1)|_B$. The left-hand bight-point of the second half-cycle is indicated by $I_L/l_2 = I_{L_2}/l_2$.

The next half-cycle (the third half-cycle), running from lower-left to upper-right, starts at the left-hand bight-point I_{L_2}/l_2 and ends at the right-hand bight-boundary r_2. The nest, to which its bight-point on the right-hand bight-boundary r_2 belongs,

† Refer to *The Braider*, Issue No. 26, pg. 592.

receives the nest-index number $I_R = I_{R_2} = |I_{R_1} + 2(A_l + A_r) + x - (r_1 + r_2 + 2l_2)|_B$. The right-hand bight-point of the third half-cycle is indicated by r_2/I_{R_2}.

The next half-cycle (the fourth half-cycle), running from lower-right to upper-left, starts at the right-hand bight-point r_2/I_{R_2} and ends at the left-hand bight-boundary l_3. The nest, to which its bight-point on the left-hand bight-boundary l_3 belongs, receives the nest-index number $I_L = I_{L_3} = |I_{L_2} + 2(A_l + A_r) + x - (l_2 + l_3 + 2r_2)|_B$. The left-hand bight-point of the fourth half-cycle is indicated by $I_L/l_3 = I_{L_3}/l_3$.

The next half-cycle (the fifth half-cycle), running from lower-left to upper-right, starts at the left-hand bight-point I_{L_3}/l_3 and ends at the right-hand bight-boundary r_3. The nest, to which its bight-point on the right-hand bight-boundary r_3 belongs, receives the nest-index number $I_R = I_{R_3} = |I_{R_2} + 2(A_l + A_r) + x - (r_2 + r_3 + 2l_3)|_B$. The right-hand bight-point of the fifth half-cycle is indicated by r_3/I_{R_3}.

And so on.

In general:

The $(2n)^{th}$ half-cycle, where $n = 1, 2, 3, \cdots$, running from lower-right to upper-left, starts at right-hand bight-point r_n/I_{R_n} and ends at the left-hand bight-boundary l_{n+1}. The nest, belonging to this bight-point on the left-hand bight-boundary l_{n+1}, receives the nest-index number $I_L = I_{L_{n+1}} = |I_{L_n} + 2(A_l + A_r) + x - (l_n + l_{n+1} + 2r_n)|_B$. The left-hand bight-point of the $(2n)^{th}$ half-cycle, where $n = 1, 2, 3, \cdots$, is indicated by $I_L/l_{n+1} = I_{L_{n+1}}/l_{n+1}$.

The $(2n+1)^{th}$ half-cycle), running from lower-left to upper-right, starts at the left-hand bight-point $I_{L_{n+1}}/l_{n+1}$ and ends at the right-hand bight-boundary r_{n+1}. The nest, to which its bight-point on the right-hand bight-boundary r_{n+1} belongs, receives the nest-index number $I_R = I_{R_{n+1}} = |I_{R_n} + 2(A_l + A_r) + x - (r_n + r_{n+1} + 2l_{n+1})|_B$. The right-hand bight-point of the $(2n+1)^{th}$ half-cycle is indicated by $r_{n+1}/I_{R_{n+1}}$.

For our Example, the Standing Ends of the five essential strings are indicated in Fig. 605. The 4^{th} component consists of two sub-components with respectively the essential strings 4 & 5.

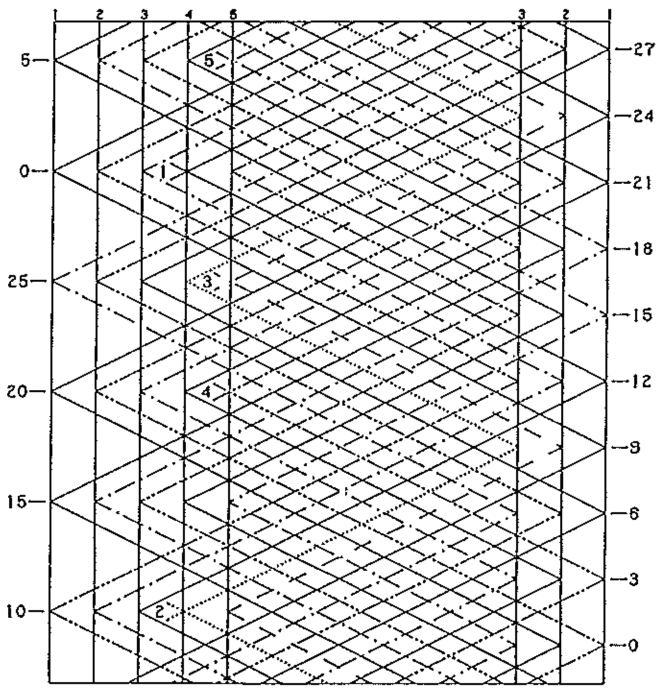


Fig. 605 — The string-runs of the five essential strings.

Thus the first-return string-runs of the four components are:

15/4 ↖ 1/27 ↗ 25/3 ↖ 1/6 ↗ 5/4 ↖ 2/18 ↗ 15/1 ↖ 1/24 ↗ 20/1 ↖ 2/0 ↗ 0/4

$$
\begin{aligned}
I_L = I_1 &= 0\,.\\
I_R = I_{1'} &= 0\,.\\
I_L = I_2 &= |I_1 + 2(A_l + A_r) + x - (l_1 + l_2 + 2r_1)|_B = \\
&\quad |0 + 2 \times 8 + 13 - (4 + 1 + 2 \times 2)|_{30} = 20\,.\\
I_R = I_{2'} &= |I_{1'} + 2(A_l + A_r) + x - (r_1 + r_2 + 2l_2)|_B = \\
&\quad |0 + 2 \times 8 + 13 - (2 + 1 + 2 \times 1)|_{30} = 24\,.\\
I_L = I_3 &= |I_2 + 2(A_l + A_r) + x - (l_2 + l_3 + 2r_2)|_B = \\
&\quad |20 + 2 \times 8 + 13 - (1 + 1 + 2 \times 1)|_{30} = 15\,.\\
I_R = I_{3'} &= |I_{2'} + 2(A_l + A_r) + x - (r_2 + r_3 + 2l_3)|_B = \\
&\quad |24 + 2 \times 8 + 13 - (1 + 2 + 2 \times 1)|_{30} = 18\,.\\
I_L = I_4 &= |I_3 + 2(A_l + A_r) + x - (l_3 + l_4 + 2r_3)|_B = \\
&\quad |15 + 2 \times 8 + 13 - (1 + 4 + 2 \times 2)|_{30} = 5\,.\\
I_R = I_{4'} &= |I_{3'} + 2(A_l + A_r) + x - (r_3 + r_4 + 2l_4)|_B = \\
&\quad |18 + 2 \times 8 + 13 - (2 + 1 + 2 \times 4)|_{30} = 6\,.\\
I_L = I_5 &= |I_4 + 2(A_l + A_r) + x - (l_4 + l_5 + 2r_4)|_B = \\
&\quad |5 + 2 \times 8 + 13 - (4 + 3 + 2 \times 1)|_{30} = 25\,.\\
I_R = I_{5'} &= |I_{4'} + 2(A_l + A_r) + x - (r_4 + r_5 + 2l_5)|_B = \\
&\quad |6 + 2 \times 8 + 13 - (1 + 1 + 2 \times 3)|_{30} = 27\,.\\
I_L = I_6 &= |I_5 + 2(A_l + A_r) + x - (l_5 + l_6 + 2r_5)|_B = \\
&\quad |25 + 2 \times 8 + 13 - (3 + 4 + 2 \times 1)|_{30} = 15\,.
\end{aligned}
$$

20/5 ↖ 1/3 ↗ 0/2 ↖ 3/12 ↗ 10/1 ↖ 3/21 ↗ 20/2 ↖ 1/0 ↗ 0/5 ↖ 3/15 ↗ 15/3 ↖ 2/27 ↗ 25/2 ↖ 2/6 ↗ 5/3 ↖ 3/18 ↗ 20/5

$$
\begin{aligned}
I_L = I_1''' &= 20\,.\\
I_R = I_{1'}''' &= |I_1''' + (l_1 + r_1) - (l_1''' + r_1''')|_B = \\
&\quad |20 + (4 + 2) - (5 + 3)|_B = |20 + 6 - 8|_{30} = 18\,.\\
I_L = I_2''' &= |I_1''' + 2(A_l + A_r) + x - (l_1''' + l_2''' + 2r_1''')|_B = \\
&\quad |20 + 2 \times 8 + 13 - (5 + 3 + 2 \times 3)|_{30} = 5\,.\\
I_R = I_{2'}''' &= |I_1''' + 2(A_l + A_r) + x - (r_1''' + r_2''' + 2l_2''')|_B = \\
&\quad |18 + 2 \times 8 + 13 - (3 + 2 + 2 \times 3)|_{30} = 6\,.\\
I_L = I_3''' &= |I_2''' + 2(A_l + A_r) + x - (l_2''' + l_3''' + 2r_2''')|_B = \\
&\quad |5 + 2 \times 8 + 13 - (3 + 2 + 2 \times 2)|_{30} = 25\,.\\
I_R = I_{3'}''' &= |I_2''' + 2(A_l + A_r) + x - (r_2''' + r_3''' + 2l_3''')|_B = \\
&\quad |6 + 2 \times 8 + 13 - (2 + 2 + 2 \times 2)|_{30} = 27\,.\\
I_L = I_4''' &= |I_3''' + 2(A_l + A_r) + x - (l_3''' + l_4''' + 2r_3''')|_B = \\
&\quad |25 + 2 \times 8 + 13 - (2 + 3 + 2 \times 2)|_{30} = 15\,.\\
I_R = I_{4'}''' &= |I_3''' + 2(A_l + A_r) + x - (r_3''' + r_4''' + 2l_4''')|_B = \\
&\quad |27 + 2 \times 8 + 13 - (2 + 3 + 2 \times 3)|_{30} = 15\,.\\
I_L = I_5''' &= |I_4''' + 2(A_l + A_r) + x - (l_4''' + l_5''' + 2r_4''')|_B = \\
&\quad |15 + 2 \times 8 + 13 - (3 + 5 + 2 \times 3)|_{30} = 0\,.\\
I_R = I_{5'}''' &= |I_4''' + 2(A_l + A_r) + x - (r_4''' + r_5''' + 2l_5''')|_B = \\
&\quad |15 + 2 \times 8 + 13 - (3 + 1 + 2 \times 5)|_{30} = 0\,.\\
I_L = I_6''' &= |I_5''' + 2(A_l + A_r) + x - (l_5''' + l_6''' + 2r_5''')|_B = \\
&\quad |0 + 2 \times 8 + 13 - (5 + 2 + 2 \times 1)|_{30} = 20\,.\\
I_R = I_{6'}''' &= |I_5''' + 2(A_l + A_r) + x - (r_5''' + r_6''' + 2l_6''')|_B = \\
&\quad |0 + 2 \times 8 + 13 - (1 + 3 + 2 \times 2)|_{30} = 21\,.\\
I_L = I_7''' &= |I_6''' + 2(A_l + A_r) + x - (l_6''' + l_7''' + 2r_6''')|_B = \\
&\quad |20 + 2 \times 8 + 13 - (2 + 1 + 2 \times 3)|_{30} = 10\,.\\
I_R = I_{7'}''' &= |I_6''' + 2(A_l + A_r) + x - (r_6''' + r_7''' + 2l_7''')|_B =
\end{aligned}
$$

$$
\begin{aligned}
& |21 + 2 \times 8 + 13 - (3 + 3 + 2 \times 1)|_{30} = 12\,. \\
I_L = I_8''' = & |I_7''' + 2(A_l + A_r) + x - (l_7''' + l_8''' + 2r_7''')|_B = \\
& |10 + 2 \times 8 + 13 - (1 + 2 + 2 \times 3)|_{30} = 0\,. \\
I_R = I_{8'}''' = & |I_7''' + 2(A_l + A_r) + x - (r_7''' + r_8''' + 2l_8''')|_B = \\
& |12 + 2 \times 8 + 13 - (3 + 1 + 2 \times 2)|_{30} = 3\,. \\
I_L = I_9''' = & |I_8''' + 2(A_l + A_r) + x - (l_8''' + l_9''' + 2r_8''')|_B = \\
& |0 + 2 \times 8 + 13 - (2 + 5 + 2 \times 1)|_{30} = 20\,.
\end{aligned}
$$

25/4
↖
3/9
↗
10/4

$$
\begin{aligned}
I_L = I_1' = & 10\,. \\
I_R = I_{1'}' = & |I_1' + (l_1 + r_1) - (l_1' + r_1')|_B = \\
& |10 + (4 + 2) - (4 + 3) - A|_B = |10 + 6 - 7|_{30} = 9\,. \\
I_L = I_2' = & |I_1' + 2(A_l + A_r) + x - (l_1' + l_2' + 2r_1')|_B = \\
& |10 + 2 \times 8 + 13 - (4 + 4 + 2 \times 3)|_{30} = 25\,.
\end{aligned}
$$

10/5
↖
2/24
↗
25/5

$$
\begin{aligned}
I_L = I_1'' = & 25\,. \\
I_R = I_{1'}'' = & |I_1'' + (l_1 + r_1) - (l_1'' + r_1'')|_B = \\
& |25 + (4 + 2) - (5 + 2)|_B = |25 + 6 - 7|_{30} = 24\,. \\
I_L = I_2'' = & |I_1'' + 2(A_l + A_r) + x - (l_1'' + l_2'' + 2r_1'')|_B = \\
& |25 + 2 \times 8 + 13 - (5 + 5 + 2 \times 2)|_{30} = 10\,.
\end{aligned}
$$

Let's assume that we braid the essential strings in their sequential numbered order (see Fig. 605). The above calculated data can then be assembled in the half-cycle pattern depicted in Fig. 606. The number of crossings on a half-cycle from lower-left to upper-right are calculated with the formula $(A_l + A_r) + x - 1 - (l_i + r_i)$ for such a half-cycle running from left-hand bight-boundary l_i to right-hand bight-boundary r_i, and the number of crossings on a half-cycle from lower-right to upper-left are calculated with the formula $(A_l + A_r) + x - 1 - (r_i + l_{i+1})$ for such a half-cycle running from right-hand bight-boundary r_i to left-hand bight-boundary l_{i+1}. In Fig. 606 the number of crossings on a half-cycle are indicated below the half-cycle number.

