THE BRAIDER

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A quarterly publication
for
the braiding artisan

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THE BRAIDER'S NOTEBOOK

In some previous issues of *The Braider* we have touched on some aspects relating to "discoveries" in the braiding arena.[†] Even today, as in the past, a braid is generally treated as a unique entity by most braiders, and consequently its associated wider picture remains invisible to them. It is, however, just this wider picture which is so important for a full understanding of the possibilities offered by a particular braid. A "newly discovered" braid is nearly always a simple braid, and before one can proceed to more involved directly or indirectly associated braids, a grid-diagram of the "newly discovered" braid is in such a process essential and hence is a must. Many braiders and especially the editors of braiding publications don't seem to comprehend the importance of grid-diagrams as is testified by the fancy and not so fancy, but in technical terms useless illustrations in their published material. Lets have a look at one of the many of such examples that may be found in the Dutch publication *Het Knoopeknauwertje*[‡].

In *Het Knoopeknauwertje*, Issue No. 6, pp. 4-5, we find a description of **the CK knot**. This is a 'flat' knot, developed by Cornelis Kooiman of Rotterdam in the Netherlands, which has to serve the purpose of 'closing' the open end of, for example, a Regular Nested Cylindrical Braid which closes insufficiently at the bight-edge desired to do so. It should be pointed out that for a purely closure purpose, no separate knot or disc of any form should be used, but instead the nesting-number (A) of the Regular Nested Cylindrical Braid at the bight-edge concerned should be large enough to achieve with four nests ($B^* = 4$) at this bight-edge a complete closure. It should also have been pointed out that the CK knot is only suitable for round braiding material, and not for normal flat material. However, such 'flat' closure-knots may be of some value when they either form an integral part with the knot which doesn't close, or are extended to a further decorative knot on the outside of it. A series of drawings by the editor, of the usual 2-dimensional lay out, accompany the CK knot in *Het Knoopeknauwertje*, and depict a way in which it may be constructed, but the all important grid-diagram of the knot has not been shown. Consequently little if anything can be learned from this knot, which is a great shame since the grid-diagram of this elementary CK knot clearly points to several other not only somewhat more decorative 'flat' closure-knots, but also to 'flat' closure-knots of a different diameter.

Below, in Fig. 505, we have shown the grid-diagram of the CK knot described in *Het Knoopeknauwertje*. The front and back appearances of the knot are shown in Fig. 506.

The knot can of course readily be constructed with the aid of its grid-diagram. Hence it would have been more fruitfull when the series of drawings accompanying this knot in *Het Knoopeknauwertje* had been replaced by its grid-diagram.

It will readily be seen that various modifications to this grid-diagram lead to other

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[†] See for example *The Braider*, Issue No. 8, pg. 162, and Issue No. 11, pg. 245.

[‡] We have selected here the publication *Het Knoopeknauwertje* since its name intends to mean *The unraveller of knot-mysteries*, but does unfortunately nothing in that respect. The reader should, however, be aware that the contents of an article in *Het Knoopeknauwertje* does not necessarily represent the true contents nor the gist of the article submitted by its author, as besides the illustrations by the editor which apparently replace those by the author, the editor states that he reserves the right to abbreviate a submitted article, but does not take any responsibility for its contents!!!

‘flat’ closure-knots for round-string braiding material, one of which is depicted in Fig. 507. The front and back appearances of this knot are shown in Fig. 508.

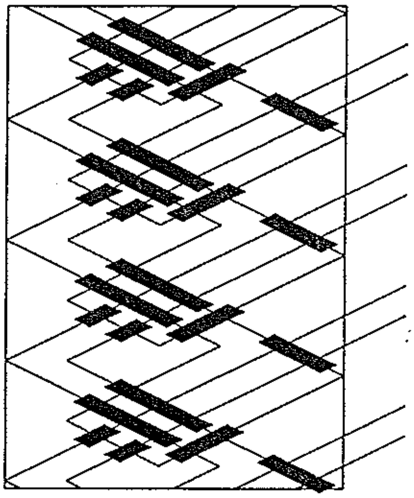

Fig. 505 — The CK knot described in *Het Knoopeknauwertje.*

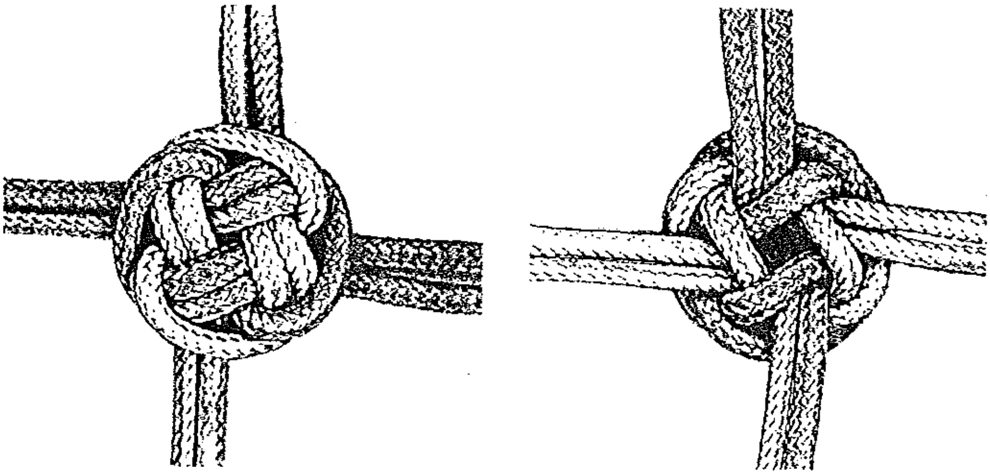

Fig. 506 — The front and the back of the knot in Fig. 505.

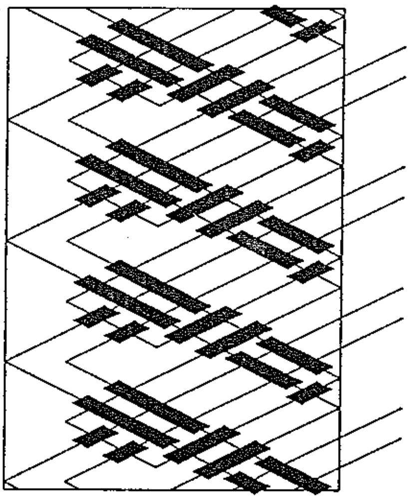

Fig. 507 — One of the several possible modifications of the diagram in Fig. 505.

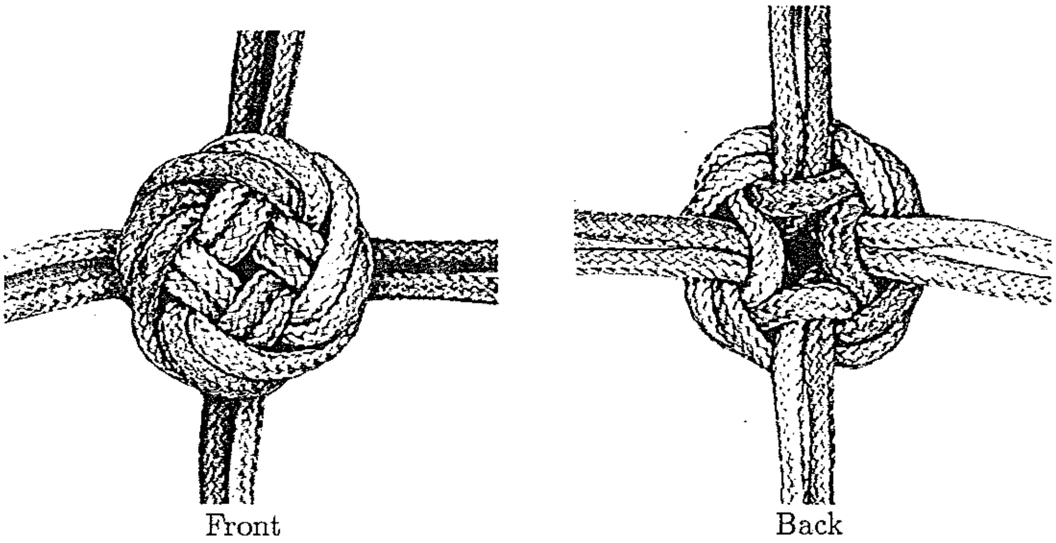

Fig. 508 — The front and the back of the knot in Fig. 507.

These 'flat' knots may be braided as cylindrical knots (which they in fact are, and which is clearly shown by their grid-diagrams), in which case the fiador knot mandrel described in *The Braider*, Issue No. 5, pg. 103, will be a handy braiding aid. After braiding these closure-knots on this mandrel, followed by decreasing their cylindrical diameter sufficiently, the string-ends should be pulled so that the knots become 'flat' and tight. They can then be used as closure-knots for closing the open end of a Regular Nested Cylindrical Braid which has too many nests or too small a nesting-number for closing its bight-edge concerned. The string-ends of a closure-knot are hereby fed through the Regular Nested Cylindrical Braid near its end, ensuring that rotational symmetry is maintained, and a further decorative knot, such as a star-knot for example, may then be braided with these ends. Maintaining rotational symmetry requires four or a multiple of four nests of bights on the bight-edge involved.

The Knot Naming Game and Knot Classification

To many people the discovery or development of a knot new to them is a great event and, without any further thought or investigation, they eagerly give their newly discovered knot a name. They don't realise that generally their "new" knot represents only a particular individual of an often large family of knots, and that in general such a family of knots consists of distinct sub-families. Hence in order to prevent the creation of chaos in the knot naming game, one should not rush into giving their "new" knot a name until a thorough investigation proves that such an action is justified. Let's look at a typical case where the naming of such a knot becomes senseless.

In the Dutch bimonthly knotting-magazine *Het Knoopeknauwertje*, No. 9, pp. 17-20, we find **the Camilla Knot** by Tom Hall.[†] It should of course have been pointed out in *Het Knoopeknauwertje* (and most likely was in Tom's original letter) that the upper and

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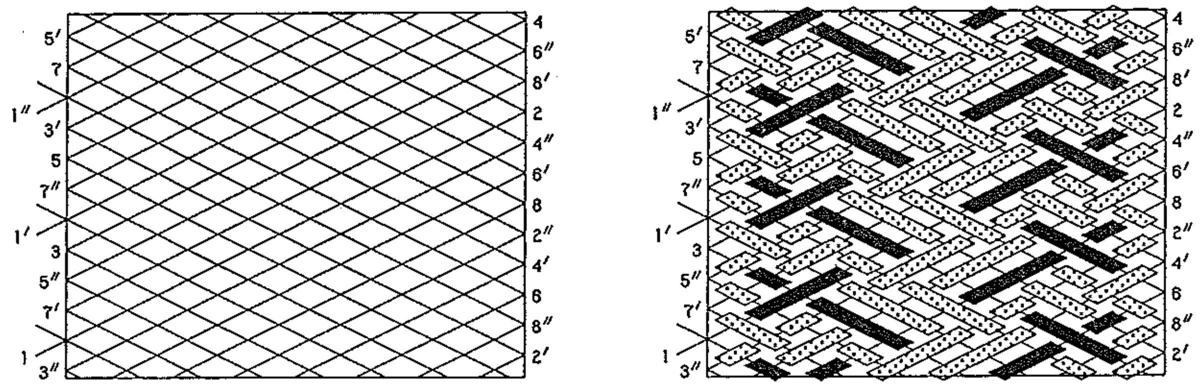
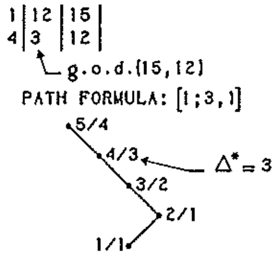
[†] From the article it is clear that Tom Hall provided also half-cycle algorithm-tables for his Camilla Knots, but instead of these being published, the editor apparently tries to impress his audience with the output of a by him generated computer-program for

lower leftmost knots on pg. 17 are similar: the two light strings in the upper drawing are the two dark strings in the lower drawing and the dark string in the upper drawing is the light string in the lower drawing. These are the knot in Fig. 509 below.





