THE BRAIDER

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SEE NOTE BELOW.

No. 31

NOTE: the first Fig. number in this Issue is 554, but should have been 584 instead. Hence the Fig. numbers 554 - 583 not only appear in *The Braider*, Issue No. 30, but the numbers 554 - 574 appear again in *The Braider*, Issue No. 31 and the numbers 575 - 583 appear again in *The Braider*, Issue No. 32. This was noted too late for rectification.

No. 32

See NOTE under No. 31 above.

Solutions to the Questions in Issue No. 30

Question on pg. 695.

Method I–1:

Since we have the cycle consisting of the successive half-cycles $1 \to 1$ and $A \leftarrow 1$, we have the cycle which consists of the successive half-cycles $A \to 2$ and $(A-1) \leftarrow 2$. We can obtain the value for y from the half-cycle $A \to 2$:

$$y = |2(A-A) + x + 2(A-2)|_{2A} = |x-4|_{2A}\,.$$

Since $\Delta = A-1$, it follows that $y = A-1$ or $y = 2A-1$. The enlargement will create only one additional nest of bights on the left-hand bight-edge and one additional nest of bights on the right-hand bight-edge, hence:

$$|(A-A) + x + 2(A-2) + (A-(A-1))|_B = 0\,,\text{ hence } |2A+x-3|_{AB^*} = 0\,.$$

(i.) For $y = A-1$:

$$x - 4 = 2n'A + A - 1\,,\text{ hence } x = (2n'+1)A + 3 = (2n-3)A + 3\,,$$

where n is a natural number (recall from *The Braider*, Issue No. 25, pg. 564 that $x \geq 2-A$).

$$|2A+x-3|_{AB^*} = |(2n-1)A|_{AB^*} = 0\,,\text{ hence } B^* = \frac{2n-1}{m}\,,$$

where m and n are natural numbers, and m divides into $2n-1$.

$$P = 2A + x - 2 = (2n-1)A + 1 = mAB^* + 1\,.$$

Since $B^*_e = B^* + 1$ it follows that $P_e = mAB^*_e + 1 = mAB^* + mA + 1 = P + mA$, and since $P_e = P + (x_e - x)$ it follows that $x_e = x + mA$.
Furthermore, $y_e = |y + (x_e - x)|_{2A} = |y + mA|_{2A} = |(m+1)A - 1|_{2A}$.

(ii.) For $y = 2A-1$:

$$x - 4 = 2n'A + 2A - 1\,,\text{ hence } x = (2n'+2)A + 3 = (2n-2)A + 3\,,$$

where n is a natural number (recall from *The Braider*, Issue No. 25, pg. 564 that $x \geq 2-A$).

$$|2A+x-3|_{AB^*} = |2nA|_{AB^*} = 0\,,\text{ hence } B^* = \frac{2n}{m}\,,$$

where m and n are natural numbers, and m divides into $2n$.

$$P = 2A + x - 2 = 2nA + 1 = mAB^* + 1\,.$$

Since $B^*_e = B^* + 1$ it follows that $P_e = mAB^*_e + 1 = mAB^* + mA + 1 = P + mA$, and since $P_e = P + (x_e - x)$ it follows that $x_e = x + mA$.
Furthermore, $y_e = |y + (x_e - x)|_{2A} = |y + mA|_{2A} = |mA - 1|_{2A}$.

Method I–2:

We have the cycle consisting of the successive half-cycles $1 \to A$ and $2 \leftarrow A$. From the half-cycle $1 \to A$ we can obtain the value for y:

$$y = |2(A-1) + x + 2(A-A)|_{2A} = |x-2|_{2A}\,.$$

Since $\Delta = 1$, it follows that $y = A+1$ or $y = 1$. The enlargement will create only one additional nest of bights on the left-hand bight-edge and one additional nest of bights on the right-hand bight-edge, hence:

$$|(A-1) + x + 2(A-A) + (A-2)|_B = 0\,,\text{ hence } |2A+x-3|_{AB^*} = 0\,.$$

(i.) For $y = A+1$:

$$x - 2 = 2n'A + A + 1\,,\text{ hence } x = (2n'+1)A + 3 = (2n-3)A + 3\,,$$

where n is a natural number (recall from *The Braider*, Issue No. 25, pg. 564 that $x \geq 2-A$).

$$|2A + x - 3|_{AB^*} = |(2n-1)A|_{AB^*} = 0\,, \text{ hence } B^* = \frac{2n-1}{m}\,,$$

where m and n are natural numbers, and m divides into $2n-1$.

$$P = 2A + x - 2 = (2n-1)A + 1 = mAB^* + 1\,.$$

Since $B^*_e = B^* + 1$ it follows that $P_e = mAB^*_e + 1 = mAB^* + mA + 1 = P + mA$, and since $P_e = P + (x_e - x)$ it follows that $x_e = x + mA$.
Furthermore, $y_e = |y + (x_e - x)|_{2A} = |y + mA|_{2A} = |(m+1)A + 1|_{2A}$.

(ii.) For $y = 1$:

$$x - 2 = 2n'A + 1\,, \text{ hence } x = 2n'A + 3 = (2n-2)A + 3\,,$$

where n is a natural number (recall from *The Braider*, Issue No. 25, pg. 564 that $x \geq 2 - A$).

$$|2A + x - 3|_{AB^*} = |2nA|_{AB^*} = 0\,, \text{ hence } B^* = \frac{2n}{m}\,,$$

where m and n are natural numbers, and m divides into $2n$.

$$P = 2A + x - 2 = 2nA + 1 = mAB^* + 1\,.$$

Since $B^*_e = B^* + 1$ it follows that $P_e = mAB^*_e + 1 = mAB^* + mA + 1 = P + mA$, and since $P_e = P + (x_e - x)$ it follows that $x_e = x + mA$.
Furthermore, $y_e = |y + (x_e - x)|_{2A} = |y + mA|_{2A} = |mA + 1|_{2A}$.

Method II–1 :

Since we have the cycle consisting of the successive half-cycles $A \rightarrow A$ and $1 \leftarrow A$, we have the cycle which consists of the successive half-cycles $1 \rightarrow (A-1)$ and $2 \leftarrow (A-1)$. We can obtain the value for y from the half-cycle $1 \rightarrow (A-1)$:

$$y = |2(A-1) + x + 2(A - (A-1))|_{2A} = |x|_{2A}\,.$$

Since $\Delta = 1$, it follows that $y = A + 1$ or $y = 1$. The enlargement will create only one additional nest of bights on the left-hand bight-edge and one additional nest of bights on the right-hand bight-edge, hence:

$$|(A-1) + x + 2(A - (A-1)) + (A-2)|_B = 0\,, \text{ hence } |2A + x - 1|_{AB^*} = 0\,.$$

(i.) For $y = A + 1$:

$$x = 2n'A + A + 1\,, \text{ hence } x = (2n'+1)A + 1 = (2n-1)A + 1\,,$$

where n is a natural number (recall from *The Braider*, Issue No. 25, pg. 564 that $x \geq 2 - A$).

$$|2A + x - 1|_{AB^*} = |(2n+1)A|_{AB^*} = 0\,, \text{ hence } B^* = \frac{2n+1}{m}\,,$$

where m and n are natural numbers, and m divides into $2n+1$.

$$P = 2A + x - 2 = (2n+1)A - 1 = mAB^* - 1\,.$$

Since $B^*_e = B^* + 1$ it follows that $P_e = mAB^*_e - 1 = mAB^* + mA - 1 = P + mA$, and since $P_e = P + (x_e - x)$ it follows that $x_e = x + mA$.
Furthermore, $y_e = |y + (x_e - x)|_{2A} = |y + mA|_{2A} = |(m+1)A + 1|_{2A}$.

(ii.) For $y = 1$:

$$x = 2n'A + 1\,, \text{ hence } x = (2n-2)A + 1\,,$$

where n is a natural number (recall from *The Braider*, Issue No. 25, pg. 564 that $x \geq 2 - A$).

$$|2A + x - 1|_{AB^*} = |2nA|_{AB^*} = 0\,, \text{ hence } B^* = \frac{2n}{m}\,,$$

where m and n are natural numbers, and m divides into $2n$.

$$P = 2A + x - 2 = 2nA - 1 = mAB^* - 1\,.$$

Since $B_e^* = B^* + 1$ it follows that $P_e = mAB_e^* - 1 = mAB^* + mA - 1 = P + mA$, and since $P_e = P + (x_e - x)$ it follows that $x_e = x + mA$.
Furthermore, $y_e = |y + (x_e - x)|_{2A} = |y + mA|_{2A} = |mA + 1|_{2A}$.

Method II–2:

We have the cycle consisting of the successive half-cycles $A \rightarrow 1$ and $(A-1) \leftarrow 1$. From the half-cycle $A \rightarrow 1$ we can obtain the value for y:

$$y = |2(A - A) + x + 2(A-1)|_{2A} = |x - 2|_{2A}.$$

Since $\Delta = A - 1$, it follows that $y = A - 1$ or $y = 2A - 1$. The enlargement will create only one additional nest of bights on the left-hand bight-edge and one additional nest of bights on the right-hand bight-edge, hence:

$$|(A - A) + x + 2(A-1) + (A - (A-1))|_B = 0\text{, hence } |2A + x - 1|_{AB^*} = 0.$$

(i.) For $y = A - 1$:

$$x = 2n'A + A + 1\text{, hence } x = (2n'+1)A + 1 = (2n-1)A + 1,$$

where n is a natural number (recall from *The Braider*, Issue No. 25, pg. 564 that $x \geq 2 - A$).

$$|2A + x - 1|_{AB^*} = |(2n+1)A|_{AB^*} = 0\text{, hence } B^* = \frac{2n+1}{m},$$

where m and n are natural numbers, and m divides into $2n+1$.

$$P = 2A + x - 2 = (2n+1)A - 1 = mAB^* - 1.$$

Since $B_e^* = B^* + 1$ it follows that $P_e = mAB_e^* - 1 = mAB^* + mA - 1 = P + mA$, and since $P_e = P + (x_e - x)$ it follows that $x_e = x + mA$.
Furthermore, $y_e = |y + (x_e - x)|_{2A} = |y + mA|_{2A} = |(m+1)A - 1|_{2A}$.

(ii.) For $y = 2A - 1$:

$$x = (2n'+2)A + 1\text{, hence } x = (2n-2)A + 1,$$

where n is a natural number (recall from *The Braider*, Issue No. 25, pg. 564 that $x \geq 2 - A$).

$$|2A + x - 1|_{AB^*} = |2nA|_{AB^*} = 0\text{, hence } B^* = \frac{2n}{m},$$

where m and n are natural numbers, and m divides into $2n$.

$$P = 2A + x - 2 = 2nA - 1 = mAB^* - 1.$$

Since $B_e^* = B^* + 1$ it follows that $P_e = mAB_e^* - 1 = mAB^* + mA - 1 = P + mA$, and since $P_e = P + (x_e - x)$ it follows that $x_e = x + mA$.
Furthermore, $y_e = |y + (x_e - x)|_{2A} = |y + mA|_{2A} = |mA - 1|_{2A}$.