1	2	3	4	5	
57/16	33/16	9/16	23/13	25/13	—25
3/18	39/15	47/15	17/15	29/12	—20
5/17	59/17	35/14	11/14	53/14	—15
41/16	49/16	19/16	21/13	27/13	—10
13/18	55/15	31/15	7/15	45/12	—5
15/17	43/17	51/14	1/14	37/14	—0

	3	2	1
27—	58/15	34/15	10/15
24—	24/13	26/13	4/18
21—	40/16	48/16	18/16
18—	30/14	6/14	60/14
15—	36/12	12/17	54/17
12—	42/15	50/15	20/15
9—	22/13	28/13	14/18
6—	56/16	32/16	8/16
3—	46/14	16/14	44/14
0—	52/12	2/17	38/17

Fig. 606 — The half-cycle pattern associated with our Example.

Let the coding, superimposed on the string-run, be as depicted in Fig. 607. Then from the half-cycle pattern in Fig. 606 we assemble the half-cycle tables in Figs. 608 & 609 for respectively the lower-left to upper-right and the lower-right to upper-left half-cycles.

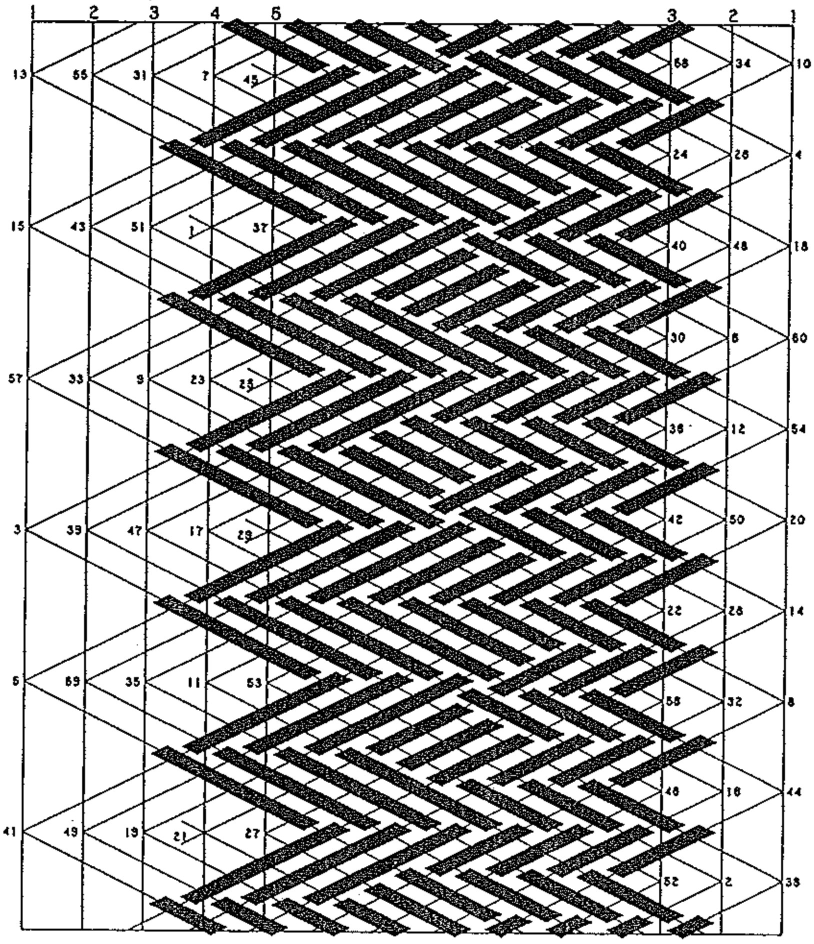


Fig. 607 — The string-run with the superimposed coding.

15	43	51	1*	37	13	55	31	7	45***	41	49	19	21**	27	5	59	35						
5	59	35	11	53	3	39	47	17	29**	57	33	9	23	25*	15	43	51						
57	33	9	23	25	15	43	51	1*	37	13	55	31	7	45**	41	49	19						
41	49	19	21	27	5	59	35	11	53	3	39	47	17	29*	57	33	9						
3	39	47	17	29**	57	33	9	23	25	15	43	51	1*	37	13	55	31						
13	55	31	7	45*	41	49	19	21	27	5	59	35	11	53	3	39	47						
u	o	o	o	o	o	u	u	u	u	u	u	u	o	o	o	u		15	5*				
u	u	o	o	o	o	o	u	o	o	o	u	u	u	o	o	o		43	59				
u	u	u	o	o	o	o	o	u	o	o	o	u	u					51*	35**				
u	u	u	u	o	o	o	o	u	u	o	o	o	u					1	11*				
u	u	u	u	u	o	o	o	u	u	u	o	o	o					37	53				
u	o	o	o	o	o	u	u	u	o	o	o	u	u	u	o	o	o			13	3*		
u	u	o	o	o	o	o	u	u	u	o	o	o	u	u						55	39		
u	u	u	o	o	o	o	o	u	u	u	o	o	o	u						31*	47**		
u	u	u	u	u	u	o	o	o	u	u	u	o	o	o						7	17*		
u	u	u	u	u	u	u	o	o	o	u	u									45	29		
u	o	o	o	o	o	u	u	u	u	u	o	o	o	u	u							41**	57***
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	u							49	33
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o							19	9*
u	u	u	u	o	o	o	u	o	o	o	u	u										21	23**
u	u	u	u	o	o	o	u	u	o	o	o	u										27*	25

Fig. 608 — The half-cycle table for the lower-left to upper-right half-cycles.

38	2	52	44	16	46	8	32	56	14	28	22	20	50	42	54	12	36										
54	12	36	60	6	30	18	48	40	4	26	24	10	34	58	38	2	52										
10	34	58	38	2	52	44	16	46	8	32	56	14	28	22	20	50	42										
20	50	42	54	12	36	60	6	30	18	48	40	4	26	24	10	34	58										
4	26	24	10	34	58	38	2	52	44	16	46	8	32	56	14	28	22										
14	28	22	20	50	42	54	12	36	60	6	30	18	48	40	4	26	24										
18	48	40	4	26	24	10	34	58	38	2	52	44	16	46	8	32	56										
8	32	56	14	28	22	20	50	42	54	12	36	60	6	30	18	48	40										
60	6	30	18	48	40	4	26	24	10	34	58	38	2	52	44	16	46										
44	16	46	8	32	56	14	28	22	20	50	42	54	12	36	60	6	30										
u	o	o	o	u	u	u	u	u	u	u	o	o	o	o	o	u		38	54								
u	u	o	o	o	u	u	u	u	u	u	u	o	o	o	o	o		2	12								
u	u	u	o	o	o	u	u	u	u	u	u							52	36								
u	o	o	o	u	u	o	o	o	o	u	u	u	u							44	60						
u	u	o	o	o	u	o	o	o	o	o	u	u	u							16	6						
u	u	u	o	o	o	u	o	o	o	o	o	u	u							46	30						
u	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u							8	18				
u	u	o	o	o	u	u	u	u	u	o	o	o	o	o	u							32	48				
u	u	u	o	o	o	u	u	u	u	u	o	o	o	o	o							56	40				
u	o	o	o	u	u	u	o	o	o	u	u	u	o	o	o	o	o							14	4		
u	u	o	o	o	u	u	o	o	o	u	u	u												28	26		
u	u	u	o	o	o	u	o	o	o	u	u	u												22	24		
u	o	o	o	u	u	u	o	o	o	o	o	u	u	u												20	10
u	u	o	o	o	u	u	u	o	o	o	o	o	u	u												50	34
u	u	u	o	o	o	u	u	u	o	o	o	o	o	u												42	58

Fig. 609 — The half-cycle table for the lower-right to upper-left half-cycles.

From the half-cycle tables we read then the half-cycle braiding algorithms for the Asymmetric Regular Nested Cylindrical Braid. Note that in the table of the odd-numbered half-cycles some of these half-cycles have one or more stars attached to them. When a half-cycle in the rightmost table-section has one or more star(s) then in its associated row of half-cycles (in the uppermost table-section) only those half-cycle numbers, equal to or smaller than the half-cycle number in the rightmost table-section, with an identical number of stars are neglected†.

Thus from the half-cycle tables in Figs. 608 & 609, associated with the Asymmetric Regular Nested Cylindrical Braid depicted in Fig. 607, we read the following half-cycle braiding algorithms:

1. $4 \longrightarrow 2$: Free run.
2. $1 \longleftarrow 2$: Free run.
3. $1 \longrightarrow 1$: Free run.
4. $1 \longleftarrow 1$: *u*.
5. $1 \longrightarrow 2$: *u*.

† The half-cycle numbers in the uppermost table-section are for crossing purposes associated with their immediately preceding even-numbered half-cycle, hence the starred half-cycle numbers in the uppermost table-section are for crossing purposes associated with the **final** (even-numbered) half-cycle of an essential string.

6. $4 \longleftarrow 2$: $2o$.
7. $4 \longrightarrow 1$: $2u$.
8. $3 \longleftarrow 1$: $o - u$.
9. $3 \longrightarrow 1$: $u - o$.
10. $4 \longleftarrow 1$: $o - u$.
11. $4 \longrightarrow 2$: $u - o$.
12. $1 \longleftarrow 2$: $3u - o$.
13. $1 \longrightarrow 1$: $o - u - 2o$.
14. $1 \longleftarrow 1$: $u - o - u - o$.
15. $1 \longrightarrow 2$: $u - o - u - 2o$.
16. $4 \longleftarrow 2$: $u - 2o - u$.
17. $4 \longrightarrow 1$: $3u - o$.
18. $3 \longleftarrow 1$: $u - o - u - 2o - u$.
19. $3 \longrightarrow 1$: $u - 2o - 2u - o$.
20. $4 \longleftarrow 1$: $o - 2u - o - u$.

---

21. $4 \longrightarrow 3$: $2u - 2o - u$.
22. $4 \longleftarrow 3$: $u - 3o - u$.
23. $4 \longrightarrow 3$: $2u - 2o - u$.
24. $4 \longleftarrow 3$: $u - 3o - u$.

---

25. $5 \longrightarrow 2$: $2u - o - 2u$.
26. $5 \longleftarrow 2$: $u - 3o - u$.
27. $5 \longrightarrow 2$: $2u - o - 2u$.
28. $5 \longleftarrow 2$: $u - 3o - u$.

---

29. $5 \longrightarrow 3$: $4u - o - u$.
30. $3 \longleftarrow 3$: $u - 3o - u - o - u$.
31. $3 \longrightarrow 2$: $u - 3o - 2u - 3o - u$.
32. $2 \longleftarrow 2$: $2u - 2o - u - 3o - u$.
33. $2 \longrightarrow 2$: $u - 2o - 2u - 3o - u$.
34. $3 \longleftarrow 2$: $u - o - 2u - 4o$.
35. $3 \longrightarrow 3$: $u - 3o - u - 2o - u$.
36. $5 \longleftarrow 3$: $2u - o - 5u$.
37. $5 \longrightarrow 1$: $3u - o - 3u - 2o$.
38. $2 \longleftarrow 1$: $2o - 5u - 2o - u$.
39. $2 \longrightarrow 3$: $u - 4o - 2u - 2o - u$.
40. $1 \longleftarrow 3$: $3u - 2o - 3u - 3o$.
41. $1 \longrightarrow 3$: $u - 3o - 2u - 3o - 2u$.
42. $2 \longleftarrow 3$: $2u - 2o - 2u - 5o$.
43. $2 \longrightarrow 1$: $u - 3o - u - 3o - 2u - 2o$.
44. $5 \longleftarrow 1$: $u - 2o - 2u - 2o - 3u$.