L → R

B	A	3	B	A	2	B	A	1	B	A	0	B	A	HALF-CYCLE			
U	O	O	O	U	U	U	U	O	O	O	U	O	U	1	3	5	7
U	U	O	O	U	U	U	O	O	O	U	U	O	O	1′	3′	5′	7′
O	U	O	U	U	O	O	O	U	O	U	U	U	O	1″	3″	5″	7″

L ← R

A	B	0	A	B	1	A	B	2	A	B	3	A	B	HALF-CYCLE			
U	O	U	O	O	O	U	U	U	U	O	O	O	U	2	4	6	8
O	O	U	U	O	O	O	U	U	U	O	O	U	U	2′	4′	6′	8′
O	U	U	U	O	U	O	O	O	U	U	O	U	O	2″	4″	6″	8″

→ L → R R → L

B	A	3	B	A	2	B	A	1	B	A	0	B	A	HALF-CYCLE				HALF-CYCLE			
U	O	O	O	U	U	U	U	O	O	O	U	O	U	1	3	5	7	2	4	6	8
U	U	O	O	U	U	U	O	O	O	U	U	O	O	1′	3′	5′	7′	2′	4′	6′	8′
O	U	O	U	U	O	O	O	U	O	U	U	U	O	1″	3″	5″	7″	2″	4″	6″	8″

Fig. 509 — $p/b = 15/12$ with colour-pattern 1.

In good braiding practice the Standing Ends of the strings of a Semi-Regular Knot should of course be distributed over its bight-boundary circumference as regularly as possible.[†] Hence the distribution of the Standing Ends should never be as in the published computer-program's output where they are placed in adjacent bight-points.

Since the string-run and the coding (**not the colour-pattern!!!**) of the knot in Fig. 509 (which is the string-run and knot in the uppermost row of diagrams in Fig. 510) is identical to the string-run and coding of the other four knots in Fig. 510, the algorithm

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Regular and Semi-Regular Knots. Such computer-programs, however, serve no useful purpose, in fact they may contain braiding-instructions, as in the case here with the component starting positions, which should not be followed.

[†] Refer also to *The Braider*, Issue No. 8, pg. 173.

diagram in Fig. 509 applies to all the knots in Fig. 510. Let the Standing End half-cycles of the consecutive strings respectively be 1, $1'$ and $1''$, with their Standing Ends S, S' and S'', then the colour-patterns of the five consecutive knots in Fig. 510 are obtained by means of the following string-colours:

1. Colour X for string S; colour Y for strings S' and S''.
2. Colour X for string S'; colour Y for strings S and S''.
3. Colour X for string S''; colour Y for strings S and S'.
4. Colour X for string S'; colour Y for string S; colour Z for string S''.
5. Colour X for strings S, S' and S''.

Note that these knots are not suitable for normal flat string since the thickness of the strings must be sufficient in order to prevent the strings from sliding over each other.

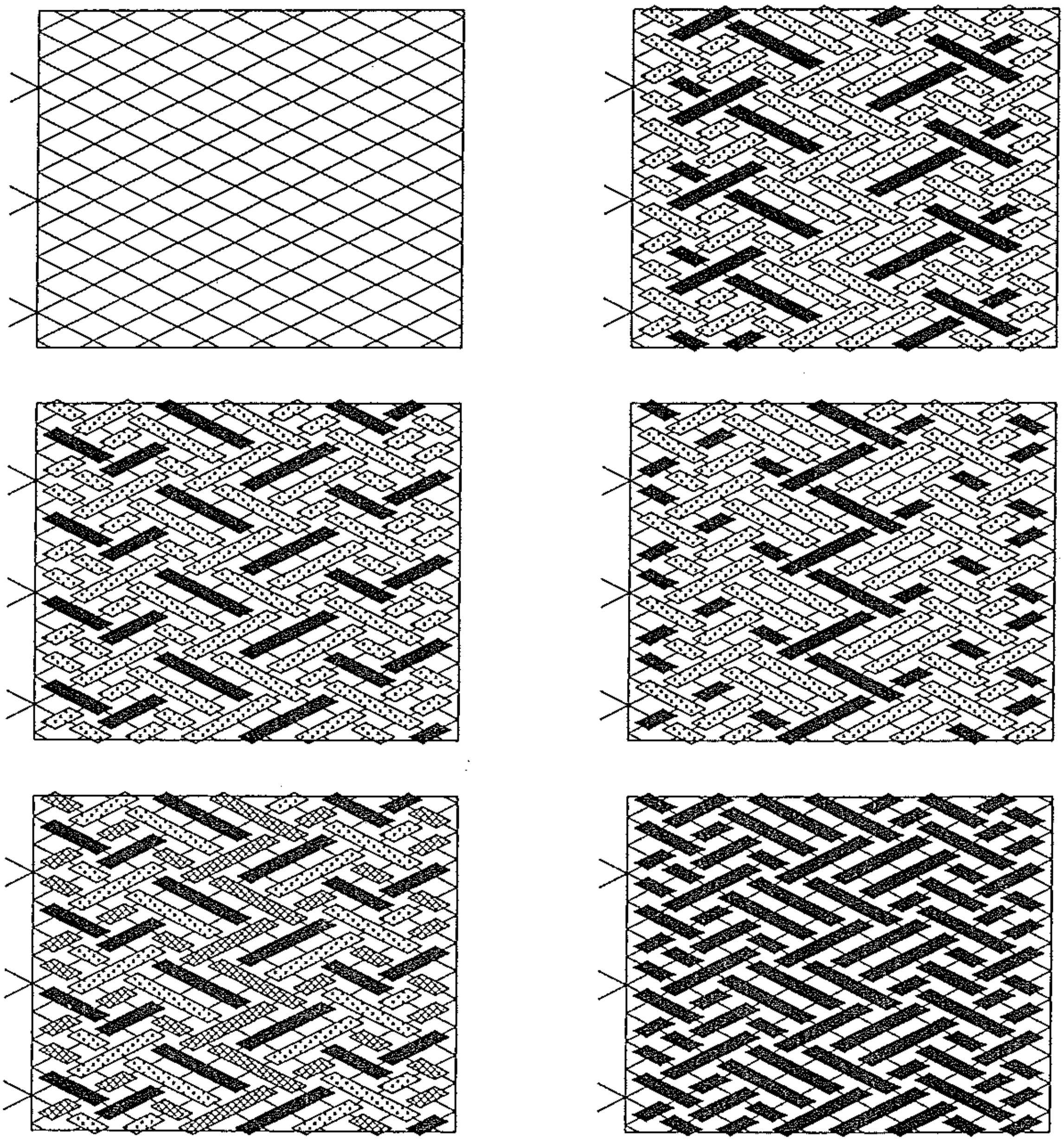


Fig. 510 — The five colour-patterns for the knot $p/b = 15/12$.

The totally dissimilar colour-patterns in Fig. 510 show that the name ‘Camilla Knot’ has a restricted meaning even for these $p/b = 15/12$ Semi-Regular Knots; one needs to attach at least a further indicator to this name in order to know which colour-pattern one is talking about. But even then the identification problems are far from being solved. Let's, however, first tabulate the half-cycle algoritms for the knots in Fig. 510:

Component 1 (starts at 1):

1. $L \longrightarrow R$: Free run.
2. $(i = 0)$ $R \longrightarrow L$: u.
3. $(i = 0)$ $L \longrightarrow R$: u.
4. $(i \leq 1)$ $R \longrightarrow L$: $o - u$.
5. $(i \leq 1)$ $L \longrightarrow R$: $o - u$.
6. $(i \leq 2)$ $R \longrightarrow L$: $u - o - u$.
7. $(i \leq 2)$ $L \longrightarrow R$: $u - o - u$.
8. $(i \leq 3)$ $R \longrightarrow L$: $o - u - o - u$.

Component 2 (starts at $1'$):

$1'$. $(i = A)$ $L \longrightarrow R$: $2u - o - u - o$.
$2'$. $(i = A, 0)$ $R \longrightarrow L$: $2u - o - 2u - o$.
$3'$. $(i = A, 0)$ $L \longrightarrow R$: $2u - o - 2u - o$.
$4'$. $(i = A, \leq 1)$ $R \longrightarrow L$: $2u - 2o - 2u - o$.
$5'$. $(i = A, \leq 1)$ $L \longrightarrow R$: $2u - 2o - 2u - o$.
$6'$. $(i = A, \leq 2)$ $R \longrightarrow L$: $3u - 2o - 2u - o$.
$7'$. $(i = A, \leq 2)$ $L \longrightarrow R$: $3u - 2o - 2u - o$.
$8'$. $(i = A, \leq 3)$ $R \longrightarrow L$: $u - o - 2u - 2o - 2u - o$.

Component 3 (starts at $1''$):

$1''$. $(i = A, B)$ $L \longrightarrow R$: $o - 3u - 3o - 2u - o$.
$2''$. $(i = A, B, 0)$ $R \longrightarrow L$: $o - 3u - 3o - 3u - o$.
$3''$. $(i = A, B, 0)$ $L \longrightarrow R$: $o - 3u - 3o - 3u - o$.
$4''$. $(i = A, B, \leq 1)$ $R \longrightarrow L$: $o - 3u - 2o - u - o - 3u - o$.
$5''$. $(i = A, B, \leq 1)$ $L \longrightarrow R$: $o - 3u - 2o - u - o - 3u - o$.
$6''$. $(i = A, B, \leq 2)$ $R \longrightarrow L$: $o - 3u - 3o - u - o - 3u - o$.
$7''$. $(i = A, B, \leq 2)$ $L \longrightarrow R$: $o - 3u - 3o - u - o - 3u - o$.
$8''$. $(i = A, B, \leq 3)$ $R \longrightarrow L$: $o - u - o - 2u - 3o - u - o - 3u - o$.

The **coding** arrangement near the bight-edges together with the 3-pass Herringbone-coding in the remaining central area requires that the number of bights b has to be a multiple of 3 (hence $b = 3y$, where y is a natural number[†]) and, when disregarding any colour-pattern requirements, that the number of parts p has also to be a multiple of 3 ($p = 15 + 6r = 3(5 + 2r) = 3x$, where $x = 5 + 2r$ with r being a whole number[‡]). Further conditions to the value of r are given by the string-colours:

When the three essential strings have all the same colour, r can be $0, 1, 2, 3, \cdots$. There is no special limit case.

When the two upper essential strings have the same colour which differs from the colour of the lowermost essential string (as in the uppermost right-hand grid-diagrams in Fig. 510 and Fig. 511), the associated general family-clan requires that r can be $1, 2, 3, 4, \cdots$. When $r = 0$ we have here a special limit case as shown by the uppermost right-hand grid-diagram in Fig. 510.

[†] Natural numbers are the numbers $1, 2, 3, \cdots$.

[‡] Whole numbers are the numbers $0, 1, 2, 3, \cdots$.

When lower and upper essential strings have the same colour which differs from the colour of the central essential string (as in the central left-hand grid-diagrams in Fig. 510 and Fig. 511), the associated general family-clan requires that r can be $1, 2, 3, 4, \cdots$. When $r = 0$ we have here a special limit case as shown by the central left-hand grid-diagram in Fig. 510.

When the two lower essential strings have the same colour which differs from the colour of the upper essential string (as in the central right-hand grid-diagrams in Fig. 510 and Fig. 511, and the central grid-diagram in Fig. 512), the associated general family-clan requires that r can be $2, 3, 4, \cdots$. We have here a special limit case when $r = 0$ as shown by the central right-hand grid-diagram in Fig. 510, and another special limit case when $r = 1$ as shown by the central right-hand grid-diagram in Fig. 511.

