THE BRAIDER

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CONTENTS

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Solving Knotty problems

Most braiders will no doubt have encountered braiding instructions which were, judging from the obtained result, somehow incorrect. The question then arrises to what was incorrect in the given braiding recipe and what it should have been instead. Sometimes, due to various possible reasons, a part of the construction steps might be missing. Nevertheless, since it is impossibe to show in a sketch every part of the braid, and although a sketch of the final braid and sketches of various intermediate construction stages might be given, one often wonders what kind of braid really is associated with the given braiding recipe. Even the final braid in hand might not clearly show what it really is, and hence it will not necessarily be able to suggest better alternatives. Only a grid-diagram enables us to depict every part of a braid, hence a grid-diagram of a finished braid will clearly tell us what kind of braid is involved, and it will also clearly suggest better alternatives if such are possible. In order to avoid a possible waste of materials and time, it is important to translate a braiding recipe into its associated grid-diagram before actually braiding commences.

In our discussion here, we shall present an example in which, due to us not having a complete photocopy of the braiding recipe, the initial construction stages were missing. Furthermore, the instructions are in Spanish, a language we are not familiar with. The photocopied instructions we received are shown on pp. 538 – 541; they come from the book: *Trenzas Gauchas*, by Mario A. Lopez Osornio.

As we see from these pages, the instructions are provided with a great number of sketches, although at least some of them lack in detail. As so often is the case, Lopez Osornio does not work in exact half-cycles, which makes the instructions without the sketches virtually impossible to follow. Our available braiding recipe does not indicate the number of parts and the number of bights of the finished knot. Although the final knot appears, from a casual observation of the final sketches, to be a Column-coded Regular Knot (1 string Column-coded Regular Cylindrical Braid), we cannot be sure about that, and as we will see in our discussion, it is in fact not such a braid.

In Fig. N° 242 the *armadura* (= foundation knot) is completed, and from it we can see that it is an over–under coded Regular Knot having 5 parts, but since we are missing the sketches and instructions before Fig. N° 239, we cannot tell how many bights this knot has. It is, however, essential to know the number of bights before we can go on, and hence we have somehow to find this out.

The first step is to employ the general layout of the algorithm diagram for a Regular Knot which has 5-parts and b-bights (see Fig. 453 on page 542). From the sequence of sketches in Fig. N° 242, Fig. N° 241, Fig. N° 240 and Fig. N° 239, we can deduce the half-cycle algorithms of the last seven half-cycles:

half-cycle 1		Free run.
⋮		⋮
half-cycle $2b-6$	(even numbered half-cycle)	2 crossings, hence 2 i-values involved.
half-cycle $2b-5$	(odd numbered half-cycle)	2 crossings, hence 2 i-values involved.
half-cycle $2b-4$	(even numbered half-cycle)	3 crossings, hence 3 i-values involved.
half-cycle $2b-3$	(odd numbered half-cycle)	3 crossings, hence 3 i-values involved.
half-cycle $2b-2$	(even numbered half-cycle)	4 crossings, hence 4 i-values involved.
half-cycle $2b-1$	(odd numbered half-cycle)	4 crossings, hence 4 i-values involved.
half-cycle $2b$	(even numbered half-cycle)	4 crossings, hence 4 i-values involved.

Fig. Nº 239. — Sobre 4. y bajo 1. la guía marcada con una (M).

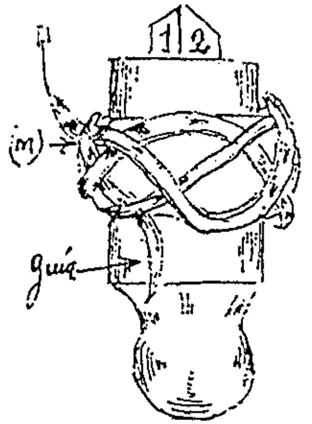


Fig. Nº 239

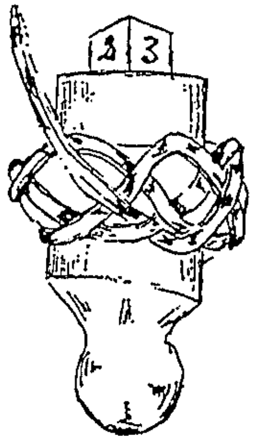


Fig. Nº 240

Fig. 240. — De arriba a abajo: Sobre dos. Esta última pasada forma el borde inferior: bajo 1. Subiendo sobre 2.

Fig. 241. — Al bajar bajo 1 y sobre 1, bajo 1 (la guía), sobre 1 y bajo 1 (borde inferior).

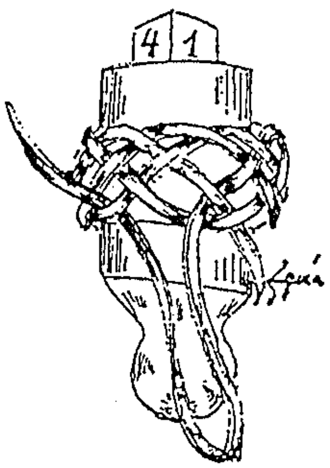


Fig. Nº 241

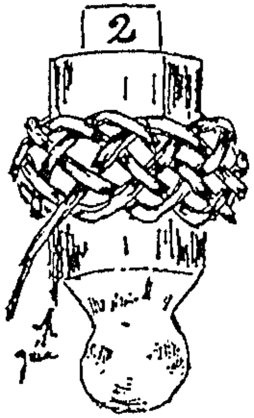


Fig. Nº 242

Fig. 242. — Subiendo: sobre 1, bajo 1, sobre 1 y bajo 1 (borde superior), bajando: sobre 1, bajo 1, sobre 1, llegando con esta pasada al fin de la *armadura*.

Primera maniobra. — Formación de pares.

Fig. Nº 243. — Bajo 1. Al dar esta puntada, la brida comienza a correr a la par de la guía y a la derecha, como podrá observarse: Sobre 1, bajo 1 y sobre 1 y bajo 2. (Al hacer estas dos últimas pasadas, el tiento que teje formó una arista del borde superior).

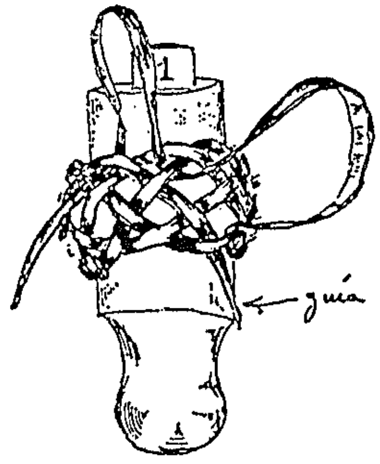


Fig. Nº 243

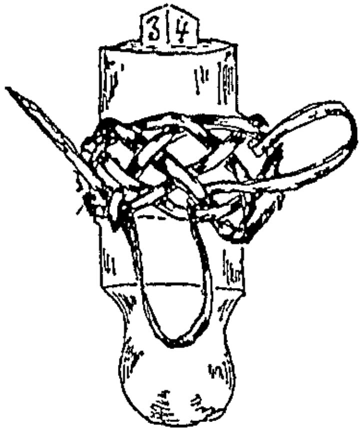


Fig. Nº 244

Fig. Nº 244. — Sobre 1, bajo 1, sobre 1 y bajo 2, formando el borde inferior.

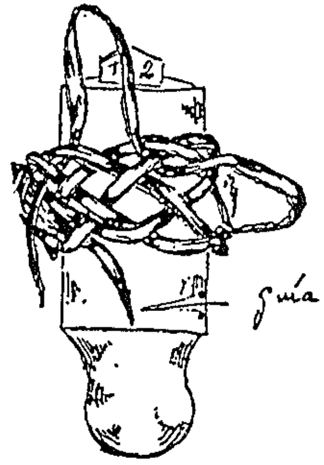


Fig. Nº 245

Fig. Nº 245. — Sobre 1, bajo 1, sobre 1, bajo 2, formando el borde superior.

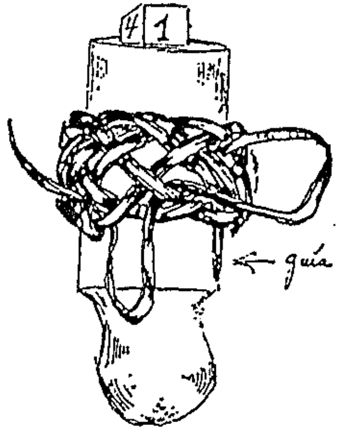


Fig. Nº 246

Fig. Nº 246. — Sobre 2, bajo 1, sobre 1 y bajo 2, formando el borde inferior.

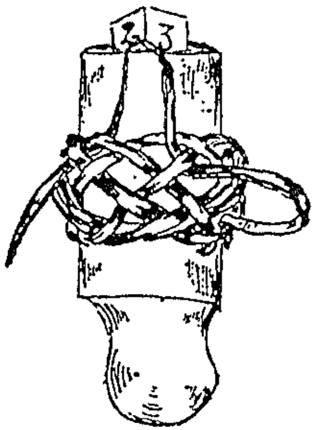


Fig. Nº 247

Fig. Nº 247. — Sobre 2, bajo 1, sobre 1 y bajo 2 (borde superior).

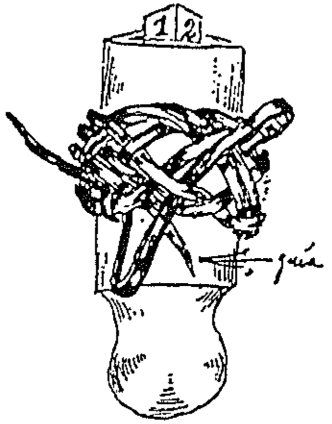


Fig. Nº 248

Fig. Nº 248. — Sobre 2, bajo 1, sobre 2, bajo 2 (borde inferior).

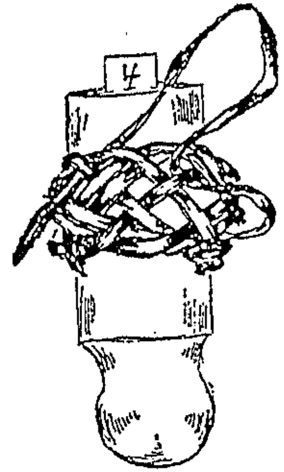


Fig. Nº 249

Fig. Nº 249. — Sobre 2, bajo 1, sobre 2, bajo 2 (borde superior).

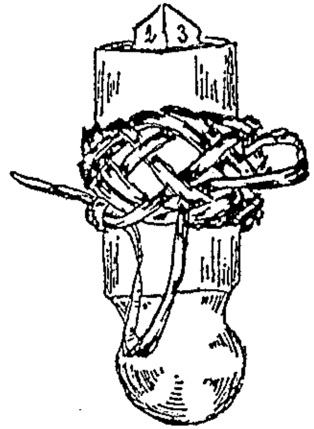


Fig. Nº 250

Fig. Nº 250. — Sobre 2, bajo 1, sobre 2, bajo 2 (borde inferior).

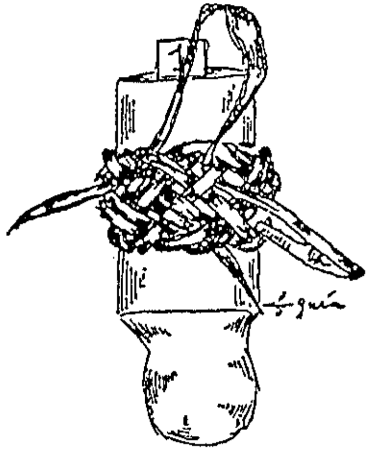


Fig. Nº 251

Fig. Nº 251. — Sobre 2, bajo 1, sobre 2, bajo 3 (borde superior).

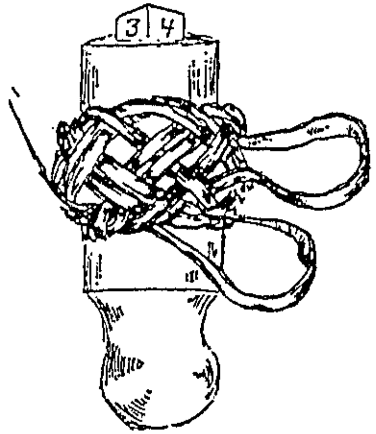


Fig. Nº 252

Fig. Nº 252. — Sobre 2, bajo 1, sobre 2, bajo 3 (borde inferior).

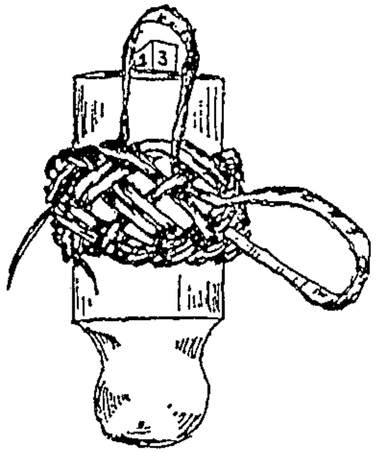


Fig. Nº 253

Fig. Nº 253. — Sobre 2, bajo 1, sobre 2, bajo 3 (borde superior).