---

45. $5 \longrightarrow 3$: $5u - 2o - 2u$.
46. $3 \longleftarrow 3$: $2u - 3o - u - 3o - u$.
47. $3 \longrightarrow 2$: $2u - 4o - 3u - 2o$.
48. $2 \longleftarrow 2$: $2u - 3o - 3u - 4o - u$.
49. $2 \longrightarrow 2$: $u - 4o - 4u - 3o - u$.

50. $3 \longleftarrow 2$: $u - 2o - 3u - 5o - u$.
51. $3 \longrightarrow 3$: $2u - 4o - u - 2o - 2u$.
52. $5 \longleftarrow 3$: $3u - 2o - 6u$.
53. $5 \longrightarrow 1$: $5u - 2o - 3u - 3o$.
54. $2 \longleftarrow 1$: $3o - 7u - 4o - u$.
55. $2 \longrightarrow 3$: $2u - 4o - 3u - 3o - 2u$.
56. $1 \longleftarrow 3$: $3u - 3o - 4u - 5o$.
57. $1 \longrightarrow 3$: $u - 5o - 4u - 3o - 2u$.
58. $2 \longleftarrow 3$: $3u - 3o - 3u - 5o - u$.
59. $2 \longrightarrow 1$: $2u - 5o - u - 3o - 3u - 3o$.
60. $5 \longleftarrow 1$: $u - 3o - 2u - 4o - 4u$.

Some braiders might find it easier to work with separate half-cycle tables for each essential string, hence similar to the procedures discussed in *The Braider*, Issue No. 31. This certainly makes the reading of the half-cycle braiding algorithms from these tables quicker and easier.

It will have been noticed that the calculation procedures for obtaining the half-cycle braiding algorithms for the Asymmetric Regular Nested Cylindrical Braids are similar to the calculation procedures of the half-cycle braiding algorithms for the Perfect and Semi-Perfect Regular Nested Cylindrical Braids (see *The Braider*, Issue No. 28 and Issue No. 29). Although these calculation procedures are very simple, they are somewhat lengthy and hence here is where suitable computer software can play an important and valuable role. Some braiders may say that the half-cycle braiding algorithms can directly be obtained (with or without the assistance of a computer) from the grid-diagram. This is certainly true, but it is easy to make mistakes without the assistance of a computer and much more cumbersome to check whether or not the obtained half-cycle braiding algorithms are correct.

Designing Interbraided Knots

The Spanish Ring Knot is one of the most commonly encountered knots in a braiding project. Generally their braid is made of a single string and hence consists of a single colour, however two colour work where V's of one colour are regularly interspersed between the V's of the other colour will in many cases enliven the braided project.

A Ring Knot can have two coding arrangements: one in which the V's point up, and one in which the V's point down. These are each others complementary coding forms (see Fig. 610).

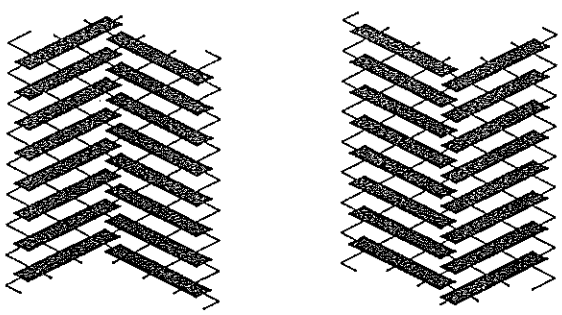


Fig. 610 — The complementary coding forms.

They are in reallity identical since a rotation through 180° of a knot having one of these coding forms produces the knot with its complementary coding form. However, since we normally start braiding from left to right and in the upwards direction, we will have to look at both the above mentioned coding forms. Each of the above mentioned coding forms provides for two ways of incorporating V's with a contrasting colour (see for example Fig. 611 where the sequence of the two sets of V's is 1 : 1).

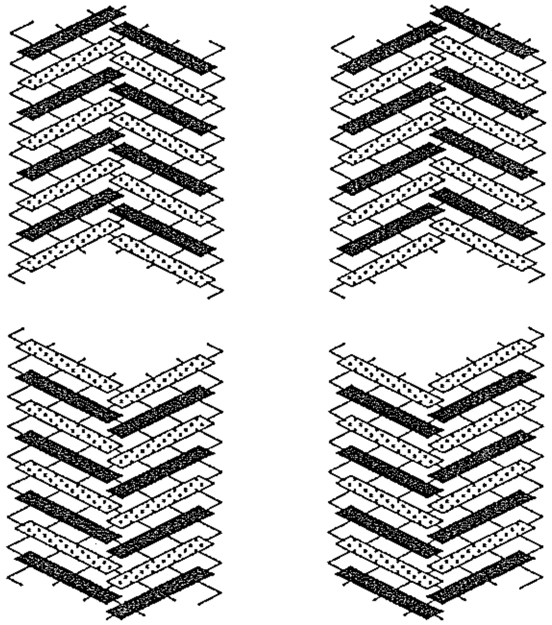


Fig. 611 — Incorporating coloured V's with the sequence 1 : 1.

It will be readily apparent from the diagrams in Fig. 611 that the string-run cannot exactly be as depicted since the coding-patterns would require the string(s) to change in colour at certain positions.

In order to obtain the pattern of V's depicted in the uppermost left-hand diagram of Fig. 611 we require the string-run and the coding to be as shown in the rightmost diagram of Fig. 612. Observe how this string-run and coding is obtained from the string-run and coding shown in the second diagram from the left, and hence how its string-run is associated with the string-run depicted in the leftmost diagram. From this leftmost string-run in Fig. 612 we can thus determine with the aid of the generalised law of the greatest common divisor† $\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right)$ the properties of the actual string-run belonging to the pattern of V's in the uppermost left-hand diagram of Fig. 611.

This leftmost string-run in Fig. 612 consists of two components; their first-return string-runs are drawn in thick heavy lines. In these first-return string-runs the bights are numbered on the right, and the bights which these first-return string-runs span in their components are numbered on the left. Hence for the lowermost first-return string-run we obtain $\alpha = 1$ and $\beta = 4$, while for the uppermost first-return string-run we obtain $\alpha = 1$ and $\beta = 3$. Let the number of bights B_c of the component associated with the lowermost first-return string-run be equal to B, then the number of bights B_c of the component associated with the uppermost first-return string-run is also equal to B. Thus for the component with the lowermost first-return string-run the number of sub-components $\lambda = \text{g.c.d.}(4, B)$, while for the component with the uppermost first-

† See *The Braider*, Issue No. 29, pg. 682, and Issue No. 30, pp. 683–690.

return string-run the number of sub-components λ = g.c.d.(3 , B) . Hence for both λ-values to be equal to 1, the value of B must be coprime with 4 and 3.

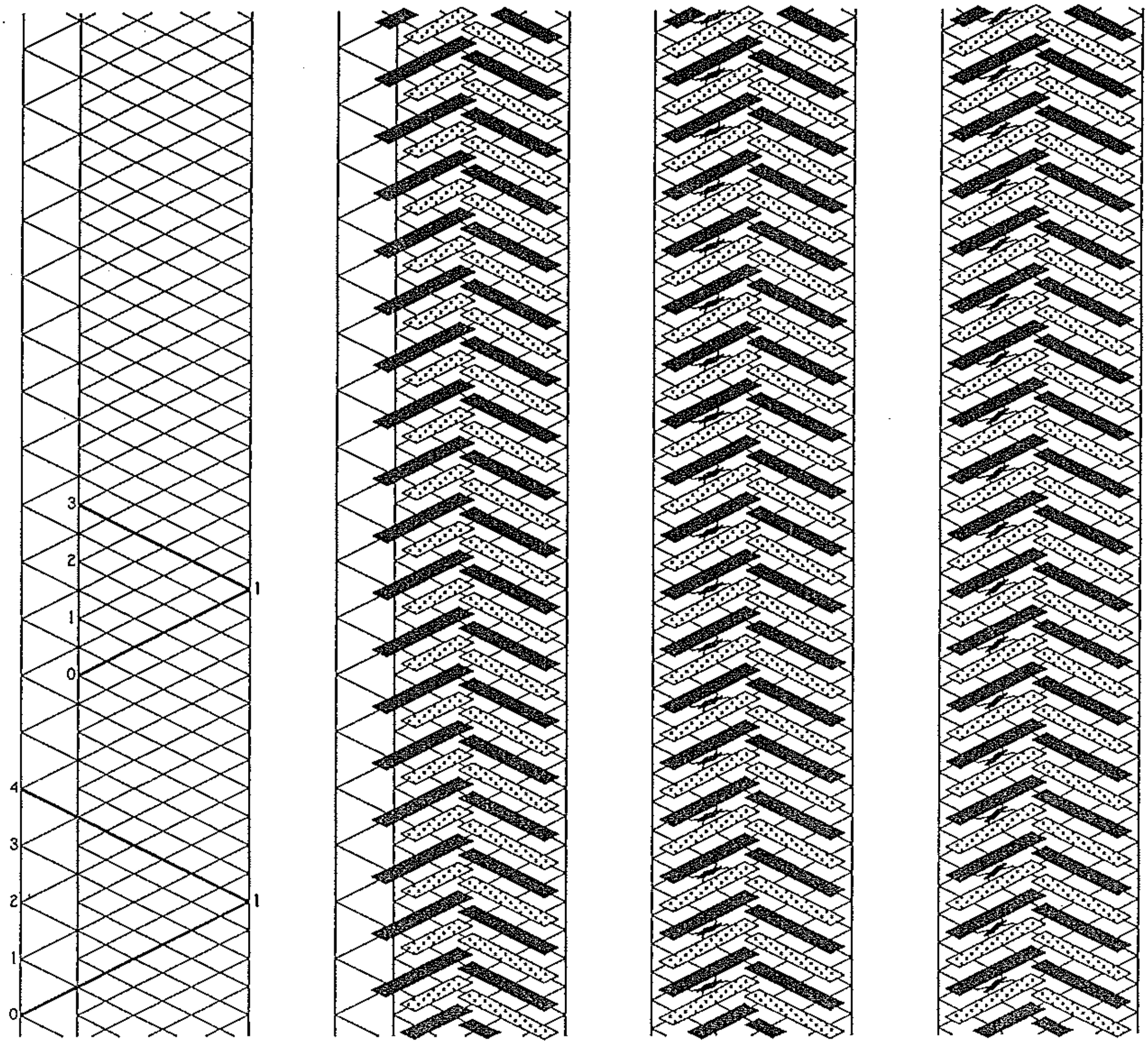


Fig. 612 — The development of the actual string-run and coding for the uppermost left-hand V pattern in Fig. 611.

If, for example, we take $B = 7$, hence the knot has 14 bights in total, we obtain the construction details shown in Fig. 613. The upper two left grid-diagrams depict respectively the foundation knot $p/b = 4/7$ and the interbraid $p'/b' = 3/7$. The foundation knot with its ultimate string-run is depicted by the uppermost right-hand grid-diagram. In the lowermost row of diagrams the three diagrams on the left depict the development of the ultimate knot depicted by the lower rightmost grid-diagram.

First we braid the foundation knot; its half-cycle braiding algorithms, read from its associated algorithm diagram, are as follows:

1.		:	Free run.
2.	$i = 0$	:	Free run.
3.	$i = 0$	:	Free run.
4.	$i = 1$	:	u .
5.	$i = 1$	:	u .

6. $i = 2$: u .
7. $i = 2$: u .
8. $i = 3$: $2u$.
9. $i = 3$: $o - u$.
10. $i = 4$: $2u$.
11. $i = 4$: $o - u$.
12. $i = 5$: $o - 2u$.
13. $i = 5$: $2o - u$.
14. $i = 6$: $o - 2u$.