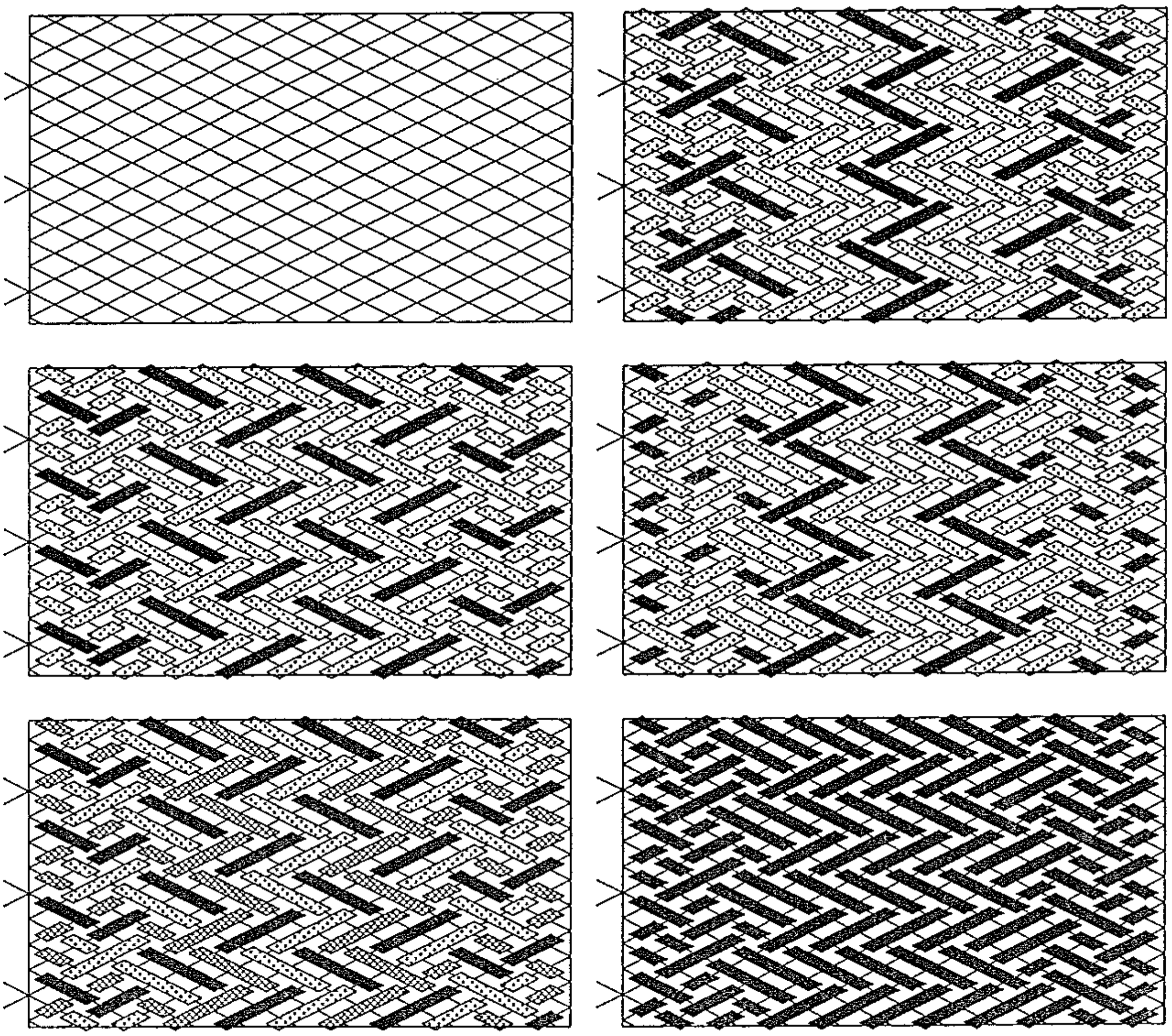


Fig. 511 — The five colour-patterns for the knot $p/b = 21/12$.

When each string has a different colour (as in the lowermost left-hand grid-diagrams in Fig. 510 and Fig. 511, and the lowermost grid-diagram in Fig. 512), the associated general family-clan requires that r can be $2, 3, 4, \cdots$. We have here a special limit case when $r = 0$ as shown by the lowermost left-hand grid-diagram in Fig. 510, and another special limit case when $r = 1$ as shown by the lowermost left-hand grid-diagram in Fig. 511.

The g.c.d. $(p, b) = 3 \times$ g.c.d. $(x, y) = 3t$. Hence a total of $3t$ strings are required.

Only when $t = 1$ and g.c.d.$(3, y) = 1$ can we distribute the string-ends regularly over a bight-boundary; in general we ensure that this is the case.

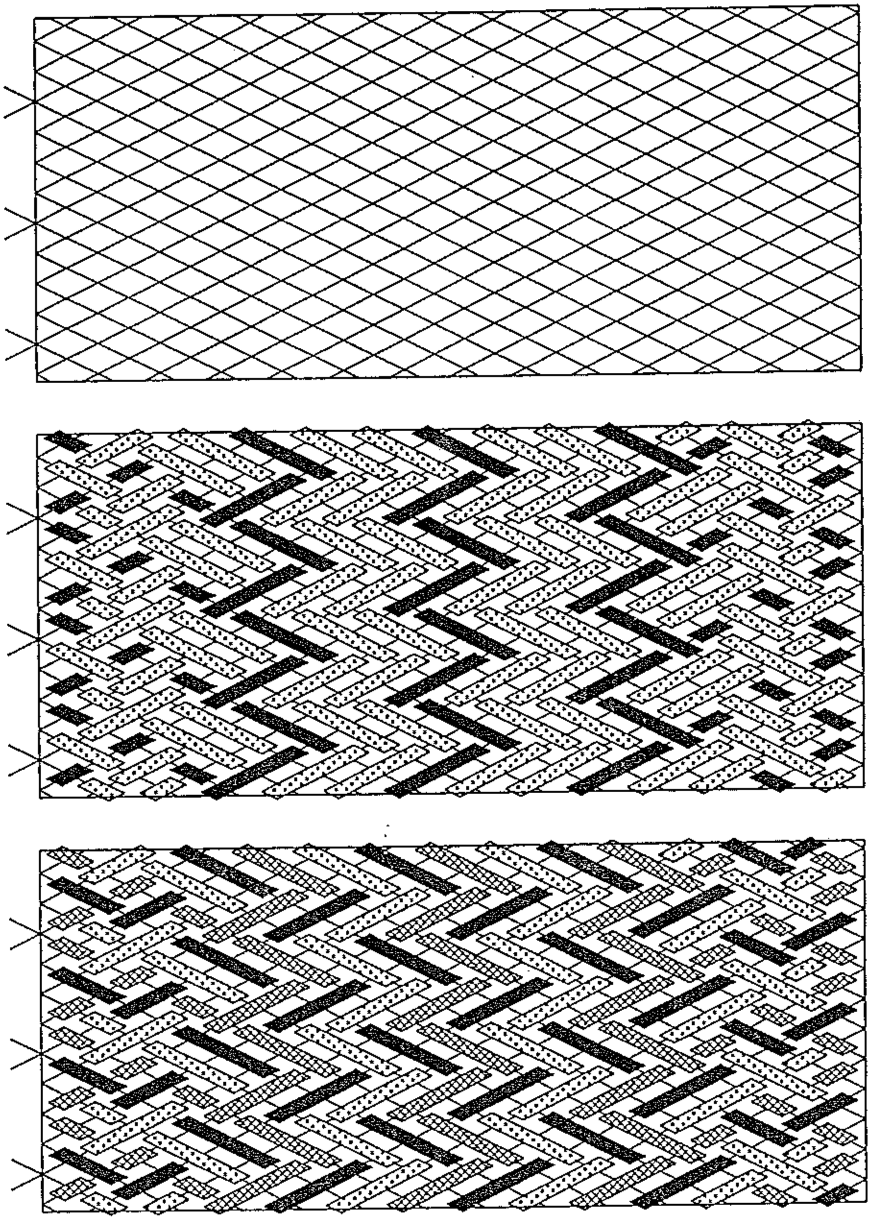


Fig. 512 — $p/b = 27/12$.

For the grid-diagrams in Fig. 511 the value of r is equal to 1, hence $p = 21$. Their associated algorithm diagram is shown in Fig. 513, and their half-cycle algorithms are:

Component 1 (starts at 1):

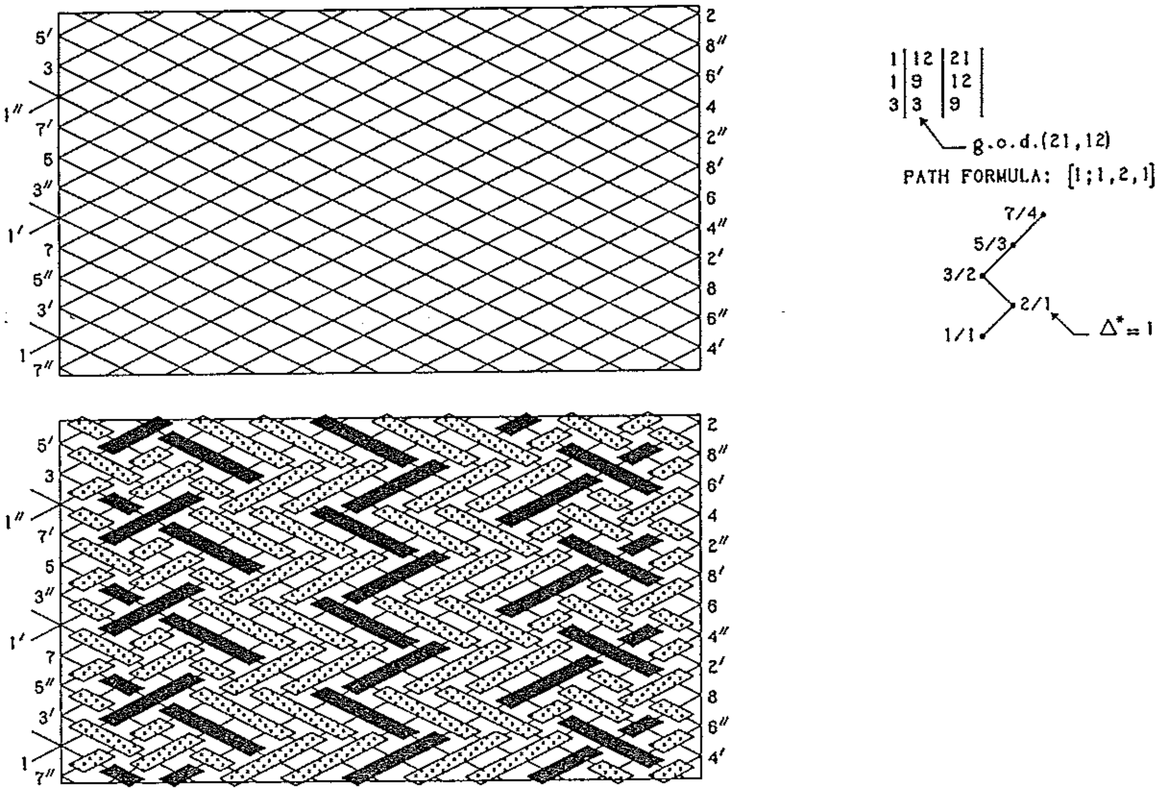
1. $L \longrightarrow R$: Free run.
2. $(i = 0)$ $R \longrightarrow L$: o.
3. $(i = 0)$ $L \longrightarrow R$: o.
4. $(i \leq 1)$ $R \longrightarrow L$: $3o$.
5. $(i \leq 1)$ $L \longrightarrow R$: $3o$.
6. $(i \leq 2)$ $R \longrightarrow L$: $o - u - 2o - u$.
7. $(i \leq 2)$ $L \longrightarrow R$: $o - u - 2o - u$.
8. $(i \leq 3)$ $R \longrightarrow L$: $o - 2u - 2o - u$.