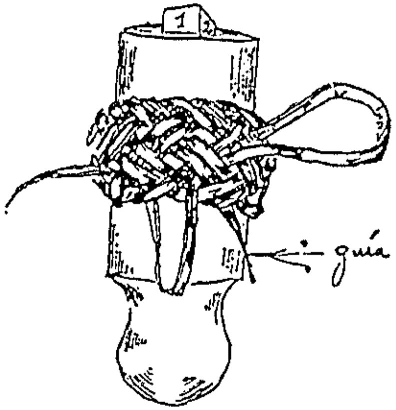


Fig. Nº 254

Fig. Nº 254.—Sobre 2, bajo 2, sobre 2, bajo 3 (borde inferior).

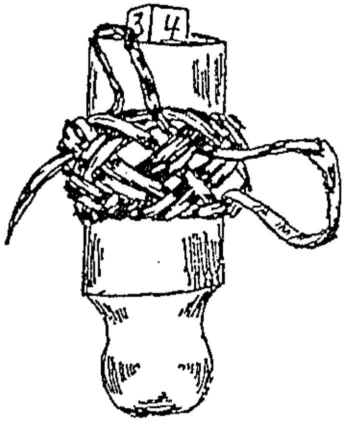


Fig. Nº 255

Fig. Nº 255.—Sobre 2, bajo 2, sobre 2, bajo 3 (borde superior).

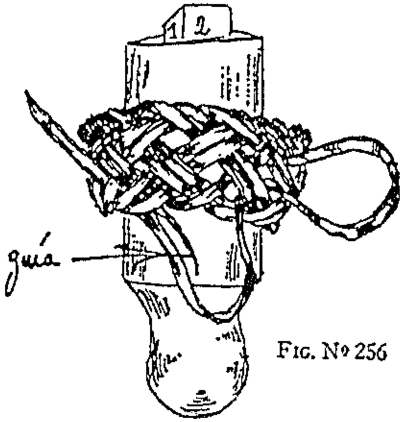


Fig. Nº 256

Fig. Nº 256. — Sobre 2, bajo 2, sobre 2 (borde inferior). Quedando con esto terminada la primer maniobra, es decir la formación de pares. De ahora en adelante la brida comienza a correr a la izquierda de la guía, bajo 3 (borde inferior).

RETEJIDO: 2ª maniobra. — Separación de pares.

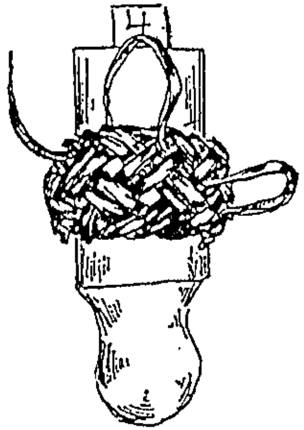


Fig. Nº 257

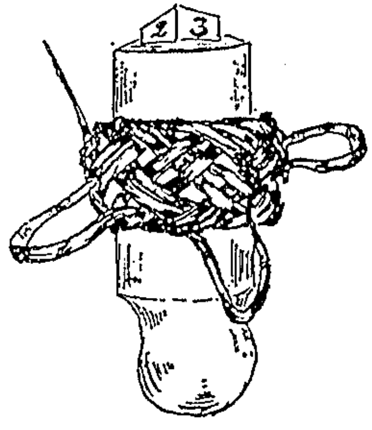


Fig. Nº 258

Fig. Nº 257. — Ya en la anterior figura hemos visto que la última pasada comenzaba separando los pares, las siguientes manipulaciones harán lo mismo, es decir, cada puntada, va separando siempre los tientos que corren juntos ya sea por un extremo o por otro o por los dos a la vez. Sobre 2, bajo 2, sobre 3, bajo 3 (borde superior).

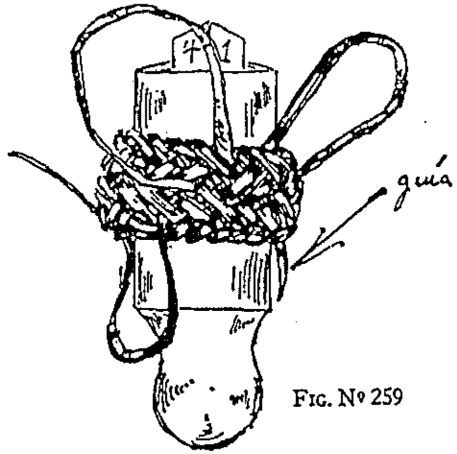


Fig. Nº 259

Fig. Nº 258. — Sobre 2, bajo 2, sobre 3, bajo 3 (borde inferior), sobre 2, bajo 2.

Fig. Nº 259. — Sobre 3, bajo 3 (borde superior), sobre 3, bajo 2, sobre 3 y bajo 3 (borde inferior).

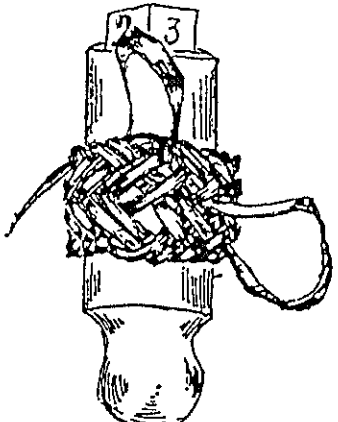


FIG. Nº 260

Fig. Nº 260. — Sobre 3, bajo 2, sobre 3, bajo 3 (borde superior).

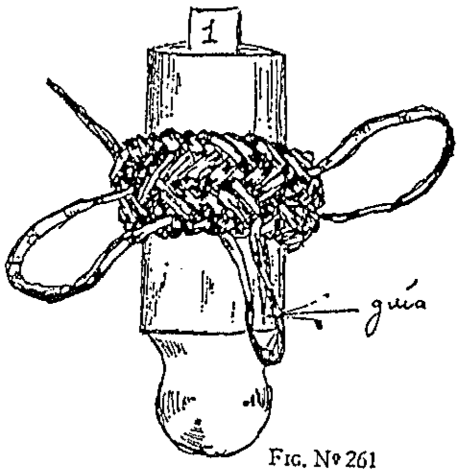


FIG. Nº 261

Fig. 261. — Sobre 3, bajo 2, sobre 4, bajo 3 (borde inferior), sobre 3, bajo 2.

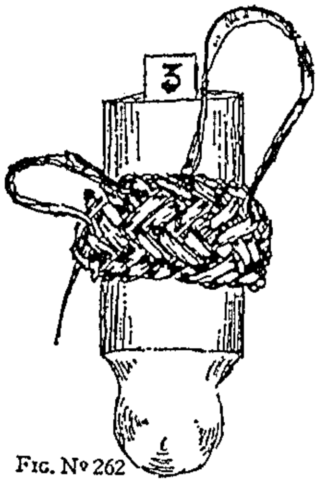


FIG. Nº 262

Fig. Nº 262. — Sobre 4, bajo 3 (borde superior), sobre 3, bajo 2.

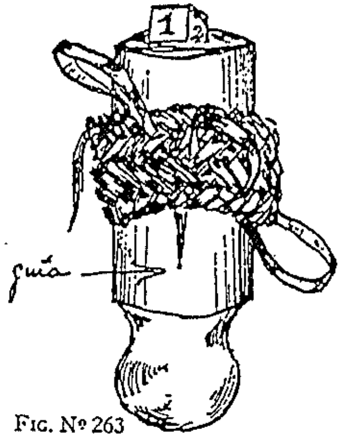


FIG. Nº 263

Fig. 263. — Sobre 4 y bajo 3 (borde inferior), sobre 3, bajo 2, sobre 4, bajo 4 (borde superior).

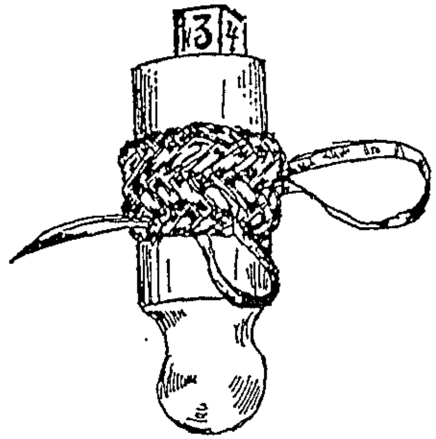


FIG. Nº 264

Fig. Nº 264. — Sobre 3, bajo 2, sobre 4, bajo 4 (borde inferior).

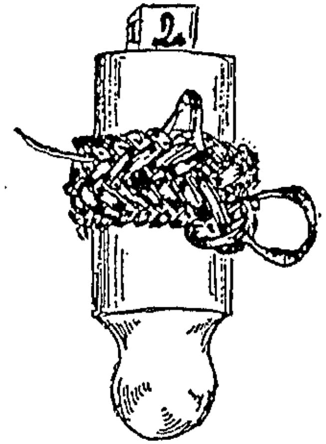


FIG. Nº 265

Fig. 265. — Sobre 3, bajo 2, sobre 4, bajo 4 (borde superior).

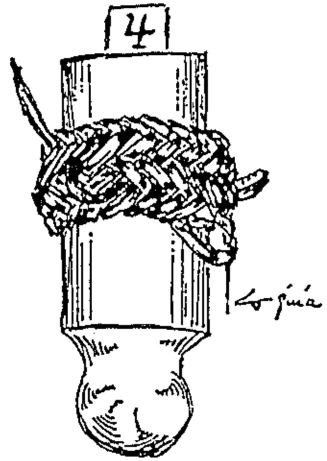


FIG. Nº 266

Fig. 266. — Sobre 3, bajo 3, sobre 4, bajo 4 (borde inferior), sobre 3, bajo 3, sobre 4 (borde superior).

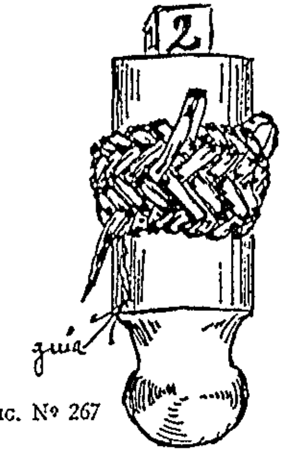


FIG. Nº 267

Fig. 267. — Bajo 4 (borde superior), sobre 3, bajo 3, sobre 4, y al esconder la hebra que teje junto a la guía, quedará terminada la bombita.

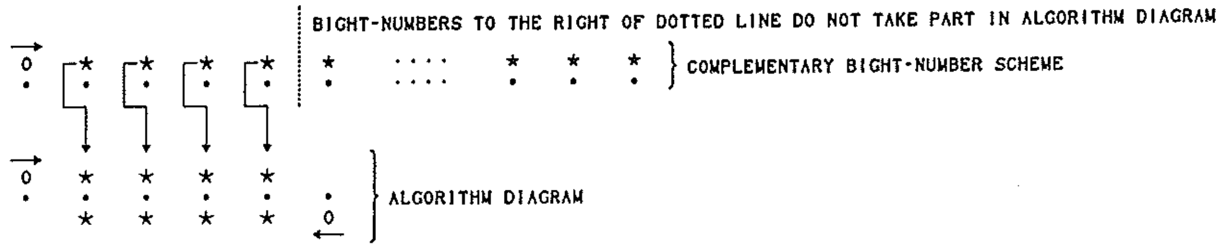


Fig. 453 — Complementary cyclic bight-number scheme and algorithm diagram for $p = 5$ and $b > p$; the bight-numbers (i-values) are indicated by a star.

From the final three half-cycles we know that the number of bights must be greater than p, hence greater than 5, because when the number of bights is smaller than p each bight-number (i-value) is associated with at least one intersection-column and hence we cannot have more than two consecutive half-cycles (an even and its consecutive odd numbered half-cycle) with the same number of crossings.

When there are 6 bights ($b = 6$), then $i = 5$ lies to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453 (see also the upper diagram in Fig. 454). Bight-number $i = 5$ is associated with the final half-cycle $2b = 12$, and $i = 4$ is associated with the half-cycles 10 (is half-cycle $2b - 2$) and 11 (is half-cycle $2b - 1$), hence each of the last three half-cycles makes the same number (4) of crossings. Thus $b = 6$ is a possibility.

When there are 7 bights ($b = 7$), then $i = 3$ and $i = 6$ lie to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453 (see also the second diagram in Fig. 454). Bight-number $i = 3$ is associated with the half-cycles 8 (is half-cycle $2b - 6$) and 9 (is half-cycle $2b - 5$), while $i = 6$ is associated with the final half-cycle $2b = 14$. Hence each of the last three half-cycles makes the same number (4) of crossings. From our tabulation at the bottom of pg. 537 we cannot tell whether or not the half-cycles $2b - 8$ and $2b - 7$ each have the same number of crossings as the half-cycles $2b - 6$ and $2b - 5$. Thus $b = 7$ is a possibility.