Question on pg. 705.

The sequence of i-values is the complementary cyclic bight-number scheme and is directly related to the cyclic bight-number scheme, the sequence in which the string-run visits the bight-points on a bight-boundary.†

For $\Delta^* = 1$ and the Standing-End half-cycle from lower-left to upper-right, the successive odd-numbered half-cycles are being laid down adjacent to each other **below** the Standing-End half-cycle. For $\Delta^* = 1$, hence $|p|_b = b-1$, we obtain $p = nb + b - 1 = (n+1)b - 1$ where n is a whole number, or $p = n'b - 1$ where n' is a natural number.

† See *The Braider*, Issue No. 5, pg. 93.

An alternative proof may be obtained from the algorithm diagram: since $\Delta^* = 1$, it follows that $|(p-1)\Delta^*|_b = b-2$ (recall that $(p-1)\Delta^* + \Delta^* = b-1$)†. Hence $|p-1|_b = b-2$, and thus $p = (n+1)b - 1$ where n is a whole number.

For $\Delta^* = b-1$ and the Standing-End half-cycle from lower-left to upper-right, the successive odd-numbered half-cycles are being laid down adjacent to each other **above** the Standing-End half-cycle. For $\Delta^* = b-1$, hence $|p|_b = 1$, we obtain $p = nb+1$ where n is a whole number (note, however, that for $n = 0$, hence for $p = 1$, no knot of practical use will be obtained).

An alternative proof may be obtained from the algorithm diagram: since $\Delta^* = b-1$, it follows that $|(p-1)\Delta^*|_b = 0$ (recall that $(p-1)\Delta^* + \Delta^* = b-1$)†. Hence $|(p-1)(b-1)|_b = 0$, and thus $p = nb+1$ where n is a whole number.

Nested Cylindrical Braids

Instead of two algorithm tables (one for the odd half-cycles and one for the even half-cycles) for a Semi-Perfect Regular Nested Cylindrical Braid as discussed in *The Braider*, Issue No. 29, pp. 671–682, some braiders might prefer to have two algorithm tables (one for the odd half-cycles and one for the even half-cycles) for each of its sub-components. This certainly makes reading of the half-cycle braiding algorithms easier. For example, in Example 5 on pp. 677–679 we can replace the two algorithm tables in Fig. 549 by the algorithm tables in Fig. 554, where the two upper algorithm tables are associated with the first sub-component (hence with the half-cycles 1 to 20), and the two lower algorithm tables are associated with the second sub-component (hence with the half-cycles 21 to 40).

The Pineapple Knot depicted in *The Braider*, Issue No. 14, Fig. 287, pg. 324 is a Compound Regular Nested Cylindrical Braid shown once again in Fig. 558. It is an interbraid of two components, each being a Perfect Regular Nested Knot.

The foundation knot (the first knot) comprises the half-cycles 1 to 16, and the second knot comprises the half-cycles 17 to 32. For the first knot $A = 2$, $x = 9$, $B^* = 4$, hence $P_{c_1} = 2A + x - 2 = 11$, and for the second knot $A = 2$, $x = 7$, $B^* = 4$, hence $P_{c_2} = 2A + x - 2 = 9$.

The first-return string-run of each component and the half-cycle pattern obtained from them is shown in Fig. 555. From the half-cycle pattern we can again assemble the two algorithm tables in Fig. 556 from which we can read the half-cycle braiding algorithms for each of the two components. These half-cycle braiding algorithms were given on pg. 324.

Instead of obtaining the half-cycle braiding algorithms from the two algorithm tables in Fig. 556, it may be preferred to obtain the half-cycle braiding algorithms of each component from its associated algorithm tables shown in Fig. 557. The upper two tables in Fig. 557 are associated with the foundation knot (the first component with the half-cycles 1 to 16), and the lower two tables in Fig. 557 are associated with the second knot (the component with the half-cycles 17 to 32).

† See *The Braider*, Issue No. 7, pg. 133: it follows from $a^* + b^* = b - 1$.

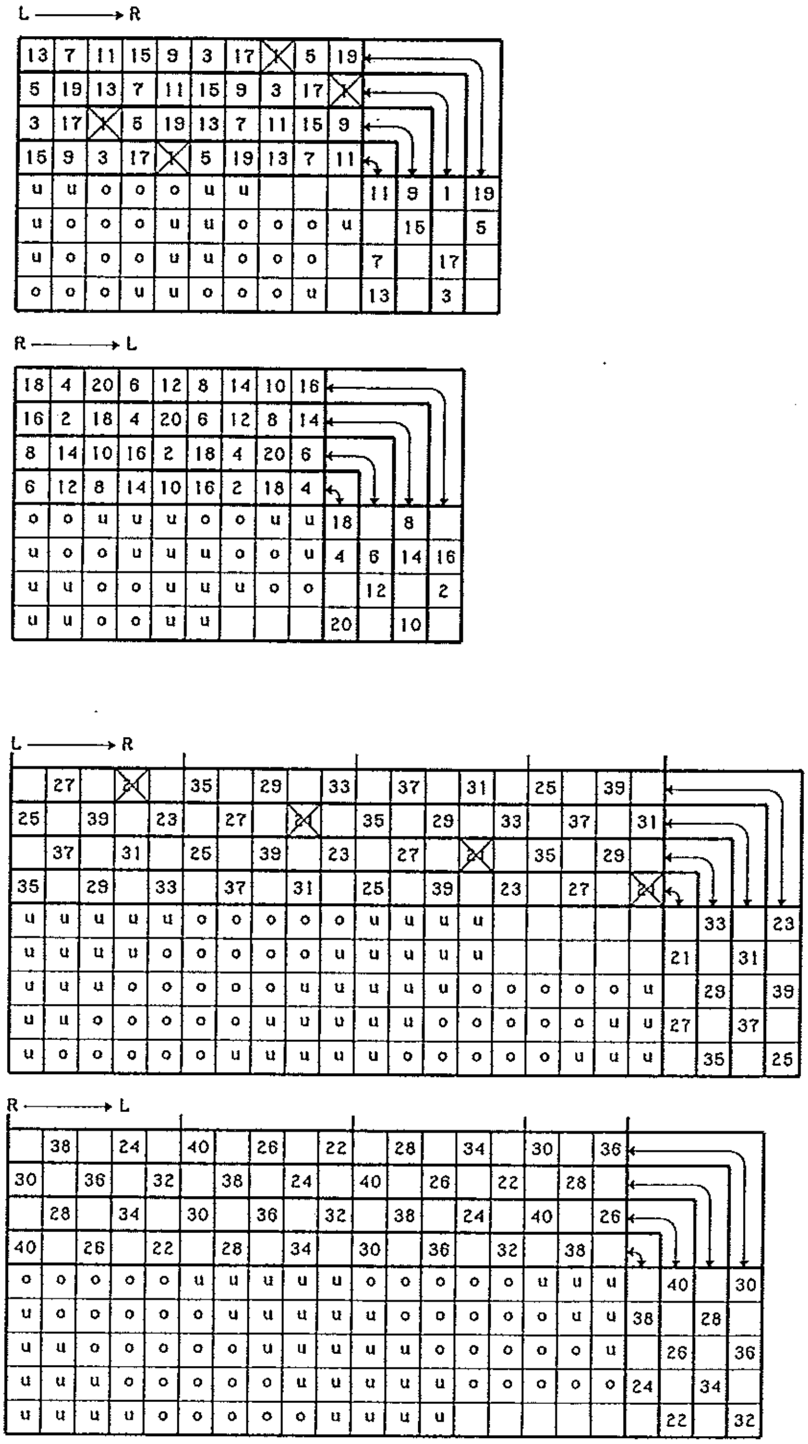


Fig. 554 — The algorithm tables of the sub-components in Example 5, pp. 677–679.

Since the bight-boundaries associated with one component are not associated with any other component, the algorithm tables of a component are easier to compile than the algorithm tables of a sub-component. Therefore compiling and working with algorithm tables of components as compared to compiling and working with algorithm tables of the overall braid offers greater advantages than the compiling and working with algorithm tables of sub-components as compared to compiling and working with algorithm tables of the overall component.

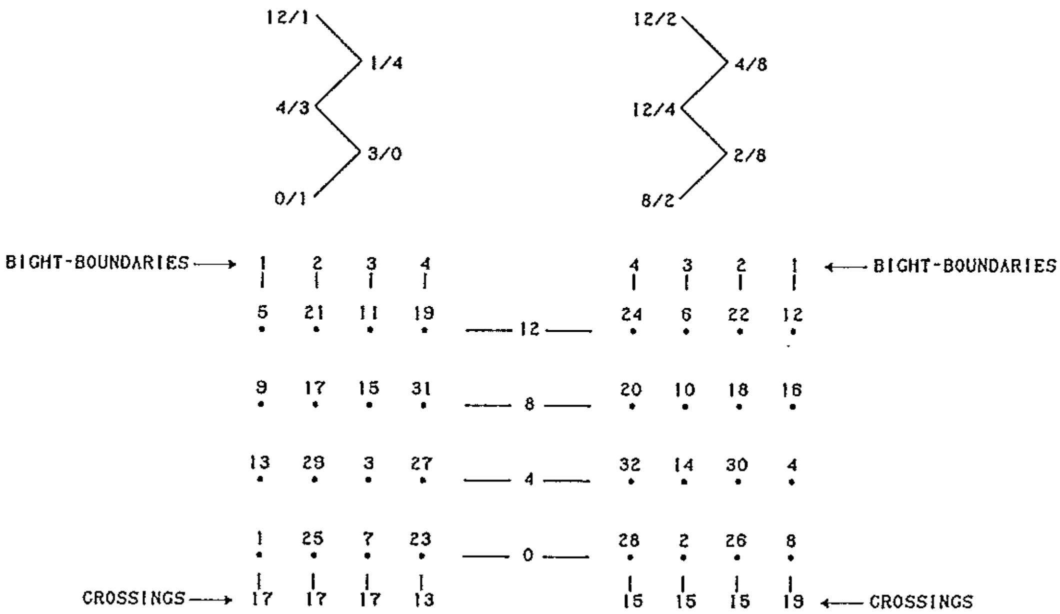


Fig. 555 — First-return string-runs and half-cycle pattern of the knot on pg. 324 and in Fig. 558.