Component 2 (starts at $1'$):

$1'$. $(i = A)$ $L \longrightarrow R$: $2u - o - u - o - u - o$.
$2'$. $(i = A, 0)$ $R \longrightarrow L$: $2u - o - 2u - o - u - o$.
$3'$. $(i = A, 0)$ $L \longrightarrow R$: $2u - o - 2u - o - u - o$.
$4'$. $(i = A, \leq 1)$ $R \longrightarrow L$: $u - o - u - o - 2u - 2o - u - o$.
$5'$. $(i = A, \leq 1)$ $L \longrightarrow R$: $u - o - u - o - 2u - 2o - u - o$.
$6'$. $(i = A, \leq 2)$ $R \longrightarrow L$: $u - o - 2u - o - 2u - 2o - 2u - o$.
$7'$. $(i = A, \leq 2)$ $L \longrightarrow R$: $u - o - 2u - o - 2u - 2o - 2u - o$.
$8'$. $(i = A, \leq 3)$ $R \longrightarrow L$: $u - o - 2u - 2o - 2u - 2o - 2u - o$.

Component 3 (starts at $1''$):

$1''$. $(i = A, B)$ $L \longrightarrow R$: $o - 3u - 2o - 2u - 3o - 2u - o$.
$2''$. $(i = A, B, 0)$ $R \longrightarrow L$: $o - 3u - 2o - 2u - 4o - 2u - o$.
$3''$. $(i = A, B, 0)$ $L \longrightarrow R$: $o - 3u - 2o - 2u - 4o - 2u - o$.
$4''$. $(i = A, B, \leq 1)$ $R \longrightarrow L$: $o - u - o - 2u - 2o - 2u - 3o - u - o - 2u - o$.
$5''$. $(i = A, B, \leq 1)$ $L \longrightarrow R$: $o - u - o - 2u - 2o - 2u - 3o - u - o - 2u - o$.
$6''$. $(i = A, B, \leq 2)$ $R \longrightarrow L$: $o - u - o - 2u - 3o - 2u - 3o - u - o - 3u - o$.
$7''$. $(i = A, B, \leq 2)$ $L \longrightarrow R$: $o - u - o - 2u - 3o - 2u - 3o - u - o - 3u - o$.
$8''$. $(i = A, B, \leq 3)$ $R \longrightarrow L$: $o - u - o - 2u - 3o - 3u - 3o - u - o - 3u - o$.



→

																				L → R				R → L			
B	A	1	B	A	2	B	A	3	B	A	0	B	A	1	B	A	2	B	A	HALF-CYCLE				HALF-CYCLE			
U	O	O	O	U	U	U	U	U	O	O	O	U	U	O	O	O	U	O	U	1	3	5	7	2	4	6	8
U	U	O	O	U	U	U	O	O	O	U	U	U	O	O	O	U	U	O	O	1′	3′	5′	7′	2′	4′	6′	8′
O	U	O	U	U	O	O	O	U	U	U	O	O	O	U	O	U	U	U	O	1″	3″	5″	7″	2″	4″	6″	8″

Fig. 513 — $p/b = 21/12$.

Hence the completely dissimilar colour-patterns, associated with the Camilla Knots, require a further classification **within** the family of Camilla Knots, and consequently the name 'Camilla Knots' can be associated with the coding-pattern only. We can therefore only speak of '**a** Camilla Knot' when a colour-pattern is being taken into account, or of 'some Camilla Knots' when some colour-patterns are being considered, but not of '**the** Camilla Knot'.

Nested Cylindrical Braids

In Issue No. 22 of *The Braider* we saw on pg. 510 the formula:

$$P_{component} = 4\alpha + \frac{\alpha \cdot x - 2\sum(l_i + r_i)}{A}.$$

for the **Regular Nested Cylindrical Braids**[†].

When α is equal to A, then there is only one first-return string-run. Consequently, $\sum(l_i + r_i) = A(A+1)$, and hence:

$$P_{component} = 4A + x - 2(A+1) = 2A + x - 2 = P_{total} = P.$$

For the **Regular Nested Cylindrical Braids** we saw on pg. 570 in Issue No. 25 of *The Braider* that $\alpha = \dfrac{A}{\gamma}$. Hence $\gamma = 1$ for $\alpha = A$. Since $\gamma = \text{g.c.d.}(\Delta, A)$, it follows that for $\alpha = A$, hence for a single first-return string-run only, Δ and A must be coprime; furthermore since $\Delta = |y|_A$, it follows that y and A must be coprime.

The Regular Nested Cylindrical Braids with $\alpha = A$ and $\text{g.c.d.}(P, B^*) = 1$ are the **Perfect Regular Nested Cylindrical Braids**, and the Regular Nested Cylindrical Braids with $\alpha = A$ and $\text{g.c.d.}(P, B^*) \neq 1$ are the **Semi-Perfect Regular Nested Cylindrical Braids**.

There are two special cases of special interest, one in which $y = A - 1$, and one in which $y = A + 1$. In these two special cases we are able to superimpose on their string-runs a **Herringbone Pineapple coding** (see Figs. 514 and 515). With $\text{g.c.d.}(P, B^*) = 1$ we have then the **Perfect Herringbone Pineapple Knots**, and with $\text{g.c.d.}(P, B^*) \neq 1$ we have then the **Semi-Perfect Herringbone Pineapple Knots.**

On pg. 567, Issue No. 25 of *The Braider* we saw that:

$$k = \left|\frac{x - y - 2}{2}\right|_A.$$

Hence for:

$$y = A - 1 \quad \rightarrow \quad k = \left|\tfrac{x-A-1}{2}\right|_A \quad \begin{cases} A = \text{odd} & \rightarrow \quad x = \text{even} \\ A = \text{even} & \rightarrow \quad x = \text{odd} \end{cases}$$

$$y = A + 1 \quad \rightarrow \quad k = \left|\tfrac{x-A-3}{2}\right|_A \quad \begin{cases} A = \text{odd} & \rightarrow \quad x = \text{even} \\ A = \text{even} & \rightarrow \quad x = \text{odd} \end{cases}$$

[†] See *The Braider*, Issue No. 22, pg. 502, for the definition of *Regular Nested Cylindrical Braids.*

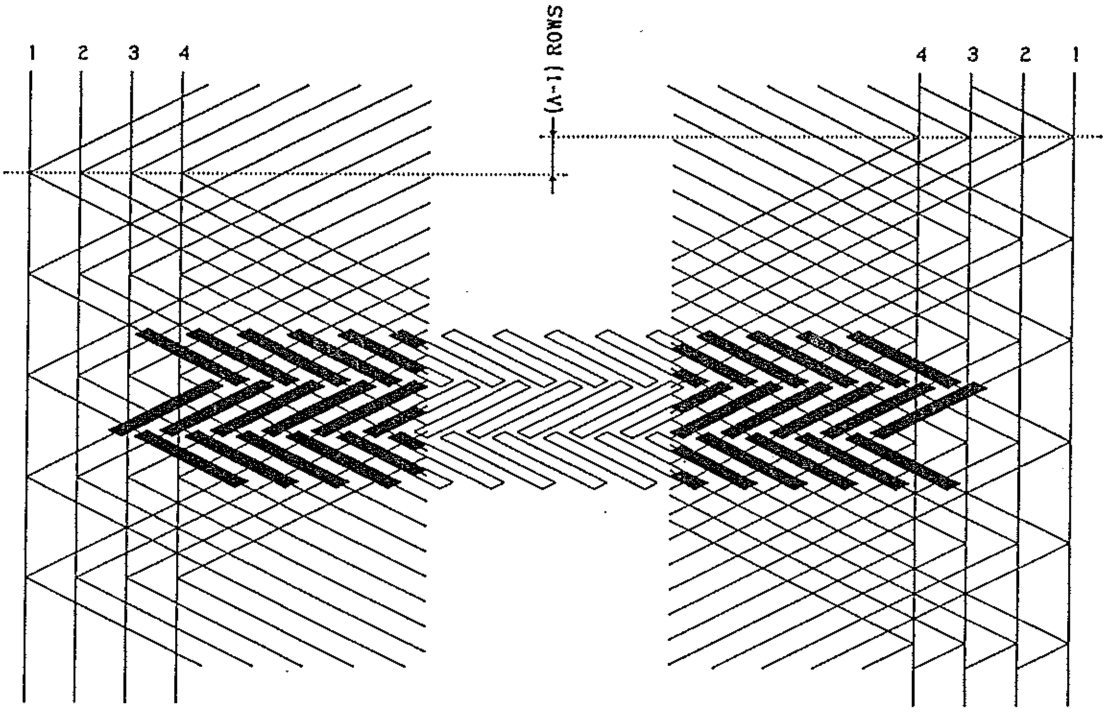


Fig. 514 — $y = A - 1$; Herringbone Pineapple coding.

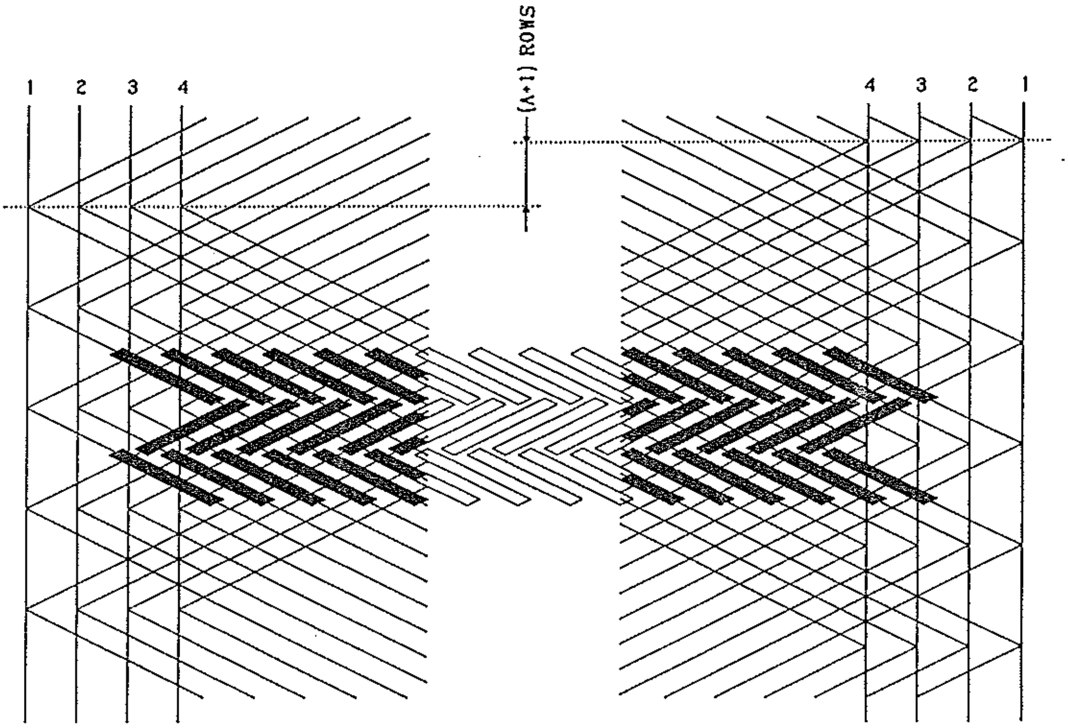


Fig. 515 — $y = A + 1$; Herringbone Pineapple coding.

In order to braid the **Perfect** and **Semi-Perfect Herringbone Pineapple Knots** we have to develop a method which enables us to obtain their half-cycle braiding algorithms in an easy way.

Note that a lower-left to upper-right half-cycle in a Regular Nested Cylindrical Braid has $2A+x-1-(l_i+r_i)$ crossings, and that a lower-right to upper-left half-cycle has $2A+x-1-(r_i+l_{i+1})$ crossings.

