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A quarterly publication
for
the braiding artisan

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Solution to the Question in Issue No. 28

Question on pg. 649.

Let's roll the first-return string-run out. Let's take any cycle, hence any odd-numbered half-cycle running from lower-left bight-boundary l_i to upper-right bight-boundary r_i and its consecutive even-numbered half-cycle running from lower-right bight-boundary r_i to upper-left bight-boundary l_{i+1}. Then the distance in bights between the l_i bight-point and the l_{i+1} bight-point is equal to:

$$\frac{2(A-l_i)+x+2(A-r_i)+2(A-r_i)+x+2(A-l_{i+1})}{2} = 4A+x-(l_i+2r_i+l_{i+1}).$$

The distance in bights between the starting-point and end-point of the first-return string-run is the sum of the bight-distances spanned by its cycles. A first-return string-run has α cycles, and since $\alpha = A$ for Perfect and Semi-Perfect Regular Nested Cylindrical Braids, the distance in bights between the starting-point and end-point of the first-return string-run is:

$$A\cdot 4A + A\cdot x - (2\sum l_i + 2\sum r_i).$$

Since $\sum l_i = \sum r_i = 1+2+3+\cdots+A = \frac{A}{2}\cdot(1+A)$, the distance in bights between the starting-point and end-point of the first-return string-run is:

$$4A^2+Ax-[A(1+A)+A(1+A)] = 2A^2+Ax-2A = 2A^2+A(x-2).$$

With the first sequence of half-cycles in a first-return string-run starting at nest-index number $I_L = 0$, the end-point of this first-return string-run, and hence the starting-point of second sequence of half-cycles in a first-return string-run will be at nest-index number

$$I_L = \left|2A^2+A(x-2)\right|_B = A|2A+x-2|_{B^*}.$$

The sequence of half-cycles between the starting-point of half-cycle h_n (≥ 1) and the starting-point of half-cycle h_n+2A ($\leq 2B$) is the sequence of half-cycles in a first-return string-run (although generally of course not starting with the same type of half-cyle), and hence when half-cycle h_n starts at nest-index number I then half-cycle h_n+2A starts at nest-index number $\left|I+2A^2+A(x-2)\right|_B$.

Some Heel Knot Constructions

Bosals[†] are a piece of braided horse equipment which is probably more commonly found in the United States and Mexico. Several braiding skills are necessary for the construction of the bosal. General construction details can readily be found in the books "Encyclopedia of Rawhide and Leather Braiding" and "How to Make Cowboy Horse Gear" both by Bruce Grant[‡] or in "Braiding Rawhide Horse Tack" by Robert Woolery[††]. The appearance of the finished product depends on the quality and size of

[†] The **bosal** *(boh-zal)* is the noseband of the **hackamore**. The **hackamore** is a type of headstall or bridle without a bit.

[‡] For further details about these books see *The Braider*, Issue No. 1, pp. 17-18.

[††] For further details about this book see *The Braider*, Issue No. 9, pp. 197-198.

the string-materials used and on the braiding and knot tying skills of the braider. There is always the opportunity to try various braids for the noseband† itself and to tie very beautiful multi-colour knots for the noseband button knot, the side buttons and the heel knot. They can also be made quite plain, yet perfectly functional, by using only single colour and heavier, coarser string-material. Quality rawhide is the material of choice. We shall restrict our discussion here to a single aspect of the bosal construction, namely some construction procedures for the **heel knot** in association with the joining of the two ends of the noseband braid, and introduce techniques which have not been explored in the above mentioned books. These techniques are not themselves new to braiding or even new to the braiding of bosals. Several bosals are known which were made by a Mexican braider in the 1940's which employ these techniques, but the methods appear to be seldom used and are perhaps largely unknown in relation to bosals. The techniques should not be dismissed as being too complicated or too time consuming or that they require more string-material; the difference is insignificant. The braider himself/herself should decide to use or not to use the techniques on their own merit. Any braider who chooses to use these methods may need to alter somewhat the techniques given here and adapt them to their own project: for the type and size of string-material being used and the number of strings in the noseband braid.

As mentioned above, we shall restrict our discussion to some methods of joining the two ends of the noseband by means of the heel knot. We therefore presuppose that the braider has already braided the noseband of a sufficient length, probably about 80 cm. (32 inches), with each string of the braid extending 45 cm. (18 inches) beyond each of the ends of the noseband braid. Furthermore, we will assume that the braider has already tied the noseband button knot and the side button knots of whatever type, colour and style desired. We are then ready to bend the two ends of the braided noseband together and join them in some secure fashion with the aid of the heel knot.

Two methods are commonly employed and both methods are described in each of the above mentioned books by Bruce Grant, which we will use as reference, while only one method is described in Robert Woolery's book.

On pg. 137 of the "Encyclopedia of Rawhide and Leather Braiding" and on pg. 44 of "How to Make Cowboy Horse Gear", fig. 1 and fig. 2 show the method by which the two ends of the noseband braid are tied together (a constrictor knot can be used for this) and subsequently a Spanish ring knot is placed over these combined ends. Then each one of the projecting strings of the noseband is wrapped around this Spanish ring knot as indicated by the dashed lines and arrows in fig. 1.

For the other method the text on pg. 134 of the "Encyclopedia of Rawhide and Leather Braiding" and on pg. 40 of "How to Make Cowboy Horse Gear", gives reference to respectively pg. 199 and pg. 100, where figs. 9, 10, 11 indicate how to use four or six strings of the combined projecting string-ends of the noseband braid‡ to braid a knot around the remaining projecting strings as indicated on respectively pg. 201 and pg. 102, by the dashed lines and arrows in figs. 9,10,11.

We can, however, also use a method which employs all the projecting strings of the noseband braid to braid a knot, or knots, which can be left as the finished heel knot or it can be used as a foundation knot over which to braid one of the many woven knots.

† Also known as the cheekpiece braid.

‡ Half from each end of the noseband braid.

Say the noseband has no core and consists of an 8-string round braid with a herringbone $2u - 2o$ coding-sequence. After the two ends of the noseband braid have been brought together we will have a total of 16 strings with which to work. If these 16 strings were to be evenly divided into 4 sets, then there would be 4 strings in each set. The 4 strings within each set could themselves then be braided into a 4-string flat braid or a 4-string round braid.

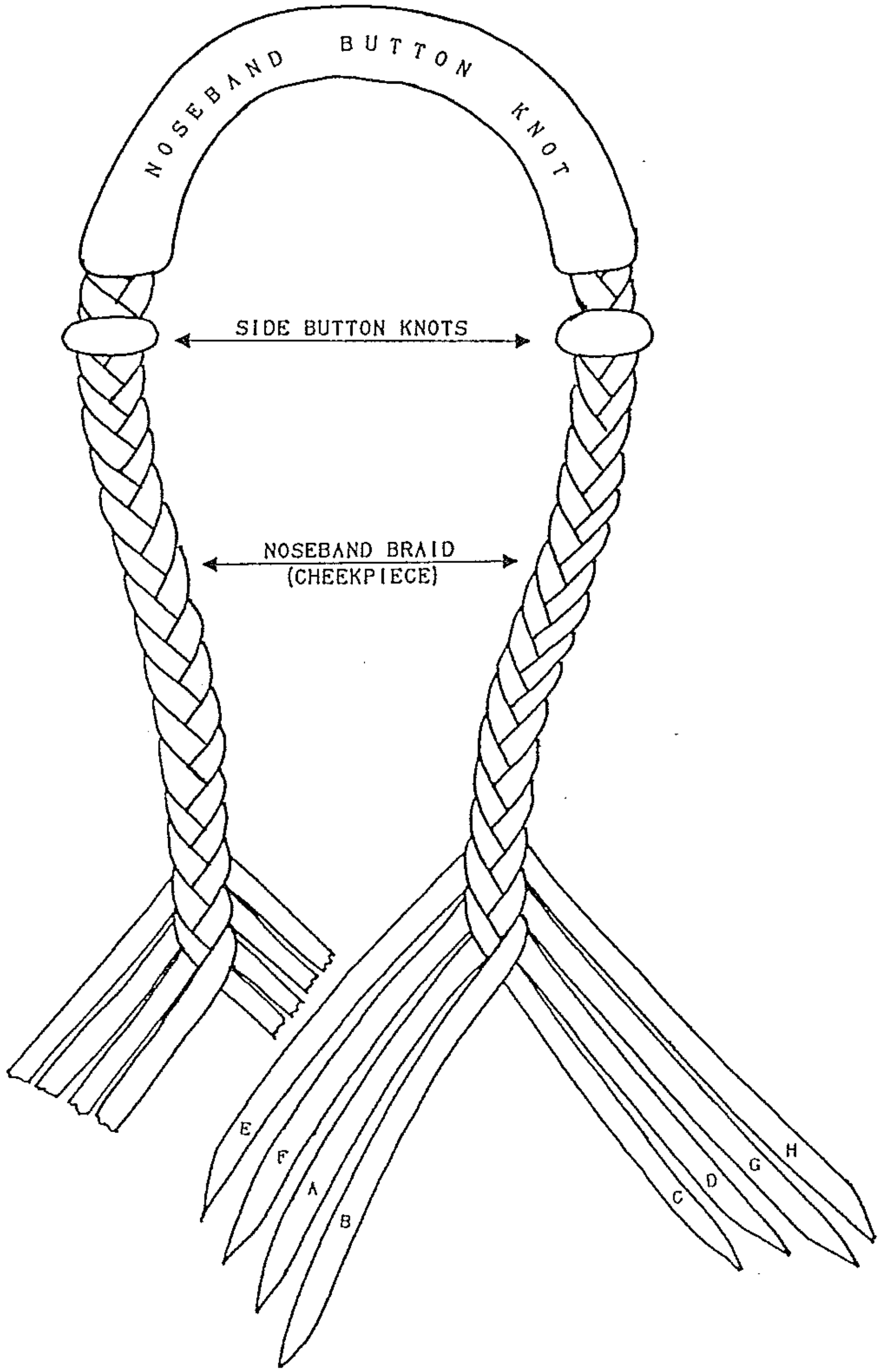

Fig. 528 — The 8-string round noseband braid with noseband button knot and side button knots.

When we have braided the 8-string round noseband to its required length, we can reach a stage at any time where the braid on either end will appear as shown in Fig. 528 with the hair-side of the strings uppermost; this stage is also shown by the leftmost grid-diagram of Fig. 529.

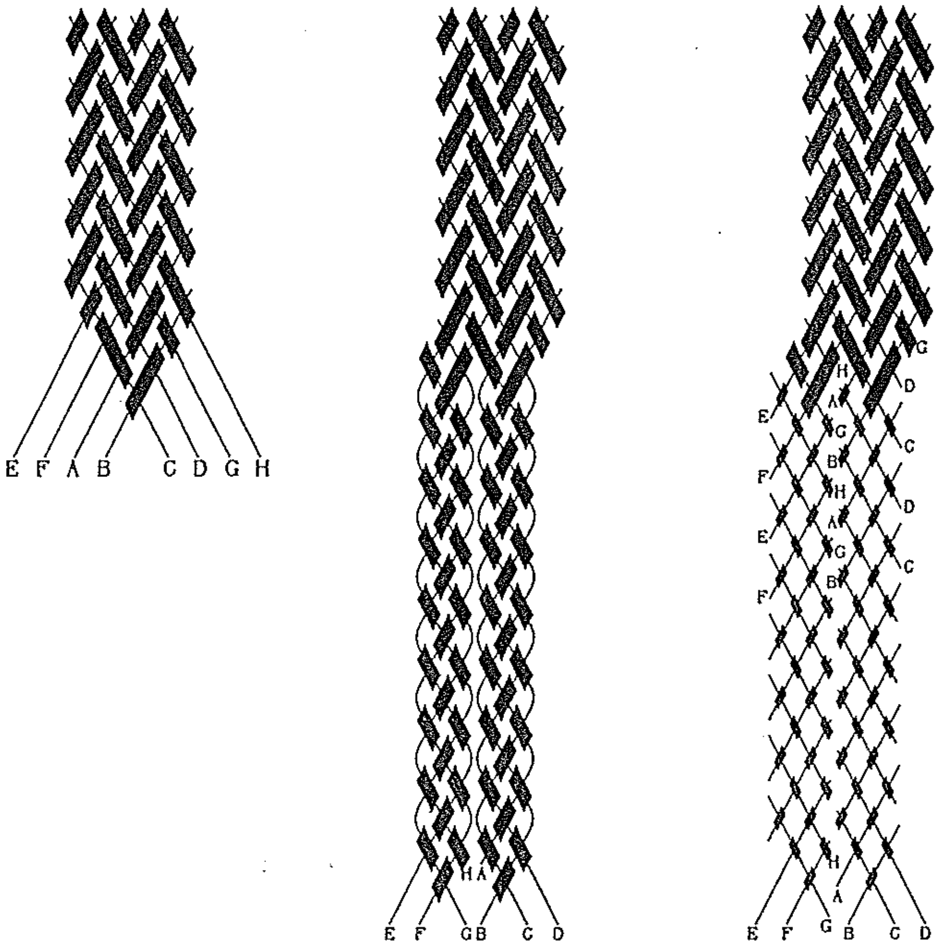

Fig. 529 — Each end of the 8-string round noseband braid.

By now taking the strings E, F, G and H of one end of the noseband braid, one at a time around behind the braid in the order H, then E, then G, then F, the front and the back of the braid will appear exactly alike and are as shown for the strings A, B, C and D in the left-hand drawing of Fig. 530. For the strings E, F, G and H, string H lies behind string A, string E lies behind string D, string G lies behind string B, and string F lies behind string C.