When there are 8 bights ($b = 8$), then $i = 2$, $i = 5$ and $i = 7$ lie to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453 (see also the third diagram in Fig. 454). Bight-number $i = 5$ is associated with the half-cycles 12 (is half-cycle $2b - 4$) and 13 (is half-cycle $2b - 3$), hence each of the half-cycles $2b - 6$, $2b - 5$, $2b - 4$, and $2b - 3$ have the same number of crossings. Since this does not agree with our tabulation, it follows that $b = 8$ is not a possibility.

When there are 9 bights ($b = 9$), then $i = 2$, $i = 4$, $i = 6$ and $i = 8$ lie to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453 (see also the fourth diagram in Fig. 454). Here bight-number $i = 6$ acts the same as bight-number $i = 5$ in the case $b = 8$, hence $b = 9$ is not a possibility.

When there are 11 bights ($b = 11$), then $i = 1$, $i = 3$, $i = 5$, $i = 7$, $i = 9$ and $i = 10$ lie to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453 (see also the fifth diagram in Fig. 454). The last 5 half-cycles have each the same number of crossings (due to the i-values 9 and 10 which lie to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453). Since this does not agree with our tabulation, it follows that $b = 11$ is not a possibility.

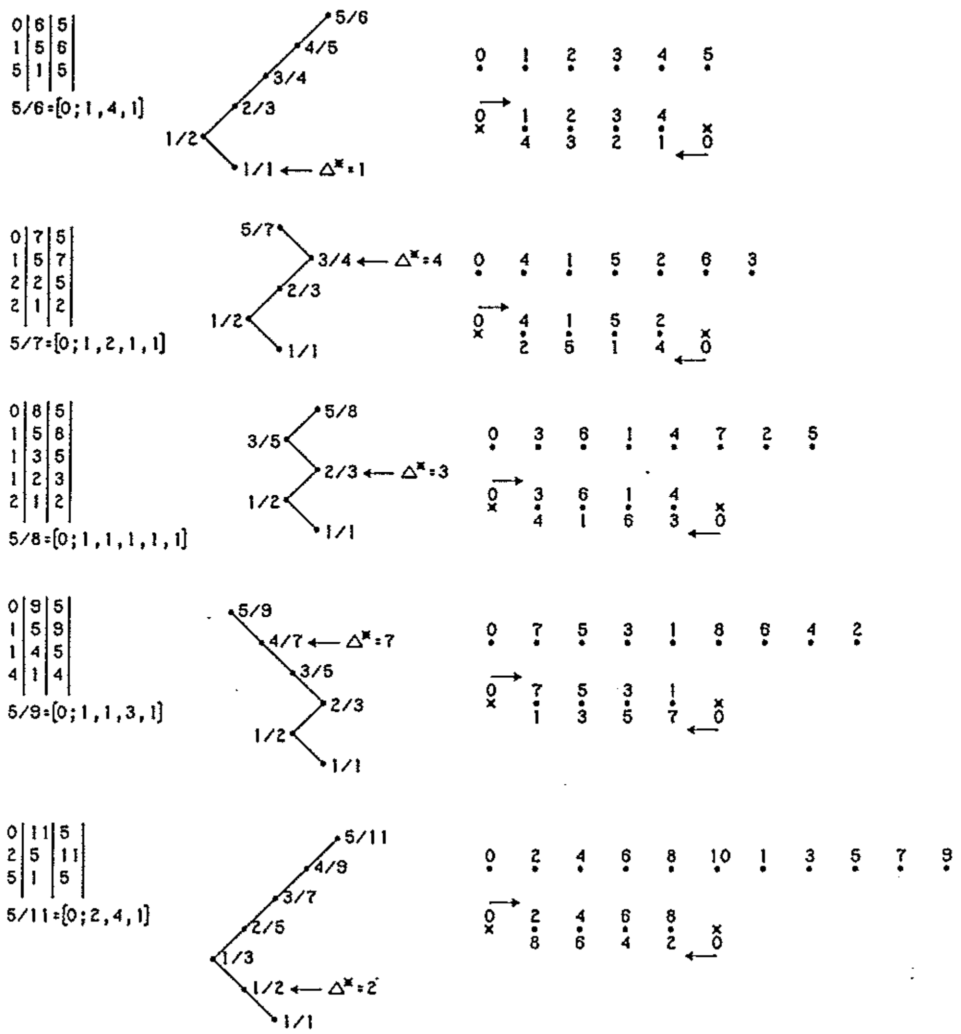


Fig. 454 — Complementary cyclic bight-number schemes and algorithm diagrams for $p/b = 5/6$ to $p/b = 5/11$.

The investigation so far not only doesn't tell us what happens when $b > 11$, but it can considerably be simplified by knowing a little more about the general outlay of a complementary cyclic bight-number scheme. Hence lets first have a look into that.

We know that p and b have to be coprime (g.c.d. $(p, b) = 1$) for a single string Regular Cylindrical Braid (a Regular Knot). Let b be greater than p, and lets find out when $|i|-p|_b|_b = p$.

$$b > p\,,$$
$$|i|-p|_b|_b = p\,,$$
$$|i(b-p)|_b = p\,,$$
$$|-ip|_b = p\,,$$
$$nb - ip = p\,; \qquad \text{where } n = \text{a natural number}\,,$$
$$(i+1)p = nb\,,$$
$$i + 1 = \frac{nb}{p}\,,$$

$$\begin{aligned} \text{since g.c.d.}\,(p,b) = 1 \quad &\rightarrow \quad n = p \text{ since } 0 \le i \le b-1\,, \\ i+1 &= b\,, \\ i &= b-1\,, \\ i &= b-1 \text{ is associated with half-cycle } 2i+2 = 2b\,. \end{aligned}$$

Thus the position p in a complementary cyclic bight-number scheme for $b > p$ carries the bight-number $i = b-1$.

When the last three half-cycles each have the same number of crossings, then $i = b-1$ is associated with a position to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453, hence with the position $= p$, and $i = b-2$ is associated with a position to the left of the dotted line in that complementary cyclic bight-number scheme, hence with a position $< p$.

$$\begin{aligned} |(b-1)|-p|_b|_b = p \quad &\rightarrow \quad |p|_b = p \text{ hence } b > p\,, \\ |(b-2)|-p|_b|_b < p \quad &\rightarrow \quad |2p|_b < p\,, \\ \text{Hence}: \quad & p < b < 2p\,. \end{aligned}$$

When the last five half-cycles each have the same number of crossings, then $(b-1) \ge i \ge (b-2)$ is associated with a position to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453, hence with a position $\ge p$, and $i = b-3$ is associated with a position to the left of the dotted line in that complementary cyclic bight-number scheme, hence with a position $< p$.

$$\begin{aligned} |(b-2)|-p|_b|_b > p \quad &\rightarrow \quad |2p|_b > p \text{ hence } b > 2p\,, \\ |(b-3)|-p|_b|_b < p \quad &\rightarrow \quad |3p|_b < p\,, \\ \text{Hence}: \quad & 2p < b < 3p\,. \end{aligned}$$

When the last $(2n+1)$ half-cycles, where n is a natural number, each have the same number of crossings, then $(b-1) \ge i \ge (b-n)$ is associated with a position to the right of the dotted line in the complementary cyclic bight-number scheme of Fig. 453, hence with a position $\ge p$, and $i = b-n-1$ is associated with a position to the left of the dotted line in that complementary cyclic bight-number scheme, hence with a position $< p$.

$$\begin{aligned} |(b-n)|-p|_b|_b > p \quad &\rightarrow \quad |np|_b > p \text{ hence } b > np\,, \\ |(b-n-1)|-p|_b|_b < p \quad &\rightarrow \quad |(n+1)p|_b < p\,, \\ \text{Hence}: \quad & np < b < (n+1)p\,. \end{aligned}$$

Thus from our tabulation at the bottom of pg. 537 we know now that since $n = 1$, the number of bights must be greater than p, hence greater than 5, but less than $2p$, hence less than 10.

When $b > p$ the overall number of half-cycles of the string-run consist of p sets of half-cycles, where all the half-cycles of a set have each the same number of crossings. The number of half-cycles in the first set (the free-run half-cycles, the half-cycles each of which intersects zero other half-cycles) is equal to:

$$2\left\lfloor \frac{b}{p} \right\rfloor + 1\,.$$

The number of half-cycles in the subsequent consecutive sets greater equal 2 and less than p, hence excepting the last set (the p^{th} set), is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor - \left\lfloor\frac{nb}{p}\right\rfloor\right), \qquad \text{where} \quad n = 1\,,\, 2\,,\, 3\,,\, \cdots\,,\, (p-2).$$

The number of half-cycles in the last set (the p^{th} set) is equal to:

$$2\left(b - \left\lfloor\frac{(p-1)b}{p}\right\rfloor\right) - 1 = 2\left\lfloor\frac{b}{p}\right\rfloor + 1\,.$$

With these formulae it is easy to prove that the sequence of the number of half-cycles of the sequential sets is palindromic†.

⋆⋆ *Prove the formulae given above for the half-cycle sets, and their palindromic nature.*

These formulae give us much essential information about the algorithm diagram associated with the Regular Knot p/b with $b > p$.

Applying these formulae to our case, we know from the formula for the p^{th} set of half-cycles (the last set of half-cycles) that:

$$2\left\lfloor\frac{b}{p}\right\rfloor + 1 = 3\,, \text{ hence } p < b < 2p\,, \text{ consequently } 5 < b < 10\,.$$

Hence also the first set of half-cycles, the free-run half-cycles, consists of 3 half-cycles, due to the palindromic nature of the sets.

For $b = 9$ the number of half-cycles in the penultimate set, for which $n = p - 2 = 5 - 2 = 3$, is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor - \left\lfloor\frac{nb}{p}\right\rfloor\right) = 2\left(\left\lfloor\frac{4\times 9}{5}\right\rfloor - \left\lfloor\frac{3\times 9}{5}\right\rfloor\right) = 2(7-5) = 4\,.$$

This does not agree with our findings on pg. 537, hence $b = 9$ is not correct.

For $b = 8$ the number of half-cycles in the penultimate set, for which $n = p - 2 = 5 - 2 = 3$, is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor - \left\lfloor\frac{nb}{p}\right\rfloor\right) = 2\left(\left\lfloor\frac{4\times 8}{5}\right\rfloor - \left\lfloor\frac{3\times 8}{5}\right\rfloor\right) = 2(6-4) = 4\,.$$

This does not agree with our findings on pg. 537, hence $b = 8$ is not correct.

For $b = 7$ the number of half-cycles in the penultimate set, for which $n = p - 2 = 5 - 2 = 3$, is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor - \left\lfloor\frac{nb}{p}\right\rfloor\right) = 2\left(\left\lfloor\frac{4\times 7}{5}\right\rfloor - \left\lfloor\frac{3\times 7}{5}\right\rfloor\right) = 2(5-4) = 2\,.$$

This does agree with our findings on pg. 537, hence $b = 7$ is a possibility.

For $b = 6$ the number of half-cycles in the penultimate set, for which $n = p - 2 = 5 - 2 = 3$, is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor - \left\lfloor\frac{nb}{p}\right\rfloor\right) = 2\left(\left\lfloor\frac{4\times 6}{5}\right\rfloor - \left\lfloor\frac{3\times 6}{5}\right\rfloor\right) = 2(4-3) = 2\,.$$

This does agree with our findings on pg. 537, hence $b = 6$ is a possibility.

For $b = 7$ the number of half-cycles in the second to last set, for which $n = p - 3 = 5 - 3 = 2$, is equal to:

---

† A palindromic sequence of numbers is a sequence of numbers which, when taken in reverse order, give the same sequence of numbers. For example: 3, 2, 4, 2, 3.

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor-\left\lfloor\frac{nb}{p}\right\rfloor\right)=2\left(\left\lfloor\frac{3\times 7}{5}\right\rfloor-\left\lfloor\frac{2\times 7}{5}\right\rfloor\right)=2(4-2)=4\,.$$

This is a possibility as far as our observations are concerned.

For $b=6$ the number of half-cycles in the second to last set, for which $n=p-3=5-3=2$, is equal to:

$$2\left(\left\lfloor\frac{(n+1)b}{p}\right\rfloor-\left\lfloor\frac{nb}{p}\right\rfloor\right)=2\left(\left\lfloor\frac{3\times 6}{5}\right\rfloor-\left\lfloor\frac{2\times 6}{5}\right\rfloor\right)=2(3-2)=2\,.$$

This is a possibility as far as our observations are concerned.

Now it is the time to turn to the algorithm diagrams associated with $b=6$ and $b=7$ in order to find out which one is applicable in our case. These algorithm diagrams are depicted in Fig. 455.

