THE BRAIDER

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CONTENTS

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for
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Solution to the Question in Issue No. 25

Question on pg. 573.

The completed cell-entries in the (A, x, y)-table for $A = 12$ are shown in Fig. 481.

y \ x	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0													12x2		1x4 11x2		2x4 10x2		3x4 9x2		4x4 8x2		5x4 7x2		6x4 6x2	
1												1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
2											2x11		1x13 1x11		2x13		1x15 1x13		2x15		1x17 1x15		2x17		1x19 1x17	
3										3x7		1x9 2x7		2x9 1x7		3x9		1x11 2x9		2x11 1x9		3x11		1x13 2x11		2x13 1x11
4									4x5		1x7 3x5		2x7 2x5		3x7 1x5		4x7		1x9 3x7		2x9 2x7		3x9 1x7		4x9	
5								1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
6							6x3		1x5 5x3		2x5 4x3		3x5 3x3		4x5 2x3		5x5 1x3		6x5		1x7 5x5		2x7 4x5		3x7 3x5	
7						1x17		1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
8					4x4		1x6 3x4		2x6 2x4		3x6 1x4		4x6		1x8 3x6		2x8 2x6		3x8 1x6		4x8		1x10 3x8		2x10 2x8	
9				3x5		1x7 2x5		2x7 1x5		3x7		1x9 2x7		2x9 1x7		3x9		1x11 2x9		2x11 1x9		3x11		1x13 2x11		2x13 1x11
10			2x7		1x9 1x7		2x9		1x11 1x9		2x11		1x13 1x11		2x13		1x15 1x13		2x15		1x17 1x15		2x17		1x19 1x17	
11		1x13		1x15		1x17		1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
12	12x1		1x3 11x1		2x3 10x1		3x3 9x1		4x3 8x1		5x3 7x1		6x3 6x1		7x3 5x1		8x3 4x1		9x3 3x1		10x3 2x1		11x3 1x1		12x3	
13		1x13		1x15		1x17		1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
14			2x7		1x9 1x7		2x9		1x11 1x9		2x11		1x13 1x11		2x13		1x15 1x13		2x15		1x17 1x15		2x17		1x19 1x17	
15				3x5		1x7 2x5		2x7 1x5		3x7		1x9 2x7		2x9 1x7		3x9		1x11 2x9		2x11 1x9		3x11		1x13 2x11		2x13 1x11
16					4x4		1x6 3x4		2x6 2x4		3x6 1x4		4x6		1x8 3x6		2x8 2x6		3x8 1x6		4x8		1x10 3x8		2x10 2x8	
17						1x17		1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
18							6x3		1x5 5x3		2x5 4x3		3x5 3x3		4x5 2x3		5x5 1x3		6x5		1x7 5x5		2x7 4x5		3x7 3x5	
19								1x19		1x21		1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
20									4x5		1x7 3x5		2x7 2x5		3x7 1x5		4x7		1x9 3x7		2x9 2x7		3x9 1x7		4x9	
21										3x7		1x9 2x7		2x9 1x7		3x9		1x11 2x9		2x11 1x9		3x11		1x13 2x11		2x13 1x11
22											2x11		1x13 1x11		2x13		1x15 1x13		2x15		1x17 1x15		2x17		1x19 1x17	
23												1x23		1x25		1x27		1x29		1x31		1x33		1x35		1x37
24													12x2		1x4 11x2		2x4 10x2		3x4 9x2		4x4 8x2		5x4 7x2		6x4 6x2	

Fig. 481 — The (A, x, y)-table for $A = 12$.

As mentioned on pg. 570 of *The Braider*, Issue No. 25, we only have to determine the cell-entries for $0 \leq y \leq A$ since they provide the cell-entries for $A < y \leq 2A$ by means of the mirror-image complementary relationship. Apart from the determination of the cell-entries for $y = A$, it is, however, not necessary to calculate the cell-entries for each y-value in the range $0 < y < A$.

Say we have determined the cell-entries for $y = y_2$, where $0 < y_2 < A$. Then for the same x, the cell-entry for $0 < y_1 = y_2 - 2n\{\text{g.c.d.}(y_2, A)\}$, where n is a natural number, is identical to the cell-entry for y_2 if and only if $\text{g.c.d.}(y_1, A) = \text{g.c.d.}(y_2, A)$. Thus in the case $A = 12$ we first determine the cell-entries for $y = A = 12$. Then we determine the cell-entries for $y_2 = 11$, and from these cell-entries we obtain directly the cell-entries for $y_1 = 7, 5, 1$ (hence $n = 2, 3, 5$ respectively). Next we determine the cell-entries for $y_2 = 10$, and from these cell-entries we obtain directly the cell-entries for $y_1 = 2$ (hence $n = 2$). Next we determine the cell-entries for $y_2 = 9$, and from these cell-entries we obtain directly the cell-entries for $y_1 = 3$ (hence $n = 1$). Next we determine the cell-entries for $y_2 = 8$; from these cell-entries we cannot obtain directly the cell-entries for a y-value which is less than 8. Next we determine the cell-entries for $y_2 = 6$; from these cell-entries we cannot obtain directly the cell-entries for a y-value which is less than 6. Next we determine the cell-entries for $y_2 = 4$; from these cell-entries we cannot obtain directly the cell-entries for a y-value which is less than 4. Next we determine the cell-entries for $y = 0$. Then we use the mirror-image complementary relationship for the cell-entries $A < y \leq 2A$, hence for $12 < y \leq 24$.

Nested Cylindrical Braids

In order to obtain an overview of the actual positions of the bights on the bight-boundaries, we attach to the bight-boundary number of a bight the **nest-index number** of the nest to which the bight belongs.

Let the Standing End half-cycle, running from lower-left to upper-right, start at the left-hand bight-boundary l_1 and end at the right-hand bight-boundary r_1. The nest, to which its left-hand bight-point belongs, receives the nest-index number $I_L = I_{L_1} = 0$, and the nest, to which its right-hand bight-point belongs, receives the nest-index number $I_R = I_{R_1} = 0$. The left-hand bight-point of the Standing End half-cycle is indicated by $I_L/l_1 = I_{L_1}/l_1 = 0/l_1$, and the right-hand bight-point of the Standing End half-cycle is indicated by $r_1/I_R = r_1/I_{R_1} = r_1/0$.

The next half-cycle (the second half-cycle), running from lower-right to upper-left, starts at the right-hand bight-point $r_1/0$ and ends at the left-hand bight-boundary l_2. The nest, to which its bight-point on the left-hand bight-boundary l_2 belongs, receives the nest-index number $I_L = I_{L_2} = |I_{L_1} + 4A + x - (l_1 + l_2 + 2r_1)|_B$. The left-hand bight-point of the second half-cycle is indicated by $I_L/l_2 = I_{L_2}/l_2$.

The next half-cycle (the third half-cycle), running from lower-left to upper-right, starts at the left-hand bight-point I_{L_2}/l_2 and ends at the right-hand bight-boundary r_2. The nest, to which its bight-point on the right-hand bight-boundary r_2 belongs, receives the nest-index number $I_R = I_{R_2} = |I_{R_1} + 4A + x - (r_1 + r_2 + 2l_2)|_B$. The right-hand bight-point of the third half-cycle is indicated by r_2/I_{R_2}.

The next half-cycle (the fourth half-cycle), running from lower-right to upper-left, starts at the right-hand bight-point r_2/I_{R_2} and ends at the left-hand bight-boundary l_3. The nest, to which its bight-point on the left-hand bight-boundary l_3 belongs, receives the nest-index number $I_L = I_{L_3} = |I_{L_2} + 4A + x - (l_2 + l_3 + 2r_2)|_B$. The left-hand bight-point of the fourth half-cycle is indicated by $I_L/l_3 = I_{L_3}/l_3$.

The next half-cycle (the fifth half-cycle), running from lower-left to upper-right, starts at the left-hand bight-point I_{L_3}/l_3 and ends at the right-hand bight-boundary r_3. The nest, to which its bight-point on the right-hand bight-boundary r_3 belongs, receives the nest-index number $I_R = I_{R_3} = |I_{R_2} + 4A + x - (r_2 + r_3 + 2l_3)|_B$. The right-hand bight-point of the fifth half-cycle is indicated by r_3/I_{R_3}.

And so on.

In general:

The $(2n)^{th}$ half-cycle, where $n = 1, 2, 3, \cdots$, running from lower-right to upper-left, starts at the right-hand bight-point r_n/I_{R_n} and ends at the left-hand bight-boundary l_{n+1}. The nest, to which its bight-point on the left-hand bight-boundary l_{n+1} belongs, receives the nest-index number $I_L = I_{L_{n+1}} = |I_{L_n} + 4A + x - (l_n + l_{n+1} + 2r_n)|_B$. The left-hand bight-point of the $(2n)^{th}$ half-cycle, where $n = 1, 2, 3, \cdots$, is indicated by $I_L/l_{n+1} = I_{L_{n+1}}/l_{n+1}$.

The $(2n+1)^{th}$ half-cycle), running from lower-left to upper-right, starts at the left-hand bight-point $I_{L_{n+1}}/l_{n+1}$ and ends at the right-hand bight-boundary r_{n+1}. The nest, to which its bight-point on the right-hand bight-boundary r_{n+1} belongs, receives the nest-index number $I_R = I_{R_{n+1}} = |I_{R_n} + 4A + x - (r_n + r_{n+1} + 2l_{n+1})|_B$. The right-hand bight-point of the $(2n+1)^{th}$ half-cycle is indicated by $r_{n+1}/I_{R_{n+1}}$.