Strings A, B, C and D can then be braided into a 4-string flat braid as depicted in the right-hand drawing of Fig. 530, and by the central grid-diagram of Fig. 529, for a length of about 30 cm. (12 iches). Strings E, F, G and H are braided similarly.

Not only are the 8-strings on the other end of the noseband braid also arranged as the strings E, F, A, B, C, D, G and H in Fig. 528 and as in the leftmost grid-diagram of Fig. 529, but they will undergo the same process as depicted for those by the right-hand drawing of Fig. 530 and the cental grid-diagram of Fig. 529. Once completed, we thus will have four 4-string flat braids: two at each end of the noseband braid.

When the two ends of the noseband braid are ready to be joined, they are brought together and the four 4-strand flat braids are used to tie any 4-strand terminal knot.

Simple terminal knots which may be used are the 3-part/4-bight *over-under* coded regular cylindrical braided open-ender, the 4-part/4-bight *over-under* coded regular cylindrical braided open-ender as depicted respectively by the left-hand and central grid-diagrams of Fig. 531.

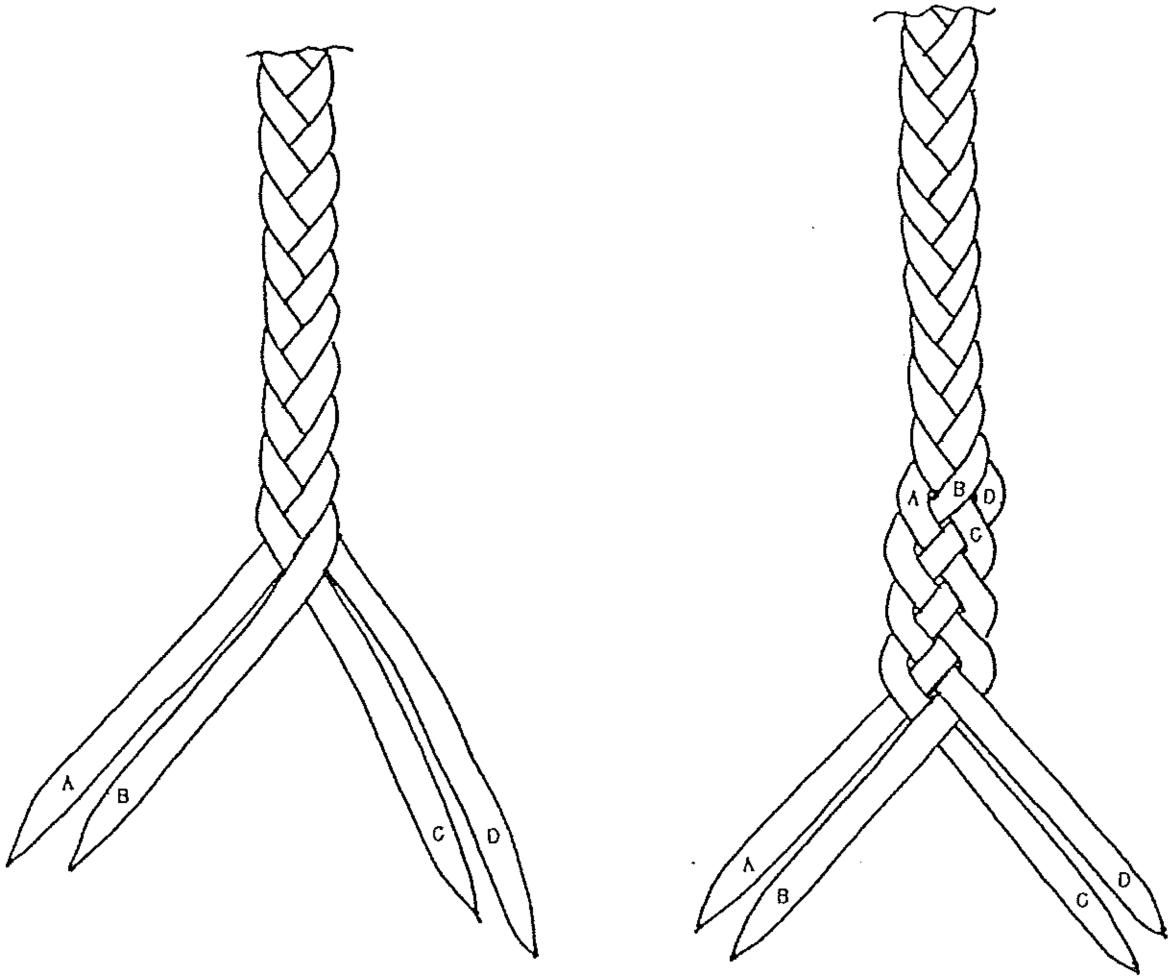

Fig. 530 — The strings A, B, C and D.

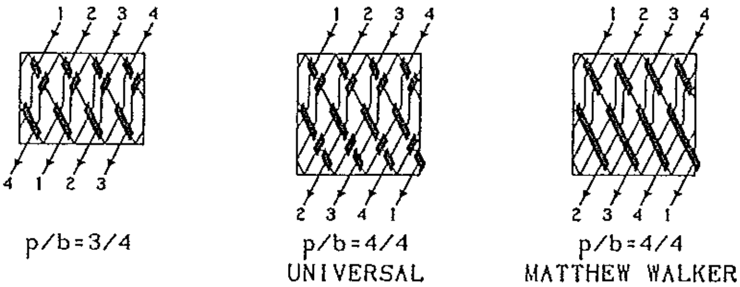

Fig. 531 — Some 4-strand terminal knots.

Instead of braiding each set of 4-strings into the flat braids as described above and shown in the right-hand drawing of Fig. 530 and the central grid-diagram of Fig. 529, we could braid each set of 4 strings into the round braids as shown by the right-hand grid-diagram of Fig. 529. The four 4-string round braids can then be tied into a terminal knot such as depicted in Fig. 531, including the 4-part/4-bight Matthew Walker coded regular cylindrical braided open-ender which is well adapted to being constructed with round cord. If these finished terminal knots are of sufficient bulk to suit the needs of the bosal, they could then be left as is to form the complete heelknot. If more bulk is required or desired then these terminal knots may be used as a foundation over which to tie any woven knot.

It is important that the ends of the noseband can not be pulled apart and out of the heelknot. Terminal knots constructed in this general fashion make for a very secure combining of the two ends of the noseband braid, whether they are left as the final heelknot or are covered with a woven knot.

Quite often the noseband is braided over a core. Steel cable, braided rawhide and twisted rawhide are the most common core-materials used. We can use the core-material in the terminal knot when a rawhide core is being used. In each of the above mentioned books the use of twisted rawhide cores is considered a very common practice and may be a best core to use.

A two strand twisted rawhide core, made with the same width and thickness of strings as used for the noseband, makes a very good core inside an 8-string round noseband braid. If, for instance, we were to need a noseband braid of 13 mm. ($\frac{1}{2}$ inch) diameter, we can use 5 mm. ($\frac{3}{16}$ inch) wide rawhide strings for both the core and the noseband braid braiding material.

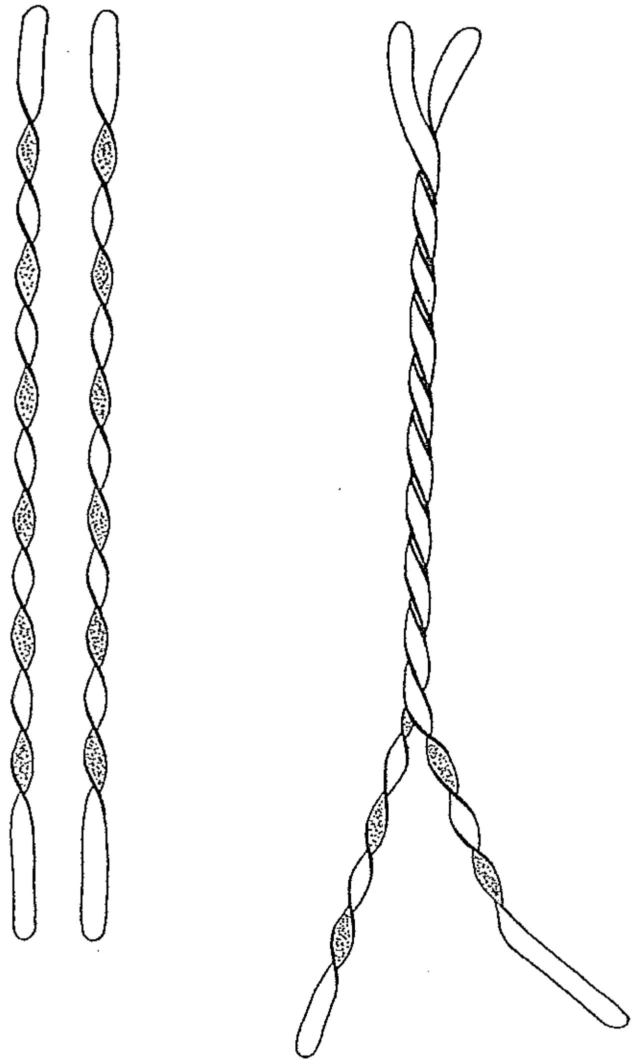

Fig. 532 — The two strand twisted core construction.

The core can be fashioned as drawn in Fig. 532, where we can twist the two tempered strands of the core separately, each in either the same right-hand or left-hand helix (a left-hand helix is depicted in Fig. 532). These twists are not done tightly since the next step will be to lay one strand into the twists of the other strand so as to intertwine them as shown at the right in Fig. 532. Once the two strands have been lain together, the twist can be tightened up and the core stetched and secured to dry. The core should be twisted for a length of about 80 cm. (32 inches) and have an additional 45 cm. (18 inches) of non-twisted string length on each end which will be used in the heelknot construction.

Over this core we braid the 8-string round herringbone noseband braid of $2u - 2o$ sequence until the braid on either end again looks like the drawing in Fig. 528 with the addition of the twisted core-strands running through the centre of the braid. The strings E, F, G and H are then each taken around the back, one at a time in the order H, then E, then G then F and the braid will appear as on the left in Fig. 530. When the core has been twisted with a left-hand helix as drawn in Fig. 532 we can now introduce one strand of the core into the braid as depicted at the left in Fig. 533 and begin a 5-string flat braid of $2o - 2u$ sequence as shown in Fig. 533. This braid is continued as depicted at the right in Fig. 533 for a length of about 30 cm. (12 inches).

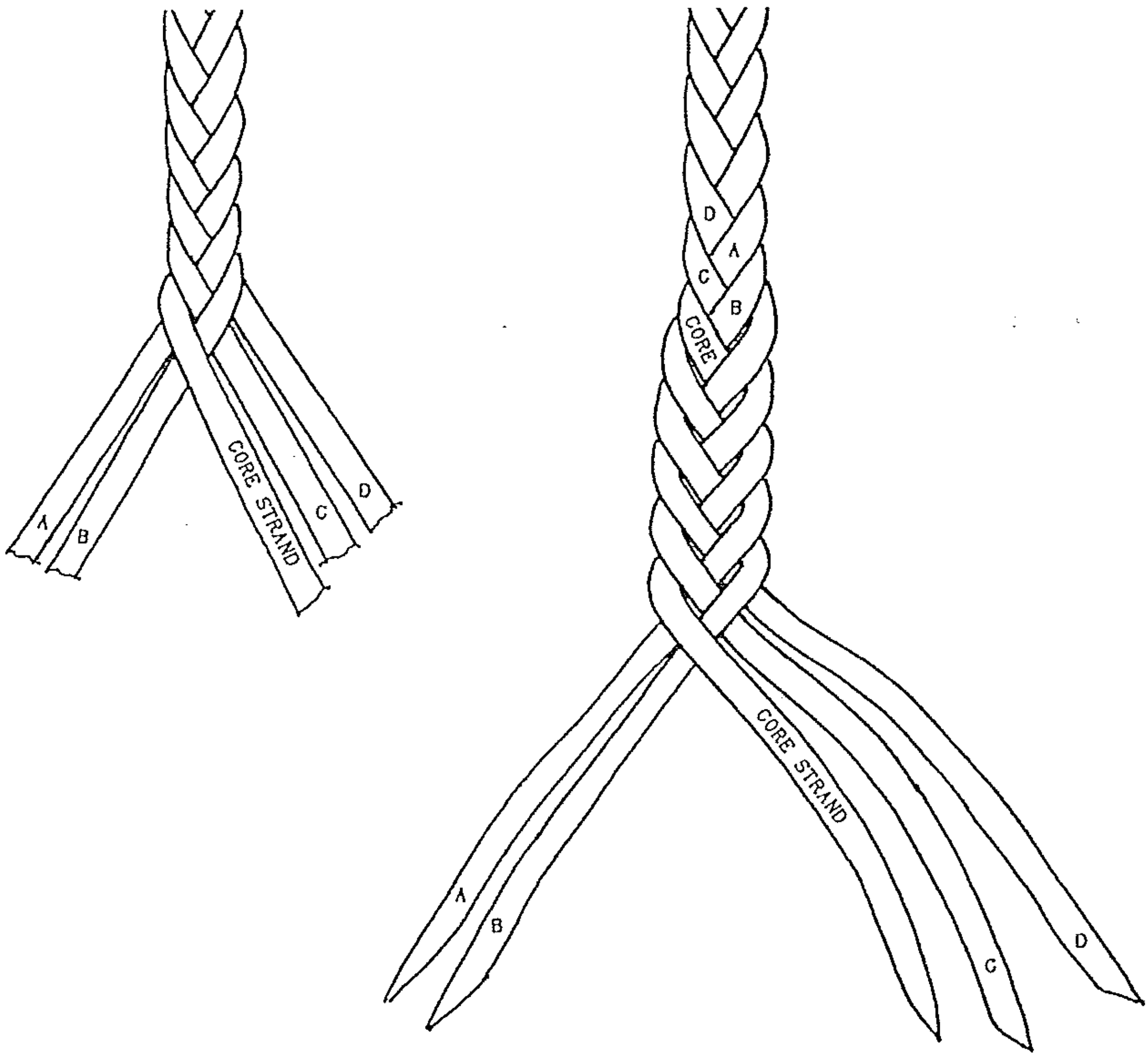

Fig. 533 — The 5-string flat braids.