L → R

1•	25	7	23	13	29	3	27	9	~~17~~	15	31	5	21	11	19	1•				
5	21	11	19	1•	25	7	23	13	29	3	27	9	~~17~~	15	31	5				
9	~~17~~	15	31	5	21	11	19	1•	25	7	23	13	29	3	27	9				
13	29	3	27	9	~~17~~	15	31	5	21	11	19	1•	25	7	23	13				
u	o	o	o	o	u	u	u	u	u	o	o	o	o	u	u	u	1	13	9	5
u	u	o	o	o	o	u	u	u	u	u	o	o	o	o	u	u	25	29	17	21
u	u	u	o	o	o	o	u	u	u	u	u	o	o	o	o	u	7	3	15	11
u	u	u	u	o	o	o	o	o	u	u	u	u					23	27	31	19

R → L

8	26	2	28	4	30	14	32	16	18	10	20	12	22	6	24	8	26	2				
12	22	6	24	8	26	2	28	4	30	14	32	16	18	10	20	12	22	6				
16	18	10	20	12	22	6	24	8	26	2	28	4	30	14	32	16	18	10				
4	30	14	32	16	18	10	20	12	22	6	24	8	26	2	28	4	30	14				
o	o	o	o	u	u	u	u	o	o	o	u	u	u	u	o	o	o	o	8	4	16	12
u	o	o	o	o	u	u	u	o	o	o	o	u	u	u					26	30	18	22
u	u	o	o	o	o	u	u	u	o	o	o	o	u	u					2	14	10	6
u	u	u	o	o	o	o	u	u	u	o	o	o	o	u					28	32	20	24

Fig. 556 — Algorithm tables of the knot on pg. 324 and in Fig. 558.

Note that the left bight-boundaries 1 & 3 and the right bight-boundaries 1 & 3 are associated with the string-run of the foundation knot (the first component), and that the left bight-boundaries 2 & 4 and the right bight-boundaries 2 & 4 are associated with the second component.

L ⟶ R

X	7	13	3	9	15	5	11	X				
5	11	X	7	13	3	9	15	5				
9	15	5	11	X	7	13	3	9				
13	3	9	15	5	11	X	7	13				
u	o	o	u	u	o	o	u	u	1	13	9	5
u	u	o	o	u	u	o	o	u	7	3	15	11

R ⟶ L

8	2	4	14	16	10	12	6	8	2				
12	6	8	2	4	14	16	10	12	6				
16	10	12	6	8	2	4	14	16	10				
4	14	16	10	12	6	8	2	4	14				
o	o	u	u	o	o	u	u	o	o	8	4	16	12
u	o	o	u	u	o	o	u			2	14	10	6

L ⟶ R

	25		23		29		27		X		31		21		19					
	21		19		25		23		29		27		X		31					
	X		31		21		19		25		23		29		27					
	29		27		X		31		21		19		25		23					
u	u	o	o	o	o	u	u	u	u	u	o	o	o	o	u	u	25	29	17	21
u	u	u	u	o	o	o	o	o	u	u	u	u					23	27	31	19

R ⟶ L

	26		28		30		32		16		20		22					
	22		24		26		28		30		32		18					
	18		20		22		24		26		28		30					
	30		32		18		20		22		24		26					
u	o	o	o	o	u	u	u	o	o	o	o	u	u	u	26	30	18	22
u	u	u	o	o	o	o	u	u	u	o	o	o	o	u	28	32	20	24

Fig. 557 — Algorithm tables for the components of the knot on pg. 324 and in Fig. 558.

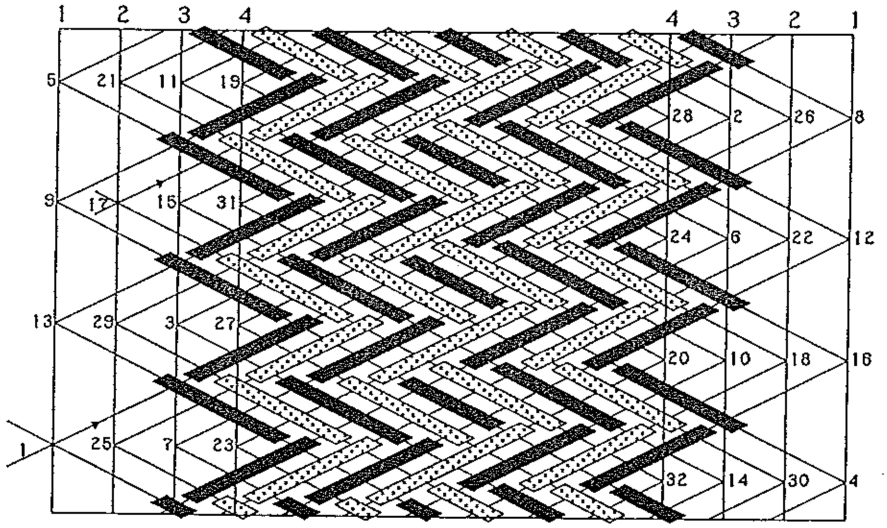
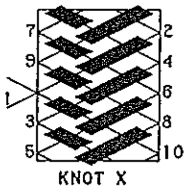


Fig. 558 — The Compound Regular Nested Cylindrical Braid on pg. 324.

Let's revisit the Regular Nested Cylindrical Braids in Issues No. 16, pg. 367, No. 13, pg. 298, and No. 12, pg. 272 of *The Braider*, and see what their associated algorithm tables are.

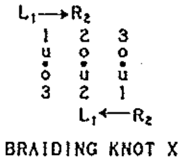
The Braider, Issue No. 16, pg. 367:

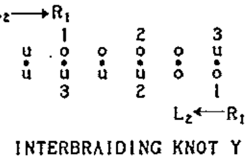
The uppermost Glen Vandy Knot on pg. 367 is an interbraid of the knots X and Y in Fig. 559. Both these knots are column-coded Regular Knots with $p/b = 4/5$, and with Euclid's algorithm, the path-formula, and the path in the RKT are as shown in Fig. 559. From the path in the RKT follows that $\Delta^* = 1$, and hence the algorithm diagram of knot X as the foundation knot is as shown in Fig. 559. Instead of its algorithm diagram we may prefer to use its depicted algorithm tables. Knot Y is the second knot, which is to be interbraided with knot X, and hence its algorithm diagram is as shown in Fig. 559. Again we may prefer to use its depicted algorithm tables. From the algorithm diagrams, or from the algorithm tables, we read and compile the uppermost half-cycle braiding algorithms shown in Fig. 313 on pg. 367.

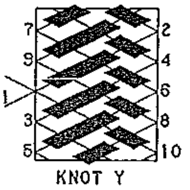


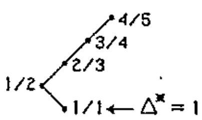
0	5	4
1	4	5
4	1	4

[0; 1, 3, 1]









BRAIDING KNOT X

$L_1 \rightarrow R_2$

1	2	3	ODD
u	o	o	HALF-CYCLES

$R_2 \rightarrow L_1$

1	2	3	EVEN
u	u	o	HALF-CYCLES

INTERBRAIDING KNOT Y

$L_2 \rightarrow R_1$

	1		2		3	ODD
u	o	o	o	o	u	HALF-CYCLES

$R_1 \rightarrow L_2$

1		2		3		EVEN
o	o	u	u	u	u	HALF-CYCLES

Fig. 559 — The two interbraided Regular Knots X and Y.

The general string-run specification for this Glen Vandy Knot is $(2/6/2)\{12/21\}B$, where $B = 8n + 2$ or $B = 8n + 6$ with n a whole number.

For $B = 8n + 2$ the interbraided Regular Knots each have $p/b = 4/(4n+1)$, while for $B = 8n + 6$ the interbraided Regular Knots each have $p/b = 4/(4n+3)$.

0	4n+1	4
n	4	4n+1
4	1	4

[0; n, 3, 1]

0	4n+3	4
n	4	4n+3
1	3	4
3	1	3

[0; n, 1, 2, 1]

BRAIDING KNOT X WITH b=4n+1

$L_1 \rightarrow R_2$

n	2n	3n	ODD
u	o	o	HALF-CYCLES

$R_2 \rightarrow L_1$

n	2n	3n	EVEN
u	u	o	HALF-CYCLES

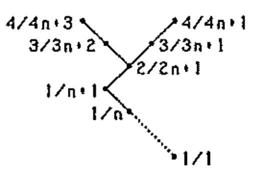
INTERBRAIDING KNOT Y WITH b=4n+1

$L_2 \rightarrow R_1$

	n		2n		3n	ODD
u	o	o	o	o	u	HALF-CYCLES

$R_1 \rightarrow L_2$

n		2n		3n		EVEN
o	o	u	u	u	u	HALF-CYCLES



BRAIDING KNOT X WITH b=4n+3

$L_1 \rightarrow R_2$

3n+2	2n+1	n	ODD
u	o	o	HALF-CYCLES

$R_2 \rightarrow L_1$

3n+2	2n+1	n	EVEN
u	u	o	HALF-CYCLES

INTERBRAIDING KNOT Y WITH b=4n+3

$L_1 \rightarrow R_2$

	3n+2		2n+1		n	ODD
u	o	o	o	o	u	HALF-CYCLES

$R_2 \rightarrow L_1$

3n+2		2n+1		n		EVEN
o	o	u	u	u	u	HALF-CYCLES

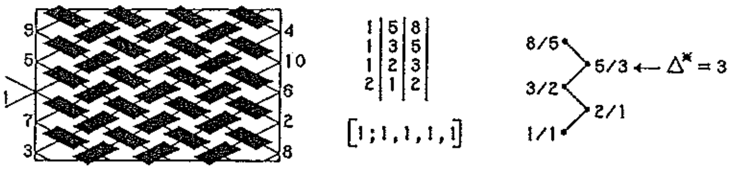
Fig. 560 — The interbraided Regular Knots for $B = 8n + 2$ and for $B = 8n + 6$.

In Fig. 560 are shown Euclid's algorithm, path-formula, and path in the RKT for $p/b = 4/(4n+1)$ and for $p/b = 4/(4n+3)$. From the paths in the RKT it follows that $\Delta^* = n$ for $p/b = 4/(4n+1)$, and $\Delta^* = 3n+2$ for $p/b = 4/(4n+3)$.

The algorithm tables associated with each interbraided pair of Regular Knots can now readily be drawn up as shown in Fig. 560.

The Braider, Issue No. 13, pg. 298:

This Regular Nested Cylindrical Braid is an interbraid of three identical over–under coded Regular Knots, each with $p/b = 8/5$. Euclid's algorithm, path-formula, and the path in the RKT for $p/b = 8/5$ are shown in Fig. 561.



