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for
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Solutions to the Questions in Issue No. 22

Question on pg. 494.

A Premature Regular Cylindrical Braid ($\delta = 0$) with b bights and p parts has $b(p-1)-(p-1) = (b-1)(p-1)$ crossings. When $b = n+1$, then $(b-1)(p-1) = n(p-1)$. When the string-run of this Premature Regular Cylindrical Braid is equivalent to the string-run of a Prime Regular Cylindrical Bihelix Braid, the number of crossings in such a Prime Regular Cylindrical Bihelix Braid must be a multiple of n. From the formulae on pp. 393 – 394 which express the sequential number of crossings in $[(n+1), n]$ Prime Regular Cylindrical Bihelix Braids we observe that those $[(n+1), n]$ Prime Regular Cylindrical Bihelix Braids in which the number of crossings are equal to:

$$\begin{aligned} &2n^2\,, \\ &2n^2 + n\,, \\ &4n^2\,, \\ &4n^2 + n\,, \\ &6n^2\,, \\ &6n^2 + n\,, \\ &8n^2\,, \\ &8n^2 + n\,, \\ &\;\;\vdots \end{aligned}$$

fulfil this requirement. Hence the string-run of a Premature Regular Cylindrical Braid in which the number of bights is equal to $b = n+1$ and the number of parts equal to $p = 2kn+1$ or $p = 2kn+2$, where $k = 1, 2, 3, \cdots$, is identical to the string-run of an $[(n+1), n]$ Prime Regular Cylindrical Bihelix Braid in which the number of crossings is equal to $2kn^2$ respectively $2kn^2 + n$. When b and p are coprime, hence g.c.d. $(b, p) = 1$, the Premature Regular Cylindrical Braid, and hence its equivalent Prime Regular Cylindrical Bihelix Braid, is a single string braid.

Question on pg. 512.

Since for the Standard Regular Nested Cylindrical Braids and the Semi-Standard Regular Nested Cylindrical Braids the number of Components is equal to A, it follows that $\alpha = 1$. Hence $l_{i+1} = l_i$ and consequently a first-return string-run is completed.

Since $l_{i+1} = |l_i + x - 2(l_i + r_i)|_A$ and hence $l_{i+1} = l_i + x - 2(l_i + r_i) + nA$, where n is an integer:

$$l_{i+1} = l_i \quad \longrightarrow \quad 0 = x - 2(l_i + r_i) + nA\,,$$
$$2r_i = x - 2l_i + nA\,,$$
$$r_i = \frac{x + nA}{2} - l_i\,,$$

but since $1 \le r_i \le A$, we obtain :

$$r_i = \left|\frac{x + nA}{2} - l_i\right|_A .$$

For $l_i = l_1 = 1$:

$$x = \text{even} \begin{cases} A = \text{odd} \quad \longrightarrow \quad r_i = r_1 = \left|\dfrac{x}{2} - 1\right|_A . \\ A = \text{even} \quad \longrightarrow \quad r_i = r_1 = \left|\dfrac{x}{2} - 1\right|_A \quad \text{and} \quad r_i = r_1 = \left|\dfrac{x + A}{2} - 1\right|_A . \end{cases}$$

$$x = \text{odd} \begin{cases} A = \text{odd} & \longrightarrow & r_i = r_1 = \left|\dfrac{x+A}{2} - 1\right|_A . \\ A = \text{even} & \longrightarrow & \text{not possible.} \end{cases}$$

For the Standard Regular Nested Cylindrical Braids and the Semi-Standard Regular Nested Cylindrical Braids the shortest vertical distance (the distance along a bight-edge) in bight-units between the apex of a left-hand nest of bights and the apex of a right-hand nest of bights is equal to:

$$\begin{aligned} &\left|\frac{2(A-l_1)+x+2(A-r_1)}{2}\right|_A \\ &= \left|\frac{x-2(l_1+r_1)}{2}\right|_A \\ &= \left|\frac{x-2(1+r_1)}{2}\right|_A \\ &= \left|\frac{x}{2}-(1+r_1)\right|_A \end{aligned}$$