Since the arrangement of the strings E, F, G and H should look similar to that shown in Fig. 530, the second strand of the twisted core can be introduced into these strings in the manner similar to that shown in Fig. 533 and a second 5-string flat braid with a $2o - 2u$ sequence is made for a length of about 30 cm. (12 inches).

The 8 strings at the other end of the noseband braid are also arranged as in Figs. 528 and 530. By using the two stands at the other end of the twisted core, we can braid two additional 5-string flat braids. Once completed we thus obtain four 5-string flat braids: two at each end of the noseband braid.

When the two ends of the noseband braid are ready to be joined, they are brought together and the four 5-string flat braids are used to tie any desired 4-strand terminal knot including, but not limited to, the knots diagrammed in Fig. 531.

When any of these terminal knots have been tied and tightened, the ends of the flat braids are cut off at the end of the terminal knot. They can be shaped into a nice round knot while the rawhide is still tempered, and once dry the knots should not come apart under normal conditions. To add bulk to the heelknot, the terminal knot may be covered with any desired woven knot.

Nested Cylindrical Braids

In the previous Issue we discussed a simple method for working out the half-cycle braiding algorithms for the Perfect Herringbone Pineapple Knots, and we gave two numerical Examples which were a mirror-imaged pair. We furthermore showed how from the half-cycle tables of a Perfect Herringbone Pineapple Knot the half-cycle tables of its mirror-image may be obtained. With respect to this latter process, the observant reader will have noticed that the two Examples were a special case in that the numeral to be added to, respectively to be subtracted from, the half-cycle number was equal to A. In general this numeral, of course, ranges from 1 to $2A-1$, and it will only be equal to A when A is **odd** and $x = 2nA + 4$, where n is a whole number; in which case $k = \frac{A+3}{2}$ for $y = A-1$, and $k = \frac{A+1}{2}$ for $y = A+1$.

Let's look at two other Examples which are each others mirror-image :

Example 1:

$A = 4; x = 19; y = A-1 = 3$, hence $\Delta = |3|_4 = 3; B^* = 4$, hence $B = A \cdot B^* = 16$.
Thus $k = \left|\frac{x-A-1}{2}\right|_A = \left|\frac{19-4-1}{2}\right|_4 = 3$.

At the upper left-hand side in Fig. 534 is depicted the first-return string-run with its associated nest-index numbers.

At the upper right-hand side in Fig. 534 are depicted the first-return string-run, the half-cycle numbers of the half-cycles, and the number of crossings which these half-cycles make in the finished knot. This layout at the upper right-hand side has been shown for additional clarification only since it can readily be dispensed with.

The depicted layout at the bottom of Fig. 534 gives the positions of the nests with respect to the nest-index numbers, the half-cycle numbers with respect to the nest-index numbers and bight-boundaries, and the number of crossings which the half-cycles make in the finished knot. This layout is the one we need for compiling the half-cycle algorithms; it is directly obtained from the first-return string-run with its associated nest-index numbers, hence from the upper left-hand layout in Fig. 534. The first step in its construction is shown in Fig. 535.

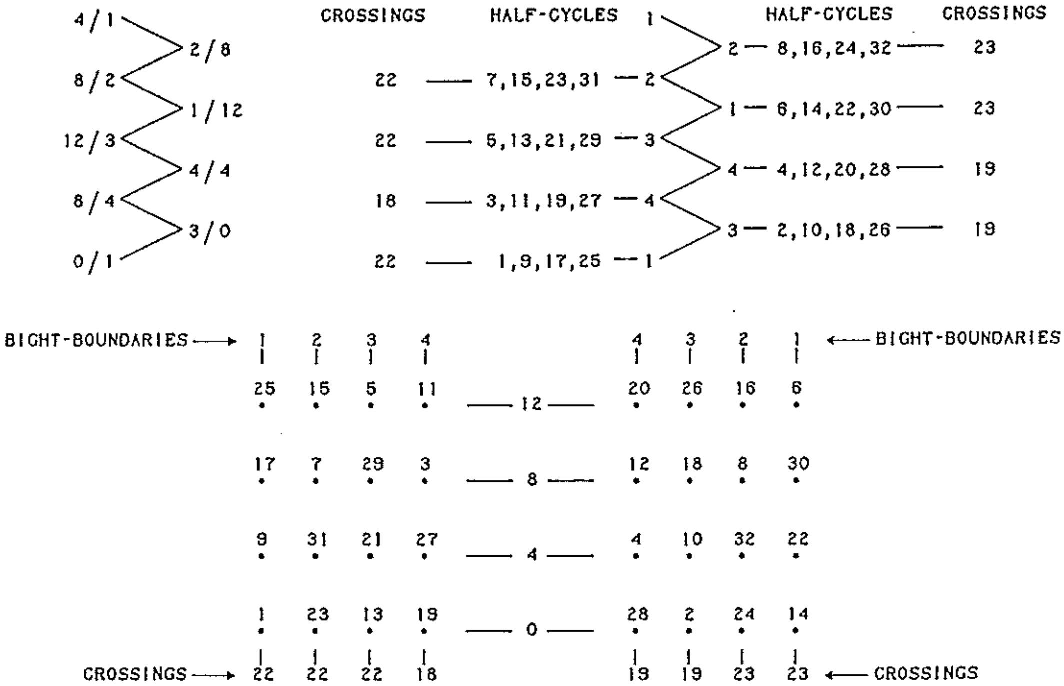

Fig. 534 — First-return string-run and half-cycle pattern.

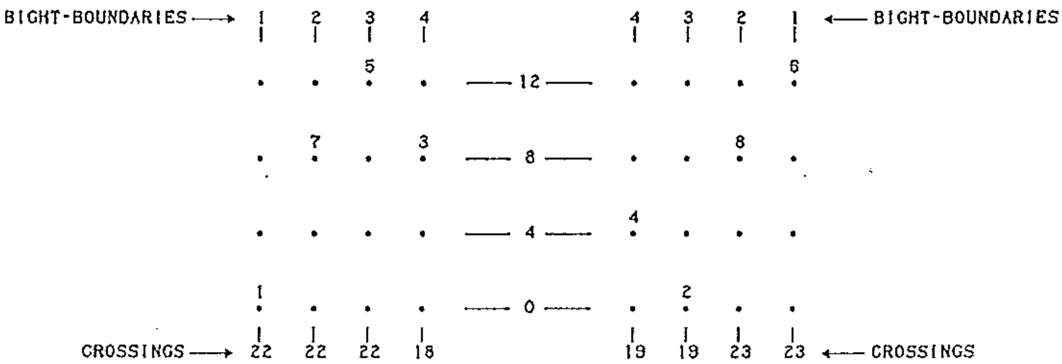

Fig. 535 — The first step in the construction of the half-cycle pattern.

The second step in the construction of the half-cycle pattern shown at the bottom of Fig. 534 consists of the determination of the half-cycle numbers in association with the nest-index numbers and bight-boundaries of their starting points.

Since the second sequence of the half-cycles in a first-return string-run starts with half-cycle number $1 + 2A = 9$ at nest-index number $I_L = A|2A + x - 2|_{B^*} = 4|25|_4 = 4$, two sequential half-cycle numbers h_n (≥ 1) and $h_n + 2A$ $(\leq 2B)$ associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I + 2A^2 + A(x-2)\right|_B = \left|I + 2 \cdot 4^2 + 4(19-2)\right|_{16} = |I + 100|_{16} = |I + 4|_{16}$. Hence we can readily complete the half-cycle pattern concerned. With the aid of the coding-sequence of the half-cycles in the finished knot with a Herringbone Pineapple coding and $y = A - 1$ (see pg. 646) we can either read the half-cycle braiding algorithms directly from the half-cycle pattern, or from the set of half-cycle tables shown in Fig. 536 which we assemble from the half-cycle pattern at the bottom of Fig. 534.

The upper table in Fig. 536 is for the odd-numbered half-cycles, hence the half-cycles from lower-left to upper-right, while the lower table in Fig. 536 is for the even-numbered half-cycles, hence the half-cycles from lower-right to upper-left.

Note that the half-cycle which ends at the start of half-cycle 1 is the very last half-cycle in the braiding process and hence will not be crossed during the braiding process, thus we leave its cell empty.

L ⟶ R

	23	13	19	9	31	21	27	17	7	29	3	25	15	5	11		23	13	19	9	31				
25	15	5	11		23	13	19	9	31	21	27	17	7	29	3	25	15	5	11		23				
17	7	29	3	25	15	5	11		23	13	19	9	31	21	27	17	7	29	3	25	15				
9	31	21	27	17	7	29	3	25	15	5	11		23	13	19	9	31	21	27	17	7				
o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	1	9	17	25
u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	23	31	7	15
u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	13	21	29	5
u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u					19	27	3	11

R ⟶ L

22	32	10	4	30	8	18	12	6	16	26	20	14	24	2	28	22	32	10	4	30	8	18				
14	24	2	28	22	32	10	4	30	8	18	12	6	16	26	20	14	24	2	28	22	32	10				
6	16	26	20	14	24	2	28	22	32	10	4	30	8	18	12	6	16	26	20	14	24	2				
30	8	18	12	6	16	26	20	14	24	2	28	22	32	10	4	30	8	18	12	6	16	26				
u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u					4	12	20	28
u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u					10	18	26	2
u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	32	8	16	24
u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	22	30	6	14

Fig. 536 — The half-cycle tables for the knot in Example 1.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 1:

1. 1 ↗ 3: Free run.
2. 4 ↖ 3: *o*.
3. 4 ↗ 4: *u*.
4. 3 ↖ 4: *u* – *o*.
5. 3 ↗ 1: *o* – *u*.
6. 2 ↖ 1: *o* – *u* – 2*o*.
7. 2 ↗ 2: *o* – 2*u* – *o*.
8. 1 ↖ 2: 2*u* – 2*o* – 2*u*.
9. 1 ↗ 3: *o* – 2*u* – 2*o*.
10. 4 ↖ 3: *u* – *o* – *u* – *o* – 2*u*.
11. 4 ↗ 4: *o* – *u* – 2*o* – *u*.
12. 3 ↖ 4: *u* – *o* – 2*u* – 2*o* – *u*.
13. 3 ↗ 1: *u* – *o* – *u* – 2*o* – 2*u* – *o*.
14. 2 ↖ 1: 2*o* – 3*u* – *o* – *u* – 2*o* – *u*.
15. 2 ↗ 2: 4*o* – 3*u* – 2*o*.
16. 1 ↖ 2: *u* – *o* – 3*u* – 3*o* – *u* – *o* – *u*.
17. 1 ↗ 3: 3*o* – *u* – *o* – 3*u* – 3*o*.
18. 4 ↖ 3: 2*u* – 2*o* – *u* – 3*o* – 3*u*.
19. 4 ↗ 4: *u* – 2*o* – 3*u* – 2*o* – 2*u*.
20. 3 ↖ 4: 2*u* – 2*o* – 3*u* – 3*o* – 2*u*.
21. 3 ↗ 1: 2*u* – 2*o* – 2*u* – 3*o* – 3*u* – 2*o*.

22. $2 \nwarrow 1$: $4o - 3u - 2o - 2u - 4o - u$.
23. $2 \nearrow 2$: $u - 2o - 2u - 3o - 4u - 2o - u$.
24. $1 \nwarrow 2$: $u - 3o - 4u - 3o - 2u - 3o - u$.
25. $1 \nearrow 3$: $3o - 2u - 3o - 4u - 3o - u$.
26. $4 \nwarrow 3$: $3u - 2o - 3u - 4o - 4u$.
27. $4 \nearrow 4$: $2u - 4o - 3u - 3o - 3u$.
28. $3 \nwarrow 4$: $3u - 3o - 4u - 4o - 2u$.
29. $3 \nearrow 1$: $2u - 3o - 3u - 4o - 4u - 3o$.
30. $2 \nwarrow 1$: $u - 4o - 4u - 3o - 4u - 4o - 2u$.
31. $2 \nearrow 2$: $u - 4o - 3u - 4o - 4u - 4o - u$.
32. $1 \nwarrow 2$: $2u - 4o - 4u - 4o - 4u - 4o - u$.

The grid-diagram of this knot is depicted in Fig. 537

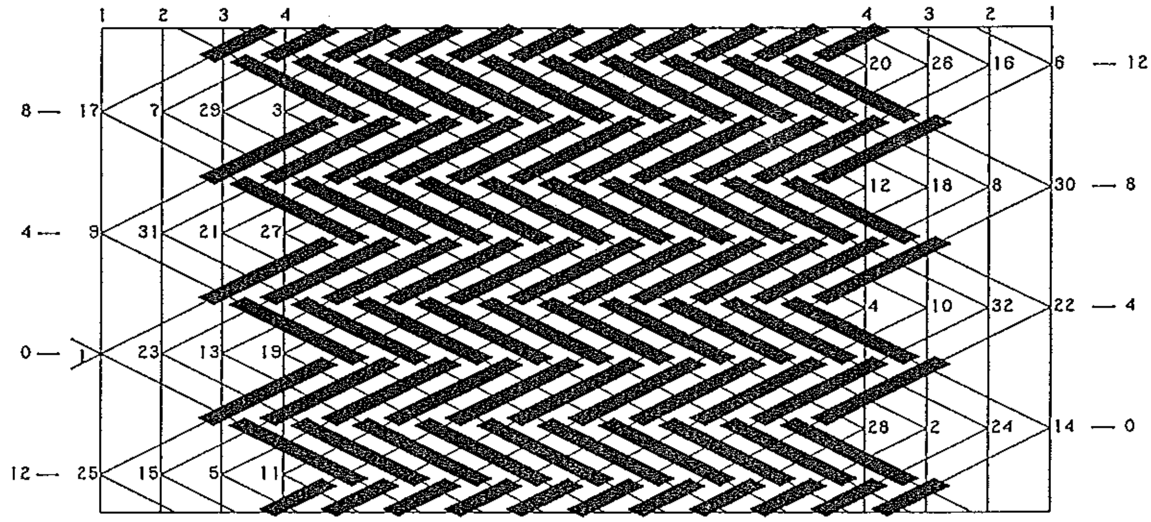

Fig. 537 — Grid-diagram of the knot in Example 1.