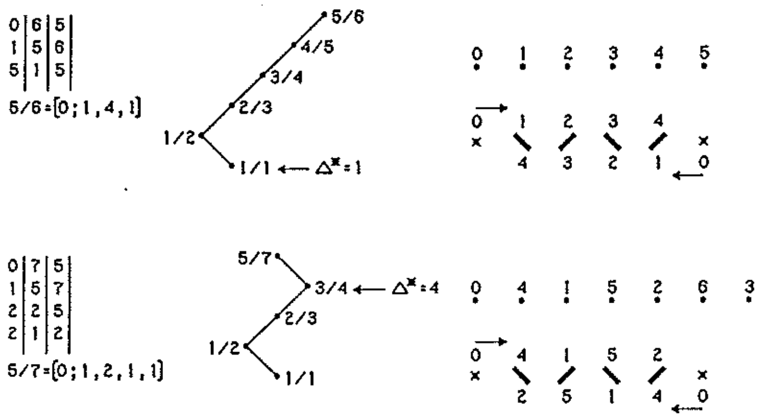


Fig. 455 — The algorithm diagrams for $p/b=5/6$ and $p/b=5/7$.

The half-cycle algorithms for $p/b=5/6$ are obtained in the usual way from its algorithm diagram:

1.		$L\longrightarrow R$	:	Free run.
2.	$i=0$	$R\longrightarrow L$	:	Free run.
3.	$i=0$	$L\longrightarrow R$	:	Free run.
4.	$i=1$	$R\longrightarrow L$	:	u.
5.	$i=1$	$L\longrightarrow R$	:	u.
6.	$i=2$	$R\longrightarrow L$	:	$u-o$.
7.	$i=2$	$L\longrightarrow R$	:	$u-o$.
8.	$i=3$	$R\longrightarrow L$	:	$u-o-u$.
9.	$i=3$	$L\longrightarrow R$	:	$u-o-u$.
10.	$i=4$	$R\longrightarrow L$	:	$u-o-u-o$.
11.	$i=4$	$L\longrightarrow R$	:	$u-o-u-o$.
12.	$i=5$	$R\longrightarrow L$	:	$u-o-u-o$.

Half-cycles 8 and 9 do not agree with Fig. № 240, which indicates for each of these two half-cycles: $u-2o$. Furthermore, half-cycles 6 and 7 do not agree with Fig. № 241, which indicates for each of these two half-cycles: $2o$. Hence $b=6$ is not correct, and $b=7$ will be the correct one. We confirm this by checking out its associated half-cycle algorithms obtained from its algorithm diagram with the sketches available to us:

1.		$L\longrightarrow R$	:	Free run.
2.	$i=0$	$R\longrightarrow L$	:	Free run.
3.	$i=0$	$L\longrightarrow R$	:	Free run.

4.	$i = 1$	$R \longrightarrow L$	:	o.
5.	$i = 1$	$L \longrightarrow R$	:	o.
6.	$i = 2$	$R \longrightarrow L$	:	$2o$.
7.	$i = 2$	$L \longrightarrow R$	:	$2o$.
8.	$i = 3$	$R \longrightarrow L$	:	$2o$.
9.	$i = 3$	$L \longrightarrow R$	:	$2o$.
10.	$i = 4$	$R \longrightarrow L$	:	$u - 2o$.
11.	$i = 4$	$L \longrightarrow R$	:	$u - 2o$.
12.	$i = 5$	$R \longrightarrow L$	:	$u - o - u - o$.
13.	$i = 5$	$L \longrightarrow R$	:	$u - o - u - o$.
14.	$i = 5$	$R \longrightarrow L$	:	$u - o - u - o$.

This agrees with the available Fig. Nº 239, 240, 241 and 242.

Now draw the grid-diagram of this foundation knot (see upper leftmost diagram in Fig. 456). Follow in this grid-diagram the instructions given by Lopez Osornio in Figs. Nº 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 and 256 till the lower bight-point, hence stop before forming this lower bight.

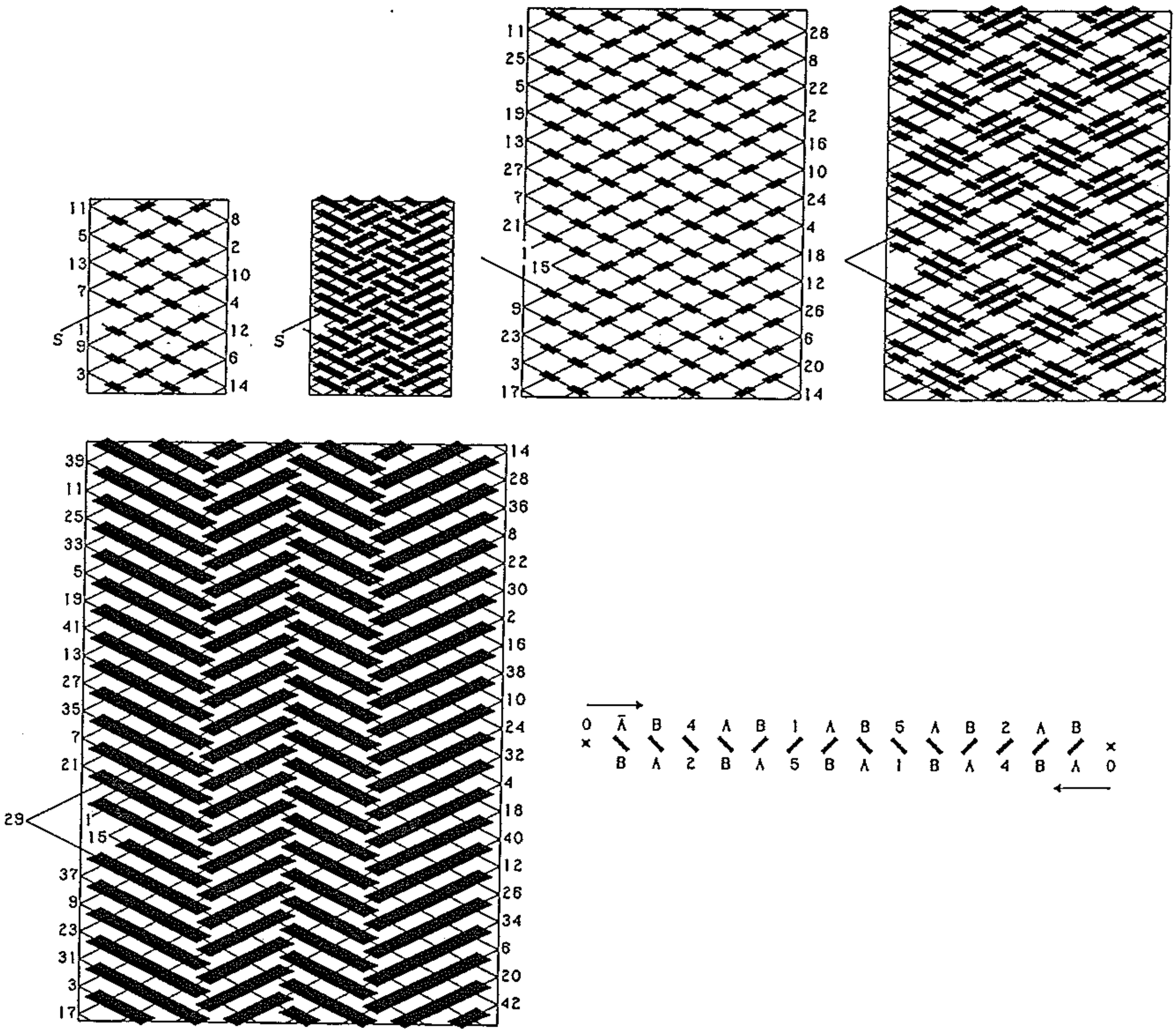


Fig. 456 — The braiding stages of the knot described by Lopez Osornio.

We thus obtain the second diagram in Fig. 456. Redraw this diagram to a larger scale; this gives the third diagram in Fig. 456. Follow in this last diagram the instructions given by Lopez Osornio in Figs. N° 256 (from lower bight up to the left), 257, 258, 259, 260, 261, 262, 263, 264, 265, 266 and 267. We thus obtain the fourth diagram (upper rightmost) in Fig. 456. Redraw this diagram to a larger scale and finish off the Working-end as descibed by Lopez Osornio in Fig. N° 267; this gives us the lower leftmost diagram in Fig. 456.

We note that the overall knot is a single string imitation $p/b = 15/21$ Column-coded Semi-Regular Knot. Obviously this knot has, due to its single string imitation form, an irregularity on the left bight-boundary. Although this irregularity might not be observed by novices or non-braiders, any braider of a reasonable standard would never use this or a similar knot. The more experienced braider can just as easy braid the single string Regular Knot $p/b = 15/22$ with the same Column-coding which would not have any irregularity at all. The less experienced braider can braid the single string Regular Knot $p/b = 15/19$ with the same Column-coding, but without any irregularities, much more easily (especially when the Standing-end is placed on the right-hand bight-boundary, and the knot is braided with the string-run from upper-right to lower-left and upper left to lower right).

Observe that the half-cycle algorithms for the by Lopez Osornio descibed imitation can readily be read from the algorithm diagram for the genuine $p/b = 15/21$ Semi-Regular Knot by omitting the first $\overline{A}$ for half-cycle 15 from lower-left to upper-right. Note that for the first stage (the foundation knot, upper leftmost diagram in Fig. 456), hence the half-cycles 1–14, the A and B positions in the algorithm diagram are neglected; for the second stage, resulting in the third diagram in Fig. 456, hence the half-cycles 15–28 (with 15=1 to 28=14 for the algorithm diagram), the B positions in the algorithm diagram are neglected and $\overline{A}$ is only neglected for half-cycle 15; for the third stage, resulting in the lower leftmost diagram in Fig. 456, hence the half-cycles 29–42 (with 29=1 to 42=14 for the algorithm diagram), all positions are taken into account. Hence we obtain:

1.		$L \longrightarrow R$	:	Free run.	
2.	$i = 0$	$R \longrightarrow L$	:	Free run.	
3.	$i = 0$	$L \longrightarrow R$	:	Free run.	
4.	$i = 1$	$R \longrightarrow L$	:	o.	
5.	$i = 1$	$L \longrightarrow R$	:	o.	
6.	$i = 2$	$R \longrightarrow L$	:	$2o$.	
7.	$i = 2$	$L \longrightarrow R$	:	$2o$.	
8.	$i = 3$	$R \longrightarrow L$	:	$2o$.	
9.	$i = 3$	$L \longrightarrow R$	:	$2o$.	
10.	$i = 4$	$R \longrightarrow L$	:	$u - 2o$.	
11.	$i = 4$	$L \longrightarrow R$	:	$u - 2o$.	
12.	$i = 5$	$R \longrightarrow L$	:	$u - o - u - o$.	
13.	$i = 5$	$L \longrightarrow R$	:	$u - o - u - o$.	
14.	$i = 6$	$R \longrightarrow L$	:	$u - o - u - o$.	
15.		$L \longrightarrow R$	:	$u - o - u - o$.	(neglect $\overline{A}$)
16.	$i = 0$	$R \longrightarrow L$	:	$2u - o - u - o$.	
17.	$i = 0$	$L \longrightarrow R$	:	$2u - o - u - o$.	

18.	$i=1$	$R \longrightarrow L$	:	$2u-2o-u-o$.
19.	$i=1$	$L \longrightarrow R$	:	$2u-2o-u-o$.
20.	$i=2$	$R \longrightarrow L$	:	$2u-2o-u-2o$.
21.	$i=2$	$L \longrightarrow R$	:	$2u-2o-u-2o$.
22.	$i=3$	$R \longrightarrow L$	:	$2u-2o-u-2o$.
23.	$i=3$	$L \longrightarrow R$	:	$2u-2o-u-2o$.
24.	$i=4$	$R \longrightarrow L$	:	$3u-2o-u-2o$.
25.	$i=4$	$L \longrightarrow R$	:	$3u-2o-u-2o$.
26.	$i=5$	$R \longrightarrow L$	:	$3u-2o-2u-2o$.
27.	$i=5$	$L \longrightarrow R$	:	$3u-2o-2u-2o$.
28.	$i=6$	$R \longrightarrow L$	:	$3u-2o-2u-2o$.
29.		$L \longrightarrow R$	:	$3u-2o-2u-3o$.
30.	$i=0$	$R \longrightarrow L$	:	$3u-2o-2u-3o$.
31.	$i=0$	$L \longrightarrow R$	:	$3u-2o-2u-3o$.
32.	$i=1$	$R \longrightarrow L$	:	$3u-3o-2u-3o$.
33.	$i=1$	$L \longrightarrow R$	:	$3u-3o-2u-3o$.
34.	$i=2$	$R \longrightarrow L$	:	$3u-3o-2u-4o$.
35.	$i=2$	$L \longrightarrow R$	:	$3u-3o-2u-4o$.
36.	$i=3$	$R \longrightarrow L$	:	$3u-3o-2u-4o$.
37.	$i=3$	$L \longrightarrow R$	:	$3u-3o-2u-4o$.
38.	$i=4$	$R \longrightarrow L$	:	$4u-3o-2u-4o$.
39.	$i=4$	$L \longrightarrow R$	:	$4u-3o-2u-4o$.
40.	$i=5$	$R \longrightarrow L$	:	$4u-3o-3u-4o$.
41.	$i=5$	$L \longrightarrow R$	:	$4u-3o-3u-4o$.
42.	$i=6$	$R \longrightarrow L$	:	$4u-3o-3u-4o$.