$L_1 \to R_3$

3	1	4	2	0	3	1	ODD
u	o	u	o	u	o	u	HALF-CYCLES

$R_3 \to L_1$

3	1	4	2	0	3	1	EVEN
o	u	o	u	o	u	o	HALF-CYCLES

$L_2 \to R_2$

	3		1		4		2		0		3		1	ODD
u	u	o	o	u	u	o	o	u	u	u	o	o	u	HALF-CYCLES

$R_2 \to L_2$

3		1		4		2		0		3		1		EVEN
o	o	u	u	o	o	u	u	o	o	u	o	o	u	HALF-CYCLES

$L_3 \to R_1$

		3			1			4			2			0			3			1	ODD
u	u	u	o	o	o	u	u	u	u	o	o	o	u	u	u	u	o	o	o	u	HALF-CYCLES

$R_1 \to L_3$

3			1			4			2			0			3			1			EVEN
o	o	o	u	u	u	o	o	o	u	u	o	o	o	u	u	o	o	o	u	u	HALF-CYCLES

Fig. 561 — The three interbraided over–under coded Regular Knots $p/b = 5/8$.

From the path in the RKT it follows that $\Delta^* = 3$, and thus we can now draw up the algorithm tables.

In Fig. 561 the algorithm tables for the foundation knot (the first knot) are at the upper-right, the algorithm tables for the second knot are at the lower-left, and the algorithm tables for the third knot are at the lower-right. We can read and compile from the algorithm tables the half-cycle braiding algorithms shown on pg. 298.

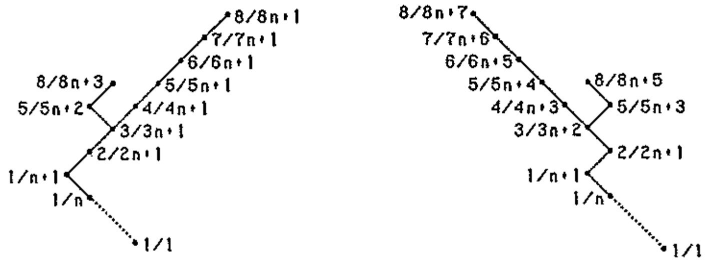
$(22/20/22)\{132/312\}B$, where $B = 24n+3$ or $B = 24n+9$ or $B = 24n+15$ or $B = 24n+21$ with n a whole number, is the general string-run specification for this Regular Nested Knot. The respective p/b-values of the three identical interbraided over–under coded Regular Knots are therefore $8/(8n+1)$ or $8/(8n+3)$ or $8/(8n+5)$ or $8/(8n+7)$. Euclid's algorithms for these p/b-values, path-fomulae, and paths in the RKT are shown in Fig. 562. From the paths in the RKT it follows that $\Delta^* = n$ for $p/b = 8/(8n+1)$, $\Delta^* = 3n+1$ for $p/b = 8/(8n+3)$, $\Delta^* = 5n+3$ for $p/b = 8/(8n+5)$, and $\Delta^* = 7n+6$ for $p/b = 8/(8n+7)$. This enables us to draw up the algorithm tables.

The upper two algorithm tables are associated with the foundation knot (the first knot), the central two algorithm tables with the second knot, and the lower two algorithm tables with the third knot.

The Braider, Issue No. 12, pg. 272:

This Regular Nested Cylindrical Braid is an interbraid of two identical over–under coded Regular Knots X with $p/b = 8/5$ and one over–under coded Regular Knot Y with $p/b = 6/5$. Euclid's algorithms, path-formulae, and the paths in the RKT for $p/b = 8/5$ and $p/b = 6/5$ are shown in Fig. 563. From the paths in the RKT it follows that $\Delta^* = 3$ for $p/b = 8/5$, and $\Delta^* = 4$ for $p/b = 6/5$. This enables us to draw up the algorithm tables.

```
0|8n+1|8    |      0|8n+3|8    |      0|8n+5|8    |      0|8n+7|8
n|8   |8n+1 |      n|8   |8n+3 |      n|8   |8n+5 |      n|8   |8n+7
8|1   |8    |      2|3   |8    |      1|5   |8    |      1|7   |8
             |      1|2   |3    |      1|3   |5    |      7|1   |7
             |      2|1   |2    |      1|2   |3    |
                                       2|1   |2    |
[0;n,7,1]          [0;n,2,1,1,1]      [0;n,1,1,1,1,1]     [0;n,1,6,1]
```



$L_1 \rightarrow R_3$

n	2n	3n	4n	5n	6n	7n	b=8n+1	ODD HALF-CYCLES
3n+1	6n+2	n	4n+1	7n+2	2n	5n+1	b=8n+3	
5n+3	2n+1	7n+4	4n+2	n	6n+3	3n+1	b=8n+5	
7n+6	6n+5	5n+4	4n+3	3n+2	2n+1	n	b=8n+7	
u	o	u	o	u	o	u		

$R_3 \rightarrow L_1$

n	2n	3n	4n	5n	6n	7n	b=8n+1	EVEN HALF-CYCLES
3n+1	6n+2	n	4n+1	7n+2	2n	5n+1	b=8n+3	
5n+3	2n+1	7n+4	4n+2	n	6n+3	3n+1	b=8n+5	
7n+6	6n+5	5n+4	4n+3	3n+2	2n+1	n	b=8n+7	
o	u	o	u	o	u	o		

$L_2 \rightarrow R_2$

	n		2n		3n		4n		5n		6n		7n	b=8n+1	ODD HALF-CYCLES
	3n+1		6n+2		n		4n+1		7n+2		2n		5n+1	b=8n+3	
	5n+3		2n+1		7n+4		4n+2		n		6n+3		n+1	b=8n+5	
	7n+6		6n+5		5n+4		4n+3		3n+2		2n+1		n	b=8n+7	
u	u	o	o	u	u	o	o	u	u	u	o	o	u		

$R_2 \rightarrow L_2$

n		2n		3n		4n		5n		6n		7n		b=8n+1	EVEN HALF-CYCLES
3n+1		6n+2		n		4n+1		7n+2		2n		5n+1		b=8n+3	
5n+3		2n+1		7n+4		4n+2		n		6n+3		n+1		b=8n+5	
7n+6		6n+5		5n+4		4n+3		3n+2		2n+1		n		b=8n+7	
o	o	u	u	o	o	u	u	o	o	u	o	o	u		

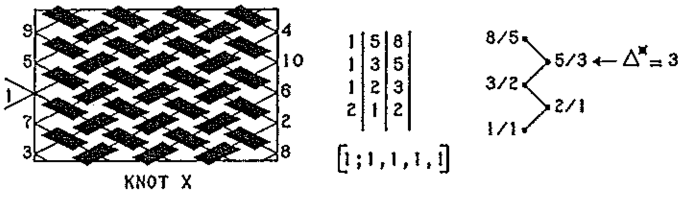
$L_3 \rightarrow R_1$

		n			2n			3n			4n			5n			6n			7n	b=8n+1	ODD HALF-CYCLES
		3n+1			6n+2			n			4n+1			7n+2			2n			5n+1	b=8n+3	
		5n+3			2n+1			7n+4			4n+2			n			6n+3			n+1	b=8n+5	
		7n+6			6n+5			5n+4			4n+3			3n+2			2n+1			n	b=8n+7	
u	u	u	o	o	o	u	u	u	u	o	o	o	u	u	u	u	o	o	o	u		

$R_1 \rightarrow L_3$

n			2n			3n			4n			5n			6n			7n			b=8n+1	EVEN HALF-CYCLES
3n+1			6n+2			n			4n+1			7n+2			2n			5n+1			b=8n+3	
5n+3			2n+1			7n+4			4n+2			n			6n+3			n+1			b=8n+5	
7n+6			6n+5			5n+4			4n+3			3n+2			2n+1			n			b=8n+7	
o	o	o	u	u	u	o	o	o	u	u	o	o	o	u	u	o	o	o	u	u		

Fig. 562 — The interbraided over–under coded Regular Knots for $B = 24n + 3$, $B = 24n + 9$, $B = 24n + 15$, and $B = 24n + 21$.

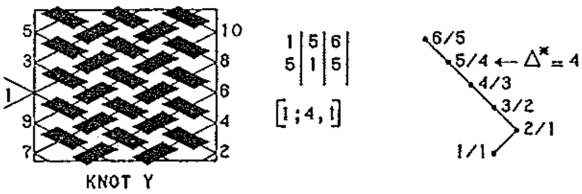


L₁ → R₂

3	1	4	2	0	3	1	ODD HALF-CYCLES
u	o	u	o	u	o	u	

R₂ → L₁

3	1	4	2	0	3	1	EVEN HALF-CYCLES
o	u	o	u	o	u	o	



$L_2 \rightarrow R_1$

	3		1		4		2		0		3		1	ODD HALF-CYCLES
u	u	o	o	u	u	u	o	o	u	u	o	o	u	

$R_1 \rightarrow L_2$

3		1		4		2		0		3		1		EVEN HALF-CYCLES
o	o	u	u	o	o	u	o	o	u	u	o	o	u	

$L_3 \rightarrow R_3$

		4			3			2			1			0			ODD HALF-CYCLES
u	u	u	o	o	o	u	o	u	o	u	o	o	o	u	u	u	

$R_3 \rightarrow L_3$

		4			3			2			1			0			EVEN HALF-CYCLES
u	u	o	o	o	u	u	o	o	o	u	u	o	o	o	u	u	

Fig. 563 — The three interbraided over–under coded Regular Knots $p/b = 8/5$, $p/b = 8/5$, and $p/b = 6/5$.

In Fig. 563 the upper-right two algorithm tables are associated with the over–under coded foundation knot $p/b = 8/5$, the lower-left two algorithm tables are associated with the second $p/b = 8/5$ over–under coded Regular Knot, and the lower-right two algorithm tables are associated with the third over–under coded $p/b = 6/5$ Regular Knot. From the algorithm tables we can read and compile the half-cycle braiding algorithms shown on pg. 272.

$(22/18/22)\{132/231\}B$, where $B = 18m + 3$ or $B = 18m + 15$ with m a whole number, is the general string-run specification for this Regular Nested Knot. The respective p/b-values of the three interbraided over–under coded Regular Knots are therefore $8/(6m+1)$, $8/(6m+1)$, and $6/(6m+1)$ or $8/(6m+5)$, $8/(6m+5)$, and $6/(6m+5)$. For these p/b-values Euclid's algorithms, path-fomulae, and paths in the RKT are shown in Fig. 564.