Example 1: (2222/10/2222){15432/34512}20

Hence:

$$\begin{aligned} A &= 5 . \\ l_1 &= 1 . \\ k = r_1 &= 3 . \\ y &= |x - 2(k+1)|_{2A} = |10 - 2(3+1)|_{10} = 2 . \\ \Delta &= |x - 2(k+1)|_A = |y|_A = |2|_5 = 2 . \end{aligned}$$

The first-return string-run is:

$$\begin{array}{ll} 1 & \\ & 5 \\ 4 & \\ & 2 \\ 2 & \\ & 4 \\ 5 & \\ & 1 \\ 3 & \\ & 3 \\ 1 & \end{array}$$

$$\begin{array}{lll} l_1 = 1 \longrightarrow 3 = r_1 & \rightarrow & l_2 = |l_1 + \Delta|_5 = |1+2|_5 = 3 . \\ l_2 = 3 \longleftarrow 3 = r_1 & \rightarrow & r_2 = |r_1 - \Delta|_5 = |3-2|_5 = 1 . \\ l_2 = 3 \longrightarrow 1 = r_2 & \rightarrow & l_3 = |l_2 + \Delta|_5 = |3+2|_5 = 5 . \\ l_3 = 5 \longleftarrow 1 = r_2 & \rightarrow & r_3 = |r_2 - \Delta|_5 = |1-2|_5 = 4 . \\ l_3 = 5 \longrightarrow 4 = r_3 & \rightarrow & l_4 = |l_3 + \Delta|_5 = |5+2|_5 = 2 . \\ l_4 = 2 \longleftarrow 4 = r_3 & \rightarrow & r_4 = |r_3 - \Delta|_5 = |4-2|_5 = 2 . \\ l_4 = 2 \longrightarrow 2 = r_4 & \rightarrow & l_5 = |l_4 + \Delta|_5 = |2+2|_5 = 4 . \\ l_5 = 4 \longleftarrow 2 = r_4 & \rightarrow & r_5 = |r_4 - \Delta|_5 = |2-2|_5 = 5 . \\ l_5 = 4 \longrightarrow 5 = r_5 & \rightarrow & l_6 = |l_5 + \Delta|_5 = |4+2|_5 = 1 = l_1 . \end{array}$$

$$P_c = 4 \cdot \alpha + \frac{\alpha \cdot x - 2\sum(l_i + r_i)}{A} = 4 \times 5 + \frac{5 \times 10 - 2\{(1+3+5+2+4)+(3+1+4+2+5)\}}{5} = 18 .$$

g.c.d. $(P_c, B^*) =$ g.c.d. $(18, 4) = 2$.

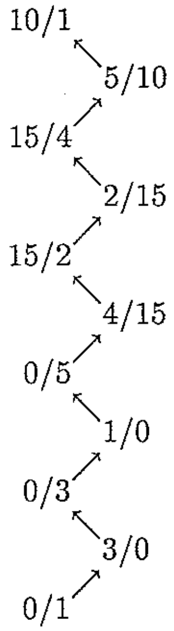
Hence:

$$P_{total} = \sum P_{component} = 18 .$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 1 .$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = \sum \text{g.c.d.}\,(P_c, B^*) = 2 .$$

Let's now attach nest-index numbers to the bight-boundary numbers of the first-return string-run. The end-points of the first half-cycle receive both the nest-index number $I = 0$.



$$
\begin{aligned}
I_L &= I_1 = 0\,.\\
I_R &= I_{1'} = 0\,.\\
I_L &= I_2 = |I_1 + 4A + x - (l_1 + l_2 + 2r_1)|_B =\\
&\qquad |0 + 4 \times 5 + 10 - (1 + 3 + 2 \times 3)|_{20} = 0\,.\\
I_R &= I_{2'} = |I_{1'} + 4A + x - (r_1 + r_2 + 2l_2)|_B =\\
&\qquad |0 + 4 \times 5 + 10 - (3 + 1 + 2 \times 3)|_{20} = 0\,.\\
I_L &= I_3 = |I_2 + 4A + x - (l_2 + l_3 + 2r_2)|_B =\\
&\qquad |0 + 4 \times 5 + 10 - (3 + 5 + 2 \times 1)|_{20} = 0\,.\\
I_R &= I_{3'} = |I_{2'} + 4A + x - (r_2 + r_3 + 2l_3)|_B =\\
&\qquad |0 + 4 \times 5 + 10 - (1 + 4 + 2 \times 5)|_{20} = 15\,.\\
I_L &= I_4 = |I_3 + 4A + x - (l_3 + l_4 + 2r_3)|_B =\\
&\qquad |0 + 4 \times 5 + 10 - (5 + 2 + 2 \times 4)|_{20} = 15\,.\\
I_R &= I_{4'} = |I_{3'} + 4A + x - (r_3 + r_4 + 2l_4)|_B =\\
&\qquad |15 + 4 \times 5 + 10 - (4 + 2 + 2 \times 2)|_{20} = 15\,.\\
I_L &= I_5 = |I_4 + 4A + x - (l_4 + l_5 + 2r_4)|_B =\\
&\qquad |15 + 4 \times 5 + 10 - (2 + 4 + 2 \times 2)|_{20} = 15\,.\\
I_R &= I_{5'} = |I_{4'} + 4A + x - (r_4 + r_5 + 2l_5)|_B =\\
&\qquad |15 + 4 \times 5 + 10 - (2 + 5 + 2 \times 4)|_{20} = 10\,.\\
I_L &= I_6 = |I_5 + 4A + x - (l_5 + l_6 + 2r_5)|_B =\\
&\qquad |15 + 4 \times 5 + 10 - (4 + 1 + 2 \times 5)|_{20} = 10\,.
\end{aligned}
$$

For each of the two sub-components, this first-return string-run with its attached nest-index numbers provides us with the necessary data for writing down the consecutive half-cycles with their associated bight-boundary numbers and nest-index numbers at the ends.

Since there are four nests on the left bight-edge, and since the bight-edge has a total length of twenty bights, it follows that the nest-index numbers are $0, 5, 10, 15$. When the Standing End of the first sub-component has the left-hand nest-index number 0, then the consecutive cycles of its string-run will be associated with the left-hand nest-index numbers: $\underline{0}; 0; 0; 15; 15; \underline{10}; 10 = |10 + 0|_B; 10 = |10 + 0|_B; 5 = |10 + 15|_B; 5 = |10 + 15|_B\,; \underline{0} = |10 + 10|_B\,.$

A first-return string-run runs between two consecutive underlined nest-index numbers. Hence the string-run of the first sub-component consists of two first-return string-runs. The first one starts at bight-boundary 1 and nest-index number 0, the second one starts at bight-boundary 1 and nest-index number 10.

Hence the second sub-component can either start at bight-boundary 1 and nest-index number 5, or at bight-boundary 1 and nest-index number 15. We shall take the first option. Note that we don't have to start the second, or for that matter the first sub-component, with the half-cycle $1 \longrightarrow 3$, but could start with any of the lower-left to upper-right half-cycles of the first-return string-run.

With the chosen start for the second sub-component at bight-boundary 1 and nest-index number 5, the consecutive cycles of its string-run will be associated with the left-hand nest-index numbers: $\underline{5}\,;\ 5 = |5 + 0|_B\,;\ 5 = |5 + 0|_B\,;\ 0 = |5 + 15|_B\,;\ 0 = |5 + 15|_B\,;\ \underline{15} = |5 + 10|_B\,;\ 15 = |15 + 0|_B\,;\ 15 = |15 + 0|_B\,;\ 10 = |15 + 15|_B\,;\ 10 = |15 + 15|_B\,;\ \underline{5} = |15 + 10|_B\,.$

Thus the string-runs of the two sub-components are:

0/1	5/1
5/5	5/10
5/4	10/4
2/5	2/10
5/2	10/2
4/5	4/10
10/5	15/5
1/10	1/15
10/3	15/3
3/10	3/15
10/1	15/1
5/10	5/0
15/4	0/4
2/15	2/0
15/2	0/2
4/15	4/0
0/5	5/5
1/0	1/5
0/3	5/3
3/0	3/5
0/1	5/1

We can now readily draw the relative bight-positions of the two sub-components (see Fig. 482), which gives us some general overview of the braid.

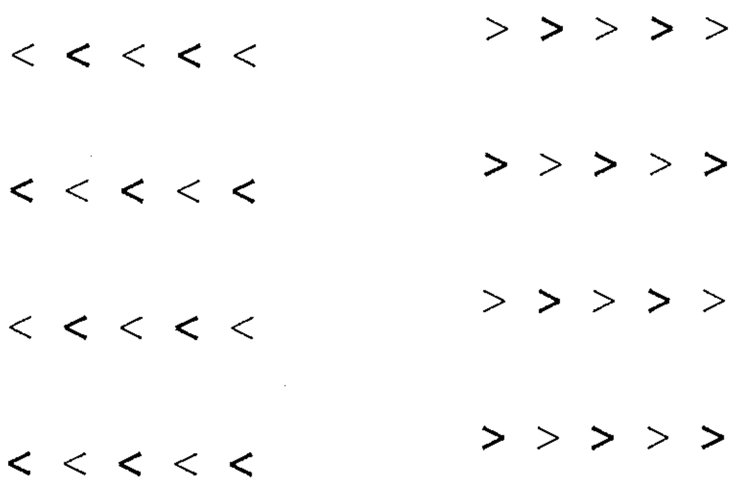


Fig. 482 — The relative bight-positions of (2222/10/2222){15432/34512}20.