Lets first give the half-cycle algorithms for the above mentioned single string $p/b = 15/22$ (see Fig. 457) and $p/b = 15/19$ (see Fig. 458) knots.

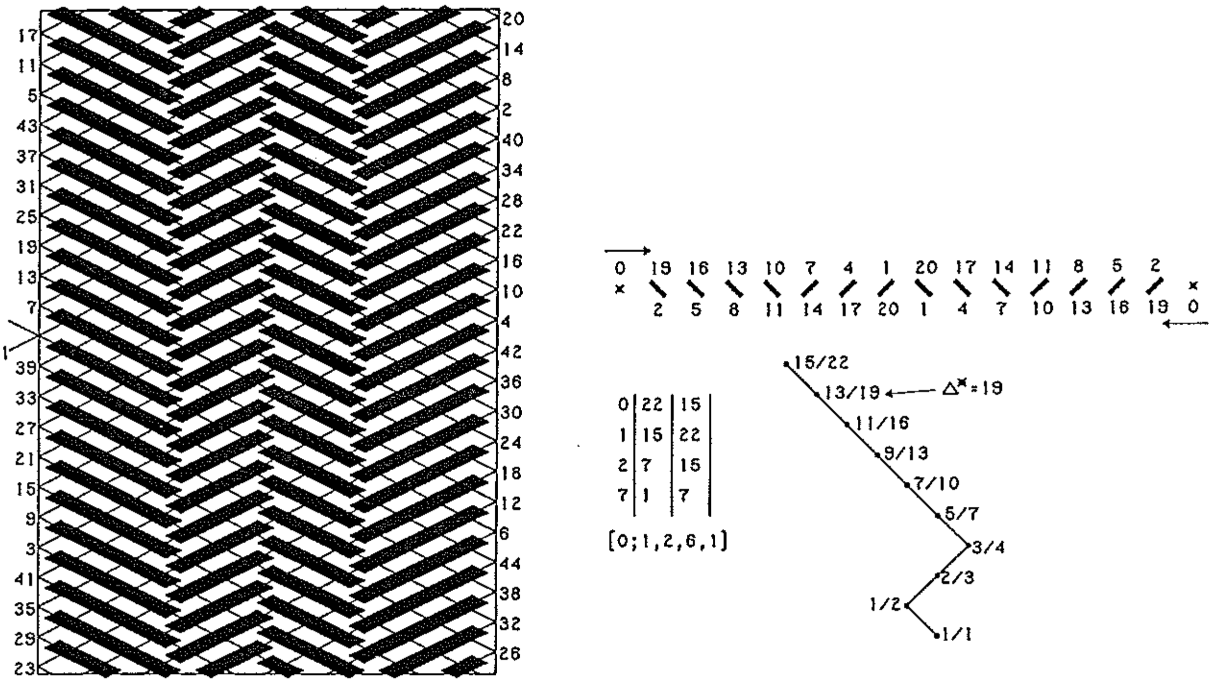


Fig. 457 — The $p/b = 15/22$ Regular Knot.

1.		$L \longrightarrow R$	:	Free run.
2.	$i = 0$	$R \longrightarrow L$	:	Free run.
3.	$i = 0$	$L \longrightarrow R$	:	Free run.
4.	$i = 1$	$R \longrightarrow L$	:	o.
5.	$i = 1$	$L \longrightarrow R$	:	o.
6.	$i = 2$	$R \longrightarrow L$	:	$2o$.
7.	$i = 2$	$L \longrightarrow R$	:	$2o$.
8.	$i = 3$	$R \longrightarrow L$	:	$2o$.
9.	$i = 3$	$L \longrightarrow R$	:	$2o$.
10.	$i = 4$	$R \longrightarrow L$	:	$3o$.
11.	$i = 4$	$L \longrightarrow R$	:	$3o$.
12.	$i = 5$	$R \longrightarrow L$	:	$4o$.
13.	$i = 5$	$L \longrightarrow R$	:	$4o$.
14.	$i = 6$	$R \longrightarrow L$	:	$4o$.
15.	$i = 6$	$L \longrightarrow R$	:	$4o$.
16.	$i = 7$	$R \longrightarrow L$	:	$5o$.
17.	$i = 7$	$L \longrightarrow R$	:	$5o$.
18.	$i = 8$	$R \longrightarrow L$	:	$6o$.
19.	$i = 8$	$L \longrightarrow R$	:	$6o$.
20.	$i = 9$	$R \longrightarrow L$	:	$6o$.
21.	$i = 9$	$L \longrightarrow R$	:	$6o$.
22.	$i = 10$	$R \longrightarrow L$	:	$u - 6o$.
23.	$i = 10$	$L \longrightarrow R$	:	$u - 6o$.
24.	$i = 11$	$R \longrightarrow L$	:	$u - 7o$.
25.	$i = 11$	$L \longrightarrow R$	:	$u - 7o$.
26.	$i = 12$	$R \longrightarrow L$	:	$u - 7o$.
27.	$i = 12$	$L \longrightarrow R$	:	$u - 7o$.
28.	$i = 13$	$R \longrightarrow L$	:	$2u - 7o$.
29.	$i = 13$	$L \longrightarrow R$	:	$2u - 7o$.
30.	$i = 14$	$R \longrightarrow L$	:	$2u - 3o - u - 4o$.
31.	$i = 14$	$L \longrightarrow R$	:	$2u - 3o - u - 4o$.
32.	$i = 15$	$R \longrightarrow L$	:	$2u - 3o - u - 4o$.
33.	$i = 15$	$L \longrightarrow R$	:	$2u - 3o - u - 4o$.
34.	$i = 16$	$R \longrightarrow L$	:	$3u - 3o - u - 4o$.
35.	$i = 16$	$L \longrightarrow R$	:	$3u - 3o - u - 4o$.
36.	$i = 17$	$R \longrightarrow L$	:	$3u - 3o - 2u - 4o$.
37.	$i = 17$	$L \longrightarrow R$	:	$3u - 3o - 2u - 4o$.
38.	$i = 18$	$R \longrightarrow L$	:	$3u - 3o - 2u - 4o$.
39.	$i = 18$	$L \longrightarrow R$	:	$3u - 3o - 2u - 4o$.
40.	$i = 19$	$R \longrightarrow L$	:	$4u - 3o - 2u - 4o$.
41.	$i = 19$	$L \longrightarrow R$	:	$4u - 3o - 2u - 4o$.
42.	$i = 20$	$R \longrightarrow L$	:	$4u - 3o - 3u - 4o$.
43.	$i = 20$	$L \longrightarrow R$	:	$4u - 3o - 3u - 4o$.
44.	$i = 21$	$R \longrightarrow L$	:	$4u - 3o - 3u - 4o$.

In order to avoid initial under-movements, we can either braid the $p/b = 15/19$ knot in Fig. 458 as shown by the left-hand grid-diagram, or we can rotate this grid-diagram through an angle of 180° about an axis perpendicular to the paper so as to obtain the right-hand grid-diagram. The knot can then be braided in accordance with this right-hand grid-diagram in the normal way.

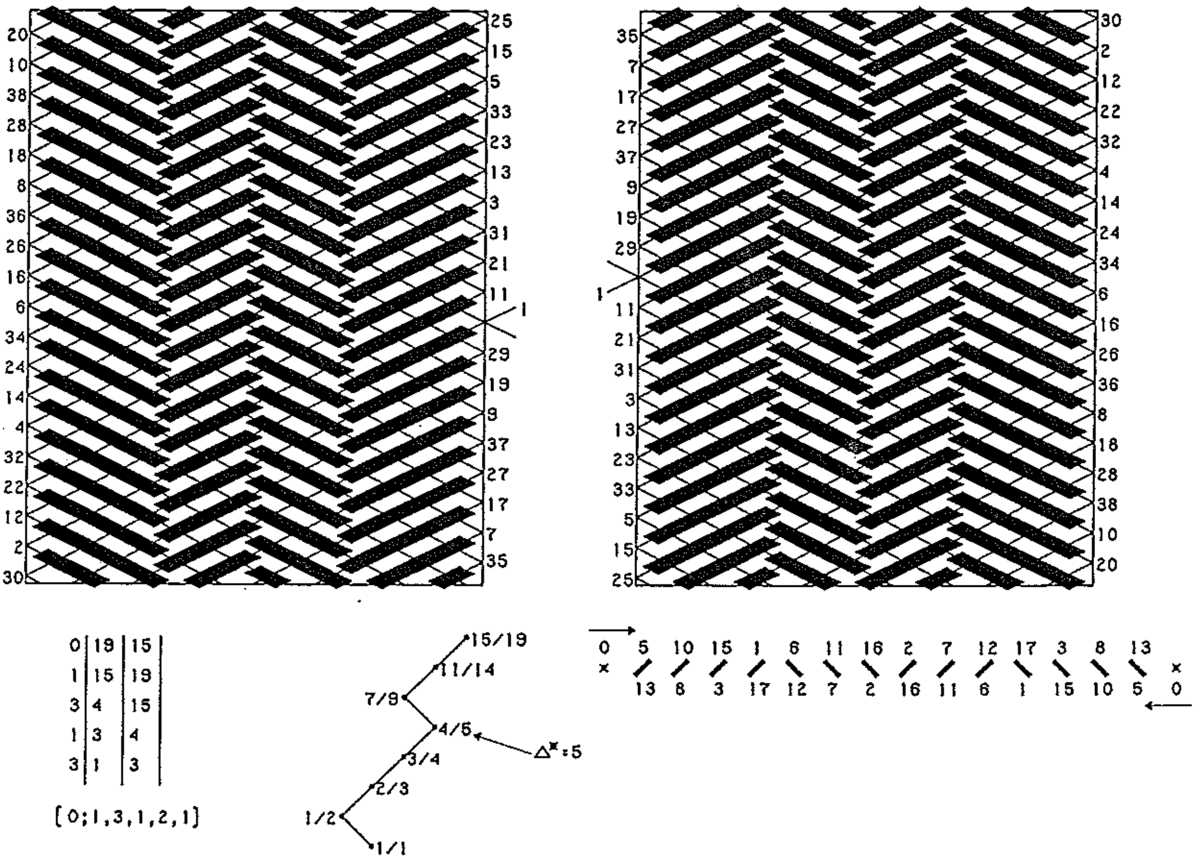


Fig. 458 — The $p/b = 15/19$ Regular Knot.

1.		$L \longrightarrow R$	:	Free run.
2.	$i = 0$	$R \longrightarrow L$	:	Free run.
3.	$i = 0$	$L \longrightarrow R$	:	Free run.
4.	$i = 1$	$R \longrightarrow L$	:	o.
5.	$i = 1$	$L \longrightarrow R$	:	o.
6.	$i = 2$	$R \longrightarrow L$	:	$2o$.
7.	$i = 2$	$L \longrightarrow R$	:	$2o$.
8.	$i = 3$	$R \longrightarrow L$	:	$2o - u$.
9.	$i = 3$	$L \longrightarrow R$	:	$2o - u$.
10.	$i = 4$	$R \longrightarrow L$	:	$2o - u$.
11.	$i = 4$	$L \longrightarrow R$	:	$2o - u$.
12.	$i = 5$	$R \longrightarrow L$	:	$3o - u$.
13.	$i = 5$	$L \longrightarrow R$	:	$3o - u$.
14.	$i = 6$	$R \longrightarrow L$	:	$2o - u - o - u$.
15.	$i = 6$	$L \longrightarrow R$	:	$2o - u - o - u$.
16.	$i = 7$	$R \longrightarrow L$	:	$2o - u - 2o - u$.
17.	$i = 7$	$L \longrightarrow R$	:	$2o - u - 2o - u$.
18.	$i = 8$	$R \longrightarrow L$	:	$2o - u - 2o - 2u$.
19.	$i = 8$	$L \longrightarrow R$	:	$2o - u - 2o - 2u$.
20.	$i = 9$	$R \longrightarrow L$	:	$2o - u - 2o - 2u$.
21.	$i = 9$	$L \longrightarrow R$	:	$2o - u - 2o - 2u$.