Hence $\left|\frac{x}{2}-(1+r_1)\right|_A = 0$ for:

$$x = \text{even} \begin{cases} A = \text{odd} & \longrightarrow & r_i = r_1 = \left|\dfrac{x}{2} - 1\right|_A . \\ A = \text{even} & \longrightarrow & r_i = r_1 = \left|\dfrac{x}{2} - 1\right|_A . \end{cases}$$

$$x = \text{odd} \begin{cases} A = \text{odd} & \longrightarrow & \text{not possible.} \\ A = \text{even} & \longrightarrow & \text{not possible.} \end{cases}$$

And $\left|\frac{x}{2}-(1+r_1)\right|_A = \frac{A}{2}$ for:

$$x = \text{even} \begin{cases} A = \text{odd} & \longrightarrow & \text{not possible.} \\ A = \text{even} & \longrightarrow & r_i = r_1 = \left|\dfrac{x+A}{2} - 1\right|_A . \end{cases}$$

$$x = \text{odd} \begin{cases} A = \text{odd} & \longrightarrow & r_i = r_1 = \left|\dfrac{x+A}{2} - 1\right|_A . \\ A = \text{even} & \longrightarrow & \text{not possible.} \end{cases}$$

There are thus for the Standard Regular Nested Cylindrical Braids and the Semi-Standard Regular Nested Cylindrical Braids two types of string-run as far as the apex positions of the left-hand nests of bights relative to the apex position of the right-hand nests of bights are concerned: one in which the apex positions of the right-hand nests of bights line up with the apex positions of the left-hand nests of bights, and one in which the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights (see Figs. 435 & 436 respectively). To the latter type belong, for example, the string-runs of the well-known **Standard Herringbone Pineapple Knots**, more commonly known as the Pineapple Knots, and the **Semi-Standard Herringbone Pineapple Knots**.

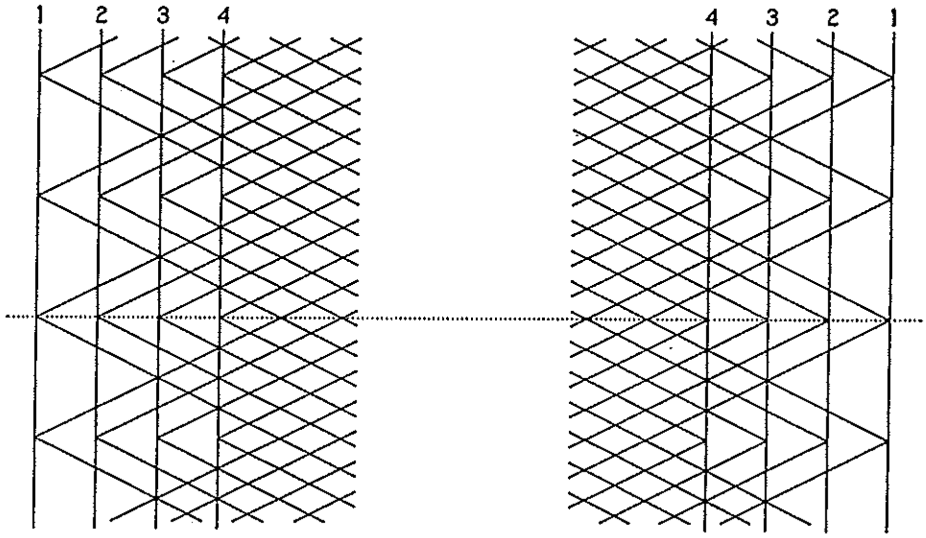


Fig. 435 — The case $\left|\frac{x}{2} - (1 + r_1)\right|_A = 0$.