The complete string-run diagram of (2222/10/2222){15432/34512}20 is shown in Fig. 483.

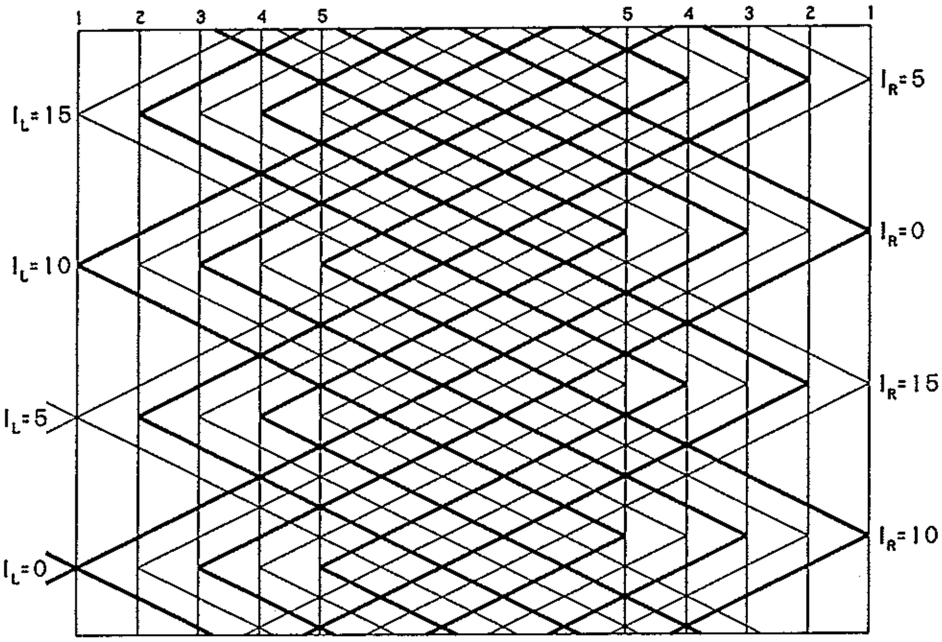


Fig. 483 — The string-run of (2222/10/2222){15432/34512}20.

The above example was a simple one in that it had only one component, hence there was only one first-return string-run and hence there was only one first half-cycle type, with the consequence that the first half-cycle of any sub-component had the same length. If we have, however, more than one component, then there are more than one first-return string-run and hence more than one first half-cycle type, with the consequence that the sub-components of different components may have their first half-cycles of a different length.

In general, let the first component have a first-return string-run in which the first half-cycle, running from lower-left to upper-right, starts at the left-hand bight-boundary $l_1 = 1$ and ends at the right-hand bight-boundary $r_1 = k$, where $1 \le k \le A$. Let furthermore the left-hand nest-index number and the right-hand nest-index number of the first half-cycle of the first sub-component of this first component be respectively equal to $I_{L_1} = 0$ and $I_{R_1} = 0$.

Let the second component have a first-return string-run in which the first half-cycle, running from lower-left to upper-right, starts at the left-hand bight-boundary l'_1 and ends at the right-hand bight-boundary r'_1. Let furthermore the left-hand nest-index number of the first half-cycle of the first sub-component of this second component be equal to I'_{L_1}. Then the right-hand nest-index number of the first half-cycle of the first sub-component of this second component is equal to $I'_{R_1} = \left|I'_{L_1} + (l_1 + r_1) - (l'_1 + r'_1)\right|_B = \left|I'_{L_1} + (k+1) - (l'_1 + r'_1)\right|_B$. In *The Braider*, Issue No. 23, on pg. 521 we have seen that for Regular Nested Cylindrical Braids $(l_i + r_i)$ is either $(k+1)$ or $(A+k+1)$ for $1 \le k < A$, and $(l_i + r_i) = (A+1)$ for $k = A$. Hence when $(l'_1 + r'_1) = (k+1)$, then $I'_{R_1} = I'_{L_1}$, and when $(l'_1 + r'_1) = (A+k+1)$ (hence only possible when $1 \le k < A$), then $I'_{R_1} = \left|I'_{L_1} - A\right|_B$.

Let the third component have a first-return string-run in which the first half-cycle, running from lower-left to upper-right, starts at the left-hand bight-boundary l''_1 and ends at the right-hand bight-boundary r''_1. Let furthermore the left-hand nest-index number of the first half-cycle of the first sub-component of this third component be equal to I''_{L_1}. Then the right-hand nest-index number of the first half-cycle of the first sub-component of this third component is equal to $I''_{R_1} = \left|I''_{L_1} + (l_1 + r_1) - (l''_1 + r''_1)\right|_B = \left|I''_{L_1} + (k+1) - (l''_1 + r''_1)\right|_B$. For Regular Nested Cylindrical Braids $(l_i + r_i)$ is either $(k+1)$ or $(A+k+1)$ for $1 \le k < A$, and $(l_i + r_i) = (A+1)$ for $k = A$. Hence when $(l''_1 + r''_1) = (k+1)$, then $I''_{R_1} = I''_{L_1}$, and when $(l''_1 + r''_1) = (A+k+1)$ (hence only possible when $1 \le k < A$), then $I''_{R_1} = \left|I''_{L_1} - A\right|_B$.

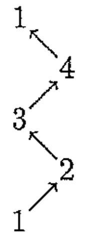
And so on.

Example 2: (222/12/222){1432/2341}16

Hence:

$$A = 4.$$
$$l_1 = 1.$$
$$k = r_1 = 2.$$
$$y = |x - 2(k+1)|_{2A} = |12 - 2(2+1)|_8 = 6.$$
$$\Delta = |x - 2(k+1)|_A = |y|_A = |6|_4 = 2.$$
$$\text{g.c.d.}\,(\Delta, A) = \text{g.c.d.}\,(2,4) = 2.$$

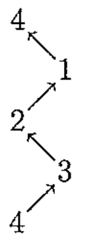
The first-return string-runs are:



$$\begin{array}{lll} l_1 = 1 \longrightarrow 2 = r_1 & \rightarrow & l_2 = |l_1 + \Delta|_4 = |1+2|_4 = 3. \\ l_2 = 3 \longleftarrow 2 = r_1 & \rightarrow & r_2 = |r_1 - \Delta|_4 = |2-2|_4 = 4. \\ l_2 = 3 \longrightarrow 4 = r_2 & \rightarrow & l_3 = |l_2 + \Delta|_4 = |3+2|_4 = 1. \end{array}$$

$$P_{c_1} = 4 \cdot \alpha + \frac{\alpha \cdot x - 2\sum(l_i + r_i)}{A} = 4 \times 2 + \tfrac{2\times 12 - 2\{(1+3)+(2+4)\}}{4} = 9.$$

$\text{g.c.d.}\,(P_{c_1}, B^*) = \text{g.c.d.}\,(9, 4) = 1.$



$$\begin{array}{lll} l'_1 = 4 \longrightarrow 3 = r'_1 & \rightarrow & l'_2 = |l'_1 + \Delta|_4 = |4+2|_4 = 2. \\ l'_2 = 2 \longleftarrow 3 = r'_1 & \rightarrow & r'_2 = |r'_1 - \Delta|_4 = |3-2|_4 = 1. \\ l'_2 = 2 \longrightarrow 1 = r'_2 & \rightarrow & l'_3 = |l'_2 + \Delta|_4 = |2+2|_4 = 4. \end{array}$$

$$P_{c_2} = 4 \cdot \alpha + \frac{\alpha \cdot x - 2\sum(l_i + r_i)}{A} = 4 \times 2 + \tfrac{2\times 12 - 2\{(4+2)+(3+1)\}}{4} = 9.$$

$\text{g.c.d.}\,(P_{c_2}, B^*) = \text{g.c.d.}\,(9, 4) = 1.$

Hence:

$$P_{total} = \sum P_{component} = P_{c_1} + P_{c_2} = 18 = 2A + x - 2.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 2.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = \sum \text{g.c.d.}\,(P_c, B^*) = 2.$$

By attaching nest-index numbers to the bight-boundary numbers of the first-return string-runs we obtain:

4/1
↖ 4/0
↗
4/3
↖ 2/0
↗
0/1

$$I_L = I_1 = 0\,.$$
$$I_R = I_{1'} = 0\,.$$
$$I_L = I_2 = |I_1 + 4A + x - (l_1 + l_2 + 2r_1)|_B = |0 + 4 \times 4 + 12 - (1 + 3 + 2 \times 2)|_{16} = 4\,.$$
$$I_R = I_{2'} = |I_{1'} + 4A + x - (r_1 + r_2 + 2l_2)|_B = |0 + 4 \times 4 + 12 - (2 + 4 + 2 \times 3)|_{16} = 0\,.$$
$$I_L = I_3 = |I_2 + 4A + x - (l_2 + l_3 + 2r_2)|_B = |4 + 4 \times 4 + 12 - (3 + 1 + 2 \times 4)|_{16} = 4\,.$$

12/4
↖ 1/8
↗
8/2
↖ 3/4
↗
8/4

$$I_L = I'_1 = 8\,.$$
$$I_R = I'_{1'} = |8 - A|_B = |8 - 4|_{16} = 4 \text{ since } l'_1 + r'_1 = A + k + 1\,.$$
$$I_L = I'_2 = |I'_1 + 4A + x - (l'_1 + l'_2 + 2r'_1)|_B = |8 + 4 \times 4 + 12 - (4 + 2 + 2 \times 3)|_{16} = 8\,.$$
$$I_R = I'_{2'} = |I'_{1'} + 4A + x - (r'_1 + r'_2 + 2l'_2)|_B = |4 + 4 \times 4 + 12 - (3 + 1 + 2 \times 2)|_{16} = 8\,.$$
$$I_L = I'_3 = |I'_2 + 4A + x - (l'_2 + l'_3 + 2r'_2)|_B = |8 + 4 \times 4 + 12 - (2 + 4 + 2 \times 1)|_{16} = 12\,.$$

Thus the string-runs of the two components are:

0/1	8/4
↖ 4/12	↖ 1/4
↗	↗
0/3	4/2
↖ 2/12	↖ 3/0
↗	↗
12/1	4/4
↖ 4/8	↖ 1/0
↗	↗
12/3	0/2
↖ 2/8	↖ 3/12
↗	↗
8/1	0/4
↖ 4/4	↖ 1/12
↗	↗
8/3	12/2
↖ 2/4	↖ 3/8
↗	↗
4/1	12/4
↖ 4/0	↖ 1/8
↗	↗
4/3	8/2
↖ 2/0	↖ 3/4
↗	↗
0/1	8/4

We can now draw the relative bight-positions of the two components (see Fig. 484), which gives us some general overview of the braid.