22. $i = 10\ R \longrightarrow L$: $3o - u - 2o - 2u$.
23. $i = 10\ L \longrightarrow R$: $3o - u - 2o - 2u$.
24. $i = 11\ R \longrightarrow L$: $3o - 2u - 2o - 2u$.
25. $i = 11\ L \longrightarrow R$: $3o - 2u - 2o - 2u$.
26. $i = 12\ R \longrightarrow L$: $3o - 2u - 3o - 2u$.
27. $i = 12\ L \longrightarrow R$: $3o - 2u - 3o - 2u$.
28. $i = 13\ R \longrightarrow L$: $3o - 2u - 3o - 3u$.
29. $i = 13\ L \longrightarrow R$: $3o - 2u - 3o - 3u$.
30. $i = 14\ R \longrightarrow L$: $3o - 2u - 3o - 3u$.
31. $i = 14\ L \longrightarrow R$: $3o - 2u - 3o - 3u$.
32. $i = 15\ R \longrightarrow L$: $4o - 2u - 3o - 3u$.
33. $i = 15\ L \longrightarrow R$: $4o - 2u - 3o - 3u$.
34. $i = 16\ R \longrightarrow L$: $4o - 3u - 3o - 3u$.
35. $i = 16\ L \longrightarrow R$: $4o - 3u - 3o - 3u$.
36. $i = 17\ R \longrightarrow L$: $4o - 3u - 3o - 4u$.
37. $i = 17\ L \longrightarrow R$: $4o - 3u - 3o - 4u$.
38. $i = 18\ R \longrightarrow L$: $4o - 3u - 3o - 4u$.

Lets return to the knot $p/b = 15/21$ by Lopez Osornio. As we mentioned on pg. 548, this knot has no merit due to the irregularity on the left-hand bight-boundary. Suppose, however, that a single string imitation of the genuine $p/b = 15/21$ Semi-Regular Knot is required, but that no irregularity should be detectable. This can, for example, readily be achieved with the string-run depicted in Fig. 459.

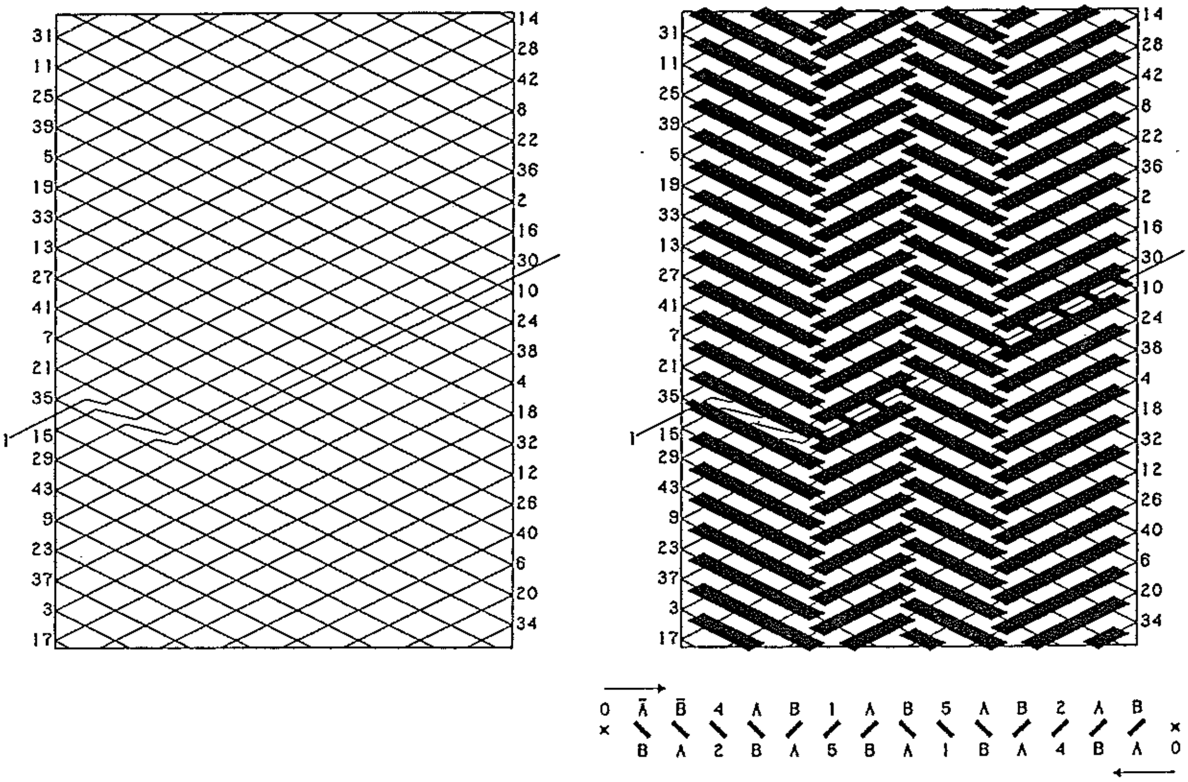


Fig. 459 — An acceptable single string imitation of the $p/b = 15/21$.

Also here can we use with a small modification the algorithm diagram of the true $p/b = 15/21$ Semi-Regular Knot. The modification required is that for the first half-cycle of the second stage, hence for half-cycle 15, $\overline{A}$ must be neglected, and that for the first half-cycle of the third stage, hence half-cycle 29, $\overline{B}$ must be neglected. Thus we obtain the half-cycle algorithms:

1.		$L \longrightarrow R$	:	Free run.	
2.	$i = 0$	$R \longrightarrow L$	:	Free run.	
3.	$i = 0$	$L \longrightarrow R$	:	Free run.	
4.	$i = 1$	$R \longrightarrow L$	:	o.	
5.	$i = 1$	$L \longrightarrow R$	:	o.	
6.	$i = 2$	$R \longrightarrow L$	:	$2o$.	
7.	$i = 2$	$L \longrightarrow R$	:	$2o$.	
8.	$i = 3$	$R \longrightarrow L$	:	$2o$.	
9.	$i = 3$	$L \longrightarrow R$	:	$2o$.	
10.	$i = 4$	$R \longrightarrow L$	:	$u - 2o$.	
11.	$i = 4$	$L \longrightarrow R$	:	$u - 2o$.	
12.	$i = 5$	$R \longrightarrow L$	:	$u - o - u - o$.	
13.	$i = 5$	$L \longrightarrow R$	:	$u - o - u - o$.	
14.	$i = 6$	$R \longrightarrow L$	:	$u - o - u - o$.	

15.		$L \longrightarrow R$	:	$u - o - u - o$.	(neglect $\overline{A}$)
16.	$i = 0$	$R \longrightarrow L$	:	$2u - o - u - o$.	
17.	$i = 0$	$L \longrightarrow R$	:	$2u - o - u - o$.	
18.	$i = 1$	$R \longrightarrow L$	:	$2u - 2o - u - o$.	
19.	$i = 1$	$L \longrightarrow R$	:	$2u - 2o - u - o$.	
20.	$i = 2$	$R \longrightarrow L$	:	$2u - 2o - u - 2o$.	
21.	$i = 2$	$L \longrightarrow R$	:	$2u - 2o - u - 2o$.	
22.	$i = 3$	$R \longrightarrow L$	:	$2u - 2o - u - 2o$.	
23.	$i = 3$	$L \longrightarrow R$	:	$2u - 2o - u - 2o$.	
24.	$i = 4$	$R \longrightarrow L$	:	$3u - 2o - u - 2o$.	
25.	$i = 4$	$L \longrightarrow R$	:	$3u - 2o - u - 2o$.	
26.	$i = 5$	$R \longrightarrow L$	:	$3u - 2o - 2u - 2o$.	
27.	$i = 5$	$L \longrightarrow R$	:	$3u - 2o - 2u - 2o$.	
28.	$i = 6$	$R \longrightarrow L$	:	$3u - 2o - 2u - 2o$.	

29.		$L \longrightarrow R$	:	$2u - 2o - 2u - 3o$.	(neglect $\overline{B}$)
30.	$i = 0$	$R \longrightarrow L$	:	$3u - 2o - 2u - 3o$.	
31.	$i = 0$	$L \longrightarrow R$	:	$3u - 2o - 2u - 3o$.	
32.	$i = 1$	$R \longrightarrow L$	:	$3u - 3o - 2u - 3o$.	
33.	$i = 1$	$L \longrightarrow R$	:	$3u - 3o - 2u - 3o$.	
34.	$i = 2$	$R \longrightarrow L$	:	$3u - 3o - 2u - 4o$.	
35.	$i = 2$	$L \longrightarrow R$	:	$3u - 3o - 2u - 4o$.	
36.	$i = 3$	$R \longrightarrow L$	:	$3u - 3o - 2u - 4o$.	
37.	$i = 3$	$L \longrightarrow R$	:	$3u - 3o - 2u - 4o$.	
38.	$i = 4$	$R \longrightarrow L$	:	$4u - 3o - 2u - 4o$.	
39.	$i = 4$	$L \longrightarrow R$	:	$4u - 3o - 2u - 4o$.	
40.	$i = 5$	$R \longrightarrow L$	:	$4u - 3o - 3u - 4o$.	
41.	$i = 5$	$L \longrightarrow R$	:	$4u - 3o - 3u - 4o$.	
42.	$i = 6$	$R \longrightarrow L$	:	$4u - 3o - 3u - 4o$.	
43.		$L \longrightarrow R$	:	$14u$.	

Note that although the string-run in Fig. 459 is quite different to the string-run in the lower-left diagram of Fig. 456, the difference in their respective half-cycle algorithms is restricted to their associated half-cycles 29 only. This clearly illustrates the importance of **always** depicting, with a half-cycle algorithm table, the associated grid-diagram.

Nested Cylindrical Braids

On pg. 502 (*The Braider*, Issue No. 22) we have seen that the Regular Nested Cylindrical Braids have the left sequence set $1A(A-1)(A-2)(A-3)\cdots 432$ and the right sequence set $k|k+1|_A|k+2|_A|k+3|_A\cdots A123\cdots|k-2|_A|k-1|_A$, where $1 \le k \le A$. Consequently for a general left and right cycle we obtained in Fig. 433 (*The Braider*, pg. 510, Issue No. 22):

$$l_{i+1} = |l_i + x - 2(l_i + r_i)|_A \,.$$
$$r_{i+1} = |r_i + x - 2(r_i + l_{i+1})|_A \,.$$

In Fig. 460 are tabulated the values for $(l_i + r_i)$.

$l_i \longrightarrow r_i$	$l_i + r_i$
$1 \longrightarrow k$	$k+1$
$A \longrightarrow k+1$	$A+k+1$
$A-1 \longrightarrow k+2$	$A+k+1$
$A-2 \longrightarrow k+3$	$A+k+1$
$\vdots$	$\vdots$
$A-(z-1) \longrightarrow k+z$	$A+k+1$
$\vdots$	$\vdots$
$k+2 \longrightarrow A-1$	$A+k+1$
$k+1 \longrightarrow A$	$A+k+1$
$k \longrightarrow 1$	$k+1$
$k-1 \longrightarrow 2$	$k+1$
$k-2 \longrightarrow 3$	$k+1$
$\vdots$	$\vdots$
$4 \longrightarrow k-3$	$k+1$
$3 \longrightarrow k-2$	$k+1$
$2 \longrightarrow k-1$	$k+1$

Fig. 460 — The values of $(l_i + r_i)$ in Regular Nested Cylindrical Braids.

Hence:

$$l_{i+1} = |l_i + x - 2(l_i + r_i)|_A = |l_i + x - 2(k+1)|_A = |l_i + \Delta|_A \,.$$
$$r_{i+1} = |r_i + x - 2(r_i + l_{i+1})|_A = |r_i - [x - 2(l_i + r_i)]|_A = |r_i - \Delta|_A \,.$$

where $\Delta = |x - 2(k+1)|_A$.

$$\Delta = |x - 2(k+1)|_A \,,$$
$$\Delta + nA = x - 2(k+1) \qquad \text{where } n = \text{an integer}\,,$$
$$2k = x - \Delta - 2 - nA\,,$$

Hence the values of k in association with A, x and Δ are:

$$A = \text{odd}\begin{cases} x = \text{odd}\begin{cases} \Delta = \text{odd} & \rightarrow \quad k = \left|\dfrac{x-\Delta-2}{2}\right|_A . \\ \Delta = \text{even} & \rightarrow \quad k = \left|\dfrac{x-\Delta-2+A}{2}\right|_A . \end{cases} \\ x = \text{even}\begin{cases} \Delta = \text{odd} & \rightarrow \quad k = \left|\dfrac{x-\Delta-2+A}{2}\right|_A . \\ \Delta = \text{even} & \rightarrow \quad k = \left|\dfrac{x-\Delta-2}{2}\right|_A . \end{cases} \end{cases}$$

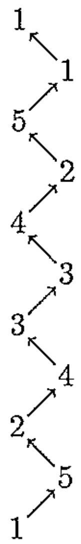
$$A = \text{even}\begin{cases} x = \text{odd}\begin{cases} \Delta = \text{odd} & \rightarrow \quad k = \left|\dfrac{x-\Delta-2}{2}\right|_A ; \; k = \left|\dfrac{x-\Delta-2+A}{2}\right|_A . \\ \Delta = \text{even} & \rightarrow \quad \text{not possible.} \end{cases} \\ x = \text{even}\begin{cases} \Delta = \text{odd} & \rightarrow \quad \text{not possible.} \\ \Delta = \text{even} & \rightarrow \quad k = \left|\dfrac{x-\Delta-2}{2}\right|_A ; \; k = \left|\dfrac{x-\Delta-2+A}{2}\right|_A . \end{cases} \end{cases}$$

Note that the above formulae for $\Delta = \text{even}$ become the formulae at the bottom of pg. 513 and top of pg. 514 when $\Delta = 0$. Hence $\Delta = 0$ is associated with the Standard Regular Nested Cylindrical Braids and the Semi-Standard Regular Nested Cylindrical Braids.