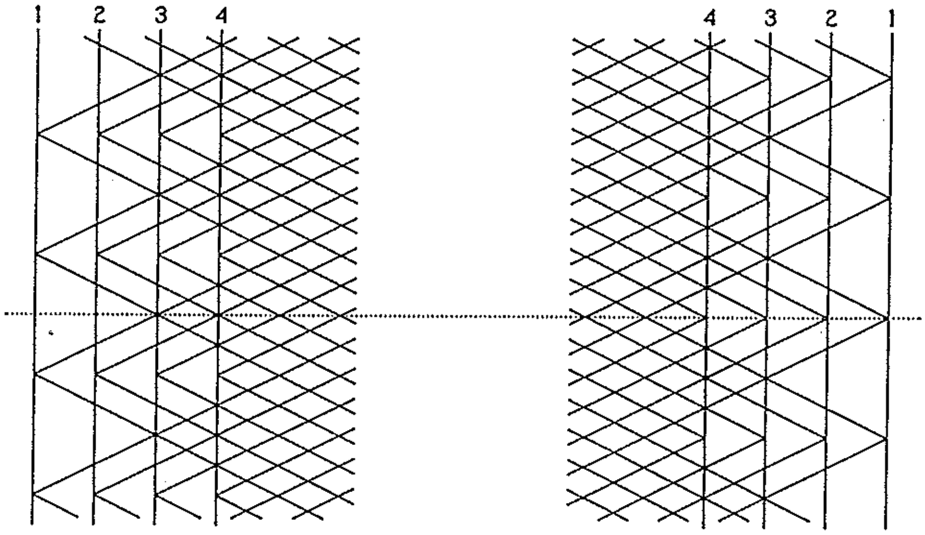


Fig. 436 — The case $\left|\frac{x}{2} - (1 + r_1)\right|_A = \frac{A}{2}$.

Column-coded Regular Cylindrical Braids
with a
balanced coding.

In *The Braider*, Issue No. 21, pp. 471 – 482, we discussed the 'classification' of the **over–under coded Regular Knots**, and in Issue No. 9, pp. 178 – 187 we discussed their 'categorisation' which can be regarded as a further refinement of their 'classification'. The reader, no doubt, has become aware that even for the most simple Regular Knots, the over–under coded type, a classification becomes a rather complicated affair. Since our knowledge about **Braids** in general is infinitely small, any attempt in setting up a proper classification system amounts to a futile exercise as we have mentioned in earlier Issues of *The Braider*. Hence we like to stress once again that our discussions about classifications and categorisation must be seen as restricted and isolated snippets in this immensely complicated field. These snippets should however not only make us appreciate the enormous difficulties associated with a classification system, but should also give us some insight about relationships which are often quite remarkable.

Since Column-coded Regular Cylindrical Braids are in fact Column-coded Regular Flat Braids in the form of a cylinder, the results in this discussion apply of course to Column-coded Regular Flat Braids also. Of special interest to braiders are those Column-coded Regular Cylindrical Braids and Column-coded Regular Flat Braids which have a balanced (symmetric) coding relative to the (circumferential) centre-line of the braid since they have the most pleasing appearance.

The tables depicting the balanced Column-coding types are shown in Fig. 437 for $2 < p \leq 10$ and in Fig. 438 for $11 \leq p \leq 12$. In these tables the c-value indicates the number of intersection-columns in the central region with identically coded crossings. The coding patterns coupled by a brace are each others complement. The c-value can only be equal to zero when p is odd (there is no central intersection-column), in which case each coding pattern is identical to its complement (we assume that the braiding-material is a flat string which has for each opposite face pair identical faces in size, shape, texture and colour).

For $p =$ **even** $= 2n$, where n is a natural number†, there are a total of 2^n balanced coding types.

The number of balanced coding types for $c = 2m - 1$, where $1 \leq m \leq n$, is:

$$\text{for} \quad 1 \leq m < n \quad \longrightarrow \quad 2^{n-m}.$$
$$\text{for} \quad m = n \quad \longrightarrow \quad 2.$$

For $p =$ **odd** $= 2n + 1$, where n is a natural number, there are a total of $3 \times 2^{n-1}$ balanced coding types.