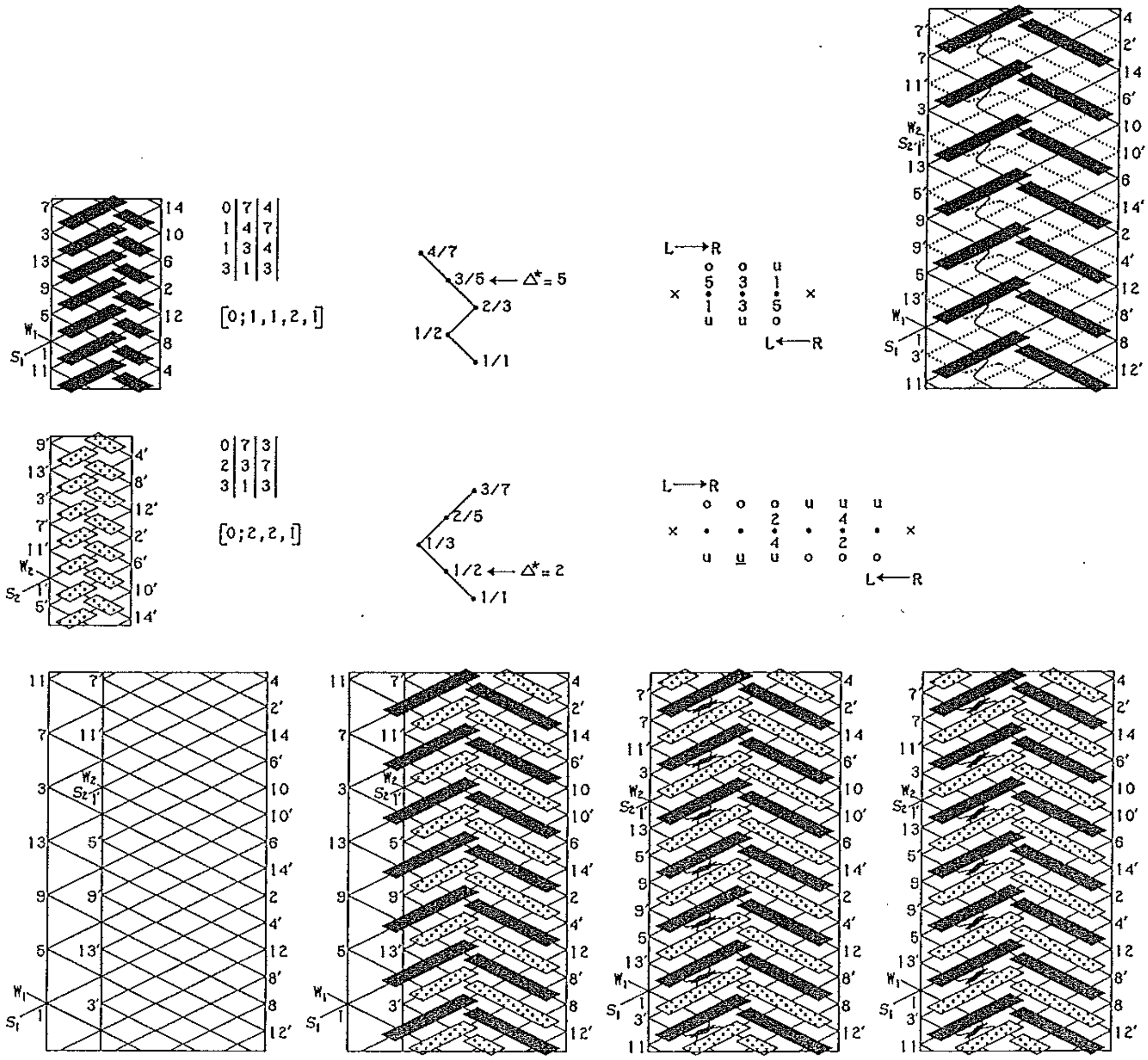


Fig. 613 — The construction details for the interbraid $p'/b' = 3/7$ with $p/b = 4/7$.

Next we interbraid the knot $p'/b' = 3/7$; its half-cycle braiding algorithms, read from its associated algorithm diagram, are as follows:

$1'$. : $2o - 2u$.
$2'$. $i = 0$: $2o - \underline{u} - u$.
$3'$. $i = 0$: $2o - 2u$.
$4'$. $i = 1$: $2o - \underline{u} - u$.

5′.	$i = 1$	:	$2o - 2u$.
6′.	$i = 2$	:	$3o - \underline{u} - u$.
7′.	$i = 2$	:	$3o - 2u$.
8′.	$i = 3$	:	$3o - \underline{u} - u$.
9′.	$i = 3$	:	$3o - 2u$.
10′.	$i = 4$	:	$3o - u - \underline{u} - u$.
11′.	$i = 4$	:	$3o - 3u$.
12′.	$i = 5$	:	$3o - u - \underline{u} - u$.
13′.	$i = 5$	:	$3o - 3u$.
14′.	$i = 6$	:	$3o - u - \underline{u} - u$.

For the other three V patterns the interbraiding development and the interbraiding procedures are shown in Figs. 614, 615, and 616.

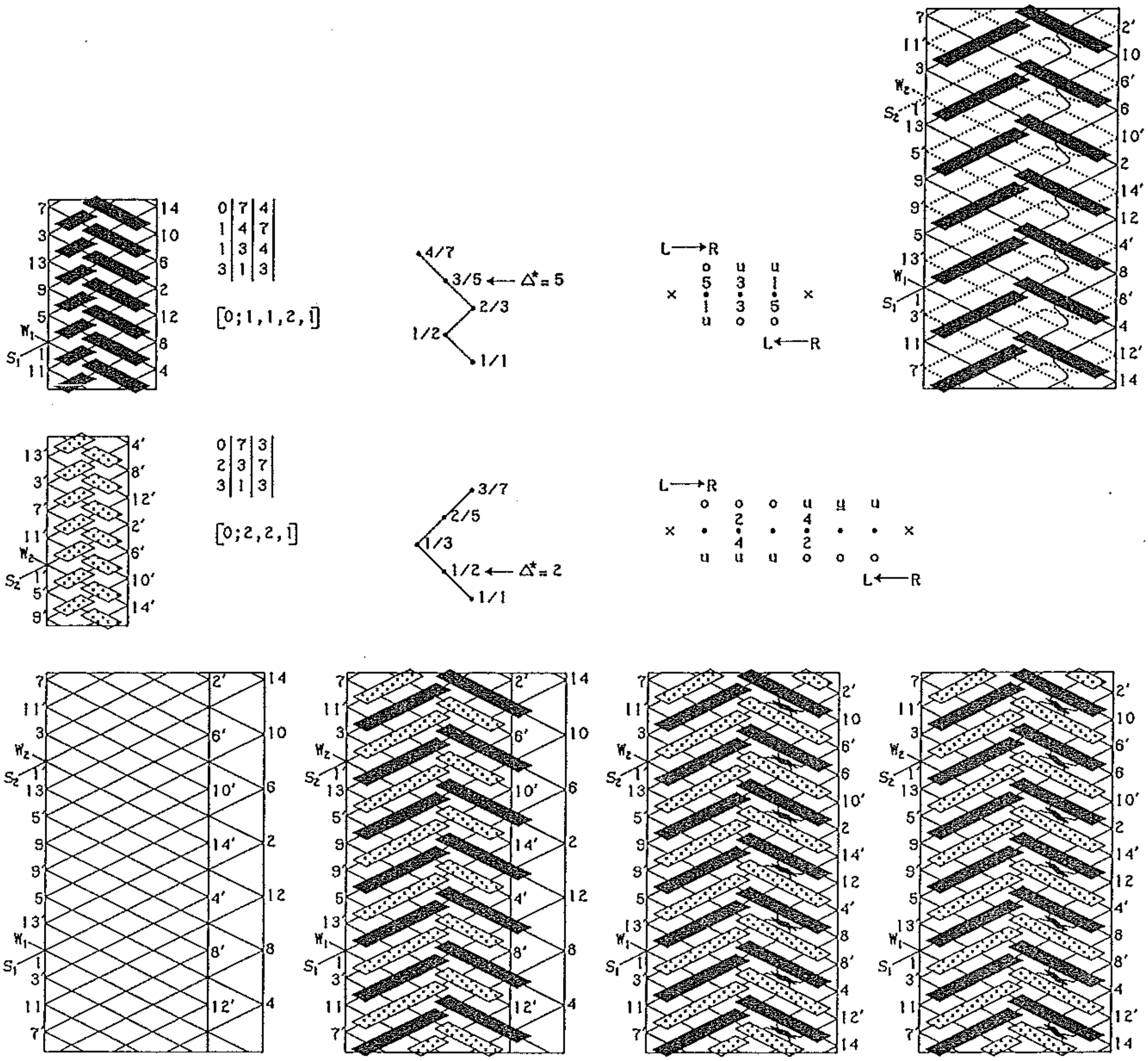


Fig. 614 — The construction details for the interbraid $p'/b' = 3/7$ with $p/b = 4/7$.

The half-cycle braiding algorithms associated with the interbraid in Fig. 614 are as follows:

1.		:	Free run.
2.	$i = 0$	:	Free run.
3.	$i = 0$	:	Free run.
4.	$i = 1$	:	u .
5.	$i = 1$	:	u .
6.	$i = 2$	:	u .
7.	$i = 2$	:	u .
8.	$i = 3$	:	$o - u$.
9.	$i = 3$	:	$2u$.
10.	$i = 4$	:	$o - u$.
11.	$i = 4$	:	$2u$.
12.	$i = 5$	:	$2o - u$.
13.	$i = 5$	:	$o - 2u$.
14.	$i = 6$	:	$2o - u$.

1′.		:	$2o - \underline{u} - u$.
2′.	$i = 0$	:	$2o - 2u$.
3′.	$i = 0$	:	$2o - \underline{u} - u$.
4′.	$i = 1$	:	$2o - 2u$.
5′.	$i = 1$	:	$2o - \underline{u} - u$.
6′.	$i = 2$	:	$3o - 2u$.
7′.	$i = 2$	:	$3o - \underline{u} - u$.
8′.	$i = 3$	:	$3o - 2u$.
9′.	$i = 3$	:	$3o - \underline{u} - u$.
10′.	$i = 4$	:	$3o - 3u$.
11′.	$i = 4$	:	$3o - u - \underline{u} - u$.
12′.	$i = 5$	:	$3o - 3u$.
13′.	$i = 5$	:	$3o - u - \underline{u} - u$.
14′.	$i = 6$	:	$3o - 3u$.

The half-cycle braiding algorithms associated with the interbraid in Fig. 615 are as follows:

1.		:	Free run.
2.	$i = 0$	:	Free run.
3.	$i = 0$	:	Free run.
4.	$i = 1$	:	o .
5.	$i = 1$	:	o .
6.	$i = 2$	:	o .
7.	$i = 2$	:	o .
8.	$i = 3$	:	$u - o$.
9.	$i = 3$	:	$2o$.
10.	$i = 4$	:	$u - o$.
11.	$i = 4$	:	$2o$.
12.	$i = 5$	:	$2u - o$.
13.	$i = 5$	:	$u - 2o$.
14.	$i = 6$	:	$2u - o$.

1′.		:	$2u - 2o$.

2′. $i=0$: $u-\underline{u}-2o$.
3′. $i=0$: $2u-2o$.
4′. $i=1$: $u-\underline{u}-2o$.
5′. $i=1$: $2u-2o$.
6′. $i=2$: $u-\underline{u}-u-2o$.
7′. $i=2$: $3u-2o$.
8′. $i=3$: $u-\underline{u}-u-2o$.
9′. $i=3$: $3u-2o$.
10′. $i=4$: $u-\underline{u}-u-3o$.
11′. $i=4$: $3u-3o$.
12′. $i=5$: $u-\underline{u}-u-3o$.
13′. $i=5$: $3u-3o$.
14′. $i=6$: $u-\underline{u}-u-3o$.