Example 2 :

$A = 4$; $x = 19$; $y = A+1 = 5$, hence $\Delta = |5|_4 = 1$; $B^* = 4$, hence $B = A \cdot B^* = 16$. Thus $k = \left|\frac{x-A-3}{2}\right|_A = \left|\frac{19-4-3}{2}\right|_4 = 2$.

At the upper left-hand side in Fig. 538 is depicted the first-return string-run with its associated nest-index numbers.

At the upper right-hand side in Fig. 538 are depicted the first-return string-run, the half-cycle numbers of the half-cycles, and the number of crossings which these half-cycles make in the finished knot. This layout at the upper right-hand side has been shown for additional clarification only since it can readily be dispensed with.

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The second step in the construction of the half-cycle pattern shown at the bottom of Fig. 538 consists of the determination of the half-cycle numbers in association with the nest-index numbers and bight-boundaries of their starting points.

Since the second sequence of the half-cycles in a first-return string-run starts with half-cycle number $1 + 2A = 9$ at nest-index number $I_L = A|2A + x - 2|_{B^*} = 4|25|_4 = 4$, two sequential half-cycle numbers h_n (≥ 1) and $h_n + 2A$ $(\leq 2B)$ associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I + 2A^2 + A(x-2)\right|_B = \left|I + 2 \cdot 4^2 + 4(19-2)\right|_{16} = |I + 100|_{16} = |I + 4|_{16}$. Hence we can readily complete the half-cycle pattern concerned. With the aid of the coding-sequence of the half-cycles in the finished knot with a Herringbone Pineapple coding and $y = A + 1$ (see pg. 646) we can either read the half-cycle braiding algorithms directly from the half-cycle pattern, or from the set of half-cycle tables shown in Fig. 540 which we assemble from the half-cycle pattern at the bottom of Fig. 538.

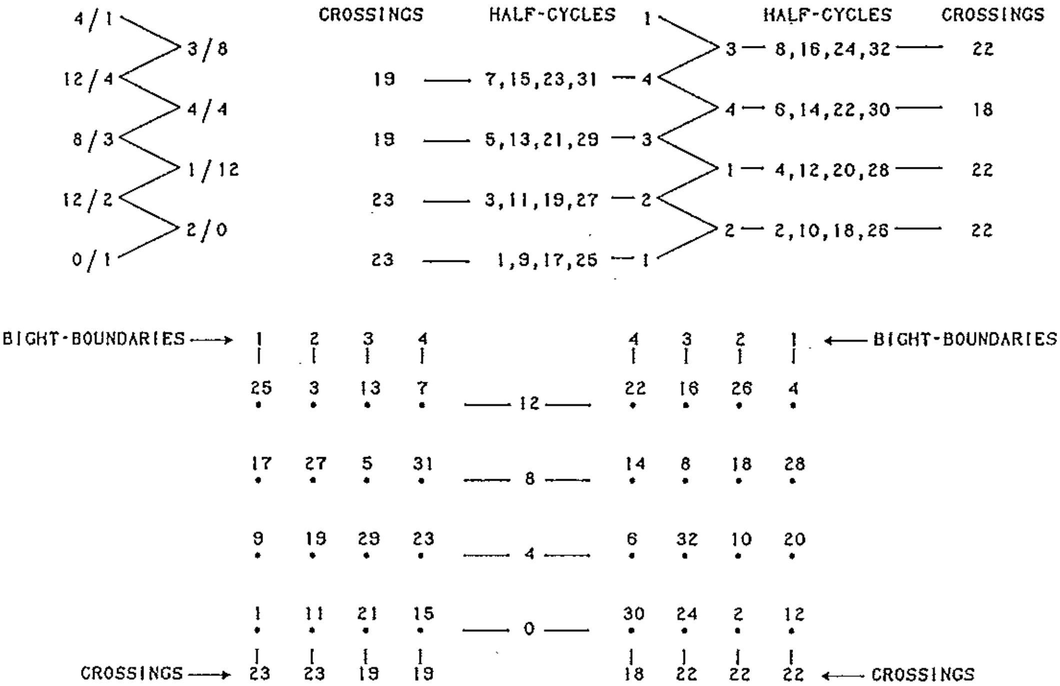

Fig. 538 — First-return string-run and half-cycle pattern.

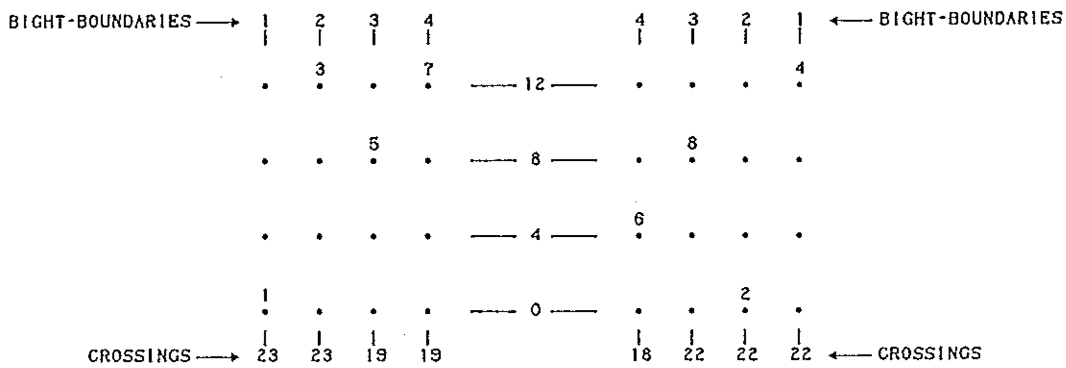

Fig. 539 — The first step in the construction of the half-cycle pattern.

The upper table in Fig. 540 is for the odd-numbered half-cycles, hence the half-cycles from lower-left to upper-right, while the lower table in Fig. 540 is for the even-numbered half-cycles, hence the half-cycles from lower-right to upper-left.

Note that the half-cycle which ends at the start of half-cycle 1 is the very last half-cycle in the braiding process and hence will not be crossed during the braiding process, thus we leave its cell empty.

L → R

25	3	13	7		11	21	15	9	19	29	23	17	27	5	31	25	3	13	7		11	21				
17	27	5	31	25	3	13	7		11	21	16	9	19	29	23	17	27	5	31	25	3	13				
9	19	29	23	17	27	5	31	25	3	13	7		11	21	15	9	19	29	23	17	27	5				
	11	21	15	9	19	29	23	17	27	5	31	25	3	13	7		11	21	15	9	19	29				
u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u					7	15	23	31
u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u					13	21	29	5
u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	3	11	19	27
u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	25	1	9	17

R → L

4	26	16	22	12	2	24	30	20	10	32	6	28	18	8	14	4	26	16	22	12	2				
28	18	8	14	4	26	16	22	12	2	24	30	20	10	32	6	28	18	8	14	4	26				
20	10	32	6	28	18	8	14	4	26	16	22	12	2	24	30	20	10	32	6	28	18				
12	2	24	30	20	10	32	6	28	18	8	14	4	26	16	22	12	2	24	30	20	10				
o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	4	12	20	28
u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	26	2	10	18
u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	16	24	32	8
u	u	u	o	o	o	o	u	u	u	u	o	o	o	o	u	u	u					22	30	6	14

Fig. 540 — The half-cycle tables for the knot in Example 2.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 2:

1. $1 \nearrow 2$: Free run.
2. $2 \nwarrow 2$: u.
3. $2 \nearrow 1$: o.
4. $3 \nwarrow 1$: $o - u - o$.
5. $3 \nearrow 4$: $u - o - u$.
6. $4 \nwarrow 4$: $o - 2u$.
7. $4 \nearrow 3$: $u - 2o$.
8. $1 \nwarrow 3$: $u - 2o - 2u - o$.
9. $1 \nearrow 2$: $o - 2u - 2o - u$.
10. $2 \nwarrow 2$: $3o - 2u - 2o$.
11. $2 \nearrow 1$: $3u - 2o - 2u$.
12. $3 \nwarrow 1$: $2o - u - o - 2u - 2o$.
13. $3 \nearrow 4$: $u - o - u - 2o - 2u$.
14. $4 \nwarrow 4$: $u - 2o - u - 2o - 2u$.
15. $4 \nearrow 3$: $u - o - 3u - 2o - u$.
16. $1 \nwarrow 3$: $2u - o - u - 3o - 3u - o$.
17. $1 \nearrow 2$: $3o - 3u - o - u - 3o - u$.
18. $2 \nwarrow 2$: $u - 2o - u - 2o - 4u - 2o - u$.
19. $2 \nearrow 1$: $u - 2o - 3u - 3o - u - 2o - u$.
20. $3 \nwarrow 1$: $3o - 2u - 2o - 3u - 3o - u$.
21. $3 \nearrow 4$: $2u - 2o - 2u - 3o - 3u$.
22. $4 \nwarrow 4$: $2u - 2o - 3u - 3o - 3u$.
23. $4 \nearrow 3$: $2u - 3o - 3u - 3o - 2u$.
24. $1 \nwarrow 3$: $2u - 2o - 3u - 4o - 3u - 2o$.
25. $1 \nearrow 2$: $4o - 3u - 2o - 3u - 4o - u$.

26.	$2 \nwarrow 2$:	$u - 3o - 2u - 4o - 4u - 3o - u$.
27.	$2 \nearrow 1$:	$2u - 3o - 4u - 3o - 3u - 3o - u$.
28.	$3 \nwarrow 1$:	$4o - 3u - 3o - 4u - 4o - 2u$.
29.	$3 \nearrow 4$:	$3u - 3o - 3u - 4o - 4u$.
30.	$4 \nwarrow 4$:	$2u - 4o - 4u - 4o - 3u$.
31.	$4 \nearrow 3$:	$4u - 3o - 4u - 4o - 3u$.
32.	$1 \nwarrow 3$:	$2u - 4o - 4u - 4o - 4u - 4o$.

The grid-diagram of this knot is depicted in Fig. 541

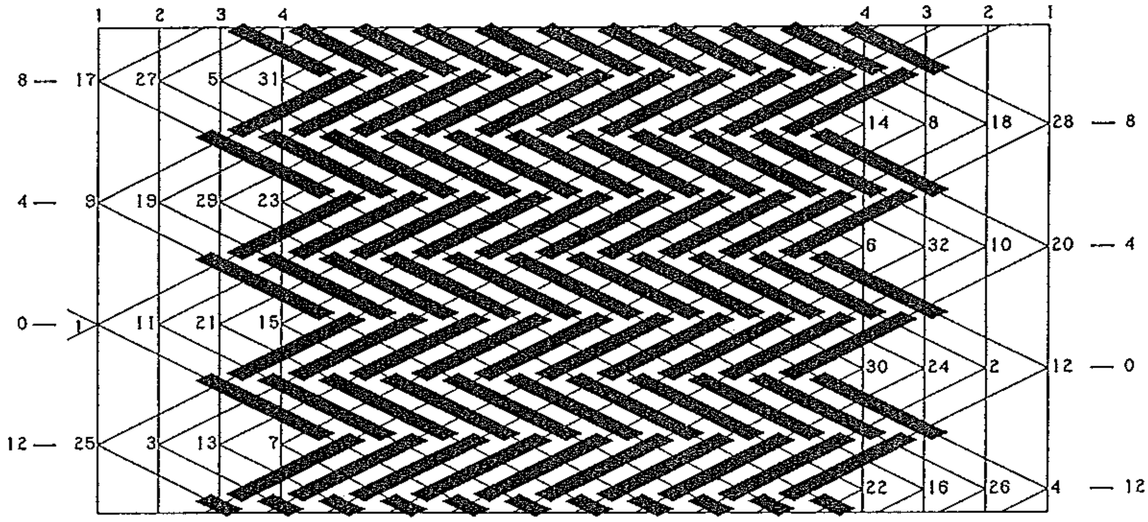

Fig. 541 — Grid-diagram of the knot in Example 2.

The string-runs of the knots in Example 1 and 2 are each others mirror-image and they have an identical number of bights. Hence when for one of such herringbone pineapple coded knots the two half-cycle tables have been determined, we can then readily derive from these tables the two half-cycle tables for the mirror-imaged knot.

Say the first-return string-run and the two half-cycle tables for the $y = A - 1$ knot are known. On its first-return string-run determine the number of half-cycles between right-hand bight-boundary 1 and left-hand bight-boundary 1 at the end of the first-return string-run (this number of half-cycles is equal to 3 for Example 1). Then replace each half-cycle number in its two half-cycle tables by

$$|\text{half-cycle number} + 3|_{2B} .$$

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A + 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A + 1$).