The number of balanced coding types for $c = 2m$, where $0 \leq m \leq n$, is:

$$\text{for} \quad m = 0 \quad \longrightarrow \quad 2^{n-1}.$$
$$\text{for} \quad 1 \leq m < n \quad \longrightarrow \quad 2^{n-m}.$$
$$\text{for} \quad m = n \quad \longrightarrow \quad 2.$$

Let's look at the position of a few well known knots:

The 2-pass Spanish Ring Knots come under $p = 5 \mid c = 0$. The 3-pass Spanish Ring Knots come under $p = 7 \mid c = 0$. The 4-pass Spanish Ring Knots come under $p = 9 \mid c = 0$. And so on.

The 2-pass Gaucho Knots come under $p = 9 \mid c = 0$; $p = 13 \mid c = 0$; $\cdots$. The 3-pass Gaucho Knots come under $p = 13 \mid c = 0$; $p = 19 \mid c = 0$; $\cdots$. The 4-pass Gaucho Knots come under $p = 17 \mid c = 0$; $p = 25 \mid c = 0$; $\cdots$. And so on.

The 2-pass Headhunter's Knots come under $p = 7 \mid c = 2$; $p = 11 \mid c = 2$; $\cdots$. The 3-pass Headhunter's Knots come under $p = 10 \mid c = 3$; $p = 16 \mid c = 3$; $\cdots$. The 4-pass Headhunter's Knots come under $p = 13 \mid c = 4$; $p = 21 \mid c = 4$; $\cdots$. And so on.

Note that there are two types for each Headhunter's Knot entry, but that there is only one type for each Spanish Ring Knot and each Gaucho Knot entry.

† **Natural numbers**, or **positive integers** are the numbers $1, 2, 3, \cdots$.