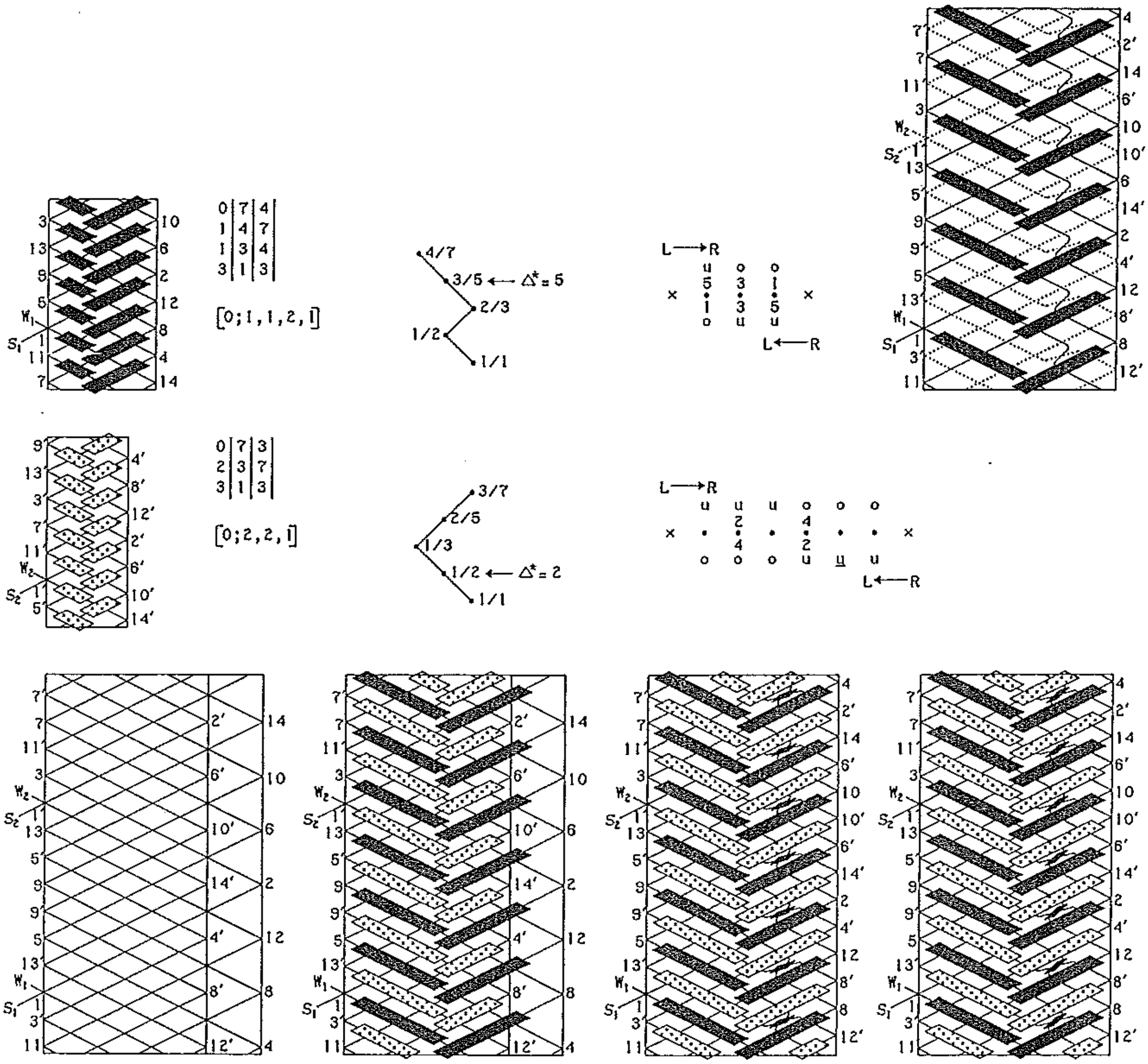


Fig. 615 — The construction details for the interbraid $p'/b' = 3/7$ with $p/b = 4/7$.

The half-cycle braiding algorithms associated with the interbraid in Fig. 616 are as follows:

1. : Free run.
2. $i = 0$: Free run.
3. $i = 0$: Free run.
4. $i = 1$: o.
5. $i = 1$: o.
6. $i = 2$: o.
7. $i = 2$: o.
8. $i = 3$: $2o$.
9. $i = 3$: $u - o$.
10. $i = 4$: $2o$.
11. $i = 4$: $u - o$.
12. $i = 5$: $u - 2o$.
13. $i = 5$: $2u - o$.
14. $i = 6$: $u - 2o$.

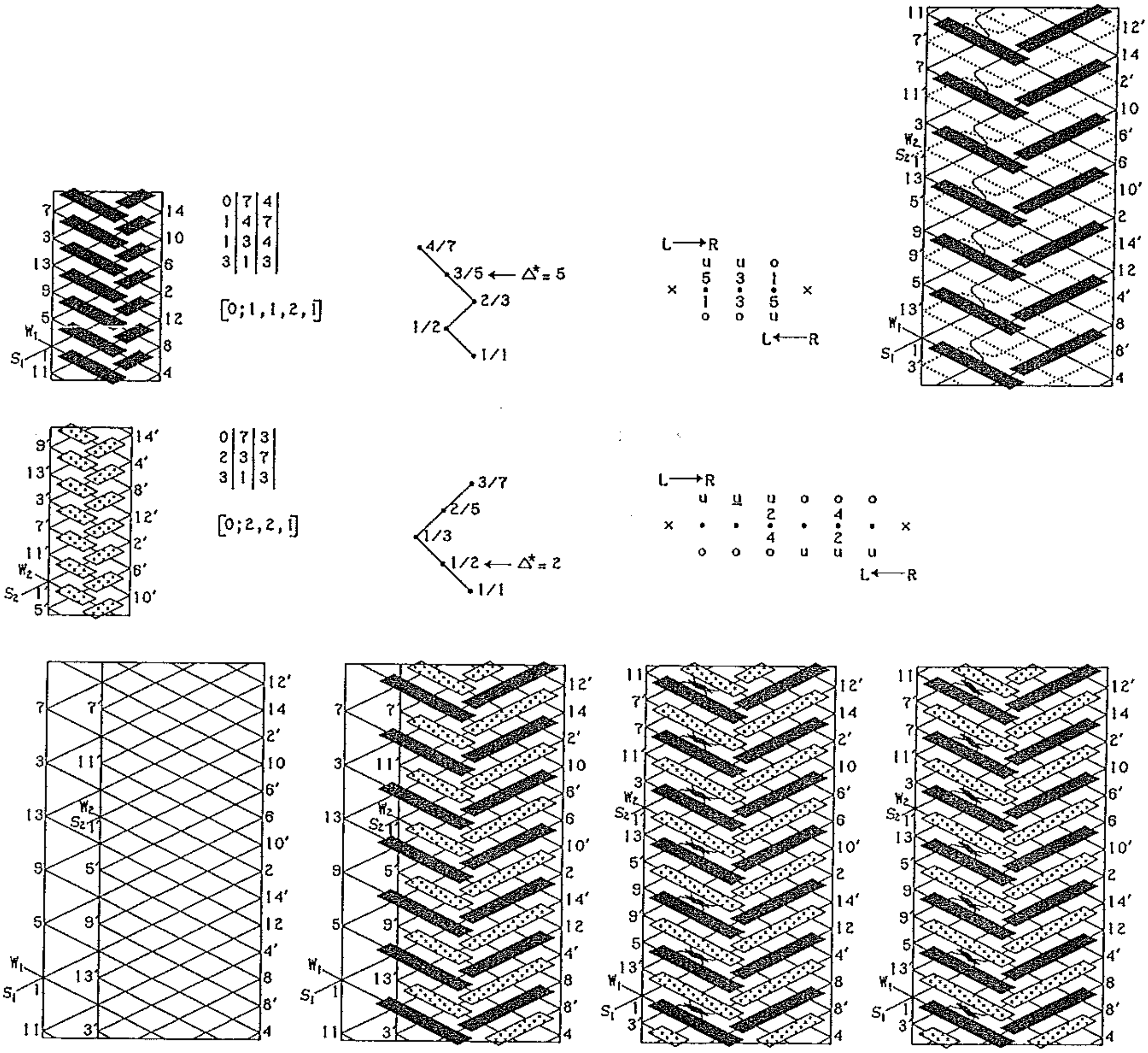


Fig. 616 — The construction details for the interbraid $p'/b' = 3/7$ with $p/b = 4/7$.

1′. : $u - \underline{u} - 2o$.

2′.	$i = 0$	:	$2u - 2o$.
3′.	$i = 0$	:	$u - \underline{u} - 2o$.
4′.	$i = 1$	:	$2u - 2o$.
5′.	$i = 1$	:	$u - \underline{u} - 2o$.
6′.	$i = 2$	:	$3u - 2o$.
7′.	$i = 2$	:	$u - \underline{u} - u - 2o$.
8′.	$i = 3$	:	$3u - 2o$.
9′.	$i = 3$	:	$u - \underline{u} - u - 2o$.
10′.	$i = 4$	:	$3u - 3o$.
11′.	$i = 4$	:	$u - \underline{u} - u - 3o$.
12′.	$i = 5$	:	$3u - 3o$.
13′.	$i = 5$	:	$u - \underline{u} - u - 3o$.
14′.	$i = 6$	:	$3u - 3o$.

Let's now have a look at the case where the sequence of two sets of V's is 2 : 1.

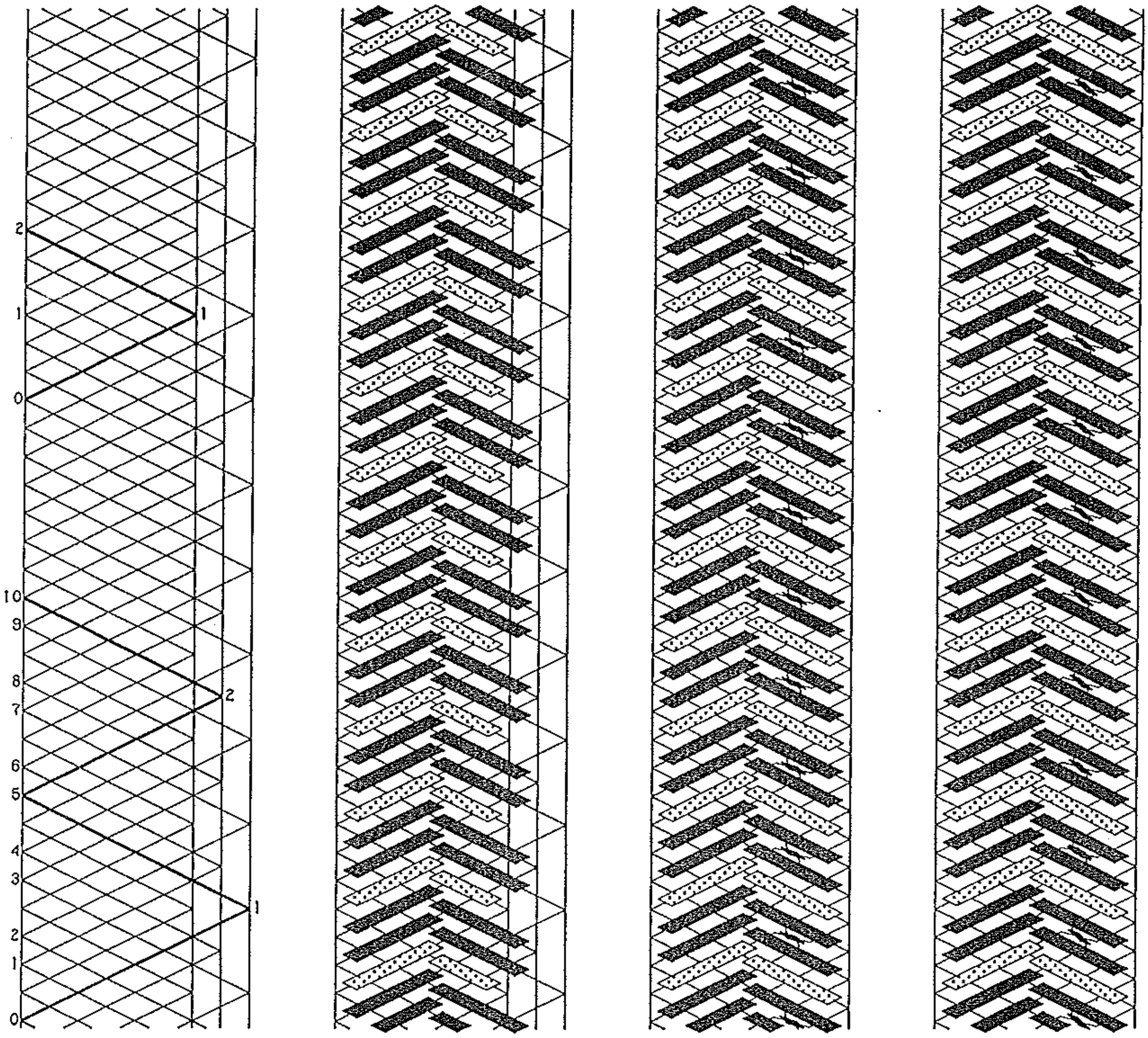


Fig. 617 — The development of the actual string-run and coding for a V pattern sequence 2 : 1.

Note again how this string-run and coding is obtained from the string-run and coding shown in the second diagram from the left, and hence how its string-run is associated with the string-run depicted in the leftmost diagram. From this leftmost string-run in Fig. 617 we can thus determine with the aid of the generalised law of the greatest common divisor $\lambda = \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right)$ the properties of the actual string-run belonging to the pattern of V's in the rightmost diagram of Fig. 617.

This leftmost string-run in Fig. 617 consists of two components; their first-return string-runs are drawn in thick heavy lines. In these first-return string-runs the bights are numbered on the right, and the bights which these first-return string-runs span in their components are numbered on the left. Hence for the lowermost first-return string-run we obtain $\alpha = 2$ and $\beta = 10$, while for the uppermost first-return string-run we obtain $\alpha = 1$ and $\beta = 2$. Let the number of bights B_c of the component associated with the lowermost first-return string-run be equal to B, then the number of bights B_c of the component associated with the uppermost first-return string-run is equal to $\frac{B}{2}$. Thus for the component with the lowermost first-return string-run the number of sub-components $\lambda = \text{g.c.d.}(5, \frac{B}{2})$, while for the component with the uppermost first-return string-run the number of sub-components $\lambda = \text{g.c.d.}(2, \frac{B}{2})$. Hence for both λ-values to be equal to 1, the value of $\frac{B}{2}$ must be coprime with 5 and 2.