Say the first-return string-run and the two half-cycle tables for the $y = A + 1$ knot are known. On its first-return string-run determine the number of half-cycles between left-hand bight-boundary 1 at the beginning of the first-return string-run and right-hand bight-boundary 1 (this number of half-cycles is equal to 3 for Example 2). Then replace each half-cycle number in its two half-cycle tables by

$$|\text{half-cycle number} - 3|_{2B} .$$

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$).

Let's now have a look at two Examples of Semi-Perfect Herringbone Pineapple Knots which are each others mirror-image.

Example 3 :

$A = 5$; $x = 14$; $y = A - 1$, hence $\Delta = A - 1 = 4$; $B^* = 4$, hence $B = A \cdot B^* = 20$.
Thus $k = \left|\frac{x-A-1}{2}\right|_A = \left|\frac{14-5-1}{2}\right|_5 = 4$.
The first-return string-run and the half-cycle pattern is shown in Fig. 542.

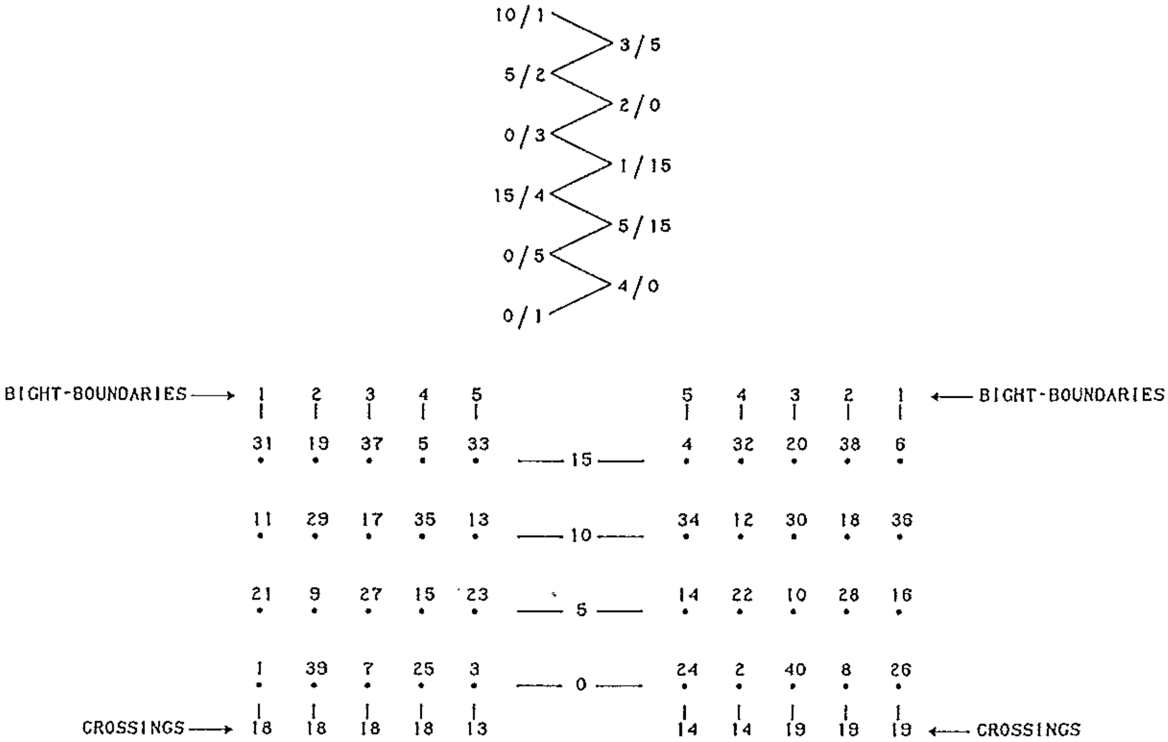

Fig. 542 — First-return string-run and half-cycle pattern.

The second sequence of half-cycles in a first-return string-run starts with half-cycle number $1 + 2A = 11$ at nest-index number $I_L = A|2A + x - 2|_{B^*} = 5|22|_4 = 10$. Hence two sequential half-cycle numbers h_n (≥ 1) and $h_n + 2A$ ($\leq 2B$) associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I + 2A^2 + A(x-2)\right|_B = \left|I + 2 \cdot 5^2 + 5(14-2)\right|_{20} = |I + 110|_{20} = |I + 10|_{20}$. Hence the last half-cycle in the second sequence of half-cycles in a first-return string-run ends at the starting-point of the first half-cycle in the first sequence of half-cycles in a first-return string-run. Since for Perfect and Semi-Perfect Regular Nested Knots the number of half-cycles in a first-return string-run is equal to $2A$ and since these knots consist of $2B = 2AB^*$ half-cycles, it follows that these knots consist of B^* first-return string-runs. Thus in our Example the knot consists of four first-return string-runs, and since in the first two consecutive first-return string-runs the end-point of the last half-cycle returns to the starting-point of the first half-cycle, the knot consists of two sub-components. The first sub-component consists of the half-cycles 1 to 20, and the second sub-component consists of the half-cycles 21 to 40. When laying down the half-cycles 1 to 20, the 1^* position in the $L \to R$ half-cycle table should **not** be read, but when laying down the half-cycles 21 to 40 it should be read and the 21 position should **not** be read.

From the half-cycle pattern in Fig. 542 we assemble again the two half-cycle tables in Fig. 543.

L ⟶ R

1•	39	7	25	3	~~21~~	9	27	15	23	11	29	17	35	13	31	19	37				
31	19	37	5	33	1•	39	7	25	3	~~21~~	9	27	15	23	11	29	17				
11	29	17	35	13	31	19	37	5	33	1•	39	7	25	3	~~21~~	9	27				
~~21~~	9	27	15	23	11	29	17	35	13	31	19	37	5	33	1•	39	7				
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	1	21	11	31
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	39	9	29	19
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	7	27	17	37
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	25	15	35	5
u	u	u	u	o	o	o	o	o	u	u	u	u						3	23	13	33

R ⟶ L

6	38	20	32	4	26	8	40	2	24	16	28	10	22	14	36	18	30	12				
36	18	30	12	34	6	38	20	32	4	26	8	40	2	24	16	28	10	22				
16	28	10	22	14	36	18	30	12	34	6	38	20	32	4	26	8	40	2				
26	8	40	2	24	16	28	10	22	14	36	18	30	12	34	6	38	20	32				
u	u	u	u	u	o	o	o	o	o	u	u	u	u						4	24	14	34
u	u	u	u	o	o	o	o	o	u	u	u	u	u						32	2	22	12
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	20	40	10	30
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	38	8	28	18
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	6	26	16	36

Fig. 543 — The half-cycle tables for the knot in Example 3.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 3:

1. $1 \nearrow 4$: Free run.
2. $5 \nwarrow 4$: Free run.
3. $5 \nearrow 5$: Free run.
4. $4 \nwarrow 5$: u.
5. $4 \nearrow 1$: o.
6. $3 \nwarrow 1$: $2o$.
7. $3 \nearrow 2$: $o - u$.
8. $2 \nwarrow 2$: $u - 2o - u$.
9. $2 \nearrow 3$: $u - 2o - u$.
10. $1 \nwarrow 3$: $o - 2u - 2o$.
11. $1 \nearrow 4$: $o - 2u - o - u$.
12. $5 \nwarrow 4$: $u - 3o - u$.
13. $5 \nearrow 5$: $u - o - 2u$.
14. $4 \nwarrow 5$: $u - 2o - 2u$.
15. $4 \nearrow 1$: $u - o - 2u - 2o$.
16. $3 \nwarrow 1$: $2o - u - 3o - u$.
17. $3 \nearrow 2$: $o - 3u - 2o - u$.
18. $2 \nwarrow 2$: $u - 2o - 2u - 3o - u$.
19. $2 \nearrow 3$: $2o - 3u - 2o - u$.
20. $1 \nwarrow 3$: $u - 3o - 2u - 3o$.
21. $1 \nearrow 4$: $3o - 2u - 3o - u$.
22. $5 \nwarrow 4$: $2u - 2o - 3u$.
23. $5 \nearrow 5$: $2u - 3o - 2u$.
24. $4 \nwarrow 5$: $4u - 2o - 2u$.
25. $4 \nearrow 1$: $u - 4o - 2u - 3o$.
26. $3 \nwarrow 1$: $u - 3o - 3u - 3o - 2u$.
27. $3 \nearrow 2$: $u - 3o - 2u - 4o - u$.
28. $2 \nwarrow 2$: $u - 2o - 4u - 4o - 2u$.
29. $2 \nearrow 3$: $3o - 3u - 5o - 2u$.
30. $1 \nwarrow 3$: $2u - 3o - 5u - 4o - u$.
31. $1 \nearrow 4$: $4o - 4u - 4o - 2u$.
32. $5 \nwarrow 4$: $3u - 5o - 4u$.
33. $5 \nearrow 5$: $3u - 4o - 4u$.
34. $4 \nwarrow 5$: $4u - 4o - 4u$.
35. $4 \nearrow 1$: $2u - 4o - 4u - 5o$.
36. $3 \nwarrow 1$: $u - 4o - 4u - 5o - 3u$.
37. $3 \nearrow 2$: $u - 4o - 5u - 5o - u$.
38. $2 \nwarrow 2$: $2u - 4o - 5u - 5o - 2u$.
39. $2 \nearrow 3$: $5o - 5u - 5o - 2u$.
40. $1 \nwarrow 3$: $3u - 5o - 5u - 5o - u$.

The grid-diagram of this knot is depicted in Fig. 544

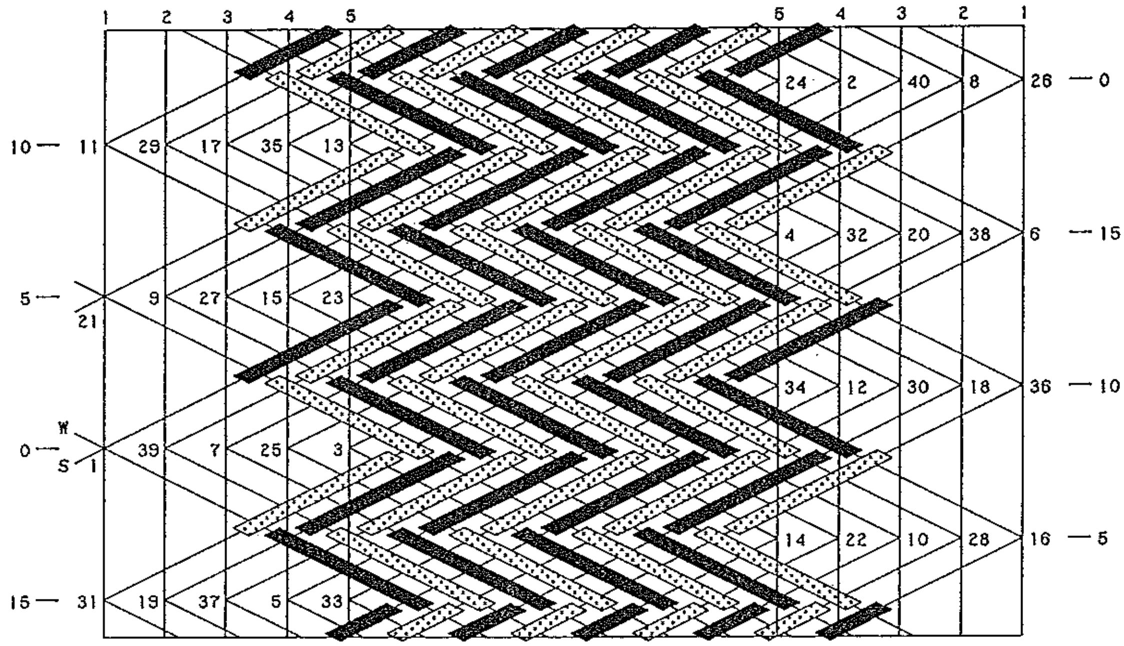

Fig. 544 — Grid-diagram of the knot in Example 3.

Example 4:

$A = 5$; $x = 14$; $y = A + 1$, hence $\Delta = 1$; $B^* = 4$, hence $B = A \cdot B^* = 20$.

Thus $k = \left|\frac{x-A-3}{2}\right|_A = \left|\frac{14-5-3}{2}\right|_5 = 3$.

The first-return string-run and the half-cycle pattern is shown in Fig. 545.

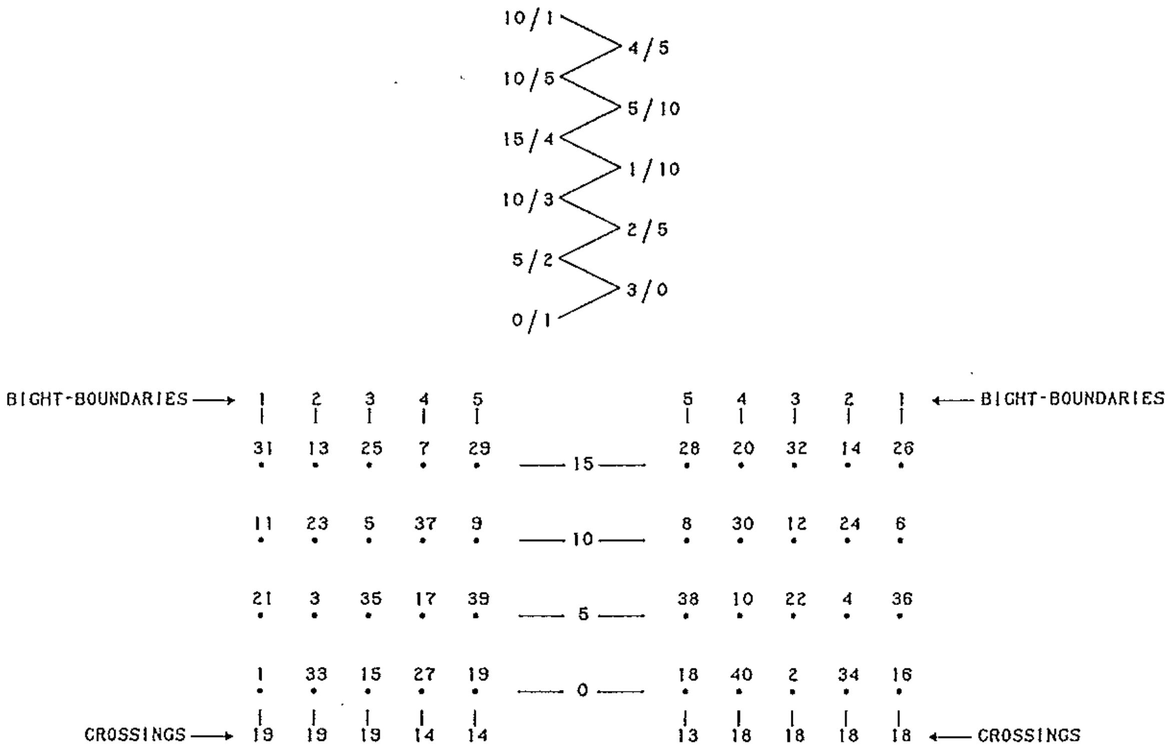

Fig. 545 — First-return string-run and half-cycle pattern.