Example 1:

Let $A = 5$; $x = 23$; $\Delta = 1$.

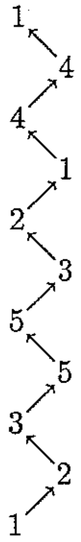
Then $k = \left|\frac{x-\Delta-2}{2}\right|_A = \left|\frac{23-1-2}{2}\right|_5 = |10|_5 = 0 = 5$. Hence the first-return string-run is:



Example 2:

Let $A = 5$; $x = 23$; $\Delta = 2$.

Then $k = \left|\frac{x-\Delta-2+A}{2}\right|_A = \left|\frac{23-2-2+5}{2}\right|_5 = |12|_5 = 2$. Hence the first-return string-run is:

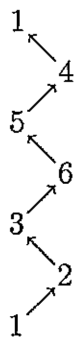


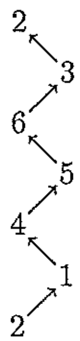
Example 3:

Let $A = 6$; $x = 20$; $\Delta = 2$.

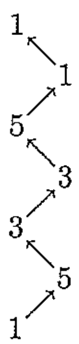
Then:

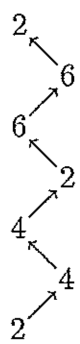
(1). $k = \left|\frac{x-\Delta-2}{2}\right|_A = \left|\frac{20-2-2}{2}\right|_6 = |8|_6 = 2$. Hence the first-return string-runs are:





(2). $k = \left|\frac{x-\Delta-2+A}{2}\right|_A = \left|\frac{20-2-2+6}{2}\right|_6 = |11|_6 = 5$. Hence the first-return string-runs are:





The Regular Nested Cylindrical Braids with $\Delta \neq 0$ can be divided into the following four families:

(i). The Perfect Regular Nested Cylindrical Braids. Here g.c.d.$(\Delta, A) = 1$, and g.c.d.$(P_c, B^*) = 1$. Hence they have one first-return string-run (thus consist of one component); this component requires only one string in its construction. It are the single string Regular Nested Cylindrical Braids.

(ii). The Semi-Perfect Regular Nested Cylindrical Braids. Here g.c.d.$(\Delta, A) = 1$, and g.c.d.$(P_c, B^*) \neq 1$. Hence they also have one first-return string-run (thus consist of one component), but this component requires more than one string in its construction.

(iii). The Compound Regular Nested Cylindrical Braids. Here g.c.d.$(\Delta, A) \neq 1$, and g.c.d.$(P_c, B^*) = 1$. Hence they have more than one first-return string-run (thus consist of more than one component), where each component requires only one string in its construction.

(iv). The Semi-Compound Regular Nested Cylindrical Braids. Here g.c.d.$(\Delta, A) \neq 1$, and at least one g.c.d.$(P_c, B^*) \neq 1$. Hence they have more than one first-return string-run (thus consist of more than one component), and at least one component requires more than one string in its construction.

For the Regular Nested Cylindrical Braids let the shortest vertical distance, in the circumferential direction of braiding (the distance along a bight-edge in the circumferential direction of braiding), from the apex of a left-hand nest of bights to the apex of a right-hand nest of bights be equal to y row-units, then:

$$\begin{aligned} y &= |2(A - l_1) + x + 2(A - r_1)|_{2A} \\ &= |2(A-1) + x + 2(A-k)|_{2A} \\ &= |x - 2(1+k)|_{2A} \,. \end{aligned}$$

Hence:

$$A = \text{odd} \begin{cases} x = \text{odd} \begin{cases} \Delta = \text{odd} & \rightarrow \quad y = |\Delta|_{2A} = \Delta \,. \\ \Delta = \text{even} & \rightarrow \quad y = |\Delta + A|_{2A} = \Delta + A \,. \end{cases} \\ x = \text{even} \begin{cases} \Delta = \text{odd} & \rightarrow \quad y = |\Delta + A|_{2A} = \Delta + A \,. \\ \Delta = \text{even} & \rightarrow \quad y = |\Delta|_{2A} = \Delta \,. \end{cases} \end{cases}$$

$$A = \text{even} \begin{cases} x = \text{odd} \begin{cases} \Delta = \text{odd} & \rightarrow \quad y = |\Delta|_{2A} = \Delta \,;\ y = |\Delta + A|_{2A} = \Delta + A \,. \\ \Delta = \text{even} & \rightarrow \quad \text{not possible} \,. \end{cases} \\ x = \text{even} \begin{cases} \Delta = \text{odd} & \rightarrow \quad \text{not possible} \,. \\ \Delta = \text{even} & \rightarrow \quad y = |\Delta|_{2A} = \Delta \,;\ y = |\Delta + A|_{2A} = \Delta + A \,. \end{cases} \end{cases}$$

In most practical applications we take $x \geq 2$, although x can be less than 2, hence can be negative. For $x \leq 2$ the following conditions apply:

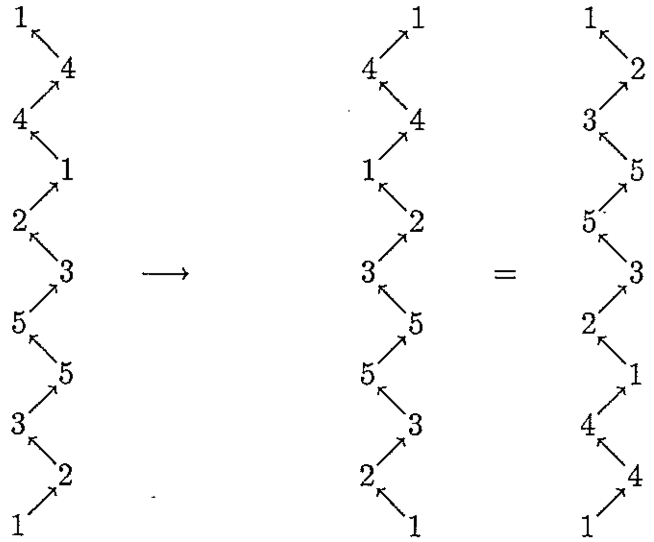
$$\begin{aligned} 2 - A &\leq x \leq 2 \,. \\ y_{min} &= 2 - x \,. \\ y_{max} &= 2A + x - 2 \,. \end{aligned}$$

⋆⋆ *Prove the above conditions for $x \leq 2$.*

There are two basic complementary forms associated with a first-return string-run:

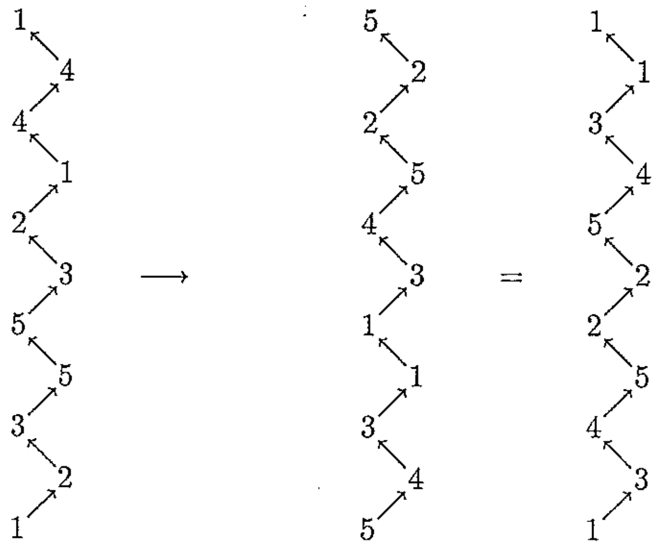
1. **The basic complementary form 1 first-return string-run is the mirror-image of a first-return string-run.** Note that the mirror-image from "left" to "right" is identical to the mirror-image from "top" to "bottom".

Example 4:



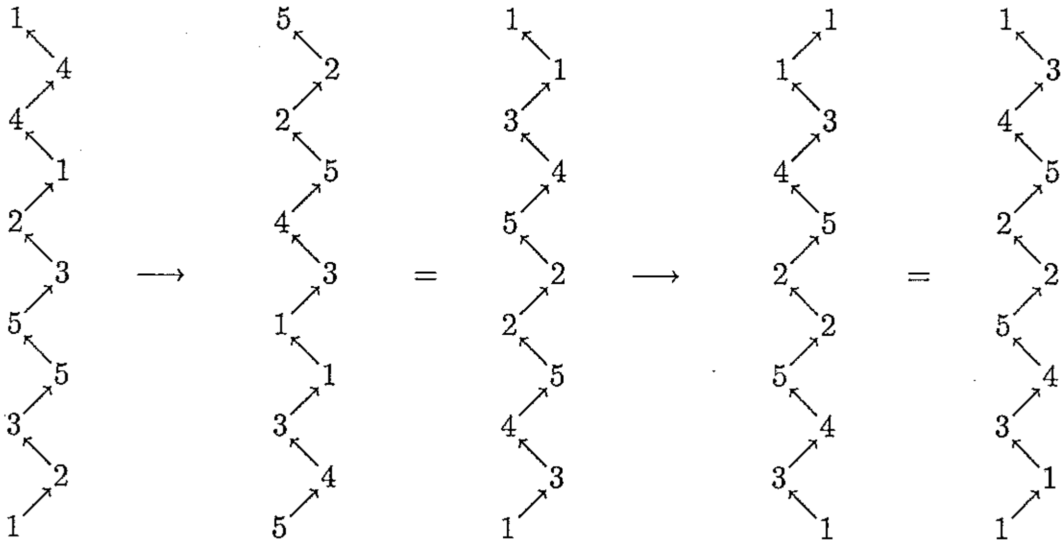
2. **The basic complementary form 2 first-return string-run involves the half-cycle complements with respect to $A_l = A$ and $A_r = A$ (the (A_l, A_r)-complement, where the complement of l_i is equal to $(A_l + 1 - l_i)$ and the complement of r_i is equal to $(A_r + 1 - r_i)$; hence in the case where $A_l = A_r = A$, the (A, A)-complement, where the complement of l_i is equal to $(A + 1 - l_i)$ and the complement of r_i is equal to $(A + 1 - r_i)$).**

Example 5:



The combination of these two basic complementary first-return string-run forms results in a compound-complementary first-return string-run which is the mirror-imaged (A_l, A_r)-complement of the given first-return string-run. Hence in the case where $A_l = A_r = A$, the combination of these two basic complementary first-return string-run forms results in a compound-complementary first-return string-run which is the mirror-imaged (A, A)-complement of the given first-return string-run.

Example 6:



At the left-hand side in Fig. 461 are depicted some of the half-cycles of the string-run of a Regular Nested Cylindrical Braid with nesting-number A, while at the right-hand side are depicted their mirror-images.

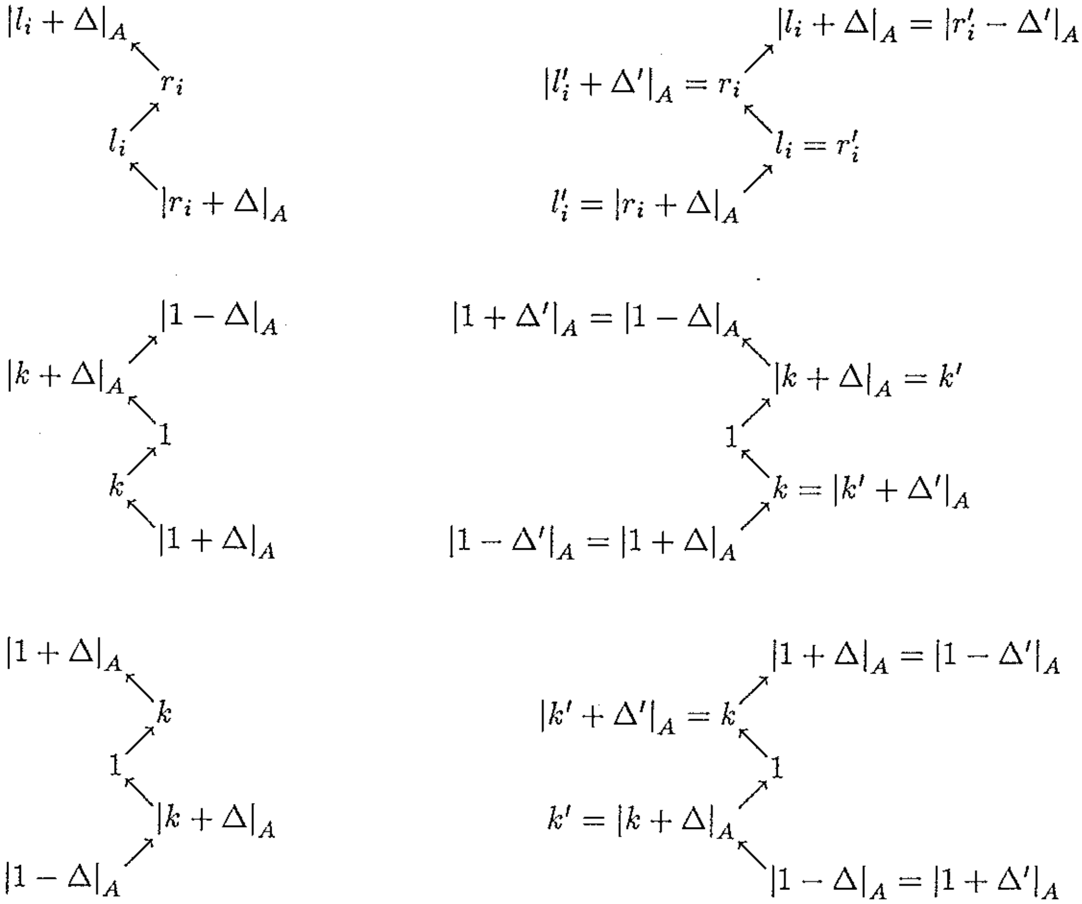


Fig. 461 — Some half-cycles of the string-run of a Regular Nested Cylindrical Braid and their mirror-image.