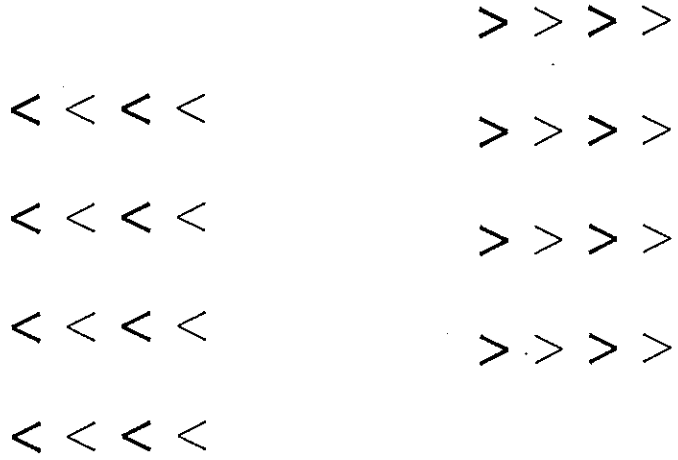


Fig. 484 — The relative bight-positions of (222/12/222){1432/2341}16.

The complete string-run diagram of (222/12/222){1432/2341}16 is depicted in Fig. 485.

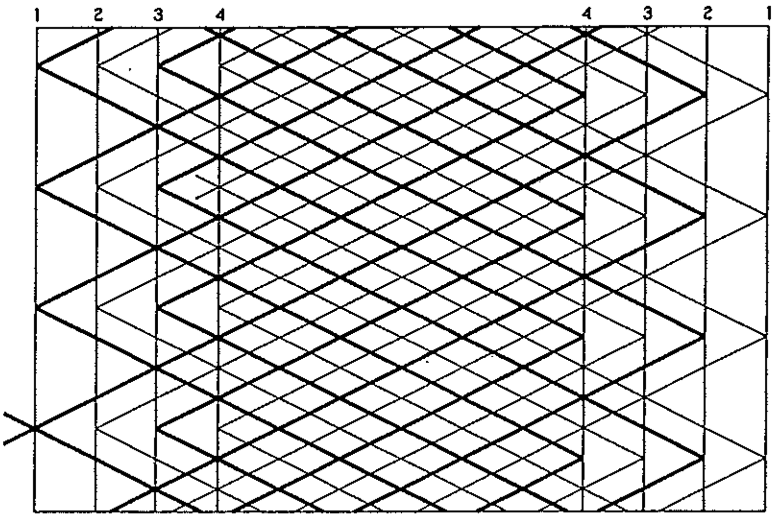


Fig. 485 — The string-run of (222/12/222){1432/2341}16.

THE BRAIDER'S NOTEBOOK

We have mentioned in the past that, if one wants to further his or her braiding skills, it is important to become thoroughly familiar with grid-diagrams. Only then will the braider really be able to design new braid-forms and hence will immediately see new possibilities resulting from a simple new knot which may have been discovered by experimentation.

From Eugene Ulrich, an eldery American knot-tyer, braider and horseman from Faith in South Dakota, we received in 1997 a letter in which he described an apparently new knot which he discovered by experimentation. In his letter he wrote:

The enclosed diagram [see Fig. 486] *is of, I think, a new knot. I can't describe the knot using your system, so will do it in the following way: When disregarding the dotted* *lines, we have a two strand Matthew Walker knot. With the dotted lines it becomes a doubled two strand Matthew Walker knot. Start at the smaller arrow points and follow each dotted line around the second time on top of itself to the end of the bigger arrow point. When working the knot down and into position, it will usually be necessary to push the strands coming from the base of the knot up and on top of the other strands and all the way around. Do this when the strands are still quite loose. This knot is at its best when tied with three or four strands and in color.*

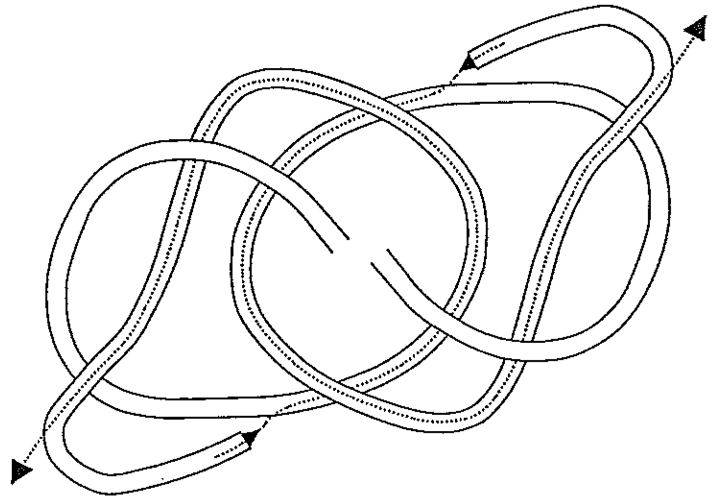


Fig. 486 — The doubled two strand Matthew Walker knot.

The grid-diagram of this doubled two strand Matthew Walker knot, which has a left-hand helix, is shown at the right in Fig. 487. The left-hand grid-diagram in Fig. 487 depicts the type with a right-hand helix. Observe that the nesting-number $A = 2$.

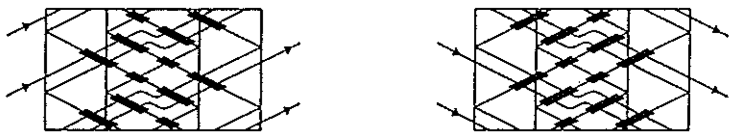


Fig. 487 — The two types of doubled two strand Matthew Walker knot ($A = 2$).

The three-strand and four-strand versions are depicted in Fig. 488.

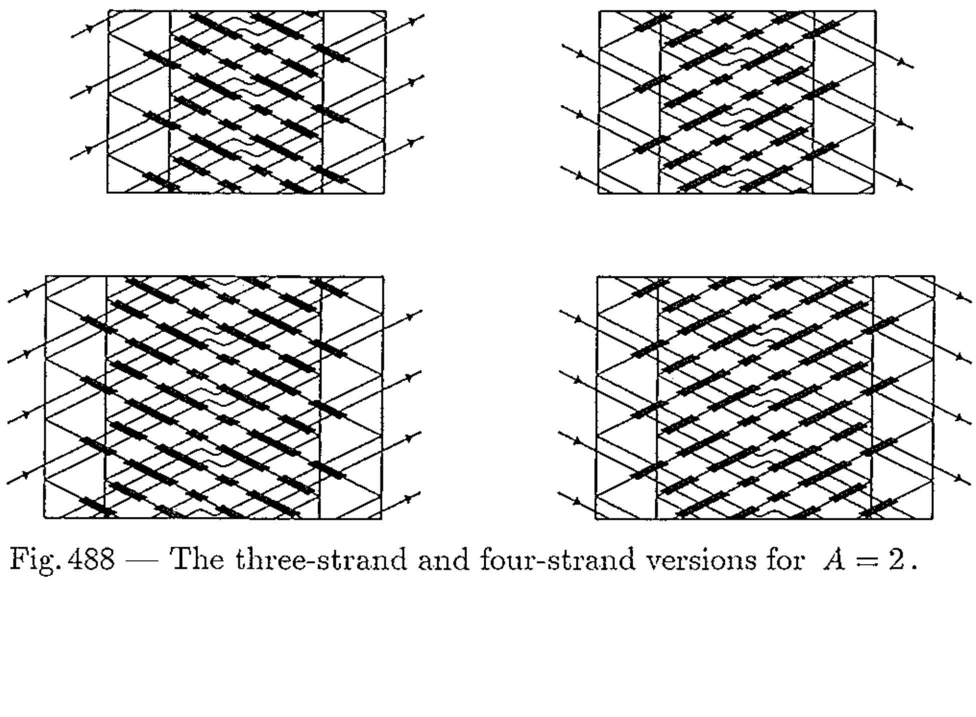


Fig. 488 — The three-strand and four-strand versions for $A = 2$.

Note that these knots with a right-hand helix are based on the string-run of the Regular Nested Cylindrical Braids $(2/2n - 1/2)\{12/12\}2n$, and that those with a left-hand helix are based on the string-run of the Regular Nested Cylindrical Braids $(2/2n - 1/2)\{12/21\}2n$, where n is the number of strands.