If, for example, we take $B = 6$, hence the knot has 9 bights in total, we obtain the construction details shown in Fig. 618.

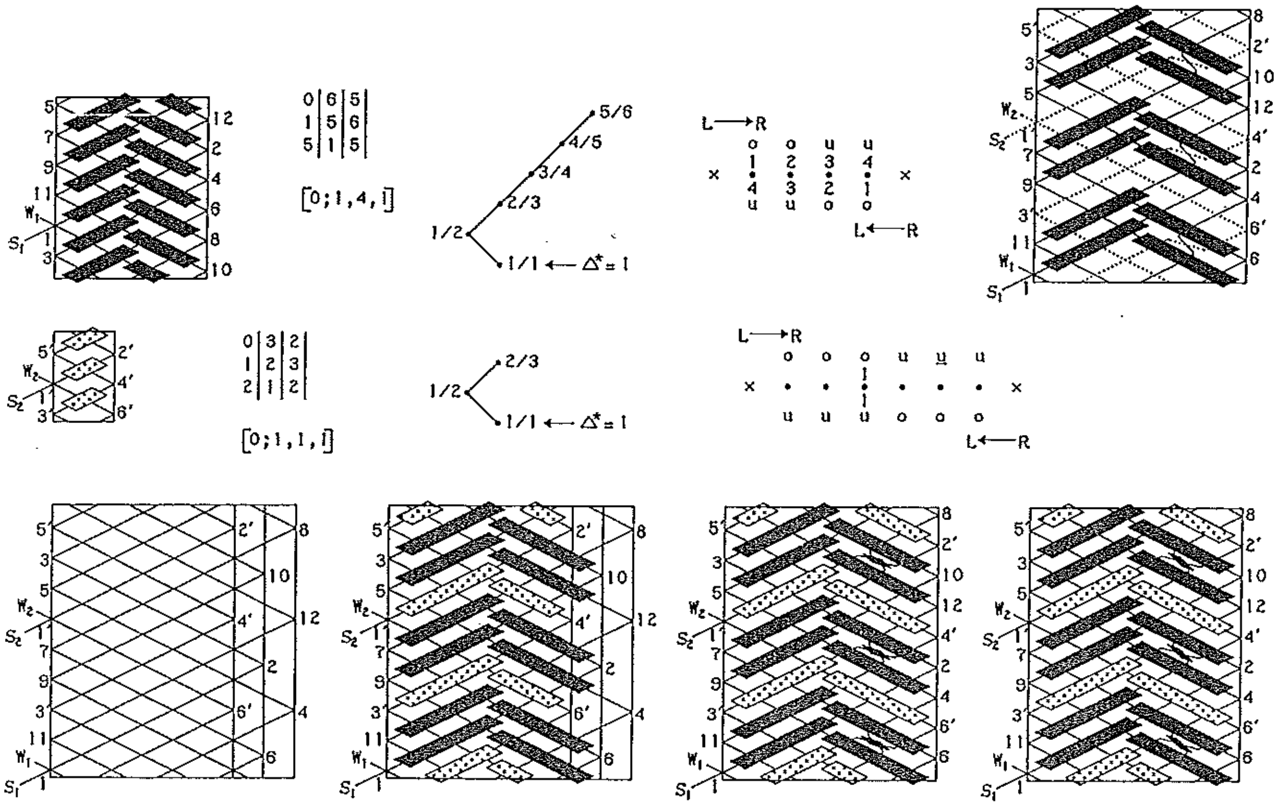


Fig. 618 — The construction details for the interbraid $p'/b' = 2/3$ with $p/b = 5/6$.

The upper two left grid-diagrams depict respectively the foundation knot $p/b = 5/6$ and the interbraid $p'/b' = 2/3$. The foundation knot with its ultimate string-run is depicted by the uppermost right-hand grid-diagram. In the lowermost row of diagrams the three diagrams on the left depict the development of the ultimate knot depicted by the lower rightmost grid-diagram.

The construction details associated with interbraiding $p/b = 5/6$ and $p'/b' = 2/3$ for the other three V patterns are shown in Figs. 619, 620, and 621.

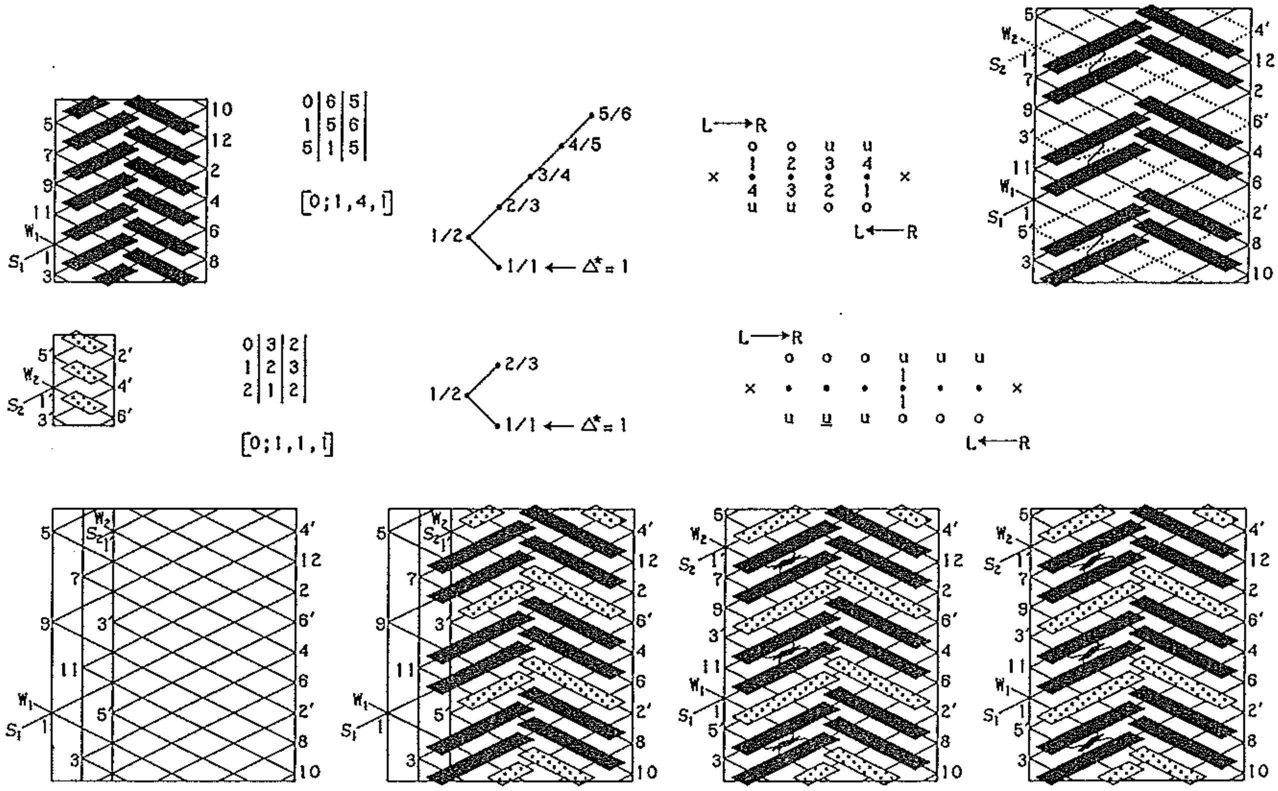


Fig. 619 — The construction details for the interbraid $p'/b' = 2/3$ with $p/b = 5/6$.

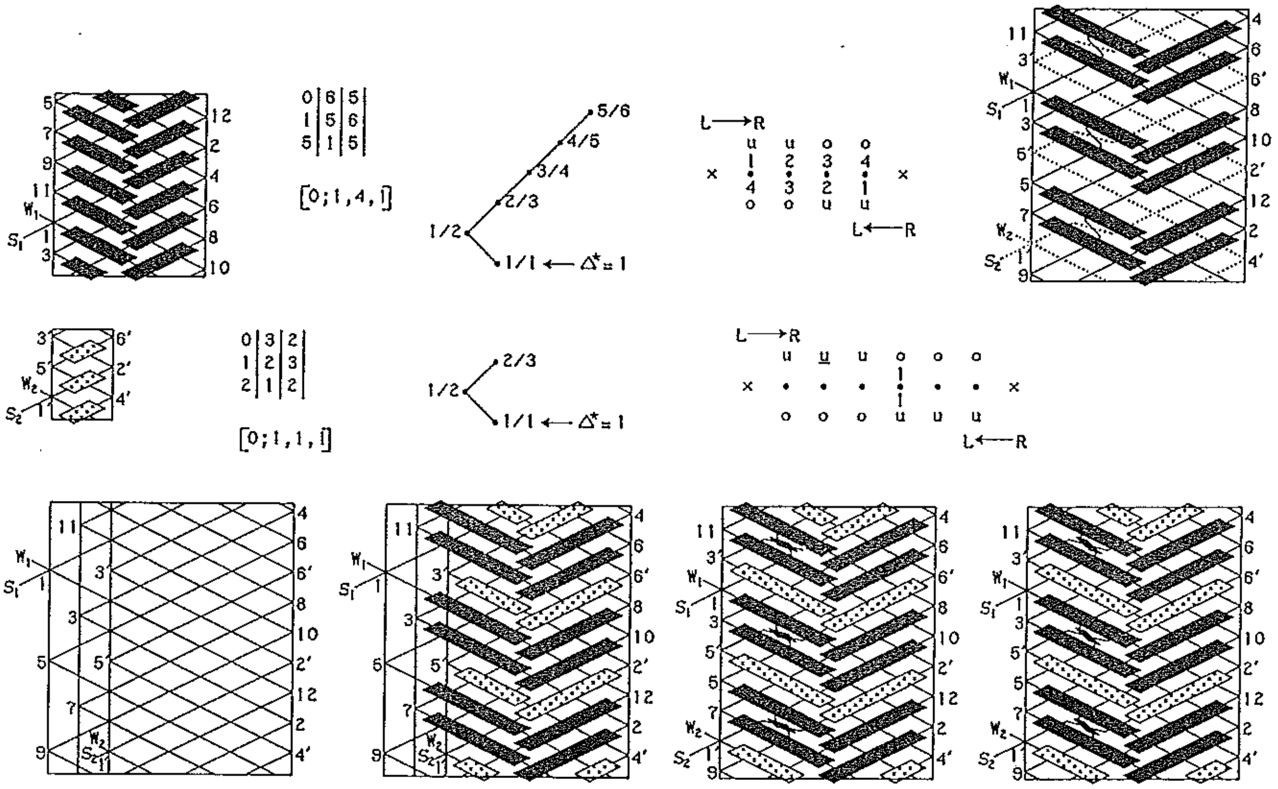


Fig. 620 — The construction details for the interbraid $p'/b' = 2/3$ with $p/b = 5/6$.