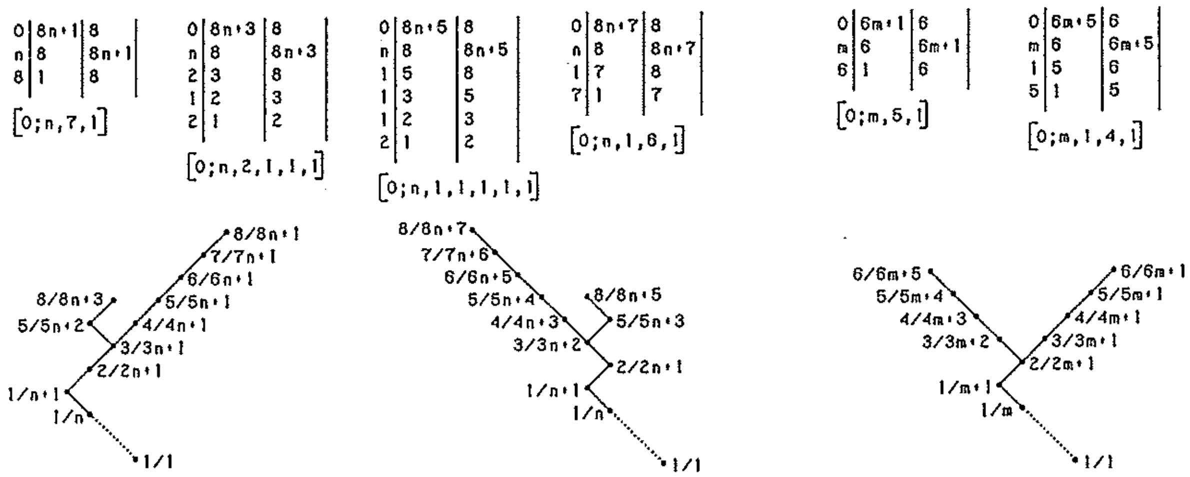
$n = \frac{6m+1-|6m+1|_8}{8}$ for $b = 6m + 1$, then from the paths in the RKT it follows that:
$\Delta^* = n$ for $p/b = 8/(6m + 1) = 8/(8n + 1)$,
$\Delta^* = 3n + 1$ for $p/b = 8/(6m + 1) = 8/(8n + 3)$,
$\Delta^* = 5n + 3$ for $p/b = 8/(6m + 1) = 8/(8n + 5)$,
$\Delta^* = 7n + 6$ for $p/b = 8/(6m + 1) = 8/(8n + 7)$,

$n = \frac{6m+5-|6m+5|_8}{8}$ for $b = 6m + 5$, then from the paths in the RKT it follows that:
$\Delta^* = n$ for $p/b = 8/(6m + 5) = 8/(8n + 1)$,
$\Delta^* = 3n + 1$ for $p/b = 8/(6m + 5) = 8/(8n + 3)$,
$\Delta^* = 5n + 3$ for $p/b = 8/(6m + 5) = 8/(8n + 5)$,
$\Delta^* = 7n + 6$ for $p/b = 8/(6m + 5) = 8/(8n + 7)$,

Furthermore it follows from the paths in the RKT that:
$\Delta^* = m$ for $p/b = 6/(6m + 1)$,
$\Delta^* = 5m + 4$ for $p/b = 6/(6m + 5)$.

This enables us to draw up the algorithm tables.

The upper two algorithm tables are associated with the foundation knot (the first knot), the central two algorithm tables with the second knot, and the lower two algorithm tables with the third knot.



$L_1 \rightarrow R_2$

n	2n	3n	4n	5n	6n	7n	b=8n+1	ODD HALF-CYCLES
3n+1	6n+2	n	4n+1	7n+2	2n	5n+1	b=8n+3	
5n+3	2n+1	7n+4	4n+2	n	6n+3	3n+1	b=8n+5	
7n+6	6n+5	5n+4	4n+3	3n+2	2n+1	n	b=8n+7	
u	o	u	o	u	o	u		

$R_2 \rightarrow L_1$

n	2n	3n	4n	5n	6n	7n	b=8n+1	EVEN HALF-CYCLES
3n+1	6n+2	n	4n+1	7n+2	2n	5n+1	b=8n+3	
5n+3	2n+1	7n+4	4n+2	n	6n+3	3n+1	b=8n+5	
7n+6	6n+5	5n+4	4n+3	3n+2	2n+1	n	b=8n+7	
o	u	o	u	o	u	o		

FOR $b' = 6m+1 \rightarrow n = \frac{6m+1-|6m+1|_8}{8} \rightarrow b = 8n + |6m+1|_8$

FOR $b' = 6m+5 \rightarrow n = \frac{6m+5-|6m+5|_8}{8} \rightarrow b = 8n + |6m+5|_8$

$L_2 \rightarrow R_1$

	n		2n		3n		4n		5n		6n		7n	b=8n+1	ODD HALF-CYCLES
	3n+1		6n+2		n		4n+1		7n+2		2n		5n+1	b=8n+3	
	5n+3		2n+1		7n+4		4n+2		n		6n+3		n+1	b=8n+5	
	7n+6		6n+5		5n+4		4n+3		3n+2		2n+1		n	b=8n+7	
u	u	o	o	u	u	u	o	o	u	u	o	o	u		

$R_1 \rightarrow L_2$

n		2n		3n		4n		5n		6n		7n		b=8n+1	EVEN HALF-CYCLES
3n+1		6n+2		n		4n+1		7n+2		2n		5n+1		b=8n+3	
5n+3		2n+1		7n+4		4n+2		n		6n+3		n+1		b=8n+5	
7n+6		6n+5		5n+4		4n+3		3n+2		2n+1		n		b=8n+7	
o	o	u	u	o	o	u	o	o	u	u	o	o	u		

$L_3 \rightarrow R_3$

		m			2m			3m			4m			5m			b'=6m+1	ODD HALF-CYCLES
		5m+4			4m+3			3m+2			2m+1			m			b'=6m+5	
u	u	u	o	o	o	u	o	u	o	u	o	o	o	u	u	u		

$R_3 \rightarrow L_3$

		m			2m			3m			4m			5m			b'=6m+1	EVEN HALF-CYCLES
		5m+4			4m+3			3m+2			2m+1			m			b'=6m+5	
u	u	o	o	o	u	u	o	o	o	u	u	o	o	o	u	u		

Fig. 564 — The interbraided over–under coded Regular Knots for $B = 18m + 3$ and $B = 18m + 15$.

Pitfalls in knot design

The Pampas Button, much in vogue in the gaucho regions of the Argentine Pampas according to Bruce Grant and described in his *Encyclopedia of Rawhide and Leather Braiding* on pp. 438–439, suffers not only from a similar central lopsided design as the small Pampas knot we discussed in *The Braider*, Issue No. 30, pp. 704–706, but has furthermore two dissimilar end-sections which, due to the small difference in their lengths, cannot be regarded as being satisfactory for an orientation purpose. The grid-diagram of this Pampas Button is shown in Fig. 565.

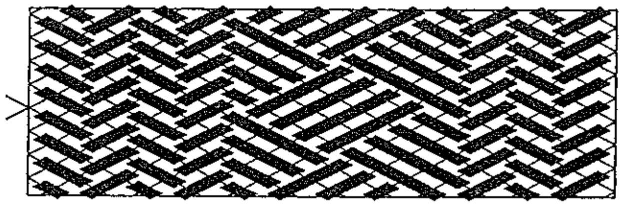


Fig. 565 — The Pampas Button in the *Encyclopedia of Rawhide and Leather Braiding.*

This Pampas Button with $p/b = 25/8$ is most likely the result of using "the running method of making woven knots", or what may be called pattern-braiding, in the design of knots (note that we have also here, as with the small Pampas knot, a typical case† of $p = nb + 1$ with $n = 3$).

The lopsided coding of the central section of the Pampas Button, shown by the leftmost diagram in Fig. 566, can readily be made symmetrical in two ways as shown by the central and rightmost diagrams in Fig. 566.

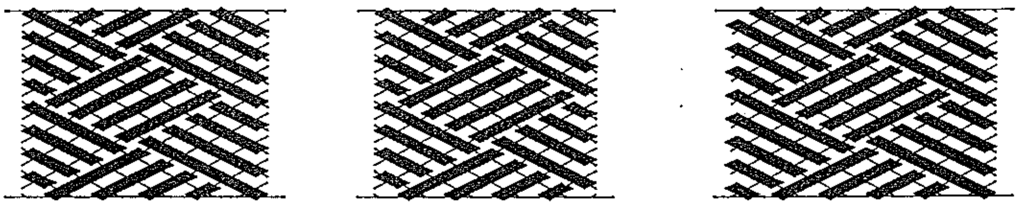


Fig. 566 — The coding of the central section of the Pampas Button and alternative modified coding arrangements.

Since the coding arrangements of the central section has a periodicity of 4 bights, we require b to be a multiple of 4. For the modified coding of the central section there are either 9 crossings (central coding arrangement in Fig. 566) or 11 crossings (rightmost coding arrangement in Fig. 566) on a half-cycle section, and hence if we make the overall knot symmetrical then the total number of crossings on a half-cycle will be odd and thus p will be even.‡ Since $b = 4n$, where n is a natural number, hence even, and since p is even with each one of the modified central coding arrangements, it follows that a single string knot is not possible (g.c.d. $(p, b) > 1$). Thus in order to obtain a single string knot we must modify the coding of the central section further.

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† Refer to *The Braider*, Issue No. 30, pg. 705.

‡ The total number of crossings on a half-cycle is equal to number of crossings on a half-cycle section in the central-section coding arrangement plus twice the number of crossings on a half-cycle section in an end-section coding arrangement. The number of parts p is equal to the total number of crossings on a half-cycle plus 1.

Instead of using the central coding arrangement with a periodicity of 4 bights, we can use a similar coding arrangement with a periodicity of 3 bights. An example of such a similar arrangement is depicted in Fig. 567. We have furthermore assured that the coding of the overall knot will be symmetrical by using suitably coded end-sections.

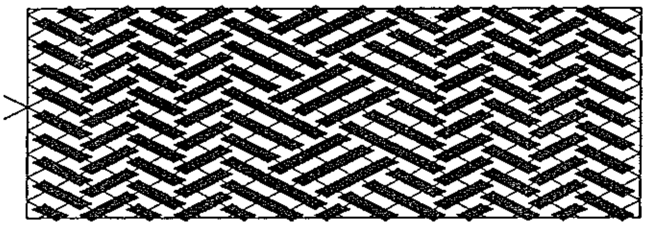

Fig. 567 — A single string Pampas type knot with a periodicity of 3 bights.

The algorithm tables of this knot are presented in Fig. 568. In these tables the i-values are above the horizontal thick line and the half-cycle numbers are to the right of the vertical thick line in line with their associated half-cycle coding sequences.