It should now have become evident that there must be similar knots with a nesting-number $A = 3$ (and with nesting-numbers greater than 3, although those are in general of little or no use in practical braidwork).

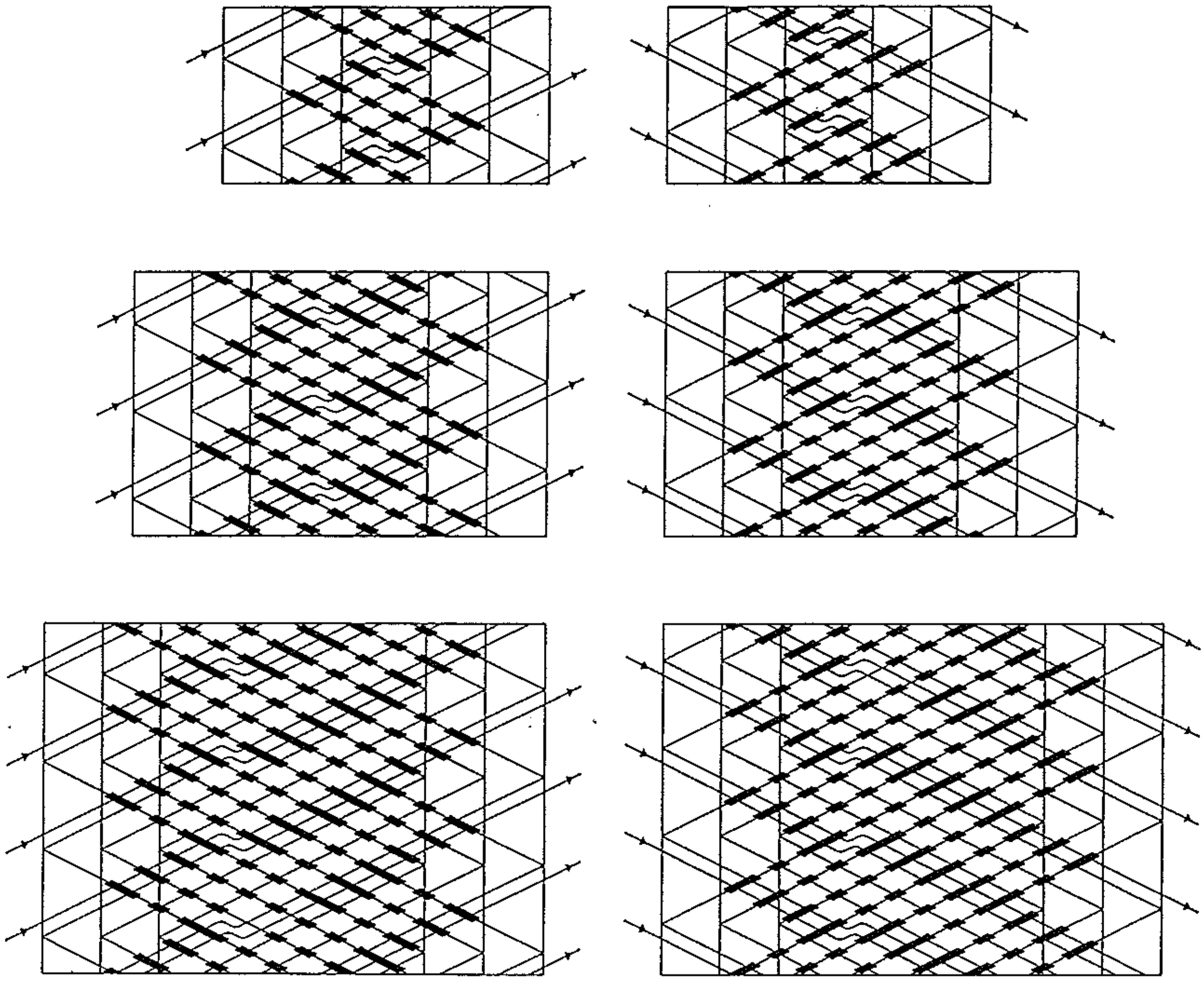


Fig. 489 — The two-, three- and four-strand versions for $A = 3$.

n \ A	2	3	4	5	6	7	8	9	10	11	12
2	3	3	3	3	3	3	3	3	3	3	3
3	5	6	7	8	9	10	11	12	13	14	15
4	7	9	11	13	15	17	19	21	23	25	27
5	9	12	15	18	21	24	27	30	33	36	39
6	11	15	19	23	27	31	35	39	43	47	51
7	13	18	23	28	33	38	43	48	53	58	63
8	15	21	27	33	39	45	51	57	63	69	75

Fig. 490 — The relationships between A, n and x.

In Fig. 490 are tabulated some (A, n, x)-values. Thus for $A = 2$ we have $x = 2n-1$; for $A = 3$ we have $x = 3n - 3$; or in general for A we have $x = A(n-2) + 3$.

Hence in general, for a nesting-number A and a number of strands equal to n, this type of knot with a right-hand helix is based on the string-run of the Regular Nested Cylindrical Braid $(222\cdots/A(n-2)+3/222\cdots)\{1A(A-1)(A-2)\cdots 2/1234\cdots A\}An$, and with a left-hand helix is based on the string-run of the Regular Nested Cylindrical Braid $(222\cdots/A(n-2)+3/222\cdots)\{1A(A-1)(A-2)\cdots 2/A123\cdots(A-1)\}An$.

We can of course design further colour-patterns. An example is shown by the grid-diagrams in Fig. 491. This type, with a number of strands equal to n, is based on the string-run of the Regular Nested Cylindrical Braid $(22/3n-2/22)\{132/231\}3n$ for a right-hand helix, and is based on the string-run of the Regular Nested Cylindrical Braid $(22/3n-2/22)\{132/312\}3n$ for a left-hand helix. The two-tone colour-pattern in the uppermost and lowermost grid-diagrams of Fig. 491 is especially suited for n = even.

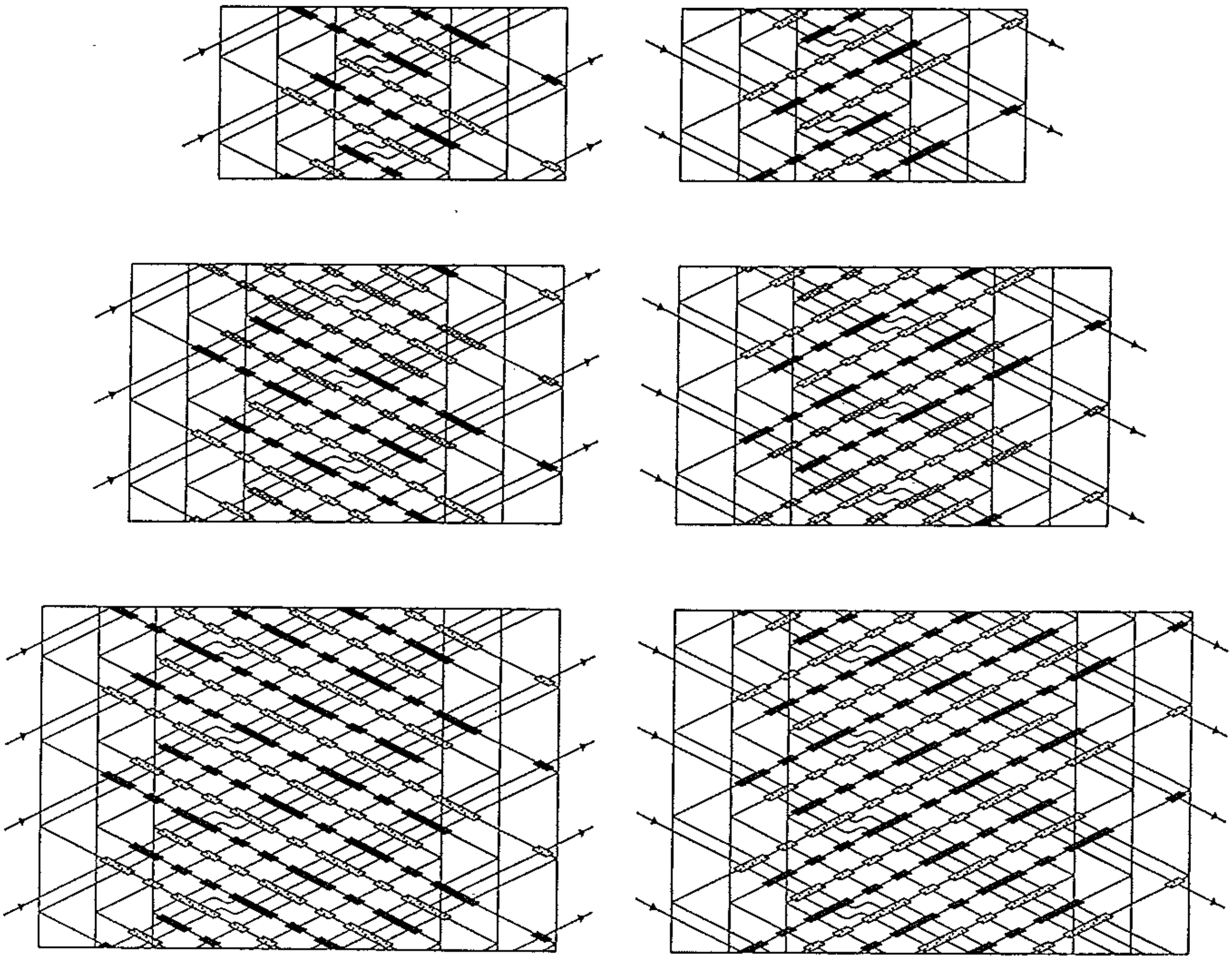


Fig. 491 — A colour-pattern example for $A = 3$.