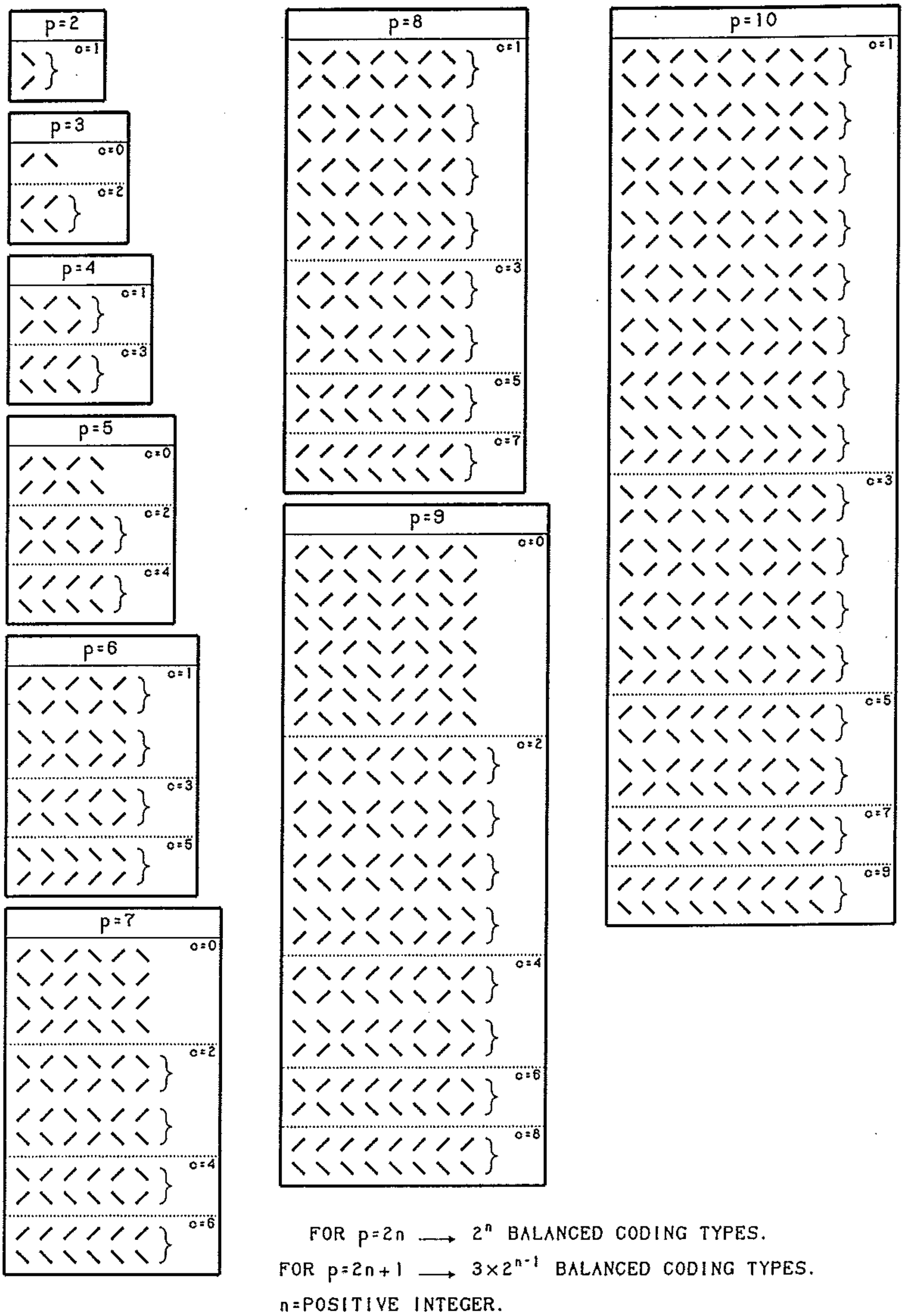


Fig. 437 — The balanced Column-coded Regular Cylindrical Braids for $2 \leq p \leq 10$.