Observe for the $y=A-1$ Herringbone Pineapple coding that a lower-left to upper-right half-cycle (hence a half-cycle from l_i to r_i) has the coding-pattern:

$$(l_i-1) \text{ under} - A \text{ over} - A \text{ under} - \cdots - A \text{ over} - (r_i-1) \text{ under},$$

and that a lower-right to upper-left half-cycle (hence a half-cycle from r_i to l_{i+1}) has the coding-pattern:

$$r_i \text{ under} - A \text{ over} - A \text{ under} - \cdots - A \text{ over} - l_{i+1} \text{ under}.$$

Observe for the $y=A+1$ Herringbone Pineapple coding that a lower-left to upper-right half-cycle (hence a half-cycle from l_i to r_i) has the coding-pattern:

$$l_i \text{ under} - A \text{ over} - A \text{ under} - \cdots - A \text{ over} - r_i \text{ under},$$

and that a lower-right to upper-left half-cycle (hence a half-cycle from r_i to l_{i+1}) has the coding-pattern:

$$(r_i-1) \text{ under} - A \text{ over} - A \text{ under} - \cdots - A \text{ over} - (l_{i+1}-1) \text{ under}.$$

With the aid of the first-return string-run and its associated **nest-index numbers**[†] we can readily draw up the half-cycle pattern of the **Perfect** or **Semi-Perfect Herringbone Pineapple Knot** and read from it the braiding sequence for each half-cycle. The following two examples should make the process clear.

Example 1:

$A=5; x=14; y=A-1=4$, hence $\Delta=A-1=4; B^*=3$, hence $B=A\cdot B^*=15$. Thus $k=\left|\frac{x-A-1}{2}\right|_A=\left|\frac{14-5-1}{2}\right|_5=4$.

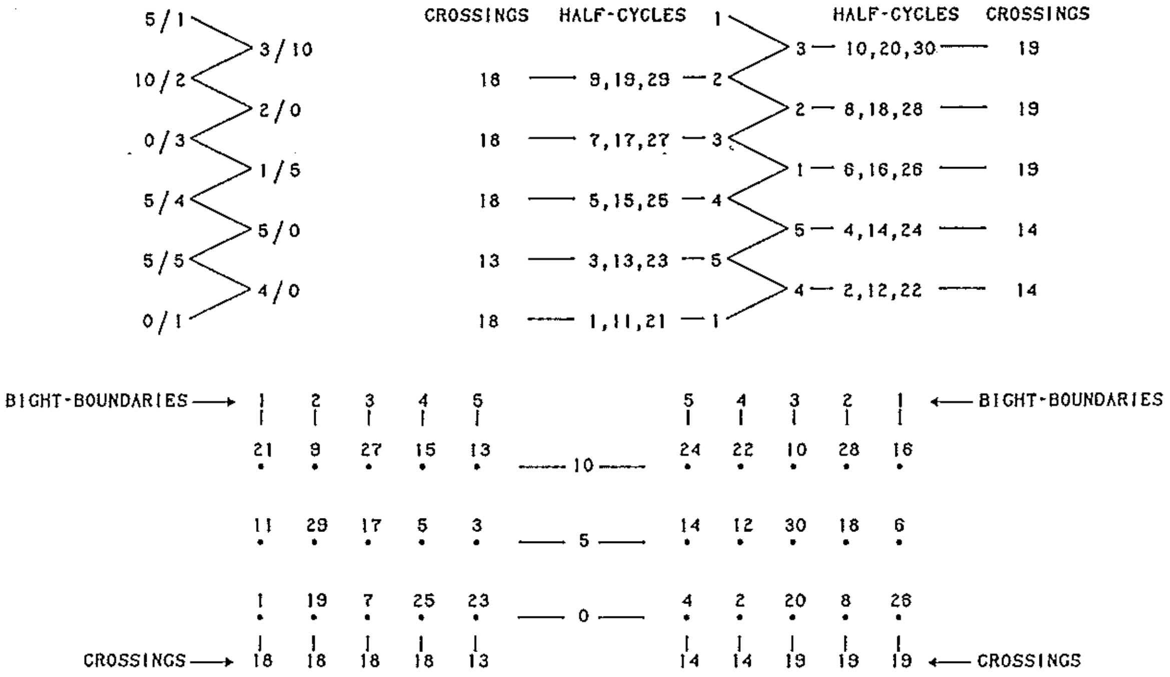


Fig. 516 — First-return string-run and half-cycle pattern.

At the upper left-hand side in Fig. 516 is depicted the first-return string-run with its associated nest-index numbers.

At the upper right-hand side in Fig. 516 are depicted the first-return string-run, the half-cycle numbers of the half-cycles, and the number of crossings which these half-cycles

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[†] See *The Braider*, Issue No. 26, pp. 592-599.

make in the finished knot. This layout at the upper right-hand side has been shown for additional clarification only since it can readily be dispensed with.

The depicted layout at the bottom of Fig. 516 gives the positions of the nests with respect to the nest-index numbers, the half-cycle numbers with respect to the nest-index numbers and bight-boundaries, and the number of crossings which the half-cycles make in the finished knot. This layout, which is the one we need for compiling the half-cycle braiding algorithms, is directly obtained from the first-return string-run with its associated nest-index numbers, hence from the upper left-hand layout in Fig. 516. The first step in its construction is shown in Fig. 517.

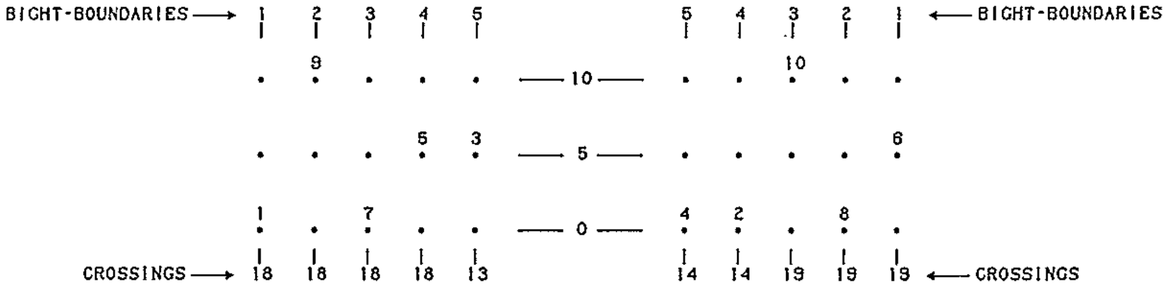


Fig. 517 — The first step in the construction of the half-cycle pattern.

At the top we set off the left-hand and right-hand bight-boundaries $1, \cdots, A$, hence $1, 2, 3, 4, 5$, then we set off the nest-index numbers $0, A, \cdots, (B^* - 1)A$, hence $0, 5, 10$, then we place the dots which represent the bight-points.

From the first-return string-run we obtain that half-cycle 1 runs from $l_i = 1$ to $r_i = 4$, and that it starts at nest-index number 0. Thus we can write 1 above the dot which lines up with left-hand bight-boundary 1 and nest-index number 0. We can calculate the number of crossings associated with a half-cycle which runs from $l_i = 1$ to $r_i = 4$ with the formula $2A + x - 1 - (l_i + r_i)$, hence the number of crossings for such a half-cycle in the finished knot is equal to 18. Since all lower-left to upper-right half-cycles starting at left-hand bight-boundary 1 end at right-hand bight-boundary 4, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 18 is placed at the bottom in line with left-hand bight-boundary 1.

From the first-return string-run we obtain that half-cycle 3 runs from $l_i = 5$ to $r_i = 5$, and that it starts at nest-index number 5. Thus we can write 3 above the dot which lines up with left-hand bight-boundary 5 and nest-index number 5. We calculate the number of crossings associated with a half-cycle which runs from $l_i = 5$ to $r_i = 5$ again with the formula $2A + x - 1 - (l_i + r_i)$, hence the number of crossings for such a half-cycle in the finished knot is equal to 13. Since all lower-left to upper-right half-cycles starting at left-hand bight-boundary 5 end at right-hand bight-boundary 5, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 13 is placed at the bottom in line with left-hand bight-boundary 5.

From the first-return string-run we obtain that half-cycle 5 runs from $l_i = 4$ to $r_i = 1$, and that it starts at nest-index number 5. Thus we can write 5 above the dot which lines up with left-hand bight-boundary 4 and nest-index number 5. We calculate the number of crossings associated with a half-cycle which runs from $l_i = 4$ to $r_i = 1$ again with the formula $2A + x - 1 - (l_i + r_i)$, hence the number of crossings for such a half-cycle in the finished knot is equal to 18. Since all lower-left to upper-right half-cycles starting at left-hand bight-boundary 4 end at right-hand bight-boundary 1, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 18 is placed at the bottom in line with left-hand bight-boundary 4.

From the first-return string-run we obtain that half-cycle 7 runs from $l_i = 3$ to $r_i = 2$, and that it starts at nest-index number 0. Thus we can write 7 above the dot which lines up with left-hand bight-boundary 3 and nest-index number 0. We calculate the number of crossings associated with a half-cycle which runs from $l_i = 3$ to $r_i = 2$ again with the formula $2A + x - 1 - (l_i + r_i)$, hence the number of crossings for such a half-cycle in the finished knot is equal to 18. Since all lower-left to upper-right half-cycles starting at left-hand bight-boundary 3 end at right-hand bight-boundary 2, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 18 is placed at the bottom in line with left-hand bight-boundary 3.

From the first-return string-run we obtain that half-cycle 9 runs from $l_i = 2$ to $r_i = 3$, and that it starts at nest-index number 10. Thus we can write 9 above the dot which lines up with left-hand bight-boundary 2 and nest-index number 10. We calculate the number of crossings associated with a half-cycle which runs from $l_i = 2$ to $r_i = 3$ again with the formula $2A + x - 1 - (l_i + r_i)$, hence the number of crossings for such a half-cycle in the finished knot is equal to 18. Since all lower-left to upper-right half-cycles starting at left-hand bight-boundary 2 end at right-hand bight-boundary 3, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 18 is placed at the bottom in line with left-hand bight-boundary 2.

The first-return string-run shows that half-cycle 2 runs from $r_i = 4$ to $l_{i+1} = 5$, and that it starts at nest-index number 0. Thus we can write 2 above the dot which lines up with right-hand bight-boundary 4 and nest-index number 0. We can calculate the number of crossings associated with a half-cycle which runs from $r_i = 4$ to $l_{i+1} = 5$ with the formula $2A + x - 1 - (r_i + l_{i+1})$, hence the number of crossings for such a half-cycle in the finished knot is equal to 14. Since all lower-right to upper-left half-cycles starting at right-hand bight-boundary 4 end at left-hand bight-boundary 5, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 14 is placed at the bottom in line with right-hand bight-boundary 4.

The first-return string-run shows that half-cycle 4 runs from $r_i = 5$ to $l_{i+1} = 4$, and that it starts at nest-index number 0. Thus we can write 4 above the dot which lines up with right-hand bight-boundary 5 and nest-index number 0. We calculate the number of crossings associated with a half-cycle which runs from $r_i = 5$ to $l_{i+1} = 4$ with the formula $2A + x - 1 - (r_i + l_{i+1})$, hence the number of crossings for such a half-cycle in the finished knot is equal to 14. Since all lower-right to upper-left half-cycles starting at right-hand bight-boundary 5 end at left-hand bight-boundary 4, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 14 is placed at the bottom in line with right-hand bight-boundary 5.

The first-return string-run shows that half-cycle 6 runs from $r_i = 1$ to $l_{i+1} = 3$, and that it starts at nest-index number 5. Thus we can write 6 above the dot which lines up with right-hand bight-boundary 1 and nest-index number 5. We calculate the number of crossings associated with a half-cycle which runs from $r_i = 1$ to $l_{i+1} = 3$ with the formula $2A + x - 1 - (r_i + l_{i+1})$, hence the number of crossings for such a half-cycle in the finished knot is equal to 19. Since all lower-right to upper-left half-cycles starting at right-hand bight-boundary 1 end at left-hand bight-boundary 3, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 19 is placed at the bottom in line with right-hand bight-boundary 1.