We hope to have shown here once again the importance of grid-diagrams in facilitating the design of new knots. Such design may be an extension to some experimental discovery as in the case discussed, or it may be a direct design without an experimental bases. Another very important aspect of grid-diagrams is that they readily enable us to generalise these designed braids into families.

Transitions from two Round Braids to one Round Braid, and vice versa

Transitions between two 6-string round braids and one 12-string round braid.

BRAIDING TYPE																	
12 STRING												6 STRING					
→						←						→			←		
U	O	U	O	U	O	O	U	O	U	O	U	U	O	U	U	O	U
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Similar to the transition between two 4-string round braids and one 8-string round braid as discussed in *The Braider*, Issue No. 25, pp. 574–578, we have here four good transition combinations. We have only depicted the type which is similar to the one on pg. 575 (see Fig. 492).

Fig. 492 :

The 6-string round braid has at one end the strings A, B, C, D, E, F and at the other end the strings 1, 2, 3, 4, 5, 6.

Form the crossings between the strings A, C, E, 2, 4, 6.

Bring 1 from the right around the back to the left, then along the front from left to right under B, over D, under F, over 2, under 4, over 6.

Bring B from the left around the back to the right, then along the front from right to left under 3, over 5, under A, over C, under E, over 1.

Bring 3 from the right around the back to the left, then along the front from left to right under D, over F, under 2, over 4, under 6, over B.

Bring D from the left around the back to the right, then along the front from right to left under 5, over A, under C, over E, under 1, over 3.

Bring 5 from the right around the back to the left, then along the front from left to right under F, over 2, under 4, over 6, under B, over D.

Bring F from the left around the back to the right, then along the front from right to left under A, over C, under E, over 1, under 3, over 5.

Bring A from the right around the back to the left, then along the front from left to right under 2, over 4, under 6, over B, under D, over F.

Bring 2 from the left around the back to the right, then along the front from right to left under C, over E, under 1, over 3, under 5, over A.

And so on.

The 12-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Bring 1 from the right around the back to the left, then along the front from left to right under 2, over 4, under 6, over 8, under 10, over 12.

Bring 2 from the left around the back to the right, then along the front from right to left under 3, over 5, under 7, over 9, under 11.

Bring 3 from the right around the back to the left, then along the front from left to right under 4, over 6, under 8, over 10, under 12.

Bring 4 from the left around the back to the right, then along the front from right to left under 5, over 7, under 9, over 11.

Bring 5 from the right around the back to the left, then along the front from left to right under 6, over 8, under 10, over 12.

Bring 6 from the left around the back to the right, then along the front from right to left under 7, over 9, under 11.

Braid the left-hand round braid of 6-strings:

Bring 1 from the right around the back to the left, then along the front from left to right under 8, over 10, under 12.

Bring 8 from the left around the back to the right, then along the front from right to left under 3, over 5, under 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 10, over 12, under 8.

Bring 10 from the left around the back to the right, then along the front from right to left under 5, over 1, under 3.

Bring 5 from the right around the back to the left, then along the front from left to right under 12, over 8, under 10.

Bring 12 from the left around the back to the right, then along the front from right to left under 1, over 3, under 5.

Bring 1 from the right around the back to the left, then along the front from left to right under 8, over 10, under 12.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 7 from the right around the back to the left, then along the front from left to right under 2, over 4, under 6.

Bring 2 from the left around the back to the right, then along the front from right to left under 9, over 11, under 7.

Bring 9 from the right around the back to the left, then along the front from left to right under 4, over 6, under 2.

Bring 4 from the left around the back to the right, then along the front from right to left under 11, over 7, under 9.

Bring 11 from the right around the back to the left, then along the front from left to right under 6, over 2, under 4.

Bring 6 from the left around the back to the right, then along the front from right to left under 7, over 9, under 11.

And so on.

BRAIDING TYPE									
12 STRING						6 STRING			
→			←			→		←	
U	O	U	U	O	U	U	O	O	U
2	2	2	2	2	2	1	2	2	1

Fig. 493:

The 6-string round braid has at one end the strings A, B, C, D, E, F and at the other end the strings 1, 2, 3, 4, 5, 6.

Form the crossings between the strings B, D, F, 2, 4, 6.

Bring 1 from the right around the back to the left, then along the front from left to right under A and C, over E and 2, under 4 and 6.

Bring A from the left around the back to the right, then along the front from right to left under 3 and 5, over B and D, under F and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under C and E, over 2 and 4, under 6 and A.

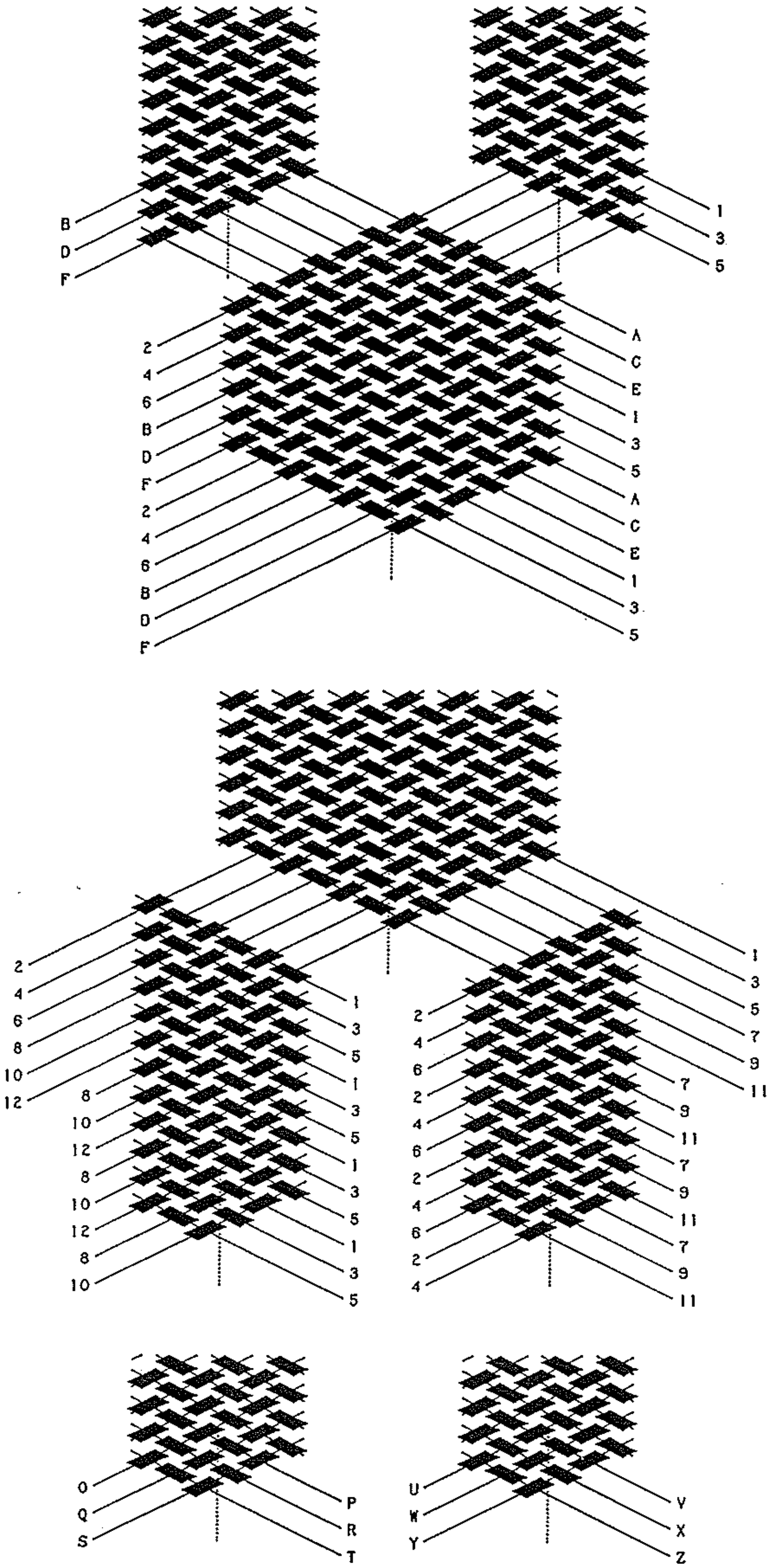


Fig. 492 — Transition between two 6-string round braids and one 12-string round braid.

Bring C from the left around the back to the right, then along the front from right to left under 5 and B, over D and F, under 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under E and 2, over 4 and 6, under A and C.

Bring E from the left around the back to the right, then along the front from right to left under B and D, over F and 1, under 3 and 5.

Bring B from the right around the back to the left, then along the front from left to right under 2 and 4, over 6 and A, under C and E.

Bring 2 from the left around the back to the right, then along the front from right to left under D and F, over 1 and 3, under 5 and B.

And so on.

The 12-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Bring 1 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6 and 8, under 10 and 12.

Bring 2 from the left around the back to the right, then along the front from right to left under 3 and 5, over 7 and 9, under 11.

Bring 3 from the right around the back to the left, then along the front from left to right under 4 and 6, over 8 and 10, under 12.