L ⟶ R BRAIDING UPWARDS

1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7			
u	u	o	o	u	u	o	o	u	u	u	o	o	o	u	u	u	o	o	u	u	o	o	u	u	1	7	13
u	u	o	o	u	u	o	o	u	u	u	u	o	o	o	u	u	o	o	u	u	o	o	u	u	3	9	15
u	u	o	o	u	u	o	o	u	u	o	o	o	u	u	u	u	o	o	u	u	o	o	u	u	5	11	17

R ⟶ L BRAIDING UPWARDS

1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7			
o	o	u	u	o	o	u	u	o	o	u	u	u	o	o	o	o	u	u	o	o	u	u	o	o	2	8	14
o	o	u	u	o	o	u	u	o	o	o	u	u	u	o	o	o	u	u	o	o	u	u	o	o	4	10	16
o	o	u	u	o	o	u	u	o	o	o	o	u	u	u	o	o	u	u	o	o	u	u	o	o	6	12	18

L ⟶ R BRAIDING DOWNWARDS

1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7			
o	o	u	u	o	o	u	u	o	o	u	u	u	o	o	o	o	u	u	o	o	u	u	o	o	1	7	13
o	o	u	u	o	o	u	u	o	o	o	u	u	u	o	o	o	u	u	o	o	u	u	o	o	3	9	15
o	o	u	u	o	o	u	u	o	o	o	o	u	u	u	o	o	u	u	o	o	u	u	o	o	5	11	17

R ⟶ L BRAIDING DOWNWARDS

1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7			
u	u	o	o	u	u	o	o	u	u	u	u	o	o	o	u	u	o	o	u	u	o	o	u	u	2	8	14
u	u	o	o	u	u	o	o	u	u	o	o	o	u	u	u	u	o	o	u	u	o	o	u	u	4	10	16
u	u	o	o	u	u	o	o	u	u	u	o	o	o	u	u	u	o	o	u	u	o	o	u	u	6	12	18

2	9	26
1	8	9
8	1	8

[2;1,7,1]

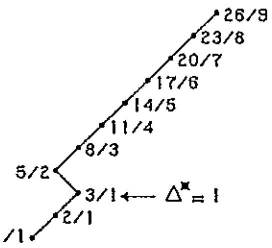

Fig. 568 — The algorithm tables of the Pampas type knot in Fig. 567.

From these algorithm tables we read the following half-cycle braiding algorithms:

1. Free run.
2. $o - u$.
3. $u - o$.
4. $3o - 2u$.
5. $3u - 2o$.
6. $5o - 2u - o$.
7. $5u - 2o - u$.
8. $2o - u - 2o - 4u - 2o$.
9. $2u - o - 4u - 2o - 2u$.
10. $2o - 2u - 3o - 4u - 2o - u$.
11. $2u - 2o - 2u - 5o - 2u - o$.
12. $2o - 2u - 5o - 4u - 2o - 2u$.
13. $2u - 2o - 4u - 5o - 2u - 2o$.
14. $2o - 2u - 4o - 3u - 2o - 2u - 2o - 2u - o$.
15. $2u - 2o - 6u - 5o - 2u - 2o - u$.
16. $2o - 2u - 2o - u - 3o - 3u - 2o - 2u - 2o - 2u - 2o$.
17. $2u - 2o - 2u - o - 2u - 3o - 3u - 2o - 2u - 2o - 2u$.
18. $2o - 2u - 2o - 2u - 4o - 3u - 2o - 2u - 2o - 2u - 2o$.

We would normally braid the knot by starting at the centre of the string-length, hence braid half the string-length upwards and the other half downwards. Thus starting at the centre of the string-length, we would braid the half-cycles 1–9 upwards, then continue with braiding the Standing End and hence the remaining half-cycles downwards:

10. $2u - 2o - 2u - 5o - 2u - o$.
11. $2o - 2u - 4o - 3u - 2o - u$.
12. $2u - 2o - 4u - 5o - 2u - 2o$.
13. $2o - 2u - 3o - 3u - o - 2u - 2o - 2u$.
14. $2u - 2o - 6u - 5o - 2u - 2o - u$.
15. $2o - 2u - 5o - 3u - o - 2u - 2o - 2u - o$.
16. $2u - 2o - 2u - o - 2u - 3o - 3u - 2o - 2u - 2o - 2u$.
17. $2o - 2u - 2o - u - 4o - 3u - o - 2u - 2o - 2u - 2o$.
18. $2u - 2o - 2u - 2o - 3u - 3o - 3u - 2o - 2u - 2o - 2u$.

In designing long knots we often 'couple' some shorter knots with symmetric coding-patterns together in series. For example, we could 'couple' in series with one of the previously discussed modified 4-bight periodic Pampas Buttons the *Three section Fan Knot or Botón Oriental* described by Bruce Grant in his *Encyclopedia of Rawhide and Leather Braiding*, pp. 430–431 and depicted below in Fig. 569.

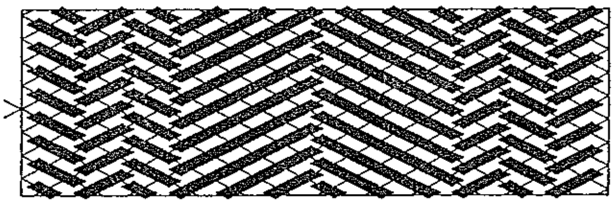

Fig. 569 — The Three section Fan Knot or Botón Oriental.

The resulting knot has then two end-sections and three sections between these end-sections. The central modified section of the Pampas Button and the central fan section of the Three section Fan Knot supply orientation to the knot and in order to accentuate this we should ensure that the central section of the overall knot (the section between the modified central section of the Pampas Button and the central section of the Three section Fan Knot) has some symmetric coding-pattern relative to its centre while the two end sections should have a balanced coding-pattern with respect to the overall knot. An easy way to achieve the latter is to make the two end sections symmetric with respect to each other, hence we could choose two-pass Headhunter's-coded end sections as those found in the Three section Fan Knot and Pampas Button, or two-pass Gaucho-coded end sections as the one found in the Pampas Button. Whichever choice we make, the combined number of intersection-columns in the two end sections, the modified central Pampas Button section and the central fan section is odd (since the number of intersection-columns is even in each end section and in the central fan section, but odd only in the modified central Pampas Button section). Thus for a single string knot the number of intersection-columns in the section between the modified central section of the Pampas Button and the central section of the Three section Fan Knot must be odd (the number of parts in the overall knot must be odd since the number of bights has to be a multiple of 4 due to the 4-bight periodicity of the modified central Pampas Button section). Hence for the central section of the overall knot we cannot use the two-pass Gaucho-coded section resulting from the two 'coupled' two-pass Headhunter's-coded end sections of the Pampas Button and the Three section Fan Knot. We must, for example, use instead a two-pass Headhunter's-coded section in which the central two intersection-columns are replaced by three intersection-columns as shown in the grid-diagrams of Figs. 570 & 571.

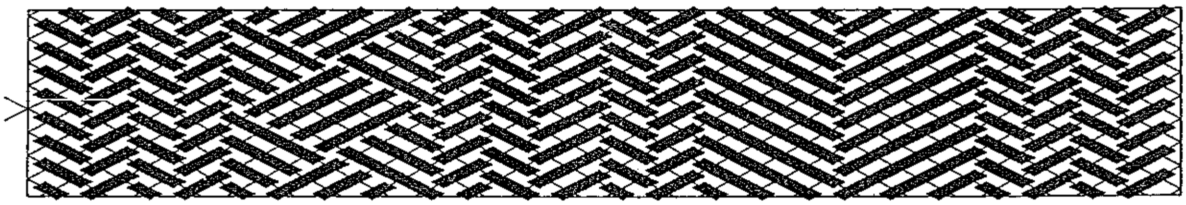

Fig. 570 — The long knot with the central depicted modification in Fig. 566.

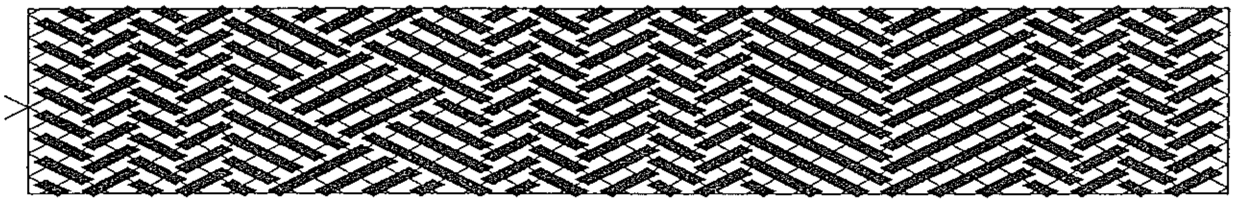

Fig. 571 — The long knot with the rightmost depicted modification in Fig. 566.

The algorithm tables of the knot in Fig. 570 are shown in Fig. 572. From these tables we read the following half-cycle braiding algorithms:

1. Free run.
2. $6o$.
3. $o - u - o - u - 2o$.
4. $6o - u - 5o$.
5. $2o - 2u - 2o - 2u - 4o$.
6. $u - 2o - u - 2o - u - 2o - 2u - 4o - u - 2o$.
7. $u - 2o - 3u - 3o - 3u - 3o - u - 2o$.
8. $2u - 2o - 2u - 2o - 2u - 3o - 2u - 5o - 2u - 2o$.
9. $2u - 6o - u - 3o - 4u - 4o - 2u - 2o$.
10. $o - 2u - 2o - 3u - 3o - 2u - 4o - 2u - o - 4u - 2o - 2u - 2o$.

11. $o - 2u - 6o - 3u - 4o - 4u - 6o - 2u - 2o$.
12. $2o - 2u - 2o - 4u - 4o - 2u - 2o - u - 2o - 2u - o - 4u - 4o - 2u - 2o$.
13. $2o - 2u - 6o - 2u - o - 2u - 5o - 4u - 8o - 2u - 2o$.
14. $u - 2o - 2u - 2o - 5u - 5o - 2u - 2o - 2u - 2o - 2u - o - 4u - 3o - u - 2o - 2u - 2o$.
15. $u - 2o - 2u - 6o - 3u - 2o - 2u - 3o - u - 2o - 5u - 6o - u - 2o - 2u - 2o$.
16. $2u - 2o - 2u - 2o - 6u - 6o - 2u - 2o - 3u - 2o - 2u - o - 4u - 4o - 2u - 2o - 2u - 2o$.

Starting at the centre of the string, braid upwards the half-cycles 1–8. Then continue with braiding the Standing End downwards:

9. $2o - 6u - o - 3u - 4o - 4u - 2o - 2u$.
10. $u - 2o - 2u - 3o - 3u - 2o - 4u - 2o - u - 4o - 2u - 2o - 2u$.
11. $u - 2o - 6u - 3o - 4u - 4o - 6u - 2o - 2u$.
12. $2u - 2o - 2u - 4o - 4u - 2o - 2u - o - 2u - 2o - u - 4o - 4u - 2o - 2u$.
13. $2u - 2o - 6u - 2o - u - 2o - 5u - 4o - 8u - 2o - 2u$.
14. $o - 2u - 2o - 2u - 5o - 5u - 2o - 2u - 2o - 2u - 2o - u - 4o - 3u - o - 2u - 2o - 2u$.
15. $o - 2u - 2o - 6u - 3o - 2u - 2o - 3u - o - 2u - 5o - 6u - o - 2u - 2o - 2u$.
16. $2o - 2u - 2o - 2u - 6o - 6u - 2o - 2u - 3o - 2u - 2o - u - 4o - 4u - 2o - 2u - 2o - 2u$.