The second sequence of half-cycles in a first-return string-run starts with half-cycle number $1 + 2A = 11$ at nest-index number $I_L = A|2A + x - 2|_{B^*} = 5|22|_4 = 10$.

Hence two sequential half-cycle numbers h_n (≥ 1) and $h_n + 2A$ $(\leq 2B)$ associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I + 2A^2 + A(x-2)\right|_B = \left|I + 2\cdot 5^2 + 5(14-2)\right|_{20} = |I + 110|_{20} = |I + 10|_{20}$. Hence the last half-cycle in the second sequence of half-cycles in a first-return string-run ends at the starting-point of the first half-cycle in the first sequence of half-cycles in a first-return string-run. Since for Perfect and Semi-Perfect Regular Nested Knots the number of half-cycles in a first-return string-run is equal to $2A$ and since these knots consist of $2B = 2AB^*$ half-cycles, it follows that these knots consist of B^* first-return string-runs. Thus in our Example the knot consists of four first-return string-runs, and since in the first two consecutive first-return string-runs the end-point of the last half-cycle returns to the starting-point of the first half-cycle, the knot consists of two sub-components. The first sub-component consists of the half-cycles 1 to 20, and the second sub-component consists of the half-cycles 21 to 40. When laying down the half-cycles 1 to 20, the 1* position in the $L \to R$ half-cycle table should **not** be read, but when laying down the half-cycles 21 to 40 it should be read and the 21 position should **not** be read.

From the half-cycle pattern in Fig. 545 we assemble again the two half-cycle tables in Fig. 546.

L → R

11	23	5	37	9	31	13	25	7	29	1*	33	15	27	19	~~21~~	3	35	17				
~~21~~	3	35	17	39	11	23	5	37	9	31	13	25	7	29	1*	33	15	27				
1*	33	15	27	19	~~21~~	3	35	17	39	11	23	5	37	9	31	13	25	7				
31	13	25	7	29	1*	33	15	27	19	~~21~~	3	35	17	39	11	23	5	37				
u	u	u	u	u	o	o	o	o	o	u	u	u	u						9	29	19	39
u	u	u	u	o	o	o	o	o	u	u	u	u	u						37	7	27	17
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	5	25	15	35
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	23	13	33	3
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	11	31	1	21

R → L

6	24	12	30	8	26	14	32	20	28	16	34	2	40	18	36	4	22				
36	4	22	10	38	6	24	12	30	8	26	14	32	20	28	16	34	2				
16	34	2	40	18	36	4	22	10	38	6	24	12	30	8	26	14	32				
26	14	32	20	28	16	34	2	40	18	36	4	22	10	38	6	24	12				
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	6	26	16	36
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	24	14	34	4
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	12	32	2	22
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	30	20	40	10
u	u	u	u	o	o	o	o	o	u	u	u	u						8	28	18	38

Fig. 546 — The half-cycle tables for the knot in Example 4.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 4:

1. $1 \nearrow 3$: Free run.
2. $2 \nwarrow 3$: u.
3. $2 \nearrow 2$: o.
4. $3 \nwarrow 2$: $o - u$.
5. $3 \nearrow 1$: $u - o$.
6. $4 \nwarrow 1$: $u - o - u$.

21. $1 \nearrow 3$: $u - 2o - 3u - 2o - 2u$.
22. $2 \nwarrow 3$: $u - 3o - 2u - 3o - u$.
23. $2 \nearrow 2$: $u - 2o - 3u - 3o - u$.
24. $3 \nwarrow 2$: $3o - 2u - 4o - 2u$.
25. $3 \nearrow 1$: $2u - 2o - 4u - 3o - u$.
26. $4 \nwarrow 1$: $3o - 3u - 4o - 2u$.

7.	$4 \nearrow 5$:	$o - u$.	27.	$4 \nearrow 5$:	$2u - 3o - 4u$.
8.	$5 \nwarrow 5$:	$o - u$.	28.	$5 \nwarrow 5$:	$2u - 4o - 3u$.
9.	$5 \nearrow 4$:	$2u$.	29.	$5 \nearrow 4$:	$4u - 2o - 3u$.
10.	$1 \nwarrow 4$:	$u - o - u - o$.	30.	$1 \nwarrow 4$:	$2u - 4o - 3u - 4o$.
11.	$1 \nearrow 3$:	$3o - u$.	31.	$1 \nearrow 3$:	$u - 3o - 3u - 4o - 3u$.
12.	$2 \nwarrow 3$:	$2u - 2o - u$.	32.	$2 \nwarrow 3$:	$u - 3o - 4u - 5o - u$.
13.	$2 \nearrow 2$:	$o - u - 3o - u$.	33.	$2 \nearrow 2$:	$u - 3o - 4u - 5o - 2u$.
14.	$3 \nwarrow 2$:	$o - 3u - 2o - u$.	34.	$3 \nwarrow 2$:	$4o - 5u - 5o - 2u$.
15.	$3 \nearrow 1$:	$u - 2o - 2u - 2o$.	35.	$3 \nearrow 1$:	$3u - 4o - 5u - 4o - u$.
16.	$4 \nwarrow 1$:	$2o - 3u - o - 2u$.	36.	$4 \nwarrow 1$:	$5o - 5u - 4o - 3u$.
17.	$4 \nearrow 5$:	$2u - 3o - u$.	37.	$4 \nearrow 5$:	$4u - 5o - 4u$.
18.	$5 \nwarrow 5$:	$2u - 2o - 2u$.	38.	$5 \nwarrow 5$:	$4u - 5o - 4u$.
19.	$5 \nearrow 4$:	$2u - 3o - 2u$.	39.	$5 \nearrow 4$:	$5u - 5o - 4u$.
20.	$1 \nwarrow 4$:	$2u - 2o - 3u - 2o$.	40.	$1 \nwarrow 4$:	$3u - 5o - 5u - 5o$.

The grid-diagram of this knot is depicted in Fig. 547

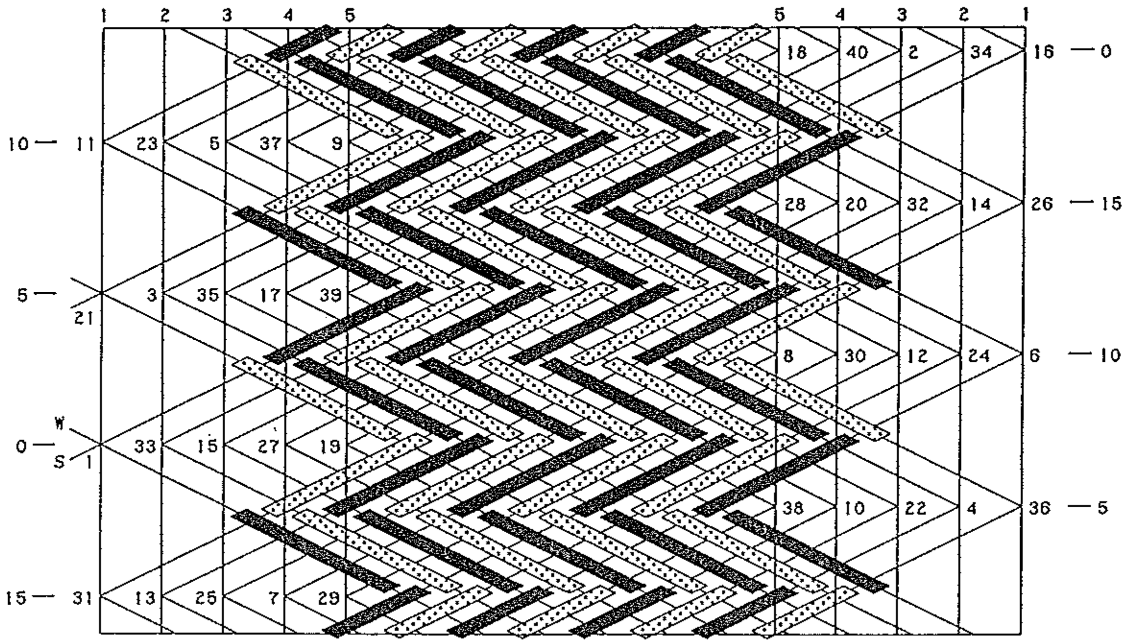

Fig. 547 — Grid-diagram of the knot in Example 4.

For a Semi-Perfect Herringbone Pineapple Knots we can also derive its half-cycle tables from the half-cycle tables of its mirror-image. Let g.c.d.$(P, B^*) = \lambda$, where $P = 2A + x - 2$. Then the Semi-Perfect Herringbone Pineapple Knot consists of λ sub-components. Each sub-component consists of $\frac{2B}{\lambda}$ half-cycles.

The first sub-component consists of the half-cycles 1 to $\frac{2B}{\lambda}$.
The second sub-component consists of the half-cycles $\left(\frac{2B}{\lambda} + 1\right)$ to $2 \cdot \frac{2B}{\lambda}$.
The third sub-component consists of the half-cycles $\left(2 \cdot \frac{2B}{\lambda} + 1\right)$ to $3 \cdot \frac{2B}{\lambda}$.

⋮

The n^{th} sub-component consists of the half-cycles $\left((n-1) \cdot \frac{2B}{\lambda} + 1\right)$ to $\left(n \cdot \frac{2B}{\lambda}\right)$.

⋮

The λ^{th} sub-component consists of the half-cycles $\left((\lambda - 1) \cdot \frac{2B}{\lambda} + 1\right)$ to $2B$.

Say that the first half-cycle of each sub-component starts at left bight-boundary 1 (as in Examples 3 and 4).

Say the first-return string-run and the two half-cycle tables for the $y = A - 1$ knot are known. On its first-return string-run determine the number of half-cycles φ between right-hand bight-boundary 1 and left-hand bight-boundary 1 at the end of the first-return string-run (this number of half-cycles is equal to $\varphi = 5$ for Example 3). Then replace in its two half-cycle tables each half-cycle number h in the range:

$1 \le h \le \frac{2B}{\lambda}$ by $|h+\varphi|_{\frac{2B}{\lambda}}$.

$\left(\frac{2B}{\lambda}+1\right) \le h \le 2\cdot\frac{2B}{\lambda}$ by $|h+\varphi|_{\frac{2B}{\lambda}} + \frac{2B}{\lambda}$.

$\left(2\cdot\frac{2B}{\lambda}+1\right) \le h \le 3\cdot\frac{2B}{\lambda}$ by $|h+\varphi|_{\frac{2B}{\lambda}} + 2\cdot\frac{2B}{\lambda}$.

$\vdots$

$\left((n-1)\cdot\frac{2B}{\lambda}+1\right) \le h \le \left(n\cdot\frac{2B}{\lambda}\right)$ by $|h+\varphi|_{\frac{2B}{\lambda}} + (n-1)\cdot\frac{2B}{\lambda}$.

$\vdots$

$\left((\lambda-1)\cdot\frac{2B}{\lambda}+1\right) \le h \le 2B$ by $|h+\varphi|_{\frac{2B}{\lambda}} + (\lambda-1)\cdot\frac{2B}{\lambda}$.

In the above formulae take $|h+\varphi|_{\frac{2B}{\lambda}} = \frac{2B}{\lambda}$ when $|h+\varphi|_{\frac{2B}{\lambda}} = 0$.

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A+1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A+1$).

Thus for the knot in Example 3 with $B = 20$; $\lambda = 2$ and $\varphi = 5$, replace in its two half-cycle tables each half-cycle number h in the range:

$1 \le h \le 20$ by $|h+5|_{20}$.

$21 \le h \le 40$ by $|h+5|_{20} + 20$.

Take $|h+5|_{20} = 20$ when $|h+5|_{20} = 0$.

Say the first-return string-run and the two half-cycle tables for the $y = A + 1$ knot are known. On its first-return string-run determine the number of half-cycles φ between left-hand bight-boundary 1 at the beginning of the first-return string-run and right-hand bight-boundary 1 (this number of half-cycles is equal to $\varphi = 5$ for Example 4). Then replace in its two half-cycle tables each half-cycle number h in the range:

$1 \le h \le \frac{2B}{\lambda}$ by $|h-\varphi|_{\frac{2B}{\lambda}}$.

$\left(\frac{2B}{\lambda}+1\right) \le h \le 2\cdot\frac{2B}{\lambda}$ by $|h-\varphi|_{\frac{2B}{\lambda}} + \frac{2B}{\lambda}$.