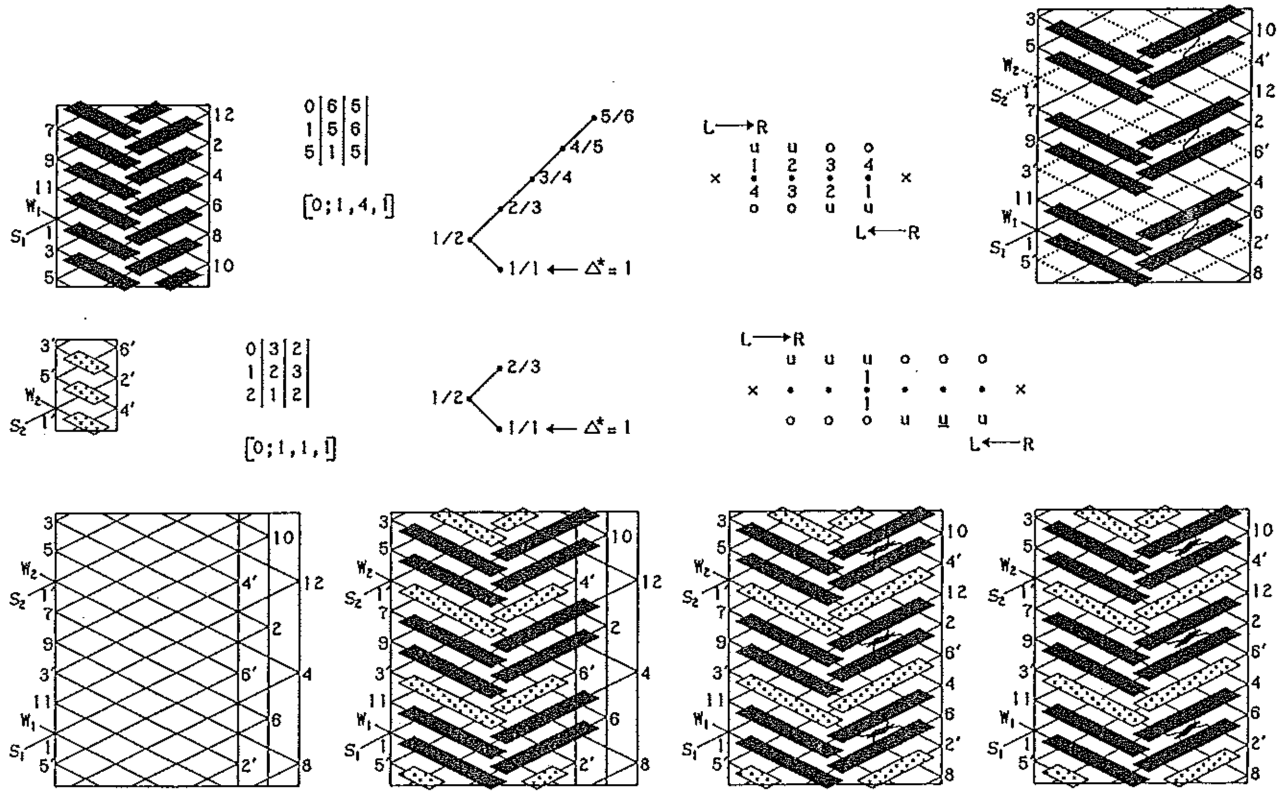


Fig. 621 — The construction details for the interbraid $p'/b' = 2/3$ with $p/b = 5/6$.

First we braid the foundation knots $p/b = 5/6$; their half-cycle braiding algorithms are again read from their associated algorithm diagrams. Note that these are 2-pass Spanish Ring Knots. Then we interbraid the knots $p'/b' = 2/3$; their half-cycle braiding algorithms are again read from their associated algorithm diagrams.

⋆⋆ *A commonly encountered 2-pass Spanish Ring Knot is the one with* $p/b = 5/8$. *Say we want to interbraid such a Spanish Ring Knot with a contrasting set of V's. Give for the four V patterns the construction details for doing this.*

In Fig. 622 we have shown the development of the actual string-run and coding for a V pattern sequence 3 : 1. In the leftmost string-run we have again drawn the two first-return string-runs in thick heavy lines, and as before, in these first-return string-runs the bights are numbered on the right, and the bights which these first-return string-runs span in their components are numbered on the left. Hence for the lowermost first-return string-run we obtain $\alpha = 1$ and $\beta = 2$, while for the uppermost first-return string-run we obtain $\alpha = 3$ and $\beta = 15$. Let the number of bights B_c of the component associated with the uppermost first-return string-run be equal to B, then the number of bights B_c of the component associated with the lowermost first-return string-run is equal to $\frac{B}{3}$. Thus for the component with the uppermost first-return string-run the number of sub-components $\lambda = \text{g.c.d.}(5\,,\,\frac{B}{3})$, while for the component with the lowermost first-return string-run the number of sub-components $\lambda = \text{g.c.d.}(2\,,\,\frac{B}{3})$. Hence for both λ-values to be equal to 1, the value of $\frac{B}{3}$ must be coprime with 5 and 2.

If, for example, we take $B = 9$, hence the knot has 12 bights in total, we obtain the construction details shown in Fig. 623.

Note that the component $p/b = 5/9$ is a 2-pass Spanish Ring Knot.

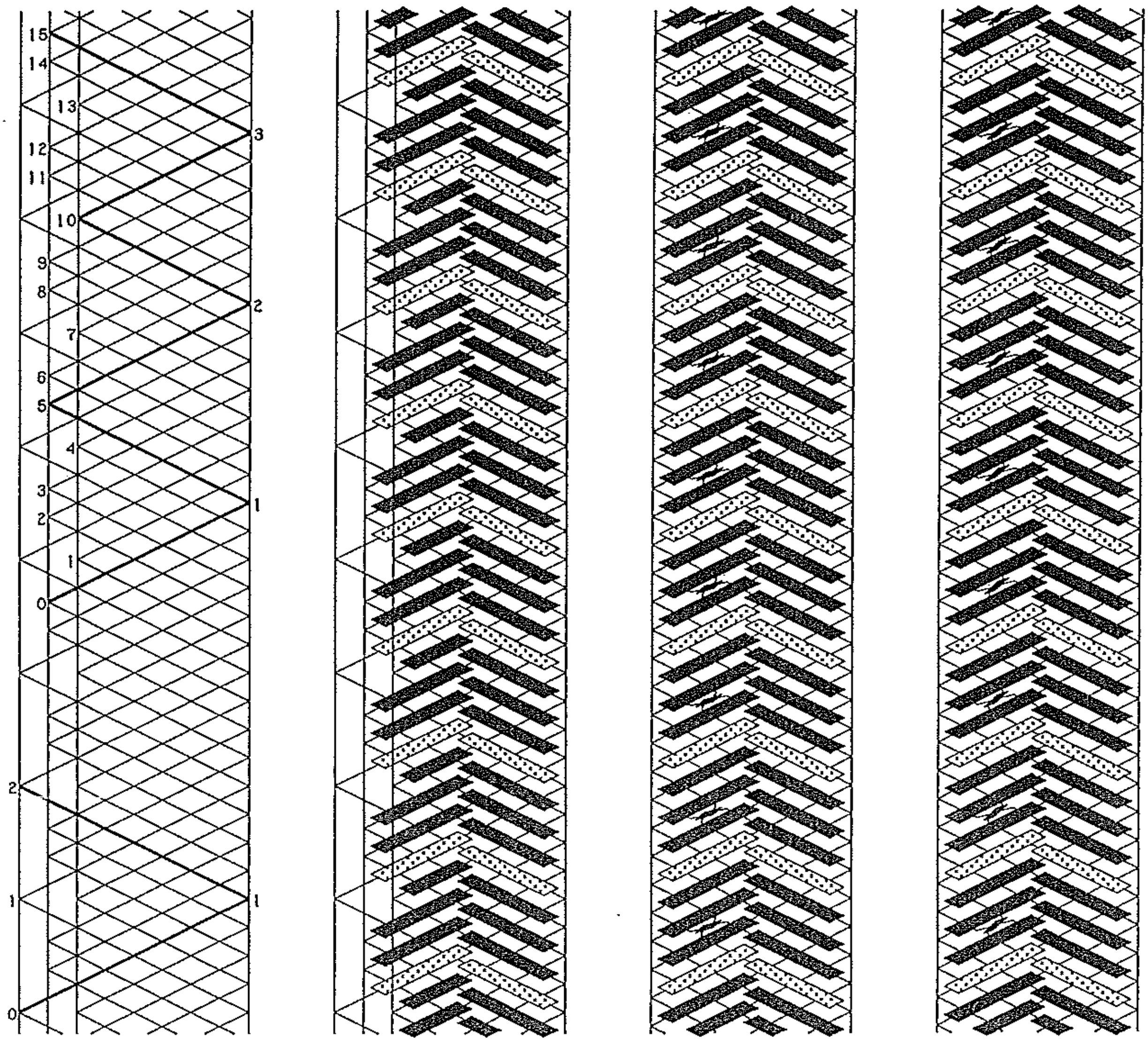


Fig. 622 — The development of the actual string-run and coding for a V pattern sequence 3 : 1 .

The construction details for the other three V patterns with a V pattern sequence of 3 : 1 are shown in Figs. 624, 625, and 626.

The half-cycle braiding algorithms are again read from the associated algorithm diagrams.

When Spanish Ring Knots with contrasting interwoven V's are employed in braiding projects it is important to ensure that the correct symmetry and/or balance is obtained.† Hence it is necessary to be able to braid any one of the four V patterns which have the same V pattern sequence. This symmetry and/or balance aspect is very important for a high braiding standard. It is therefore unfortunate that many braiders tend to neglect it. This neglect is nearly always due to a lack in braiding skills.

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† The balance and/or symmetry of similar components in a project depends on the configuration of the project, hence is project specific. It is therefore not possible to give here general guidelines for symmetry and/or balance.

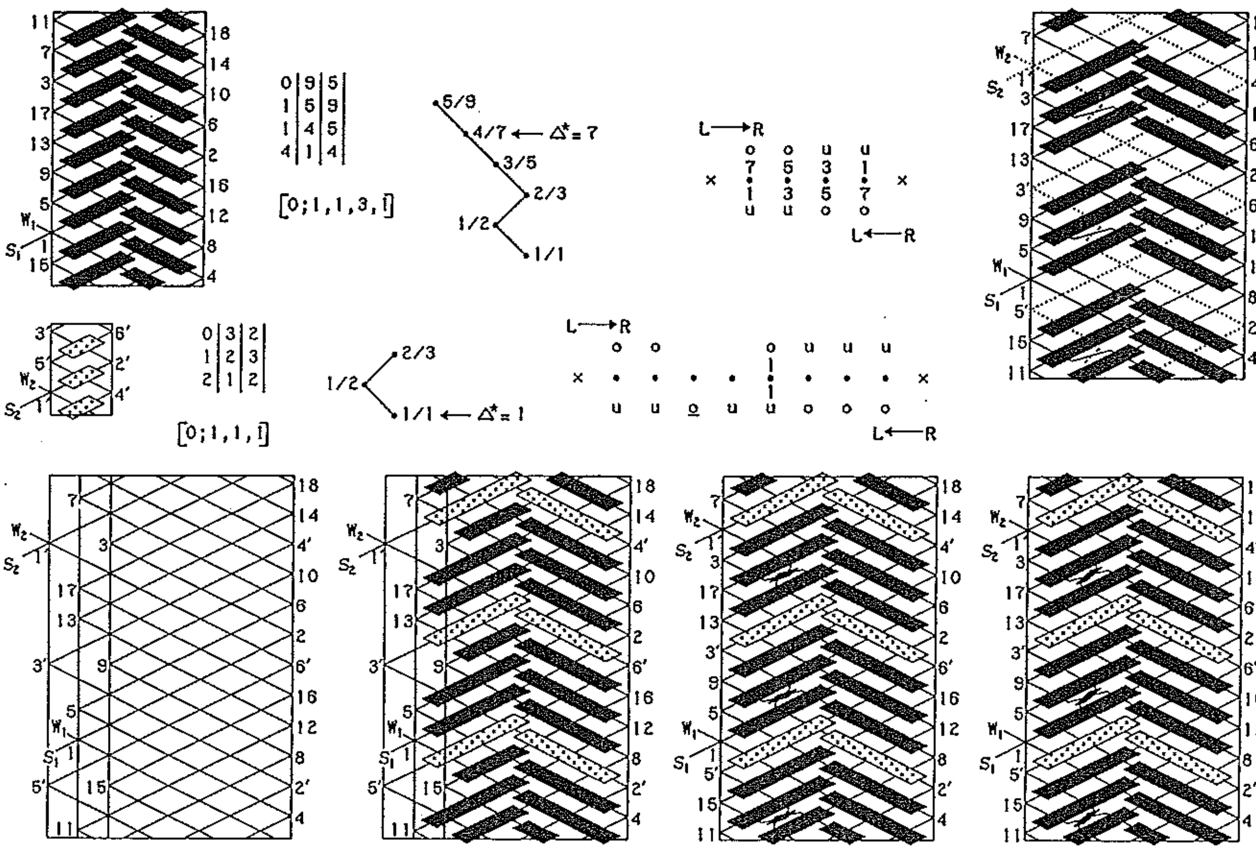

Fig. 623 — The construction details for the interbraid $p'/b' = 2/3$ with $p/b = 5/9$.