Bring 4 from the left around the back to the right, then along the front from right to left under 5 and 7, over 9 and 11.

Bring 5 from the right around the back to the left, then along the front from left to right under 6 and 8, over 10 and 12.

Bring 6 from the left around the back to the right, then along the front from right to left under 7 and 9, over 11.

Braid the left-hand round braid of 6-strings:

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3 and 5.

Bring 1 from the right around the back to the left, then along the front from left to right under 10, over 12 and 8.

Bring 10 from the left around the back to the right, then along the front from right to left under 3, over 5 and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 12, over 8 and 10.

Bring 12 from the left around the back to the right, then along the front from right to left under 5, over 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under 8, over 10 and 12.

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3 and 5.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 7 from the right around the back to the left, then along the front from left to right under 2, over 4, under 6.

Bring 2 from the left around the back to the right, then along the front from right to left under 9 and 11, over 7.

Bring 9 from the right around the back to the left, then along the front from left to right under 4, over 6, under 2.

Bring 4 from the left around the back to the right, then along the front from right

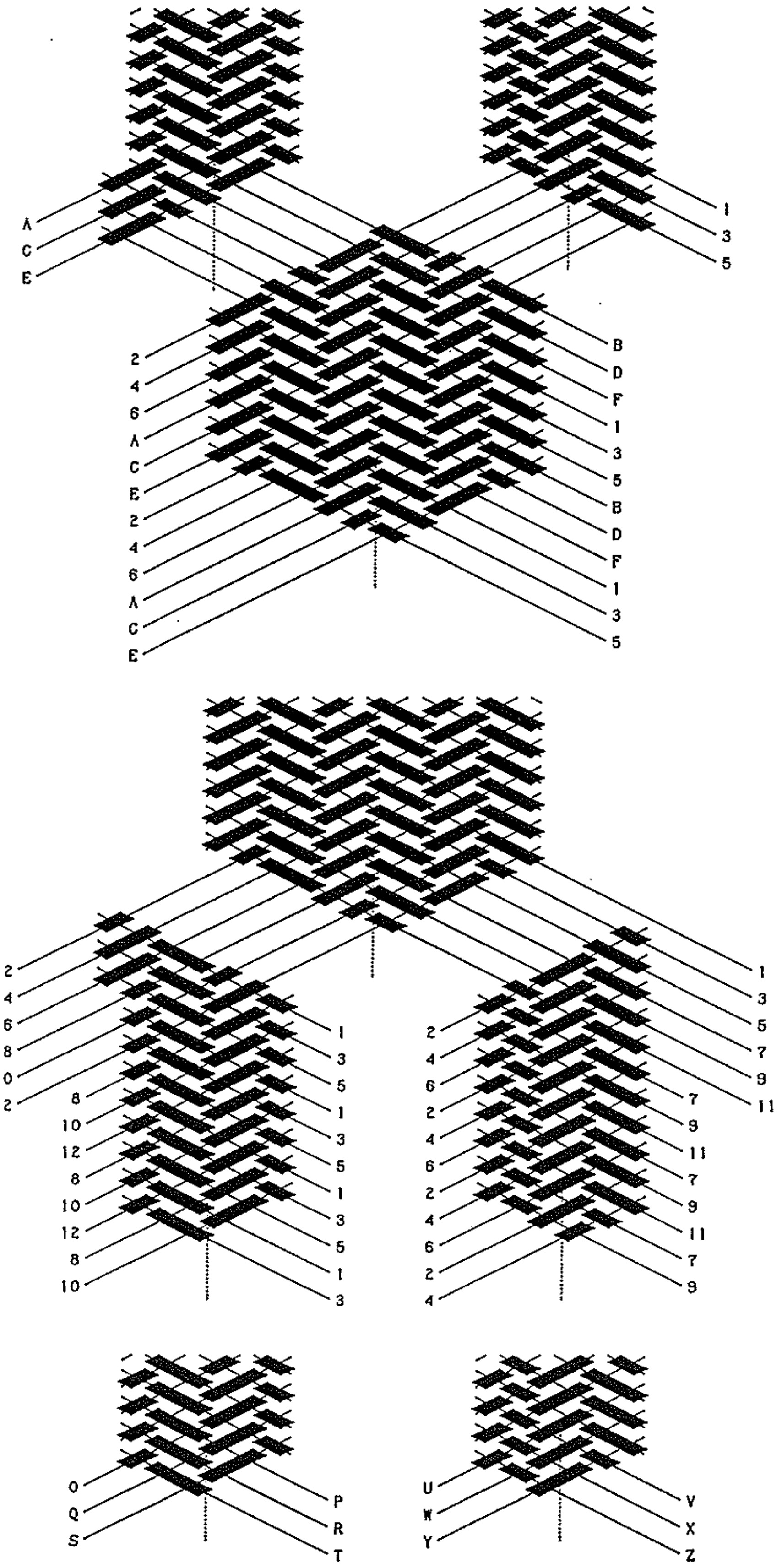


Fig. 493 — Transition between two 6-string round braids and one 12-string round braid.

to left under 11 and 7, over 9.

Bring 11 from the right around the back to the left, then along the front from left to right under 6, over 2, under 4.

Bring 6 from the left around the back to the right, then along the front from right to left under 7 and 9, over 11.

And so on.

BRAIDING TYPE									
12 STRING						6 STRING			
→			←			→		←	
U	O	U	U	O	U	U	O	O	U
2	2	2	2	2	2	2	1	2	1

Fig. 494:

The 6-string round braid has at one end the strings A, B, C, D, E, F and at the other end the strings 1, 2, 3, 4, 5, 6. Bring 2 from the left around the back to the right, then along the front from right to left under 1 and 3, over 5 as indicated by arrow.

Form the crossings between the strings B, D, F, 4, 6, 2.

Bring 1 from the right around the back to the left, then along the front from left to right under A and C, over E and 4, under 6 and 2.

Bring A from the left around the back to the right, then along the front from right to left under 3 and 5, over B and D, under F and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under C and E, over 4 and 6, under 2 and A.

Bring C from the left around the back to the right, then along the front from right to left under 5 and B, over D and F, under 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under E and 4, over 6 and 2, under A and C.

Bring E from the left around the back to the right, then along the front from right to left under B and D, over F and 1, under 3 and 5.

Bring B from the right around the back to the left, then along the front from left to right under 4 and 6, over 2 and A, under C and E.

Bring 4 from the left around the back to the right, then along the front from right to left under D and F, over 1 and 3, under 5 and B.

And so on.

The 12-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Bring 1 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6 and 8, under 10 and 12.

Bring 2 from the left around the back to the right, then along the front from right to left under 3 and 5, over 7 and 9, under 11.

Bring 3 from the right around the back to the left, then along the front from left to right under 4 and 6, over 8 and 10, under 12.

Bring 4 from the left around the back to the right, then along the front from right to left under 5 and 7, over 9 and 11.

Bring 5 from the right around the back to the left, then along the front from left to right under 6 and 8, over 10 and 12.

Bring 6 from the left around the back to the right, then along the front from right to left under 7 and 9, over 11.