The first-return string-run shows that half-cycle 8 runs from $r_i = 2$ to $l_{i+1} = 2$, and that it starts at nest-index number 0. Thus we can write 8 above the dot which lines up with right-hand bight-boundary 2 and nest-index number 0. We calculate the number of crossings associated with a half-cycle which runs from $r_i = 2$ to $l_{i+1} = 2$ with the formula $2A + x - 1 - (r_i + l_{i+1})$, hence the number of crossings for such a half-cycle in the finished knot is equal to 19. Since all lower-right to upper-left half-cycles starting at right-hand bight-boundary 2 end at left-hand bight-boundary 2, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 19 is placed at the bottom in line with right-hand bight-boundary 2.

The first-return string-run shows that half-cycle 10 runs from $r_i = 3$ to $l_{i+1} = 1$, and that it starts at nest-index number 10. Thus we can write 10 above the dot which lines up with right-hand bight-boundary 3 and nest-index number 10. We calculate the number of crossings associated with a half-cycle which runs from $r_i = 3$ to $l_{i+1} = 1$ with the formula $2A + x - 1 - (r_i + l_{i+1})$, hence the number of crossings for such a half-cycle in the finished knot is equal to 19. Since all lower-right to upper-left half-cycles starting at right-hand bight-boundary 3 end at left-hand bight-boundary 1, these half-cycles make each the same number of crossings in the finised knot. Hence the number of crossings 19 is placed at the bottom in line with right-hand bight-boundary 3.

The second step in the construction of the half-cycle pattern, shown at the bottom of Fig. 516, consists of the determination of the further half-cycle numbers in association with the nest-index numbers and bight-boundaries of their starting points.

Since the second sequence of the half-cycles in a first-return string-run starts with half-cycle number $1 + 2A = 11$ at nest-index number $I_L = A|2A + x - 2|_{B*} = 5|22|_3 = 5$, two sequential half-cycle numbers h_n (≥ 1) and $h_n + 2A$ $(\leq 2B)$ associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I + 2A^2 + A(x-2)\right|_B = \left|I + 2 \cdot 5^2 + 5(14-2)\right|_{15} = |I + 110|_{15} = |I + 5|_{15}$.

$\star\star$ *Prove the above formulae* $I_L = A|2A + x - 2|_{B*}$ *and* $\left|I + 2A^2 + A(x-2)\right|_B$.

We can now readily complete the half-cycle pattern concerned.

Let's illustrate for the half-cycles 17, 23, 8 and 12 how this half-cycle pattern may be used.

For the determination of the braiding sequence of half-cycle 17 we start at the extreme left on the line immediately above (in cyclic order) half-cycle 17 (the first crossing on half-cycle 17 in the finished knot is with half-cycle 20 (= 21 − 1)). To arrive at the finish, count the number of crossings on half-cycle 17 (= 18) in the finished knot from left to right in cyclic order.

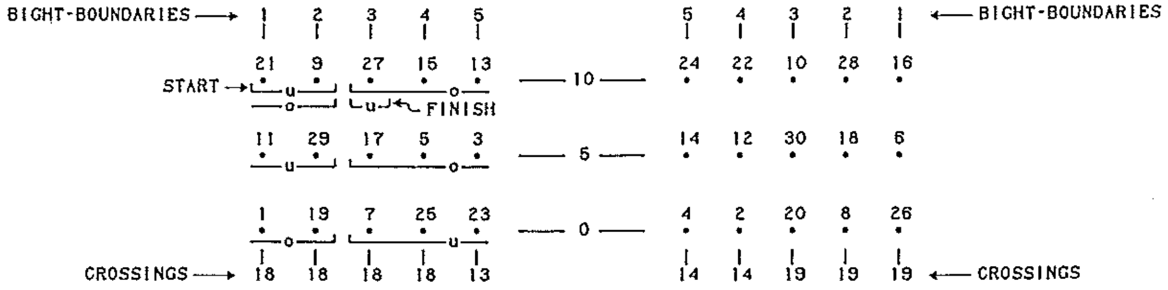


Fig. 518 — Determination of the braiding sequence for half-cycle 17.

The first $l_i - 1$ crossings $(= 3 - 1 = 2)$ on half-cycle 17 in the finished knot are

under crossings. Next we have A **over** crossings (= 5), next A **under** crossings (= 5), next A **over** crossings (= 5), and finally 1 **under** crossing. When braiding half-cycle 17, it is only possible for half-cycle 17 to cross the half-cycles which end at the start of the half-cycles greater 1 and less or equal 17. Hence when braiding half-cycle 17, it may cross the half-cycles which end at the start of the half-cycles greater or equal 3 and less or equal 17, hence it crosses the half-cycles 9 ; 15, 13 ; 7, 11 ; 17, 5, 3, 9. Thus the braiding sequence of half-cycle 17 is $u - 2o - 2u - 4o$.

For the determination of the braiding sequence of half-cycle 23 we start at the extreme left on the line immediately above (in cyclic order) half-cycle 23 (the first crossing on half-cycle 23 in the finished knot is with half-cycle 10 (= 11 − 1)). To arrive at the finish, count the number of crossings on half-cycle 23 (= 13) in the finished knot from left to right in cyclic order.

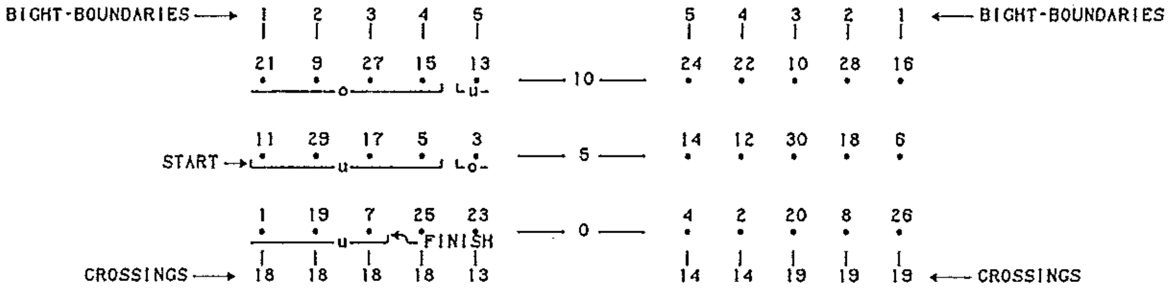


Fig. 519 — Determination of the braiding sequence for half-cycle 23.

The first $l_i - 1$ crossings (= 5 − 1 = 4) on half-cycle 23 in the finished knot are **under** crossings. Next we have A **over** crossings (= 5), and finally 4 **under** crossing. When braiding half-cycle 23, it is only possible for half-cycle 23 to cross the half-cycles which end at the start of the half-cycles greater 1 and less or equal 23. Hence when braiding half-cycle 23, it may cross the half-cycles which end at the start of the half-cycles greater or equal 3 and less or equal 23, hence it crosses the half-cycles 11, 17, 5 ; 3, 21, 9, 15 ; 13, 19, 7. Thus the braiding sequence of half-cycle 23 is $3u - 4o - 3u$.

For the determination of the braiding sequence of half-cycle 8 we start at the extreme right on the line immediately above (in cyclic order) half-cycle 8 (the first crossing on half-cycle 8 in the finished knot is with half-cycle 5 (= 6 − 1)). To arrive at the finish, count the number of crossings on half-cycle 8 (= 19) in the finished knot from right to left in cyclic order.

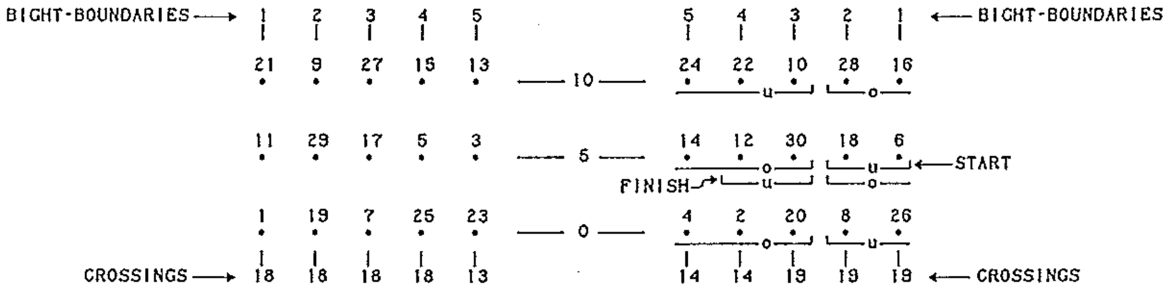


Fig. 520 — Determination of the braiding sequence for half-cycle 8.

The first r_i crossings (= 2) on half-cycle 8 in the finished knot are **under** crossings. Next we have A **over** crossings (= 5), then A **under** crossings (= 5), then A **over** crossings (= 5), and finally 2 **under** crossing. When braiding half-cycle 8, it is only possible for half-cycle 8 to cross the half-cycles which end at the start of the half-cycles greater or equal 2 and less or equal 8, hence it crosses the half-cycles 6 ; 8 ; 2 , 4 , 6 . Thus the braiding sequence of half-cycle 8 is $2u - 3o$.

For the determination of the braiding sequence of half-cycle 12 we start at the extreme right on the line immediately above (in cyclic order) half-cycle 12 (the first crossing on half-cycle 12 in the finished knot is with half-cycle 15 (= 16 − 1)). To arrive at the finish, count the number of crossings on half-cycle 12 (= 14) in the finished knot from right to left in cyclic order.

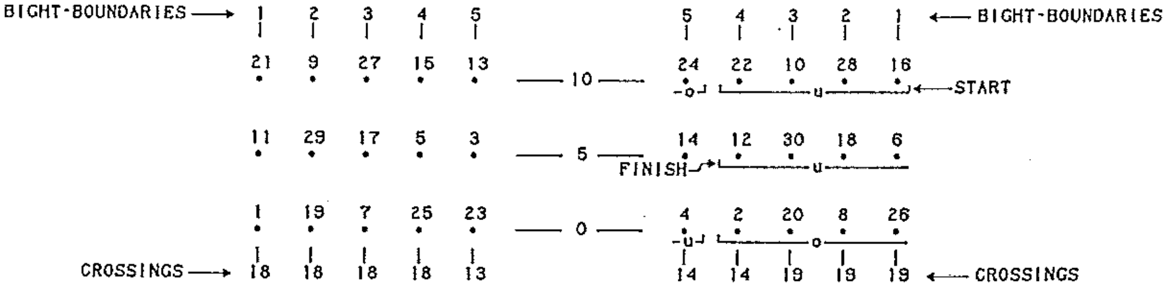
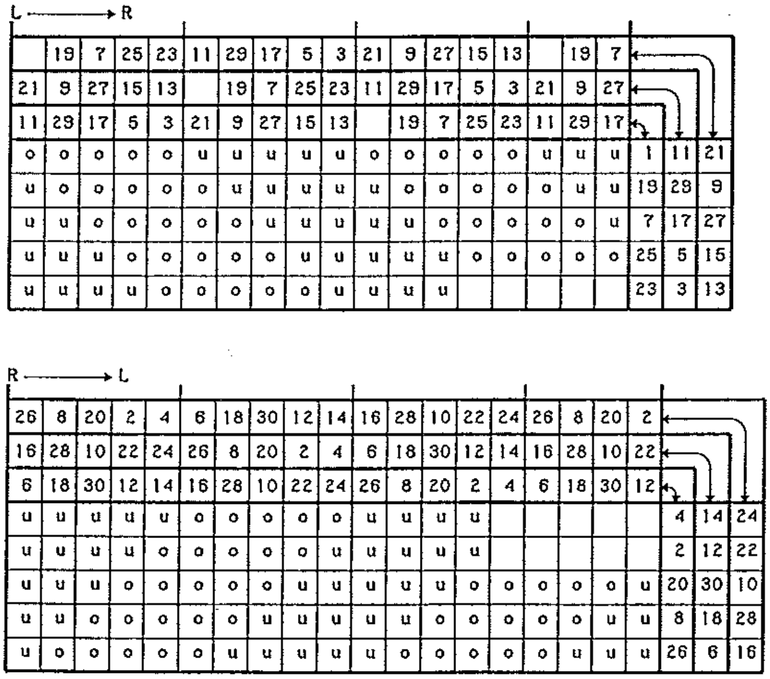


Fig. 521 — Determination of the braiding sequence for half-cycle 12.