L → R

BRAIDING UPWARDS

7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0		
u	u	o	o	u	u	o	o	u	u	u	u	o	o	o	o	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	o	o	1	9
u	u	o	o	u	u	o	o	u	u	u	o	o	o	o	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	o	o	3	11
u	u	o	o	u	u	o	o	u	u	o	o	o	o	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	o	o	5	13
u	u	o	o	u	u	o	o	u	o	o	o	o	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	o	o	7	15

R → L

BRAIDING UPWARDS

7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0		
u	u	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	u	u	u	u	o	u	u	o	o	u	u	o	o	2	10
u	u	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	u	u	u	u	o	o	u	u	o	o	u	u	o	o	4	12
u	u	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	u	u	u	u	o	o	o	u	u	o	o	u	u	o	o	6	14
u	u	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	u	u	u	u	o	o	o	o	u	u	o	o	u	u	o	o	8	16

L → R

BRAIDING DOWNWARDS

7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0		
o	o	u	u	o	o	u	u	o	o	o	o	u	u	u	u	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	u	u	1	9
o	o	u	u	o	o	u	u	o	o	o	u	u	u	u	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	u	u	3	11
o	o	u	u	o	o	u	u	o	o	u	u	u	u	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	u	u	5	13
o	o	u	u	o	o	u	u	o	u	u	u	u	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	u	u	7	15

R → L

BRAIDING DOWNWARDS

7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0	7	6	5	4	3	2	1	0		
o	o	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	o	o	o	o	u	o	o	u	u	o	o	u	u	2	10
o	o	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	o	o	o	o	u	u	o	o	u	u	o	o	u	u	4	12
o	o	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	o	o	o	o	u	u	u	o	o	u	u	o	o	u	u	6	14
o	o	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	o	o	o	o	u	u	u	u	o	o	u	u	o	o	u	u	8	16

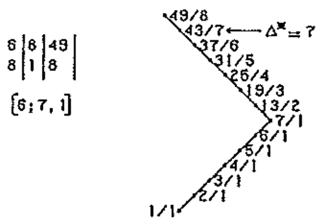

Fig. 572 — The algorithm tables of the knot in Fig. 570.

The algorithm tables of the knot in Fig. 571 are shown in Fig. 573.

L⟶R BRAIDING UPWARDS

5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2		
u	u	o	o	u	u	o	o	u	u	u	u	u	o	o	o	o	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	o	o	1	9
u	u	o	o	u	u	o	o	u	u	o	o	o	o	u	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	o	o	3	11
u	u	o	o	u	u	o	o	u	u	u	o	o	o	o	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	o	o	5	13
u	u	o	o	u	u	o	o	u	u	u	u	o	o	o	o	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	o	o	7	15

R⟶L BRAIDING UPWARDS

5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2		
u	u	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	u	u	u	u	o	o	o	u	u	o	o	u	u	o	o	2	10
u	u	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	o	u	u	u	u	o	o	u	u	o	o	u	u	o	o	4	12
u	u	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	u	u	u	u	o	o	o	o	o	u	u	o	o	u	u	o	o	6	14
u	u	o	o	u	u	o	o	u	u	u	u	u	u	o	o	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	u	u	u	u	o	o	o	o	u	u	o	o	u	u	o	o	8	16

L⟶R BRAIDING DOWNWARDS

5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2		
o	o	u	u	o	o	u	u	o	o	o	o	u	u	u	u	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	u	u	1	9
o	o	u	u	o	o	u	u	o	o	o	o	o	u	u	u	u	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	u	u	3	11
o	o	u	u	o	o	u	u	o	o	u	u	u	u	o	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	u	u	5	13
o	o	u	u	o	o	u	u	o	o	o	u	u	u	u	o	o	o	o	u	u	o	o	u	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	u	u	7	15

R⟶L BRAIDING DOWNWARDS

5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2	7	4	1	6	3	0	5	2		
o	o	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	o	o	o	o	u	u	u	u	o	o	u	u	o	o	u	u	2	10
o	o	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	o	o	o	o	u	u	u	o	o	u	u	o	o	u	u	4	12
o	o	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	u	u	u	o	o	o	o	u	u	o	o	u	u	o	o	u	u	6	14
o	o	u	u	o	o	u	u	o	o	o	o	o	o	u	u	u	u	u	u	o	o	u	u	o	o	o	u	u	o	o	u	u	o	o	o	o	u	u	u	u	u	o	o	u	u	o	o	u	u	8	16

6	8	51
2	3	8
1	2	3
2	1	2

[6;2,1,1,1]

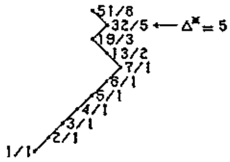

Fig. 573 — The algorithm tables of the knot in Fig. 571.

From these tables we read the following half-cycle braiding algorithms:

1. Free run.
2. $5o - u$.
3. $o - u - o - u - o - u$.
4. $u - o - u - o - u - 3o - 2u - o - u$.
5. $u - 2o - u - 3o - u - 3o - u$.
6. $2u - o - 2u - 2o - u - o - u - 2o - 2u - 3o - u - o$.
7. $2u - o - u - 2o - u - 4o - 2u - 4o - u - o$.
8. $2u - 2o - 2u - 3o - u - 2o - u - o - u - 2o - u - 4o - 2u - o$.
9. $2u - 2o - 2u - 2o - u - o - u - 3o - 3u - 5o - 2u - o$.
10. $u - o - u - 2o - 3u - 4o - u - 2o - u - 2o - u - 2o - 3u - 2o - u - o - 2u - o$.
11. $u - o - u - 2o - u - 2o - 3u - 2o - u - 2o - u - o - 4u - 4o - u - o - 2u - o$.
12. $2u - o - u - 2o - 4u - 5o - u - 2o - 2u - 2o - u - 4o - 3u - 2o - u - o - 2u - 2o$.
13. $2u - o - u - 2o - 2u - 3o - 3u - 2o - u - 3o - u - o - 5u - 5o - u - o - 2u - 2o$.
14. $2u - o - 2u - 2o - 5u - 5o - 2u - 2o - 2u - 2o - 2u - 2o - 3u - 5o - u - 2o - 2u - 2o$.
15. $2u - o - 2u - 2o - 3u - 4o - 2u - 2o - 2u - 3o - u - 2o - 5u - 6o - u - 2o - 2u - 2o$.
16. $2u - 2o - 2u - 2o - 6u - 6o - 2u - 2o - 3u - 2o - 2u - 3o - 4u - 4o - 2u - 2o - 2u - 2o$.

Starting at the centre of the string, braid upwards the half-cycles 1–8. Then continue with braiding the Standing End downwards:

9. $2o - 2u - o - 3u - o - u - o - 3u - 3o - 5u - 2o - u$.
10. $o - u - o - 2u - 3o - 4u - o - 2u - o - 2u - o - 2u - 2o - 3u - o - u - 2o - u$.
11. $o - u - o - 2u - 3o - 2u - o - 2u - o - 2u - o - u - 4o - 4u - o - u - 2o - u$.
12. $2o - u - o - 2u - 4o - 5u - o - 2u - 2o - 2u - o - 3u - 3o - 3u - o - u - 2o - 2u$.
13. $2o - u - o - 2u - 2o - 2u - 4o - 2u - o - 3u - o - u - 5o - 5u - o - u - 2o - 2u$.
14. $2o - u - 2o - 2u - 5o - 5u - 2o - 2u - 2o - 2u - 2o - 4u - 4o - 2u - o - 2u - 2o - 2u$.
15. $2o - u - 2o - 2u - 2o - 4u - 3o - 2u - 2o - 3u - o - 2u - 5o - 6u - o - 2u - 2o - 2u$.
16. $2o - 2u - 2o - 2u - 6o - 6u - 2o - 2u - 3o - 2u - 2o - 2u - 4o - 5u - 2o - 2u - 2o - 2u$.

An 8-bight knot will in general be of little value, hence a greater multiple of 4 for the number of bights is normally required in order to cover the intended object properly. A greater multiple of 4 gives the modified central Pampas Button section also a much better appearance. For the knot in Fig. 570 with $p = 49$ we can take for b the values $4, 8, 12, 16, 20, 24, 32, 36, 40, 44, 48, 52, 60, \cdots$, and for the knot in Fig. 571 with $p = 51$ we can take for b the values $4, 8, 16, 20, 28, 32, 40, 44, 52, 56, 64, \cdots$. Note that in the algorithm tables the o/u bodies of the tables do not change and that only the i-values change due to the Δ^*-values and the half-cycle numbers are extended. This makes the algorithm tables of much greater value than a list of half-cycle braiding algorithms, consequently the more experienced braider may prefer to braid directly from the algorithm tables rather then from half-cycle braiding algorithms.

THE BRAIDER'S NOTEBOOK

At times one comes across a general statement which is, besides not being quite true, an obvious extrapolation of a relatioship found for a few cases. Invariably one looks in vain for an explanation and/or proof, and often such a general statement is incomplete. A typical case of such a general statement can be found in an article by Barney Belford entitled *"The 7 Part, 5 Bight Turk's-head"* published in *The Australian Whipmaker, No. 42*. In that article we come across the following lines: *"As there are 2 more parts than bights over half the knot is tied before having to take the working end under at all. The 5 part 3 bight turkshead is quick to tie for this reason and so is the 9 part 7 bight one"*, it then continues with the erroneous statement: *"but none of these can be made into larger turksheads in the way standard knots can"*. Although what *"standard knots"* are has not been defined, it can only be assumed that these also are *"turksheads"* or more correctly called "over–under coded Regular Knots". In the quoted knots and in the extrapolated statement (concerning over–under coded Regular Knots with an **odd** number of bights) it are exactly the first half (and hence **not** over half) of the total number of half-cycles which are being laid down without having to take the Working End under at all. Barney Belford's general statement (concerning over–under coded Regular Knots with an **odd** number of bights) does not mention the case where there are two less parts than bights, in which case over half the knot is tied before having to take the Working End under at all. Let's therefore have a look at both cases: the case where the number of parts is 2 less than the number of bights and the case where the number of parts is 2 more than the number of bights; in both cases the number of bights is **odd** as required for a single string string-run.

Details concerning the over–under coded Regular Knot $p/b = (x-2)/x$, where x is an odd positive integer, are shown at the left in Fig. 574, and details concerning the over–under coded Regular Knot $p/b = (x+2)/x$, where x is an odd positive integer, are shown at the right in Fig. 574.