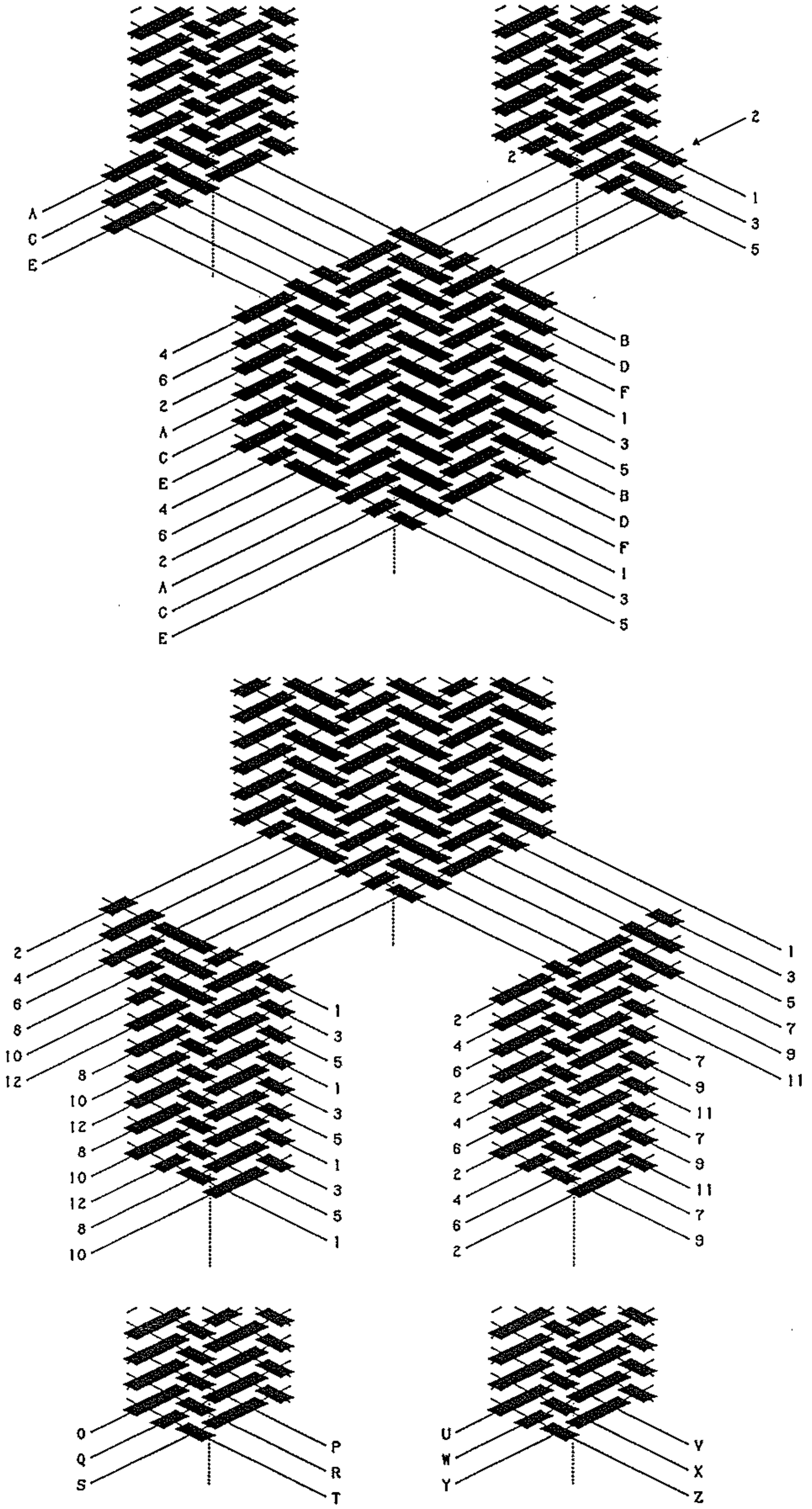


Fig. 494 — Transition between two 6-string round braids and one 12-string round braid.

Braid the left-hand round braid of 6-strings:

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3 and 5.

Bring 1 from the right around the back to the left, then along the front from left to right under 10 and 12, over 8.

Bring 10 from the left around the back to the right, then along the front from right to left under 3, over 5 and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under 12 and 8, over 10.

Bring 12 from the left around the back to the right, then along the front from right to left under 5, over 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under 8 and 10, over 12.

Bring 8 from the left around the back to the right, then along the front from right to left under 1, over 3 and 5.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 7 from the right around the back to the left, then along the front from left to right under 2, over 4, under 6.

Bring 9 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6.

Bring 2 from the left around the back to the right, then along the front from right to left under 11, over 7 and 9.

Bring 11 from the right around the back to the left, then along the front from left to right under 4 and 6, over 2.

Bring 4 from the left around the back to the right, then along the front from right to left under 7, over 9 and 11.

Bring 7 from the right around the back to the left, then along the front from left to right under 6 and 2, over 4.

And so on.

Fig. 495:

The 6-string round braid has at one end the strings A, B, C, D, E, F and at the other end the strings 1, 2, 3, 4, 5, 6.

Form the crossings between the strings B, D, F, 2, 4, 6.

Bring A from the left around the back to the right, then along the front from right to left under 1 and 3, over 5 and B, under D and F.

Bring 1 from the right around the back to the left, then along the front from left to right under C and E, over 2 and 4, under 6 and A.

Bring C from the left around the back to the right, then along the front from right to left under 3 and 5, over B and D, under F and 1.

Bring 3 from the right around the back to the left, then along the front from left to right under E and 2, over 4 and 6, under A and C.

Bring E from the left around the back to the right, then along the front from right to left under 5 and B, over D and F, under 1 and 3.

Bring 5 from the right around the back to the left, then along the front from left to right under 2 and 4, over 6 and A, under C and E.

Bring 2 from the left around the back to the right, then along the front from right

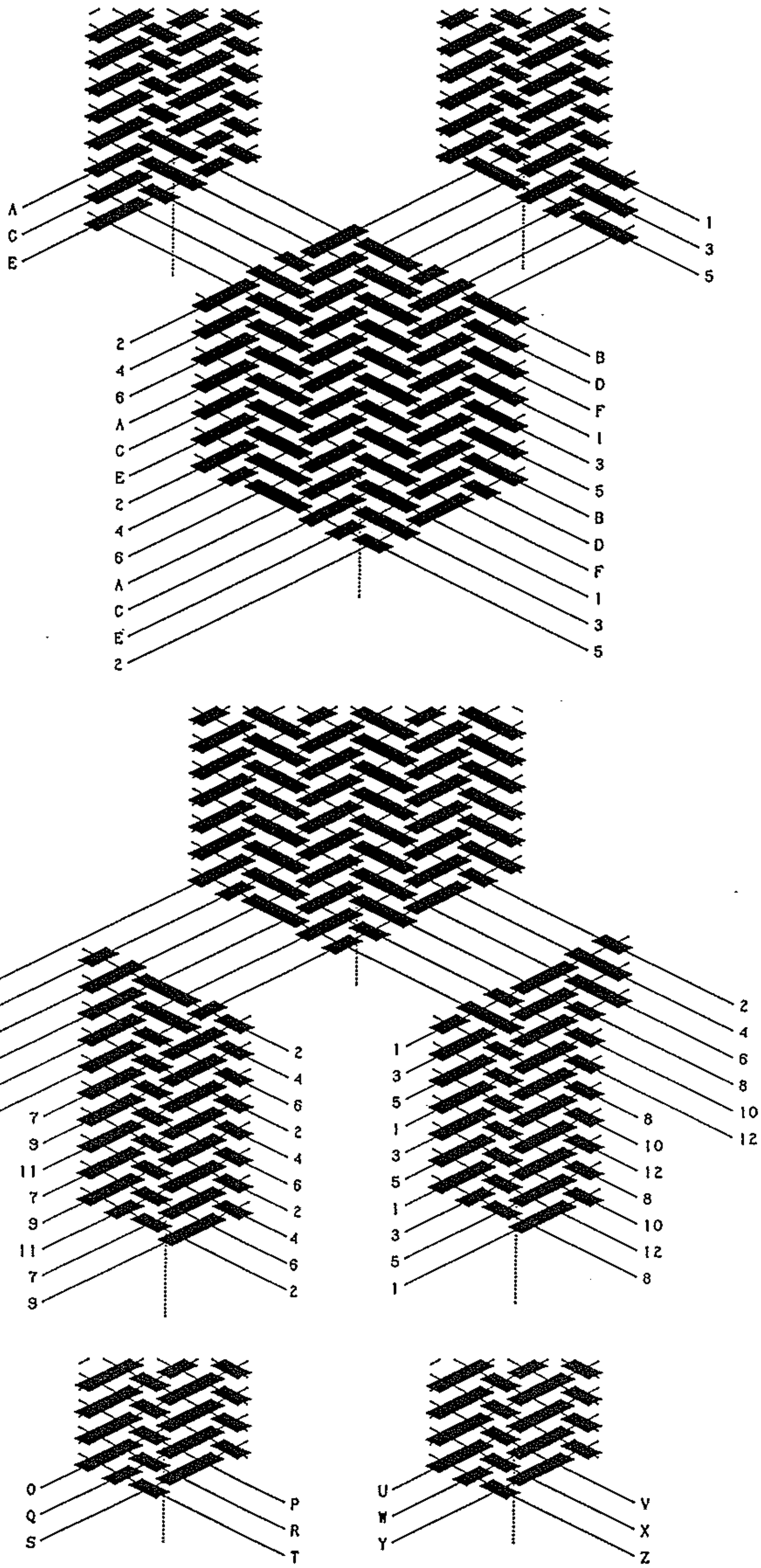


Fig. 495 — Transition between two 6-string round braids and one 12-string round braid.

to left under B and D, over F and 1, under 3 and 5.

Bring B from the right around the back to the left, then along the front from left to right under 4 and 6, over A and C, under E and 2.

And so on.

The 12-string round braid has the strings 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Bring 1 from the left around the back to the right, then along the front from right to left under 2 and 4, over 6 and 8, under 10 and 12.

Bring 2 from the right around the back to the left, then along the front from left to right under 3 and 5, over 7 and 9, under 11.

Bring 3 from the left around the back to the right; then along the front from right to left under 4 and 6, over 8 and 10, under 12.

Bring 4 from the right around the back to the left, then along the front from left to right under 5 and 7, over 9 and 11.

Bring 5 from the left around the back to the right, then along the front from right to left under 6 and 8, over 10 and 12.

Bring 6 from the right around the back to the left, then along the front from left to right under 7 and 9, over 11.

Braid the left-hand round braid of 6-strings:

Bring 7 from the left around the back to the right, then along the front from right to left under 2, over 4 and 6.

Bring 2 from the right around the back to the left, then along the front from left to right under 9 and 11, over 7.

Bring 9 from the left around the back to the right, then along the front from right to left under 4, over 6 and 2.

Bring 4 from the right around the back to the left, then along the front from left to right under 11 and 7, over 9.

Bring 11 from the left around the back to the right, then along the front from right to left under 6, over 2 and 4.

Bring 6 from the right around the back to the left, then along the front from left to right under 7 and 9, over 11.

Bring 7 from the left around the back to the right, then along the front from right to left under 2, over 4 and 6.

And so on.

Braid the right-hand round braid of 6-strings:

Bring 8 from the right around the back to the left, then along the front from left to right under 1 and 3, over 5.

Bring 1 from the left around the back to the right, then along the front from right to left under 10, over 12 and 8.

Bring 10 from the right around the back to the left, then along the front from left to right under 3 and 5, over 1.

Bring 3 from the left around the back to the right, then along the front from right to left under 12, over 8 and 10.

Bring 12 from the right around the back to the left, then along the front from left to right under 5 and 1, over 3.

Bring 5 from the left around the back to the right, then along the front from right to left under 8, over 10 and 12.

And so on.