The first r_i crossings (= 4) on half-cycle 12 in the finished knot are **under** crossings. Next we have A **over** crossings (= 5), and finally 5 **under** crossing. When braiding half-cycle 12, it is only possible for half-cycle 12 to cross the half-cycles which end at the start of the half-cycles greater or equal 2 and less or equal 12, hence it crosses the half-cycles 10; 8 , 2; 4 , 6 , 12 . Thus the braiding sequence of half-cycle 12 is $u - 2o - 3u$.

Although, as shown above, we can read the half-cycle braiding algorithms directly from the half-cycle pattern, it is easier to determine the half-cycle braiding algorithms from the half-cycle tables shown in Fig. 522 which we assemble from the half-cycle pattern at the bottom of Fig. 516.



	19	7	25	23	11	29	17	5	3	21	9	27	15	13		19	7			
21	9	27	15	13		19	7	25	23	11	29	17	5	3	21	9	27			
11	29	17	5	3	21	9	27	15	13		19	7	25	23	11	29	17			
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	1	11	21
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	19	29	9
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	7	17	27
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	25	5	15
u	u	u	u	o	o	o	o	o	u	u	u	u						23	3	13

26	8	20	2	4	6	18	30	12	14	16	28	10	22	24	26	8	20	2			
16	28	10	22	24	26	8	20	2	4	6	18	30	12	14	16	28	10	22			
6	18	30	12	14	16	28	10	22	24	26	8	20	2	4	6	18	30	12			
u	u	u	u	u	o	o	o	o	o	u	u	u	u						4	14	24
u	u	u	u	o	o	o	o	o	u	u	u	u	u						2	12	22
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	20	30	10
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	8	18	28
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	26	6	16

Fig. 522 — The half-cycle tables for the knot in Example 1.

The upper table in Fig. 522 is for the odd-numbered half-cycles, hence the half-cycles from lower-left to upper-right, while the lower table in Fig. 522 is for the even-numbered half-cycles, hence the half-cycles from lower-right to upper-left. The upper table is assembled as follows:

Half-cycles 1, 19, 7, 25 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

11, 29, 17, 5, 3, 21, 9, 27, 15, 13, –, 19, 7, 25, 23, 11, 29, 17,

and half-cycle 23 crosses in the finished knot sequentially the half-cycles which end at the start of the half-cycles

11, 29, 17, 5, 3, 21, 9, 27, 15, 13, –, 19, 7.

Half-cycles 21, 9, 27, 15 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

–, 19, 7, 25, 23, 11, 29, 17, 5, 3, 21, 9, 27, 15, 13, –, 19, 7,

and half-cycle 13 crosses in the finished knot sequentially the half-cycles which end at the start of the half-cycles

–, 19, 7, 25, 23, 11, 29, 17, 5, 3, 21, 9, 27.

Half-cycles 11, 29, 17, 5 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

21, 9, 27, 15, 13, –, 19, 7, 25, 23, 11, 29, 17, 5, 3, 21, 9, 27,

and half-cycle 3 crosses in the finished knot sequentially the half-cycles which end at the start of the half-cycles

21, 9, 27, 15, 13, –, 19, 7, 25, 23, 11, 29, 17.

Note that the half-cycle which ends at the start of half-cycle 1 is the very last half-cycle in the braiding process and hence will not be crossed during the braiding process, thus we leave its cell empty.

The lower table is assembled in a way which enables us to read the braiding half-cycle algorithms from left to right, hence:

Half-cycles 4, 2 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

6, 18, 30, 12, 14, 16, 28, 10, 22, 24, 26, 8, 20, 2,

and half-cycles 20, 8, 26 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

6, 18, 30, 12, 14, 16, 28, 10, 22, 24, 26, 8, 20, 2, 4, 6, 18, 30, 12.

Half-cycles 14, 12 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

16, 28, 10, 22, 24, 26, 8, 20, 2, 4, 6, 18, 30, 12,

and half-cycles 30, 18, 6 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

16, 28, 10, 22, 24, 26, 8, 20, 2, 4, 6, 18, 30, 12, 14, 16, 28, 10, 22.

Half-cycles 24, 22 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

26, 8, 20, 2, 4, 6, 18, 30, 12, 14, 16, 28, 10, 22,

and half-cycles 10, 28, 16 cross in the finished knot sequentially the half-cycles which end at the start of the half-cycles

26, 8, 20, 2, 4, 6, 18, 30, 12, 14, 16, 28, 10, 22, 24, 26, 8, 20, 2.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 1:

1. $1 \nearrow 4$: Free run.
2. $5 \nwarrow 4$: u.
3. $5 \nearrow 5$: Free run.
4. $4 \nwarrow 5$: u.
5. $4 \nearrow 1$: $2o$.
6. $3 \nwarrow 1$: $3u$.
7. $3 \nearrow 2$: $3o$.
8. $2 \nwarrow 2$: $2u - 3o$.
9. $2 \nearrow 3$: $o - 2u - o - u$.
10. $1 \nwarrow 3$: $u - 3o - u - o - u$.
11. $1 \nearrow 4$: $o - u - 3o - u$.
12. $5 \nwarrow 4$: $u - 2o - 3u$.
13. $5 \nearrow 5$: $u - 2o - 2u$.
14. $4 \nwarrow 5$: $u - 3o - 2u$.
15. $4 \nearrow 1$: $u - o - 3u - 3o$.
16. $3 \nwarrow 1$: $4o - 3u - o - 2u$.
17. $3 \nearrow 2$: $u - 2o - 2u - 4o$.
18. $2 \nwarrow 2$: $u - 2o - 4u - 3o - u$.
19. $2 \nearrow 3$: $u - 3o - 3u - 3o - u$.
20. $1 \nwarrow 3$: $2u - 4o - 2u - 4o - u$.
21. $1 \nearrow 4$: $2o - 4u - 4o - 2u$.
22. $5 \nwarrow 4$: $3u - 4o - 4u$.
23. $5 \nearrow 5$: $3u - 4o - 3u$.
24. $4 \nwarrow 5$: $4u - 4o - 3u$.
25. $4 \nearrow 1$: $2u - 4o - 4u - 4o$.
26. $3 \nwarrow 1$: $u - 4o - 4u - 5o - 2u$.
27. $3 \nearrow 2$: $u - 4o - 5u - 4o - u$.
28. $2 \nwarrow 2$: $2u - 5o - 4u - 5o - 2u$.
29. $2 \nearrow 3$: $u - 4o - 5u - 5o - 2u$.
30. $1 \nwarrow 3$: $3u - 5o - 5u - 5o - u$.

The grid-diagram of this knot is depicted in Fig. 523

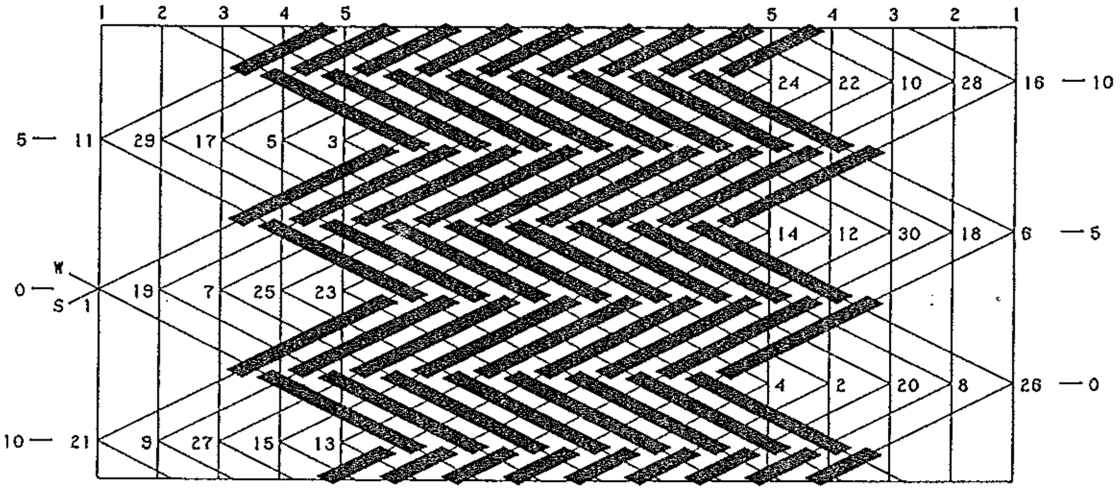


Fig. 523 — Grid-diagram of the knot in Example 1.

Example 2:

$A = 5$; $x = 14$; $y = A+1 = 6$, hence $\Delta = |6|_5 = 1$; $B^* = 3$, hence $B = A \cdot B^* = 15$. Thus $k = \left|\frac{x-A-3}{2}\right|_A = \left|\frac{14-5-3}{2}\right|_5 = 3$.

At the upper left-hand side in Fig. 524 is depicted the first-return string-run with its associated nest-index numbers.

At the upper right-hand side in Fig. 524 are depicted the first-return string-run, the half-cycle numbers of the half-cycles, and the number of crossings which these half-cycles make in the finished knot. This layout at the upper right-hand side has been shown for additional clarification only since it can readily be dispensed with.

The depicted layout at the bottom of Fig. 524 gives the positions of the nests with respect to the nest-index numbers, the half-cycle numbers with respect to the nest-index numbers and bight-boundaries, and the number of crossings which the half-cycles make in the finished knot. This layout is again the one we need for compiling the half-cycle algorithms; it is directly obtained from the first-return string-run with its associated nest-index numbers, hence from the upper left-hand layout in Fig. 524. The first step in its construction is shown in Fig. 525.

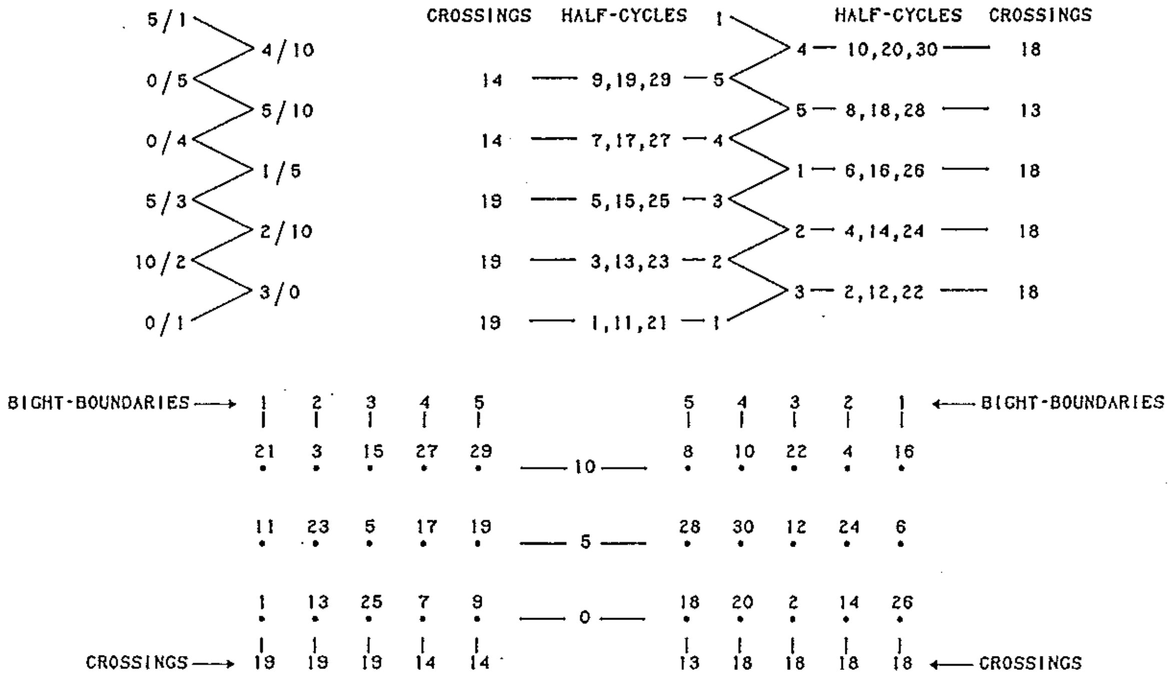


Fig. 524 — First-return string-run and half-cycle pattern.