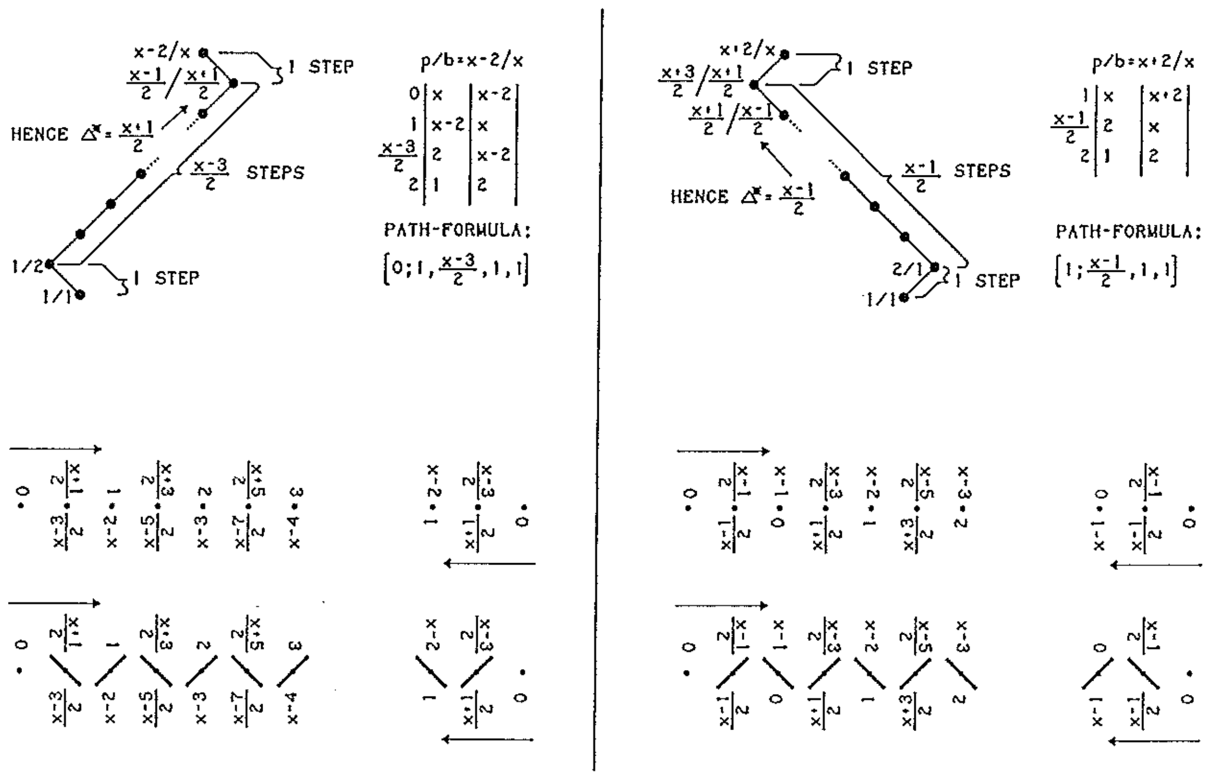

Fig. 574 — The over–under coded Regular Knots $p/b = (x-2)/x$ and $p/b = (x+2)/x$.

With the aid of Euclid's algorithm we determine the path-formula which enables us to plot the path in the RKT. From the path in the RKT we establish the Δ^*-value with which we calculate the i-values in the algorithm diagram. Next we place on the inner $(p-1)$ dots of the algorithm diagram (the intersection-columns of the knot) the codings for the consecutive half-cycles, using for each half-cycle **over** codings where free dots are available and fulfil the requirement of an over–under coded Regular Knot.

The case $p = x - 2$**:**

$$\Delta^* = \frac{x+1}{2}.$$

$$2\Delta^* = \left|\frac{2(x+1)}{2}\right|_b = \left|\frac{2(x+1)}{2}\right|_x = |x+1|_x = 1.$$

$$3\Delta^* = \left|\frac{3(x+1)}{2}\right|_b = \left|\frac{3(x+1)}{2}\right|_x = \left|\frac{3x+3}{2}\right|_x = \frac{x+3}{2}.$$

$$4\Delta^* = \left|\frac{4(x+1)}{2}\right|_b = \left|\frac{4(x+1)}{2}\right|_x = |2x+2|_x = 2\,.$$

$$5\Delta^* = \left|\frac{5(x+1)}{2}\right|_b = \left|\frac{5(x+1)}{2}\right|_x = \left|\frac{5x+5}{2}\right|_x = \frac{x+5}{2}\,.$$

$$\vdots$$

Thus the last free dot in the algorithm diagram for a half-cycle from lower-right to upper-left, able to receive an over coding, is the dot above $i = |(p-1)\Delta^*|_b = \left|(x-3)\left(\frac{x+1}{2}\right)\right|_x = \left|(x+1)\left(\frac{x-3}{2}\right)\right|_x = \frac{x-3}{2}$, and the last free dot for a half-cycle from lower-left to upper-right, able to receive an over coding, is the dot below $i = |(p-1)\Delta^*|_b = \left|(x-3)\left(\frac{x+1}{2}\right)\right|_x = \frac{x-3}{2}$. The i-value $\frac{x-1}{2}$, the next greater i-value above $\frac{x-3}{2}$, does not appear in the algorithm diagram since it would be associated with the dot for intersection-column $(p+1)$ which of couse does not exist in a Regular Knot having p parts. Hence the last half-cycle with all over crossings is half-cycle $h_o = 2\left\{\frac{x-1}{2}\right\} + 3 = x+2$. Since the knot has $2x$ half-cycles, the first half plus two of the total number of half-cycles can thus be laid down without having to take the Working End under any already laid down half-cycle.

The case $p = x+2$:

$$\Delta^* = \frac{x-1}{2}\,.$$

$$2\Delta^* = \left|\frac{2(x-1)}{2}\right|_b = \left|\frac{2(x-1)}{2}\right|_x = |x-1|_x = x-1\,.$$

$$3\Delta^* = \left|\frac{3(x-1)}{2}\right|_b = \left|\frac{3(x-1)}{2}\right|_x = \left|\frac{3x-3}{2}\right|_x = \frac{x-3}{2}\,.$$

$$4\Delta^* = \left|\frac{4(x-1)}{2}\right|_b = \left|\frac{4(x-1)}{2}\right|_x = |2x-2|_x = x-2\,.$$

$$5\Delta^* = \left|\frac{5(x-1)}{2}\right|_b = \left|\frac{5(x-1)}{2}\right|_x = \left|\frac{5x-5}{2}\right|_x = \frac{x-5}{2}\,.$$

$$\vdots$$

Thus the last free dot in the algorithm diagram for a half-cycle from lower-right to upper-left, able to receive an over coding, is the dot above $i = |3\Delta^*|_b = \left|3\left(\frac{x-1}{2}\right)\right|_x = \frac{x-3}{2}$, and the last free dot for a half-cycle from lower-left to upper-right, able to receive an over crossing, is the dot below $i = |3\Delta^*|_b = \left|3\left(\frac{x-1}{2}\right)\right|_x = \frac{x-3}{2}$. Note that the outermost intersection columns have the same i-value for both the lower-right to upper-left and the lower-left to upper-right half-cycles ($i = |(p-1)\Delta^*|_b = \left|(x+1)\left(\frac{x-1}{2}\right)\right|_x = \frac{x-1}{2} = \Delta^*$), hence cannot be given over codings and at the same time fulfil the coding requirements of an over–under coded Regular Knot. Thus the last half-cycle with all over crossings is half-cycle $h_o = 2\left\{\frac{x-3}{2}\right\} + 3 = x$. Since the knot has $2x$ half-cycles, the first half of the total number of half-cycles can thus be laid down without having to take the Working End under any already laid down half-cycle.

Note that in the algorithm diagram the i-value associated with the dot representing the last intersection-column for the half-cycles from lower-right to upper-left and for the half-cycles from lower-left to upper-right can more readily be calculated from the relationship $a^* + b^* = b - 1 = x - 1$ (see *The Braider*, Issue No. 7, pg. 133).

Besides the above way at looking at the two cases of single string over–under coded Regular Knots where respectively the number of parts is 2 less than the number of bights and the number of parts is 2 more than the number of bights, we can also look at them in the following way:

The case $p = x - 2$:

From the path in the RKT, which we could plot with the aid of the path-formula obtained with Euclid's algorithm, we see that the over–under coded $p/b = (x-2)/x$ Regular Knot is obtained from the $p/b = 1/1$ Regular Knot by an n-order Method I enlargement (where $n = 1 + \frac{x-3}{2} = \frac{x-1}{2}$) followed by a first-order Method I enlargement. The first enlargement stage (the n-order Method I enlargement) brings us from $p/b = 1/1$ to $p/b = \left(\frac{x-1}{2}\right) / \left(\frac{x+1}{2}\right)$. This are $(x+1)$ half-cycles in which the Working End is not taken under any previous laid down half-cycle (assuming that we want the least number of 'unders' possible; refer to *The Braider*, Issue No. 7). The first track-splitting half-cycle is then also without 'unders', hence overall we have the first $(x+2)$ half-cycles without 'unders'. Since the total number of half-cycles in the $p/b = (x-2)/x$ over–under coded Regular Knot is equal to $2x$, we see that the first half plus two of these are without 'unders'.

The case $p = x + 2$:

From the path in the RKT, which we could plot with the aid of the path-formula obtained with Euclid's algorithm, we see that the over–under coded $p/b = (x+2)/x$ Regular Knot is obtained from the $p/b = 1/1$ Regular Knot by an n-order Method II enlargement (where $n = 1 + \frac{x-1}{2} = \frac{x+1}{2}$) followed by a first-order Method II enlargement. The first enlargement stage (the n-order Method II enlargement) brings us from $p/b = 1/1$ to $p/b = \left(\frac{x+3}{2}\right) / \left(\frac{x+1}{2}\right)$. This are $(x+1)$ half-cycles, however, the last of these half-cycles has as the last crossing an 'under', hence we have the first x half-cycles without 'unders' (assuming that we want the least number of 'unders' possible; refer to *The Braider*, Issue No. 7). Since the total number of half-cycles in the $p/b = (x+2)/x$ over–under coded Regular Knot is equal to $2x$, we see that the first half of these are without 'unders'.

It will be obvious that too many initial half-cycles without any 'unders' besides 'overs' makes the braiding of the knot without the aid of a mandrel with pins cumbersome. For example, the $p/b = 7/72$ over–under coded Regular Knot may be regarded as being impossible to braid without the aid of a mandrel with pins since the first 83 half-cycles are without any 'unders', assuming that we want the least number of 'unders' possible. Only small knots with a relative large number of initial half-cycles which do not have both 'overs' and 'unders' don't give any trouble in braiding them, but even slightly bigger ones become difficult to braid without a mandrel with pins due to the initial lack of formfastness.