$\left(2\cdot\frac{2B}{\lambda}+1\right) \le h \le 3\cdot\frac{2B}{\lambda}$ by $|h-\varphi|_{\frac{2B}{\lambda}} + 2\cdot\frac{2B}{\lambda}$.

$\vdots$

$\left((n-1)\cdot\frac{2B}{\lambda}+1\right) \le h \le \left(n\cdot\frac{2B}{\lambda}\right)$ by $|h-\varphi|_{\frac{2B}{\lambda}} + (n-1)\cdot\frac{2B}{\lambda}$.

$\vdots$

$\left((\lambda-1)\cdot\frac{2B}{\lambda}+1\right) \le h \le 2B$ by $|h-\varphi|_{\frac{2B}{\lambda}} + (\lambda-1)\cdot\frac{2B}{\lambda}$.

In the above formulae take $|h-\varphi|_{\frac{2B}{\lambda}} = \frac{2B}{\lambda}$ when $|h-\varphi|_{\frac{2B}{\lambda}} = 0$.

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$).

Thus for the knot in Example 4 with $B = 20$; $\lambda = 2$ and $\varphi = 5$, replace in its two half-cycle tables each half-cycle number h in the range:

$1 \le h \le 20$ by $|h - 5|_{20}$.
$21 \le h \le 40$ by $|h - 5|_{20} + 20$.
Take $|h - 5|_{20} = 20$ when $|h - 5|_{20} = 0$.

In many applications it is much better **not** to have the Standing-End and Working-End at bight-boundary 1. In general we furthermore like to distribute the Standing and Working Ends of the sub-components as regularly as possible over the circumference of a bight-edge. Note that when the Standing and Working Ends of the two sub-components are placed on bight-boundary 1 as in the above Examples 3 and 4, such a regular distribution is not possible.

Let's assume that for practical reasons we like to have the Standing and Working Ends on the inner bight-boundaries and that we want them to be regularly distributed over the circumference of the bight-edge. Thus we could place the Standing-End of the first sub-component on bight-boundary $l_1 = 5$ and nest-index number $I_{L_1} = 0$, and the Standing-End of the second sub-component on bight-boundary $l'_1 = 4$ and nest-index number $I'_{L_1} = 10$. Hence half-cycle 1 starts at left bight-boundary $l_1 = 5$ and nest-index number $I_{L_1} = 0$, while half-cycle 21 starts at left bight-boundary $l'_1 = 4$ and nest-index number $I'_{L_1} = 10$.

Say that we are going to work out the half-cycle tables for the Semi-Perfect Herringbone Pineapple Knot for which $y = A + 1 = 5 + 1 = 6$ (hence $\Delta = |6|_5 = 1$):

Example 5 :

$A = 5; x = 14; l_1 = 5, I_{L_1} = 0; l'_1 = 4, I'_{L_1} = 10; y = A+1$, hence $\Delta = 1; B^* = 4$, hence $B = A \cdot B^* = 20$.

Thus

$$r_1 = \left|\frac{x - A - 1 - 2l_1}{2}\right|_A = \left|\frac{14 - 5 - 1 - 10}{2}\right|_5 = |-1|_5 = 4,$$
$$I_{R_1} = I_{L_1} = 0;$$
$$r'_1 = \left|\frac{x - A - 1 - 2l'_1}{2}\right|_A = \left|\frac{14 - 5 - 1 - 8}{2}\right|_5 = |0|_5 = 5,$$
$$I'_{R_1} = \left|I'_{L_1} + (l_1 + r_1) - (l'_1 + r'_1)\right|_B = |10 + (5 + 4) - (4 + 5)|_{20} = 10.$$

The first-return string-run and the half-cycle pattern is shown in Fig. 548. Although there is only one type of first-return string-run, the sequential half-cycle pattern for each of the two sub-components is shown. The reader be reminded that the 1^{st} half-cycle, which starts at the left-hand bight-boundary $l_1 = 5$ and nest-index number $I_{L_1} = 0$, is the first half-cycle of the first sub-component, and that the 21^{st} half-cycle, which starts at the left-hand bight-boundary $l'_1 = 4$ and nest-index number $I'_{L_1} = 10$, is the first half-cycle of the second sub-component.

From this half-cycle pattern in Fig. 548 we assemble in the usual way the half-cycle tables in Fig. 549.

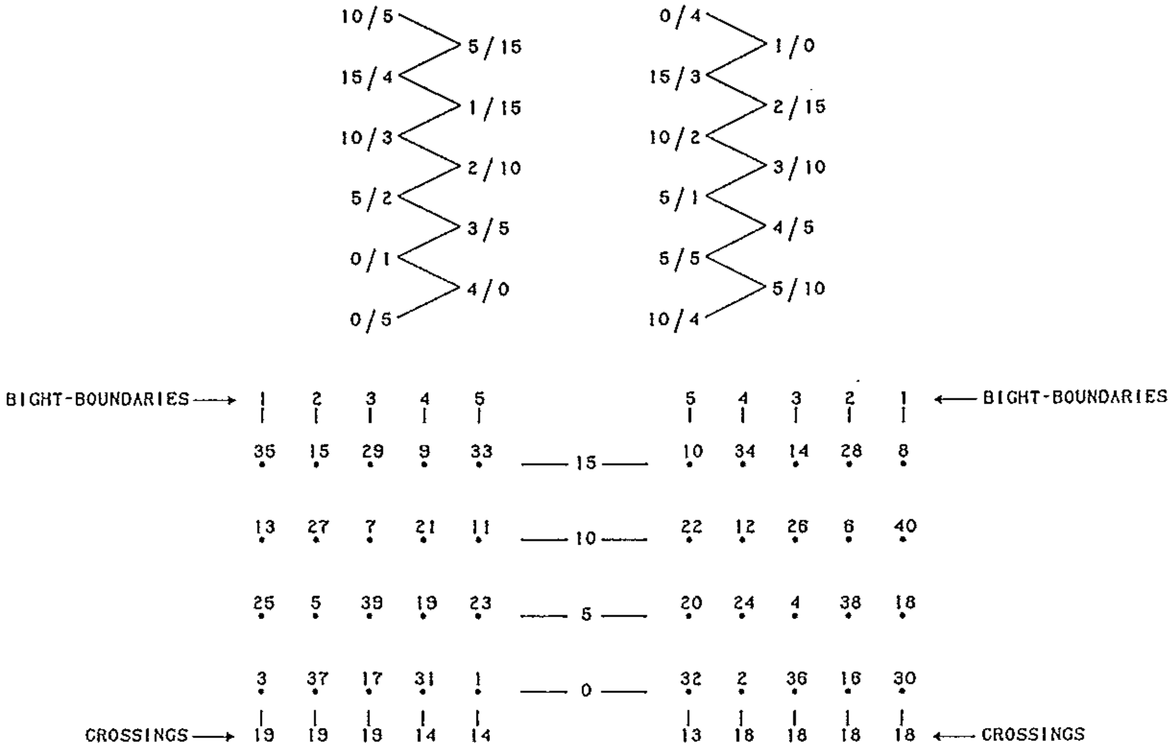

Fig. 548 — First-return string-run and half-cycle pattern.

L ⟶ R

13	27	7	~~21~~	11	35	15	29	9	33	3	37	17	31	1•	25	5	39	19				
25	5	39	19	23	13	27	7	~~21~~	11	35	15	29	9	33	3	37	17	31				
3	37	17	31	1•	25	5	39	19	23	13	27	7	~~21~~	11	35	15	29	9				
35	15	29	9	33	3	37	17	31	1•	25	5	39	19	23	13	27	7	~~21~~				
u	u	u	u	u	o	o	o	o	o	u	u	u	u						11	33	1	23
u	u	u	u	o	o	o	o	o	u	u	u	u	u						21	9	31	19
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	7	29	17	39
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	27	15	37	5
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	13	35	3	25

R ⟶ L

18	38	4	24	20	40	6	26	12	22	8	28	14	34	10	30	16	36				
30	16	36	2	32	18	38	4	24	20	40	6	26	12	22	8	28	14				
8	28	14	34	10	30	16	36	2	32	18	38	4	24	20	40	6	26				
40	6	26	12	22	8	28	14	34	10	30	16	36	2	32	18	38	4				
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	18	40	8	30
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	38	6	28	16
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	4	26	14	36
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	24	12	34	2
u	u	u	u	o	o	o	o	o	u	u	u	u						20	22	10	32

Fig. 549 — The half-cycle tables for the knot in Example 5.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 5:

1. 5 ↗ 4: Free run.
2. 1 ↖ 4: Free run.

21. 4 ↗ 5: $2u - 2o - 3u$.
22. 5 ↖ 5: $2u - 3o - 2u$.

3.	$1 \nearrow 3$:	o.	23.	$5 \nearrow 4$:	$3u - 2o - 2u$.
4.	$2 \nwarrow 3$:	$o - u$.	24.	$1 \nwarrow 4$:	$u - 4o - 2u - 3o$.
5.	$2 \nearrow 2$:	$u - o$.	25.	$1 \nearrow 3$:	$u - 2o - 3u - 3o - 2u$.
6.	$3 \nwarrow 2$:	$u - o - u$.	26.	$2 \nwarrow 3$:	$u - 3o - 2u - 4o - u$.
7.	$3 \nearrow 1$:	$o - u - o$.	27.	$2 \nearrow 2$:	$u - 2o - 4u - 4o - u$.
8.	$4 \nwarrow 1$:	$o - u - o - u$.	28.	$3 \nwarrow 2$:	$3o - 3u - 5o - 2u$.
9.	$4 \nearrow 5$:	$u - o - u$.	29.	$3 \nearrow 1$:	$2u - 3o - 5u - 3o - u$.
10.	$5 \nwarrow 5$:	$u - o - u$.	30.	$4 \nwarrow 1$:	$4o - 4u - 4o - 2u$.
11.	$5 \nearrow 4$:	$u - o - u$.	31.	$4 \nearrow 5$:	$3u - 4o - 4u$.
12.	$1 \nwarrow 4$:	$u - o - 2u - o$.	32.	$5 \nwarrow 5$:	$3u - 4o - 4u$.
13.	$1 \nearrow 3$:	$4o - u$.	33.	$5 \nearrow 4$:	$4u - 4o - 3u$.
14.	$2 \nwarrow 3$:	$o - 2u - 2o - u$.	34.	$1 \nwarrow 4$:	$2u - 4o - 4u - 5o$.
15.	$2 \nearrow 2$:	$u - o - u - 3o - u$.	35.	$1 \nearrow 3$:	$u - 4o - 4u - 4o - 3u$.
16.	$3 \nwarrow 2$:	$o - 3u - 2o - u$.	36.	$2 \nwarrow 3$:	$u - 4o - 5u - 5o - u$.
17.	$3 \nearrow 1$:	$u - 2o - 2u - 3o$.	37.	$2 \nearrow 2$:	$2u - 4o - 4u - 5o - 2u$.
18.	$4 \nwarrow 1$:	$2o - 3u - 2o - 2u$.	38.	$3 \nwarrow 2$:	$5o - 5u - 5o - 2u$.
19.	$4 \nearrow 5$:	$2u - 3o - 2u$.	39.	$3 \nearrow 1$:	$3u - 4o - 5u - 5o - u$.
20.	$5 \nwarrow 5$:	$2u - 2o - 2u$.	40.	$4 \nwarrow 1$:	$5o - 5u - 5o - 3u$.

The grid-diagram of this knot is depicted in Fig. 550

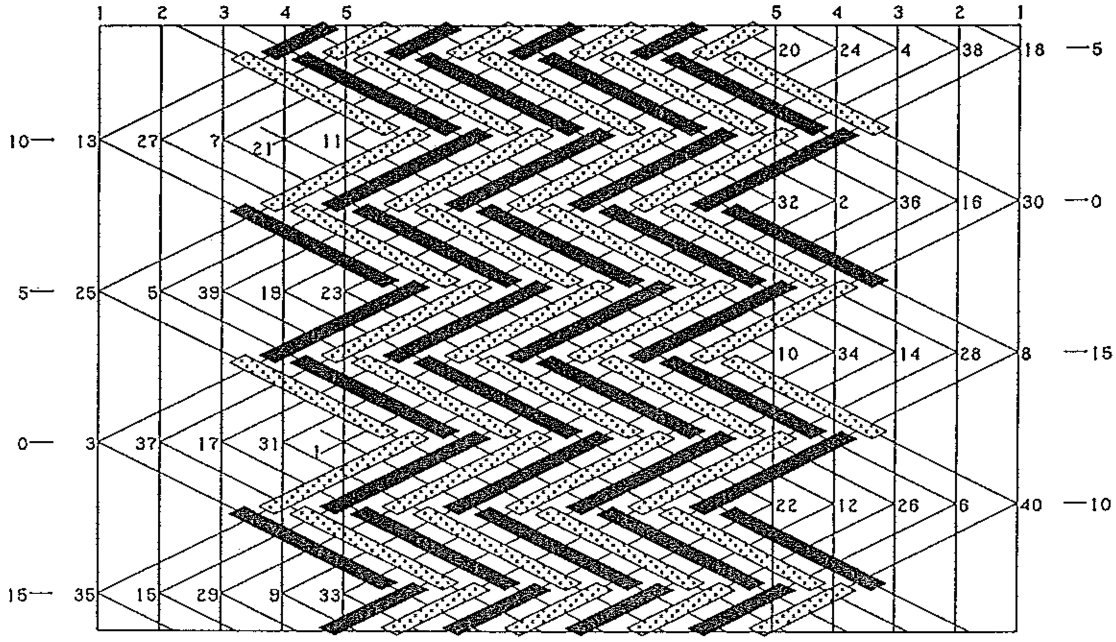

Fig. 550 — Grid-diagram of the knot in Example 5.