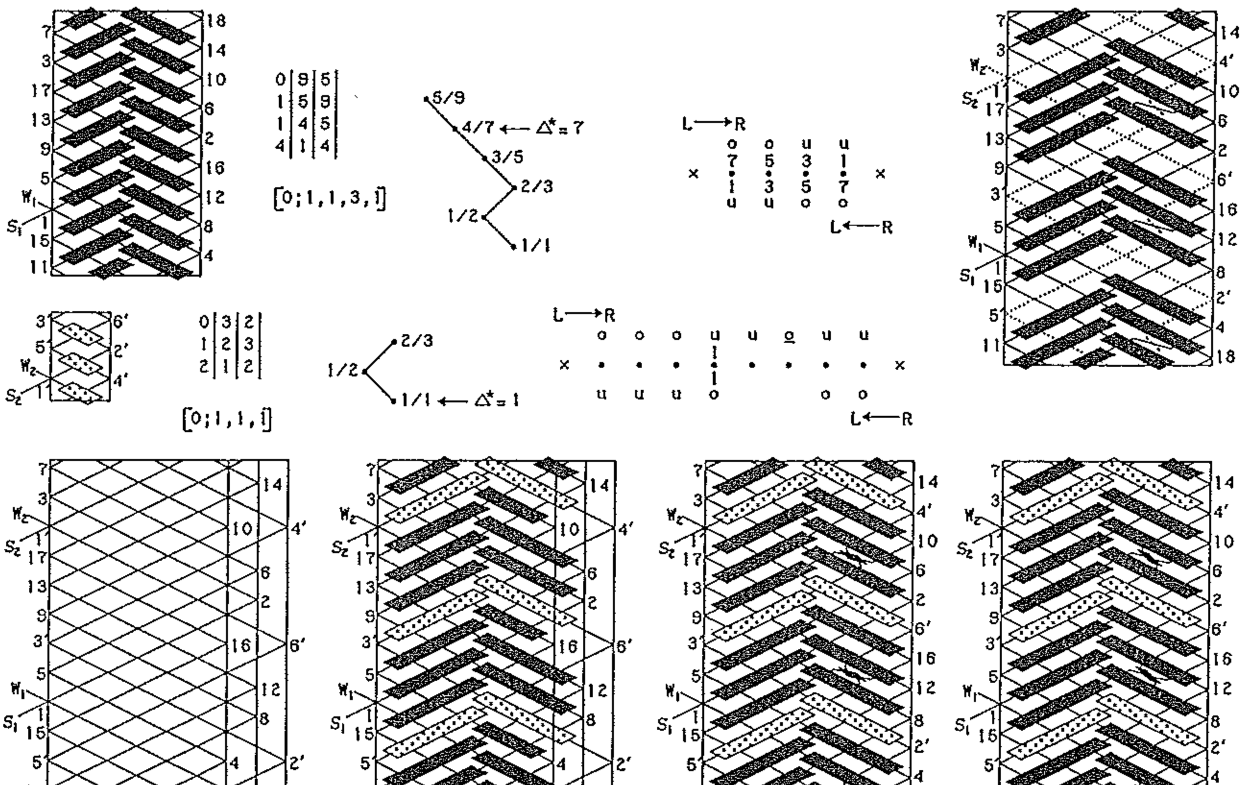

Fig. 624 — The construction details for the interbraid $p'/b' = 2/3$ with $p/b = 5/9$.

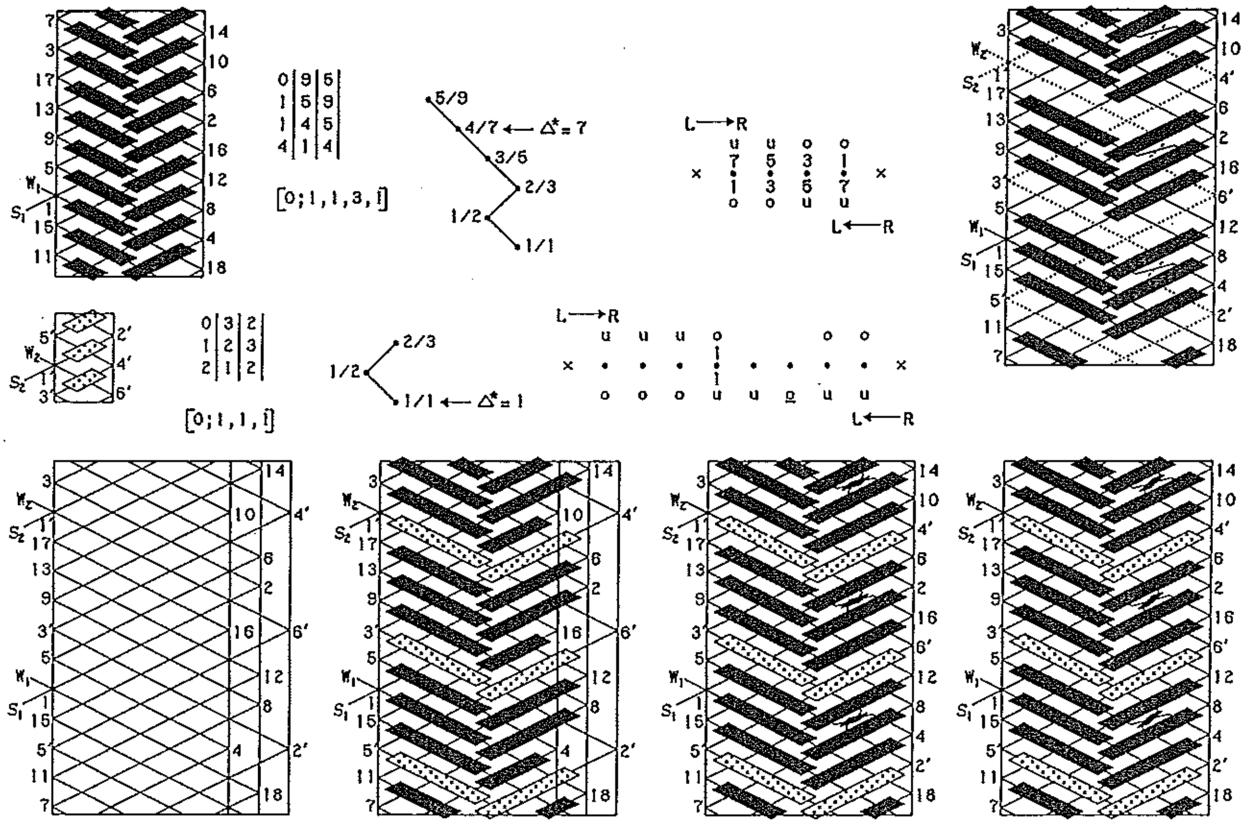

Fig. 625 — The construction details for the interbraid $p'/b' = 2/3$ with $p/b = 5/9$.

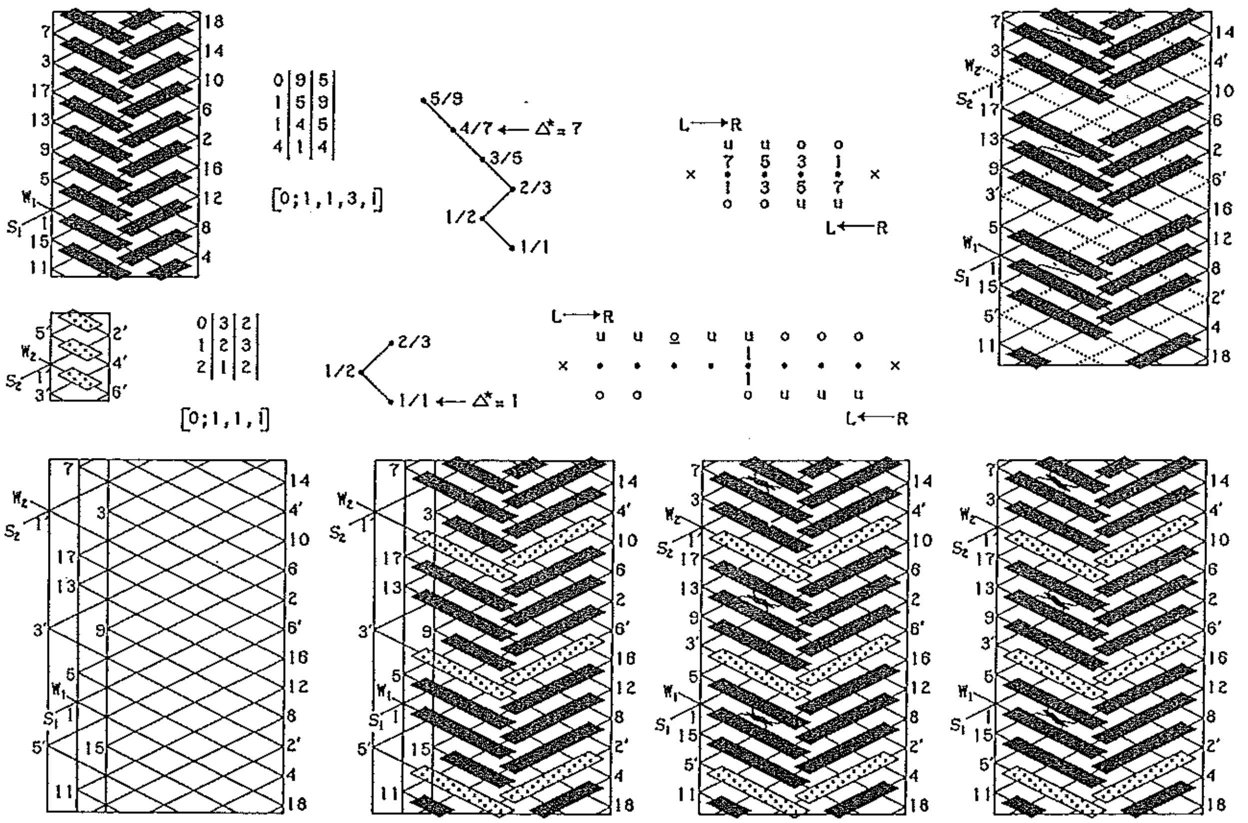

Fig. 626 — The construction details for the interbraid $p'/b' = 2/3$ with $p/b = 5/9$.

Historical events and their fictitious renderings.

In the last decade or so we have seen many fictitious renderings of historical events. Fictitious renderings of historical events are often difficult if not impossible to be recognised as such since the lapsed timespan between the event and the present may be too great, but in some circles the misrepresentsation of facts has become a habit with the result that the magnitude of lapsed timespans are neglected; this then enables us to identify such circles and file their stories in the garbage bin.

A few years ago (1998) we found on the World Wide Web (WWW) "Het Knoopeknauwertje's Homepage" with, amongst others, **The Head Hunter Ring** by Dean Westervelt. The story of **The Head Hunter Ring** by Dean Westervelt reads quite different to its translation in *Het Knoopeknauwertje*, Issue No. 9, pp. 4-5 (the more a story gets retold, the more it changes). The translation in *Het Knoopeknauwertje* reads:

> In 1948 I visited the renowned Ripley's Believe It or Not Museum. There, a descendant of a headhunter tribe was tying a ring from two strands. After returning home, I puzzled a little while over the way he did it. In due course I found the procedure. Years later I found in The Encyclopedia of Knots and Fancy Ropework by Graumont and Hensel a similar ring with the name Head hunter Ring (pg. 526, Fig. 428). They gave, however, no method for tying it since this was a secret. Below is my method which is now no secret any longer. Take two strands of equal length. Preferably of two different colours *A* and *B*. Secure the starting-ends with a rubber band and follow the drawings, the algorithm below or a combination of both.

The text on the Web-site reads:

> In 1948 I visited the famous Believe It or Not Museum in New York City. There, a descendant of a Phillipine Headhunter Tribe was tying finger rings using two strands of split bamboo. The ring was woven in a coding pattern of over-three, under-three, over three. I puzzled over this very attractive knot for several months before I discovered a procedure for tying it. The cryptic text accompanying a photo of this same knot in Graumont and Hensel's Encyclopedia of Knots and Fancy Rope Work (page 524,-5, plate 286, No. 428) was of little help.

According to the Web-site text it is clear that Dean did possess the quoted Encyclopedia by Graumont and Hensel before he solved the construction procedure, but it did not make him any wiser. In the Web-site version we cannot find 'the algorithm below', wheras it is present on pg. 4 in Het Knoopeknauwertje. There is little doubt however that the method employed by the descendant of the Headhunter Tribe was not the original tying method, but rather a 'stage act' to impress the audience. It should be noted that the 3-pass Headhunters knots $p/b = 10/12$ and $10/22$ are interweaves of respectively two over–under coded $p/b = 5/6$ Regular Knots and two over–under coded $p/b = 5/11$ Regular Knots. The $p/b = 5/6$ over–under coded Regular Knot is the 1^{st} order Method II enlargement of the over–under coded $p/b = 3/4$ Regular Knot. The $p/b = 5/11$ over–under coded Regular Knot is the 1^{st} order Method II enlargement of the over–under coded $p/b = 3/7$ Regular Knot which is easily obtained from the over–under coded $p/b = 3/4$ Regular Knot. It should be noted that a 3-pass Headhunters knot $p/b = 10/(10n + 2)$ is the interweave of two over–under coded $p/b = 5/(5n + 1)$ over–under coded Regular Knots. Hence the foundation knot of such a Headhunters knot is the 1^{st} order Method II enlargement of the over–under coded $p/b = 3/(3n + 1)$ Regular Knot. Furthermore, observe that such a foundation knot and the associated 3-pass Headhunters knot are knots of the "braiding to pattern" variety. Hence it is more likely that the original tying method was via the over–under coded $p/b = 3/(3n + 1)$ Regular Knots together with the "braiding to pattern" construction method.