Let x, y and Δ be associated with the string-run of a Regular Nested Cylindrical

Braid with nesting-number A, and let x', y' and Δ' be associated with its mirror-imaged string-run. Then:

$$\begin{aligned}
y &= |x - 2(k+1)|_{2A}\,. \\
\Delta &= |x - 2(k+1)|_{A} = |y|_{A}\,. \\
2\Delta &= 2|x - 2(k+1)|_{A} = |2\{x - 2(k+1)\}|_{2A}\,. \\
k' &= |k + \Delta|_{A}\,. \\
\Delta' &= |-\Delta|_{A}\,.
\end{aligned}$$

$$\begin{aligned}
y' &= |x' - 2(k'+1)|_{2A} \\
&= |x' - 2|k+\Delta|_{A} - 2|_{2A} \\
&= |x' - |2(k+\Delta)|_{2A} - 2|_{2A} \\
&= |x' - 2k - 2\Delta - 2|_{2A} \\
&= |x' - 2(k+1) - 2\Delta|_{2A} \\
&= |x' - 2(k+1) - 2x + 4(k+1)|_{2A} \\
&= |x' - 2x + 2(k+1)|_{2A} \\
\text{for}\quad x' = x \;\rightarrow\quad y' &= |-\{x - 2(k+1)\}|_{2A} \\
&= |-|x - 2(k+1)|_{2A}|_{2A} \\
&= |-y|_{2A}\,.
\end{aligned}$$

$$\text{Thus :}\qquad y' = |-y|_{2A}\;;\; \Delta' = |-\Delta|_{A}\;;\; k' = |k+\Delta|_{A}\,.$$

The relationship $y' = |-y|_{2A}$ follows, with $\Delta' = |-\Delta|_{A}$, also immediately from the relationships shown on pg. 557 between A, x, Δ and y.

At the left-hand side in Fig. 462 are depicted some of the half-cycles of the string-run of a Regular Nested Cylindrical Braid with nesting-number A, while at the right-hand side are depicted their mirror-imaged (A, A)-complements.

Let x, y and Δ be associated with the string-run of a Regular Nested Cylindrical Braid with nesting-number A, and let x'', y'' and Δ'' be associated with its mirror-imaged (A, A)-complementary string-run. Then:

$$\begin{aligned}
y &= |x - 2(k+1)|_{2A}\,. \\
\Delta &= |x - 2(k+1)|_{A} = |y|_{A}\,. \\
k'' &= |-k - \Delta|_{A}\,. \\
\Delta'' &= \Delta\,.
\end{aligned}$$

$$\begin{aligned}
\Delta'' &= |x'' - 2(k''+1)|_{A} \\
&= |x'' + 2k + 2\Delta - 2|_{A} \\
&= |x'' + 2k + \Delta + x - 2k - 2 - 2|_{A} \\
&= |x'' + x - 4 + \Delta|_{A}\,. \\
nA &= x'' + x - 4\,.
\end{aligned}$$

$$\frac{x'' + x}{2} = 2 + \frac{nA}{2}\,, \qquad \text{where } n = \text{whole number.}$$

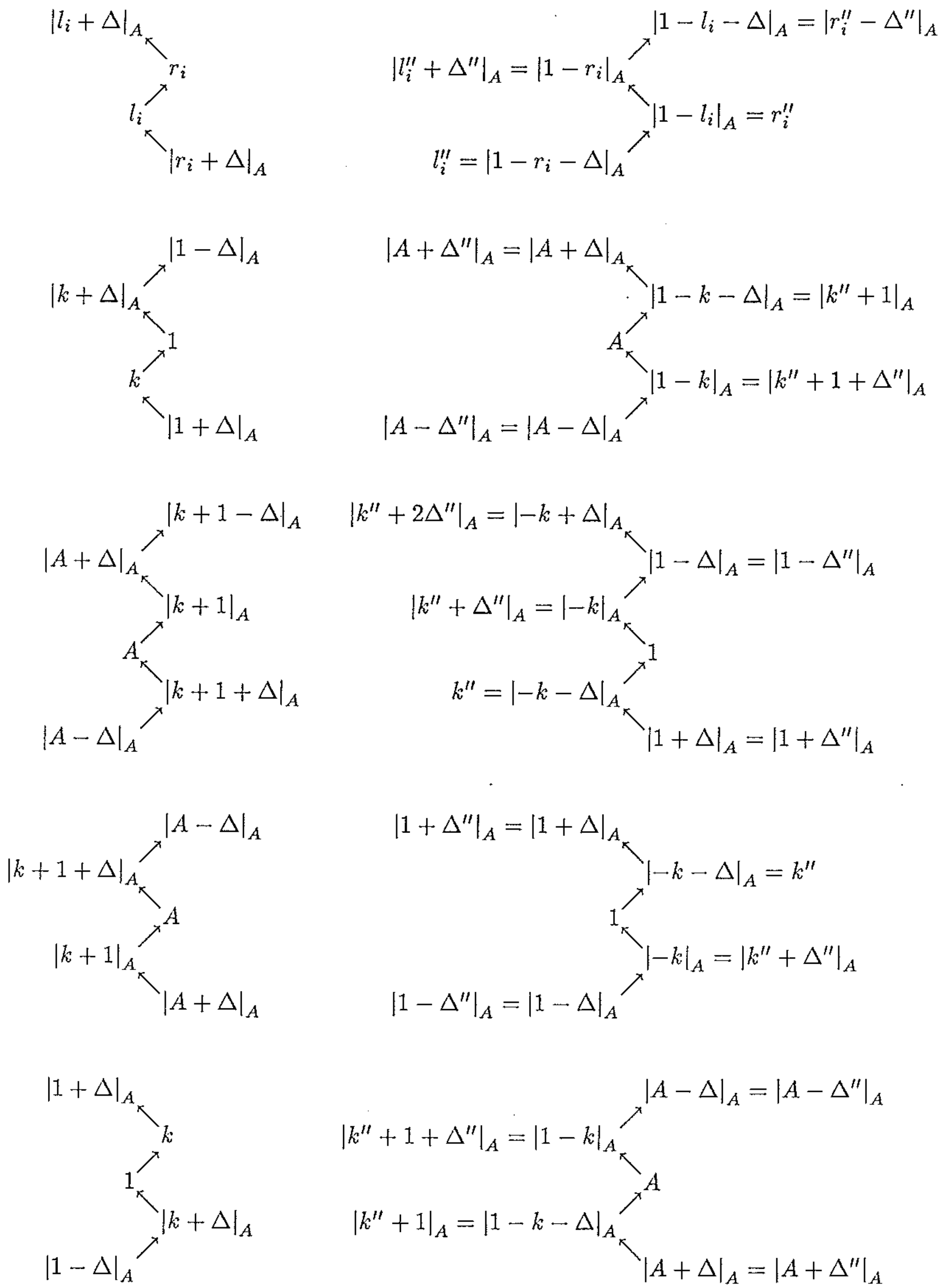


Fig. 462 — Some half-cycles of the string-run of a Regular Nested Cylindrical Braid and their mirror-imaged (A, A)-complements.

$$\begin{aligned} y'' &= |x'' - 2(k'' + 1)|_{2A} \\ &= |x'' + 2k + 2\Delta - 2|_{2A} \\ &= |x'' + 2k + 2x - 4k - 4 - 2|_{2A} \\ &= |x'' + x + y - 4|_{2A} \end{aligned}$$

for $y'' = y \rightarrow \quad \dfrac{x'' + x}{2} = 2 + \dfrac{2mA}{2}$, where $m =$ whole number.

Hence : $\quad n = 2m$

The table in Fig. 463 tabulates the Regular Nested Cylindrical Braids for A, x and y. From the foregoing it follows that:

The first-return string-runs belonging to cell F_1 and the cell F_2 are the mirror-image of each other. The first-return string-runs belonging to cell F_3 and the cell F_4 are the mirror-image of each other.

The first-return string-runs belonging to cell F_1 and the cell F_3 are the mirror-imaged (A, A)-complements of each other. The first-return string-runs belonging to cell F_2 and the cell F_4 are the mirror-imaged (A, A)-complements of each other.

The first-return string-runs belonging to cell F_1 and the cell F_4 are the (A, A)-complements of each other. The first-return string-runs belonging to cell F_2 and the cell F_3 are the (A, A)-complements of each other.

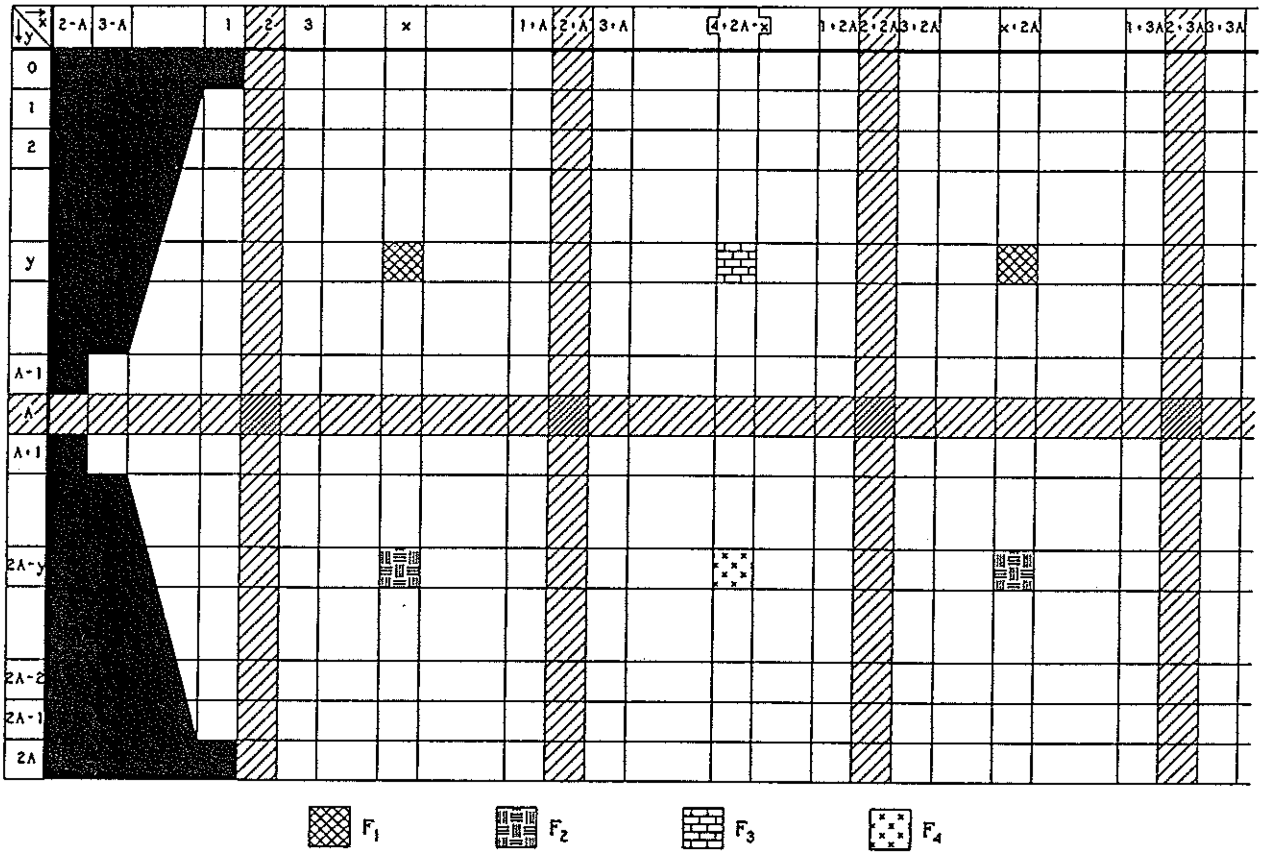


Fig. 463 — The A, x, y table.