The half-cycle tables for the mirror-imaged knot, hence the Semi-Perfect Herringbone Pineapple Knot with $A = 5$; $x = 14$; $l_1 = 5$, $I_{L_1} = 0$; $l'_1 = 4$, $I'_{L_1} = 10$; $y = A - 1 = 4$; $B^* = 4$, may now be derived from the half-cycle tables in Fig. 549. Again, the new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$).

Thus for the knot in Example 5 with $B = 20$, $\lambda = 2$, $\varphi = 9$ for the first sub-component (the number of half-cycles on its first-return string-run between left-hand bight-boundary 5 at the beginning of the first-return string-run and right-hand bight-boundary 5), hence for the half-cycles $1 \leq h \leq 20$, and $\varphi = 3$ for the second sub-component (the number of half-cycles on its first-return string-run between left-hand bight-boundary 4 at the beginning of the first-return string-run and right-hand bight-boundary 4), hence for the half-cycles $21 \leq h \leq 40$, replace in its two half-cycle tables each half-cycle number h in the range:

$1 \leq h \leq 20$ by $|h-9|_{20}$.

$21 \leq h \leq 40$ by $|h-3|_{20} + 20$.

Take $|h-9|_{20} = 20$ when $|h-9|_{20} = 0$.

The thus obtained half-cycle tables of the mirror-imaged knot, hence of the knot with $y = A - 1$, are shown in Fig. 551.

L ⟶ R

9	35	15	~~21~~	11	37	17	23	3	39	19	25	5	31	1•	27	7	33				
27	7	33	13	29	9	35	15	~~21~~	11	37	17	23	3	39	19	25	5				
19	25	5	31	1•	27	7	33	13	29	9	35	15	~~21~~	11	37	17	23				
37	17	23	3	39	19	25	5	31	1•	27	7	33	13	29	9	35	15				
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	9	37	19	27
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	35	17	25	7
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	15	23	5	33
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	21	3	31	13
u	u	u	u	o	o	o	o	o	u	u	u	u						11	39	1	29

R ⟶ L

4	24	18	38	2	32	6	26	20	30	14	34	8	28	12	22	16	36	10				
22	16	36	10	40	4	24	18	38	2	32	6	26	20	30	14	34	8	28				
14	34	8	28	12	22	16	36	10	40	4	24	18	38	2	32	6	26	20				
32	6	26	20	30	14	34	8	28	12	22	16	36	10	40	4	24	18	38				
u	u	u	u	u	o	o	o	o	o	u	u	u	u						2	30	12	40
u	u	u	u	o	o	o	o	o	u	u	u	u	u						38	20	28	10
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	18	26	8	36
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	24	6	34	16
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	4	32	14	22

Fig. 551 — The half-cycle tables for the mirror-imaged knot.

Example 6 shows how these half-cycle tables may also be obtained from the half-cycle pattern of the mirror-imaged knot.

Example 6:

$A = 5$; $x = 14$; $l_1 = 5$, $I_{L_1} = 0$; $l'_1 = 4$, $I'_{L_1} = 10$; $y = A - 1 = 4$, hence $\Delta = 4$; $B^* = 4$, hence $B = A \cdot B^* = 20$.

Thus:

$$r_1 = \left| \frac{x - A + 1 - 2l_1}{2} \right|_A = \left| \frac{14 - 5 + 1 - 10}{2} \right|_5 = |0|_5 = 5\,,$$

$$I_{R_1} = I_{L_1} = 0\,;$$

$$r'_1 = \left| \frac{x - A + 1 - 2l'_1}{2} \right|_A = \left| \frac{14 - 5 + 1 - 8}{2} \right|_5 = |1|_5 = 1\,,$$

$$I'_{R_1} = \left| I'_{L_1} + (l_1 + r_1) - (l'_1 + r'_1) \right|_B = |10 + (5 + 5) - (4 + 1)|_{20} = 15\,.$$

The first-return string-run and half-cycle pattern is shown in Fig. 552

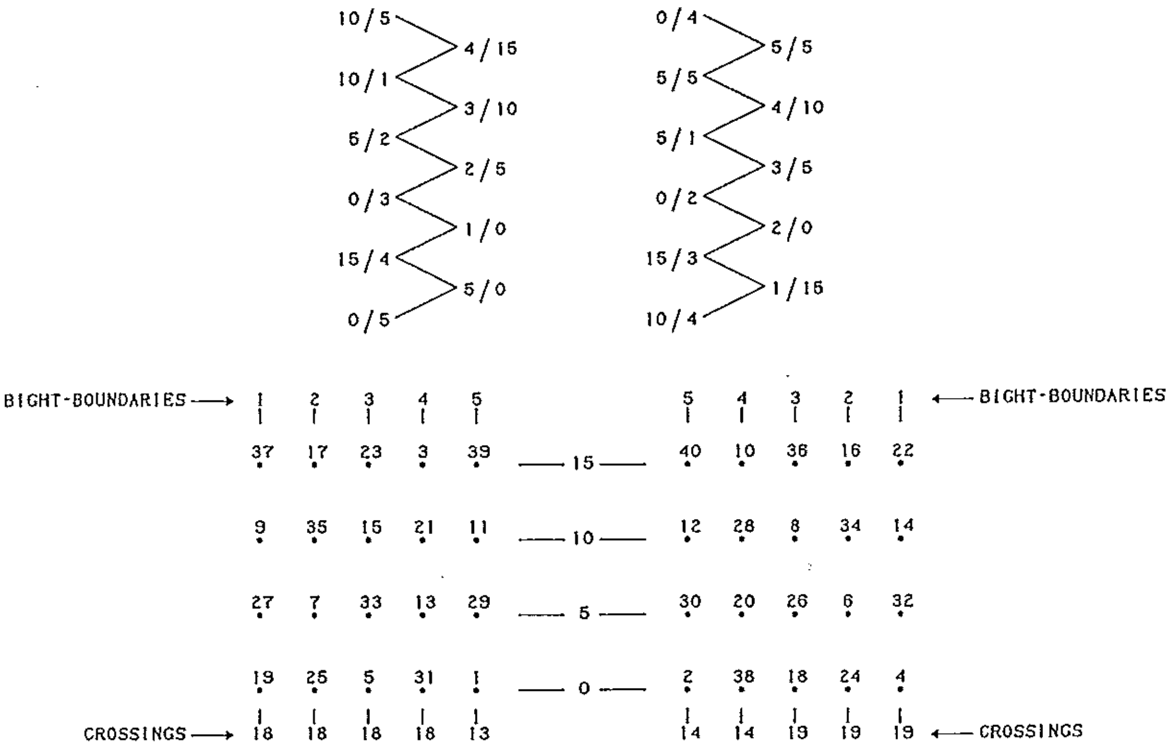

Fig. 552 — First-return string-run and half-cycle pattern.

The grid-diagram of this knot is depicted in Fig. 553

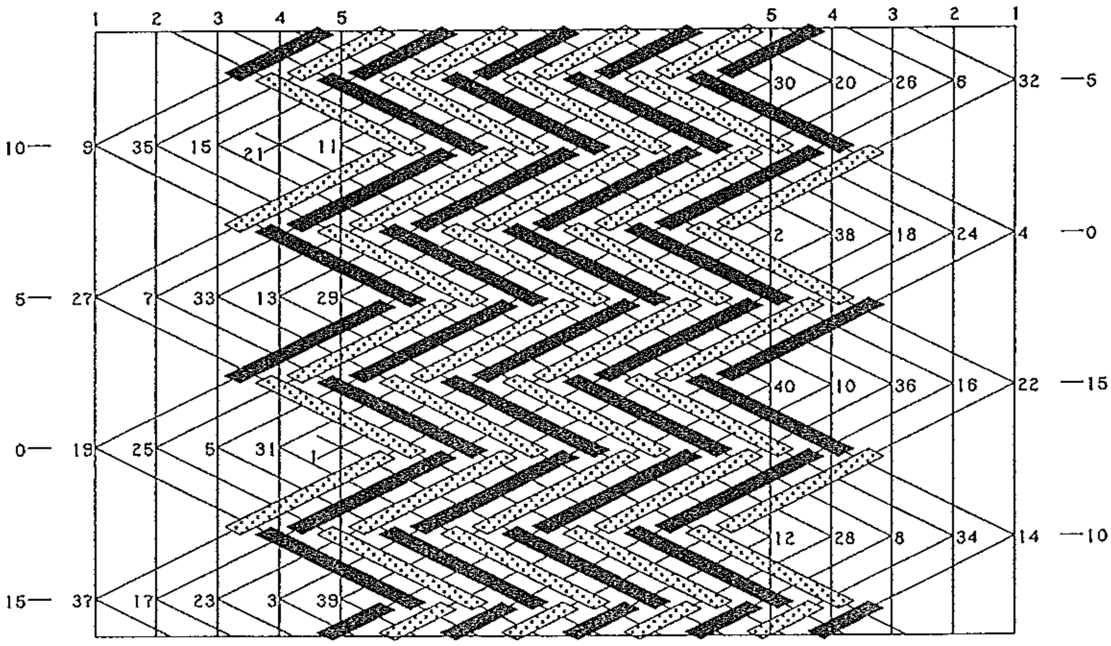

Fig. 553 — Grid-diagram of the knot in Example 6.

From the half-cycle tables in Fig. 551 we obtain its braiding half-cycle algorithms:

1. 5 ↗ 5: Free run.
2. 4 ↖ 5: Free run.
3. 4 ↗ 1: Free run.
4. 3 ↖ 1: o.
5. 3 ↗ 2: $o - u$.

21. 4 ↗ 1: $u - 3o - 2u - 3o$.
22. 3 ↖ 1: $u - 2o - 3u - 3o - 2u$.
23. 3 ↗ 2: $u - 3o - 2u - 3o - u$.
24. 2 ↖ 2: $u - 2o - 4u - 3o - u$.
25. 2 ↗ 3: $3o - 2u - 4o - 2u$.

6.	2 ↖ 2:	$u - 2o$.
7.	2 ↗ 3:	$u - o - u$.
8.	1 ↖ 3:	$o - 2u - o$.
9.	1 ↗ 4:	$o - u - o - u$.
10.	5 ↖ 4:	$u - 2o - u$.
11.	5 ↗ 5:	$u - o - u$.
12.	4 ↖ 5:	$u - 2o - u$.
13.	4 ↗ 1:	$u - o - 2u - o$.
14.	3 ↖ 1:	$2o - u - 2o - u$.
15.	3 ↗ 2:	$o - 2u - 2o - u$.
16.	2 ↖ 2:	$u - 2o - u - 3o - u$.
17.	2 ↗ 3:	$o - 3u - 2o - u$.
18.	1 ↖ 3:	$u - 2o - 2u - 3o$.
19.	1 ↗ 4:	$2o - 3u - 2o - 2u$.
20.	5 ↖ 4:	$2u - 3o - 2u$.
26.	1 ↖ 3:	$2u - 3o - 4u - 3o - u$.
27.	1 ↗ 4:	$3o - 3u - 4o - 2u$.
28.	5 ↖ 4:	$3u - 3o - 4u$.
29.	5 ↗ 5:	$2u - 4o - 3u$.
30.	4 ↖ 5:	$4u - 3o - 3u$.
31.	4 ↗ 1:	$2u - 4o - 3u - 4o$.
32.	3 ↖ 1:	$u - 4o - 3u - 4o - 3u$.
33.	3 ↗ 2:	$u - 3o - 4u - 5o - u$.
34.	2 ↖ 2:	$2u - 3o - 4u - 5o - 2u$.
35.	2 ↗ 3:	$4o - 5u - 5o - 2u$.
36.	1 ↖ 3:	$3u - 4o - 5u - 5o - u$.
37.	1 ↗ 4:	$5o - 5u - 4o - 3u$.
38.	5 ↖ 4:	$4u - 5o - 5u$.
39.	5 ↗ 5:	$4u - 5o - 4u$.
40.	4 ↖ 5:	$5u - 5o - 4u$.

THE BRAIDER'S NOTEBOOK

We have seen that in the Nested Cylindrical Braids the **first-return string-runs** play a very important role:

1. The number of **components** in a Nested Cylindrical Braid is equal to the number of different first-return string-runs in that braid.

2. The number of **parts** P_c of a component in a Nested Cylindrical Braid is determined by the first-return string-run of that component $\left[P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} \right]$.

3. The first-return string-run of a component is a function in the determination of the number of sub-components in that component, hence is a function in the determination of the number of essential strings λ required for that component $[\lambda = \text{g.c.d.}(P_c, B_c)]$.

Last point **3.** clearly indicates the great importance of the first-return string-run in the determination of the number of essential strings for a component. Hence let's look at a cylindrical braid in general (hence not necessarily a nested one) which consists of **one** component (hence has one first-return string-run).

Say this component consists of λ sub-components (hence λ essential strings are required for this component), and since each sub-component consists of a sequence of the same number of identical first-return string-runs, let this number of identical first-return string-runs in each sub-component be n_1. Say that the first-return string-run has α bights and spans β bights in the component. Let the component have B_c bights. Then:

$$n_1\beta = n_2 B_c \quad \rightarrow \quad \text{g.c.d.}(n_1, n_2) = 1,$$
$$\lambda n_1 \alpha = B_c .$$

Thus:

$$\left.\begin{aligned} n_1 &= \frac{B_c}{\lambda\alpha}, \\ n_2 &= \frac{n_1\beta}{B_c} = \frac{\beta}{\lambda\alpha}. \end{aligned}\right\} \quad \rightarrow \quad \text{g.c.d.}\left(\frac{\beta}{\alpha}, \frac{B_c}{\alpha}\right) = \lambda .$$