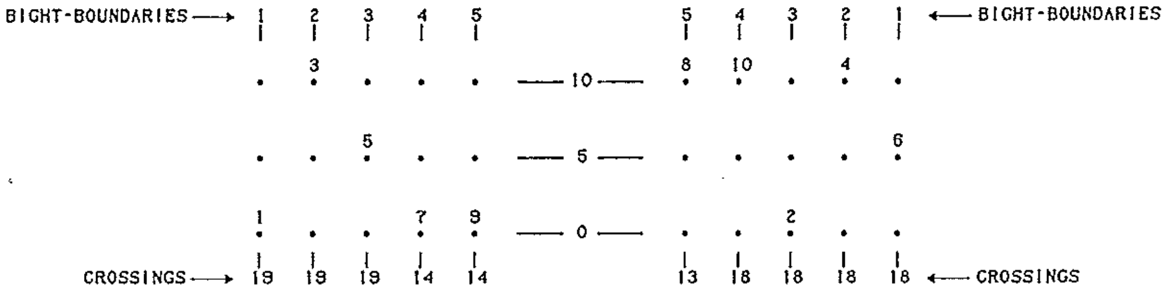


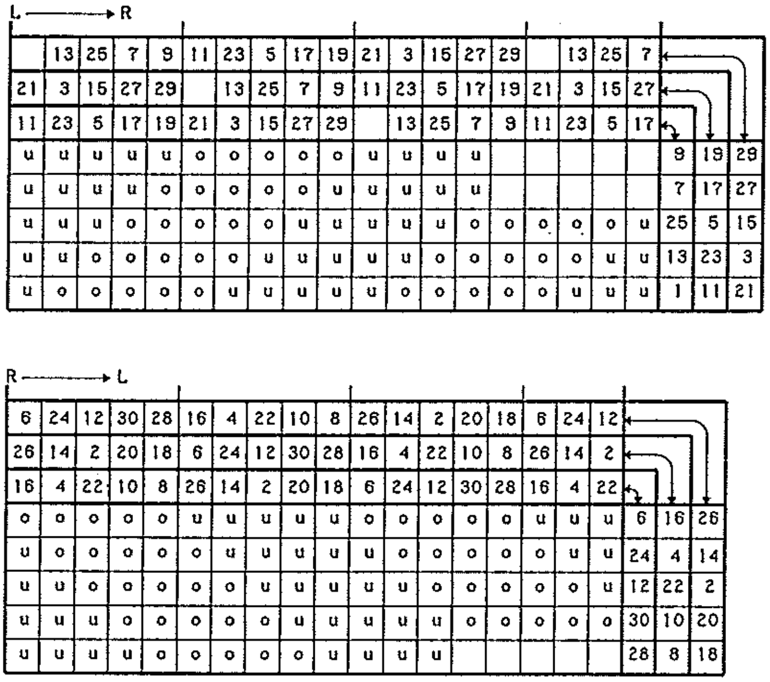
Fig. 525 — The first step in the construction of the half-cycle pattern.

The second step in the construction of the half-cycle pattern shown at the bottom of Fig. 524 consists of the determination of the half-cycle numbers in association with the nest-index numbers and bight-boundaries of their starting points.

Since the second sequence of the half-cycles in a first-return string-run starts with half-cycle number $1+2A=11$ at nest-index number $I_L = A|2A+x-2|_{B^*} = 5|22|_3 = 5$, two sequential half-cycle numbers h_n (≥ 1) and $h_n + 2A$ $(\leq 2B)$ associated with the same bight-boundary are associated respectively with nest-index number I and nest-index number $\left|I+2A^2+A(x-2)\right|_B = \left|I+2\cdot 5^2+5(14-2)\right|_{15} = |I+110|_{15} = |I+5|_{15}$. Hence we can readily complete, similar as in Example 1, the half-cycle pattern concerned. With the aid of the coding-sequence of the half-cycles in the finished knot with a Herringbone Pineapple coding and $y=A+1$ (see pg. 646) we can either read the half-cycle braiding algorithms again directly from the half-cycle pattern, or from the set of half-cycle tables shown in Fig. 526 which we assemble from the half-cycle pattern at the bottom of Fig. 524.

The upper table in Fig. 526 is for the odd-numbered half-cycles, hence the half-cycles from lower-left to upper-right, while the lower table in Fig. 526 is for the even-numbered half-cycles, hence the half-cycles from lower-right to upper-left.

Note that the half-cycle which ends at the start of half-cycle 1 is the very last half-cycle in the braiding process and hence will not be crossed during the braiding process, thus we leave its cell empty.



L → R

	13	25	7	9	11	23	5	17	19	21	3	15	27	29		13	25	7			
21	3	15	27	29		13	25	7	9	11	23	5	17	19	21	3	15	27			
11	23	5	17	19	21	3	15	27	29		13	25	7	9	11	23	5	17			
u	u	u	u	u	o	o	o	o	o	u	u	u	u						9	19	29
u	u	u	u	o	o	o	o	o	u	u	u	u	u						7	17	27
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	25	5	15
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	13	23	3
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	1	11	21

R → L

6	24	12	30	28	16	4	22	10	8	26	14	2	20	18	6	24	12			
26	14	2	20	18	6	24	12	30	28	16	4	22	10	8	26	14	2			
16	4	22	10	8	26	14	2	20	18	6	24	12	30	28	16	4	22			
o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	u	6	16	26
u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	u	24	4	14
u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	u	12	22	2
u	u	u	o	o	o	o	o	u	u	u	u	u	o	o	o	o	o	30	10	20
u	u	u	u	o	o	o	o	o	u	u	u	u						28	8	18

Fig. 526 — The half-cycle tables for the knot in Example 2.

From these tables we read then the braiding half-cycle algorithms for the knot in Example 2:

1. $1 \nearrow 3$: Free run.
2. $2 \nwarrow 3$: o.
3. $2 \nearrow 2$: u.
4. $3 \nwarrow 2$: $2o - u$.
5. $3 \nearrow 1$: $2u - o$.
6. $4 \nwarrow 1$: $o - u - o - u$.
7. $4 \nearrow 5$: $u - o - u$.
8. $5 \nwarrow 5$: $u - o - u$.
9. $5 \nearrow 4$: $u - o - u$.
10. $1 \nwarrow 4$: $u - o - u - 3o$.
11. $1 \nearrow 3$: $o - 3u - o - u$.
12. $2 \nwarrow 3$: $u - 2o - 2u - 2o$.
13. $2 \nearrow 2$: $u - 2o - u - 3o - u$.
14. $3 \nwarrow 2$: $u - o - 3u - 3o - u$.
15. $3 \nearrow 1$: $u - 4o - 2u - o - u$.
16. $4 \nwarrow 1$: $2o - 2u - 4o - 2u$.
17. $4 \nearrow 5$: $2u - 2o - 4u$.
18. $5 \nwarrow 5$: $2u - 3o - 3u$.
19. $5 \nearrow 4$: $2u - 3o - 3u$.
20. $1 \nwarrow 4$: $2u - 2o - 4u - 4o$.
21. $1 \nearrow 3$: $4o - 4u - 2o - 2u$.
22. $2 \nwarrow 3$: $u - 4o - 3u - 4o - u$.
23. $2 \nearrow 2$: $2u - 2o - 4u - 5o - u$.
24. $3 \nwarrow 2$: $u - 4o - 5u - 3o - 2u$.
25. $3 \nearrow 1$: $3u - 5o - 2u - 5o - u$.
26. $4 \nwarrow 1$: $3o - 5u - 5o - 3u$.
27. $4 \nearrow 5$: $3u - 5o - 5u$.
28. $5 \nwarrow 5$: $4u - 5o - 4u$.
29. $5 \nearrow 4$: $4u - 5o - 4u$.
30. $1 \nwarrow 4$: $3u - 5o - 5u - 5o$.

The grid-diagram of this knot is depicted in Fig. 527

Although the **nest-index numbers** I_R may be calculated as shown in *The Braider*, Issue No. 26, pp. 594 and 598, the following method is a little quicker:

Let an odd-numbered half-cycle run from left bight-boundary l_i and nest-index number I_L to right bight-boundary r_i and nest-index number I_R. Then $I_R = I_L$ when $l_i + r_i = k + 1$, and $I_R = |I_L - A|_B$ when $l_i + r_i = A + k + 1$.

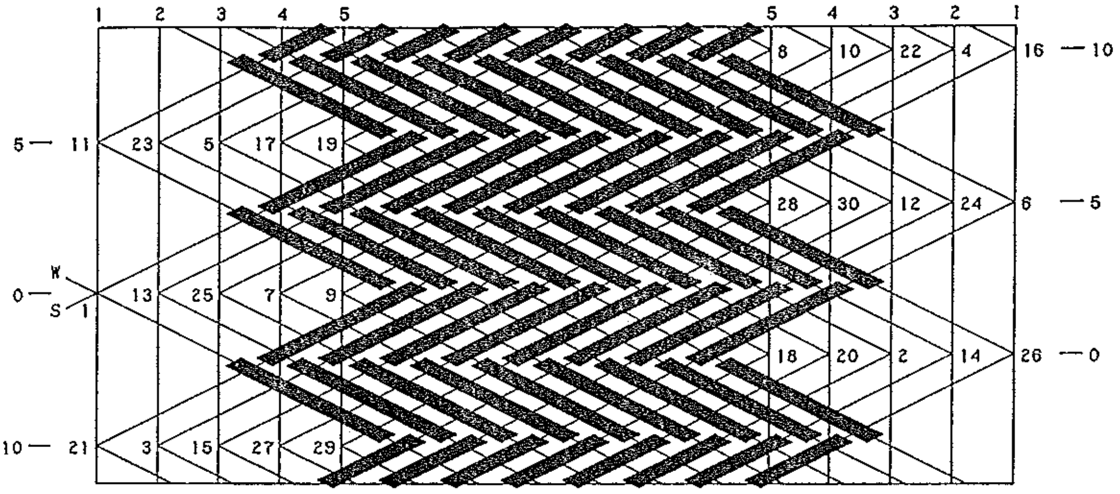


Fig. 527 — Grid-diagram of the knot in Example 2.

Note that the string-runs of the knots in Example 1 and 2 are each others mirror-image (their first-return string-runs are each others mirror-image (see *The Braider*, Issue No. 24, pg. 562, and Issue No. 25, pg. 568) and they have an identical number of bights). Hence when for one of such herringbone pineapple coded knots the two half-cycle tables have been determined, we can then readily derive from these tables the two half-cycle tables for the mirror-imaged knot.

Say the first-return string-run and the two half-cycle tables for the $y = A - 1$ knot are known. On its first-return string-run determine the number of half-cycles between right-hand bight-boundary 1 and left-hand bight-boundary 1 at the end of the first-return string-run (this number of half-cycles is equal to 5 for Example 1). Then replace each half-cycle number in its two half-cycle tables by

$$|\text{half-cycle number} + 5|_{2B} .$$

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A + 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A + 1$).

Say the first-return string-run and the two half-cycle tables for the $y = A + 1$ knot are known. On its first-return string-run determine the number of half-cycles between left-hand bight-boundary 1 at the beginning of the first-return string-run and right-hand bight-boundary 1 (this number of half-cycles is equal to 5 for Example 2). Then replace each half-cycle number in its two half-cycle tables by

$$|\text{half-cycle number} - 5|_{2B} .$$

The new table derived from the table for the odd-numbered half-cycles will now be the half-cycle table for the even-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$), and the new table derived from the table for the even-numbered half-cycles will now be the half-cycle table for the odd-numbered half-cycles of the mirror-imaged knot (the knot with $y = A - 1$).