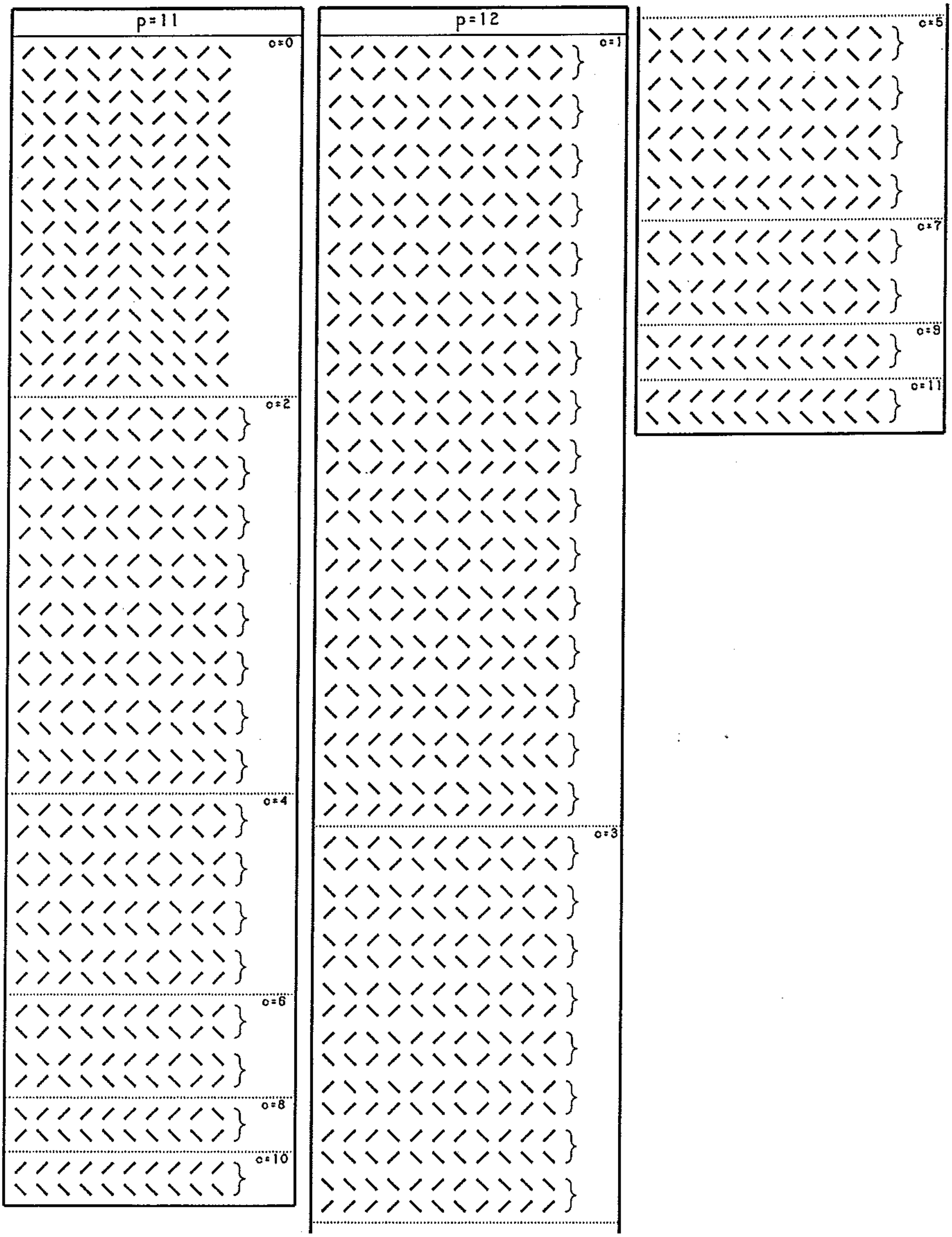


Fig. 438 — The balanced Column-coded Regular Cylindrical Braids for $11 \leq p \leq 12$.

Nested Cylindrical Braids

The number of Components is equal to A for the Standard Regular Nested Cylindrical Braids (g.c.d.$(P_c, B^*) = 1$) and Semi-Standard Regular Nested Cylindrical Braids (g.c.d.$(P_c, B^*) \neq 1$). For these braids there are two types of string-run with respect to the relative apex positions of the left-hand and the right-hand nests of bights: one in which the apex positions of the right-hand and the left-hand nests of bights line up exactly, and one in which the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights (see this Issue, pp. 513 – 515). The following Example illustrates the there obtained results:

Example 1:

Let $x = 10$ and $A = 7$. Then:

$$r_1 = \left|\tfrac{x}{2} - 1\right|_A = 4\,.$$

The associated string-run diagram is depicted in Fig. 439.

Let $x = 10$ and $A = 6$. Then:

(1) $$r_1 = \left|\tfrac{x}{2} - 1\right|_A = 4\,.$$

The associated string-run diagram is depicted at the left in Fig. 440.

(2) $$r_1 = \left|\tfrac{x+A}{2} - 1\right|_A = 1\,.$$

The associated string-run diagram is depicted at the right in Fig. 440.

Let $x = 11$ and $A = 5$. Then:

$$r_1 = \left|\tfrac{x+A}{2} - 1\right|_A = 2\,.$$

The associated string-run diagram is depicted in Fig. 441.

Note that for the Nested Cylindrical Braids the first-return string-runs do not change with $x = |x|_{\text{l.c.m.}(A_l, A_r)} + n \cdot \text{l.c.m.}(A_l, A_r)$, where n is a whole number.† Hence for the Standard and Semi-Standard Regular Nested Cylindrical Braids the first-return string-runs do not change with $x = |x|_A + nA$, while the first-return string-runs and their type do not change with $x = |x|_{2A} + 2nA$.

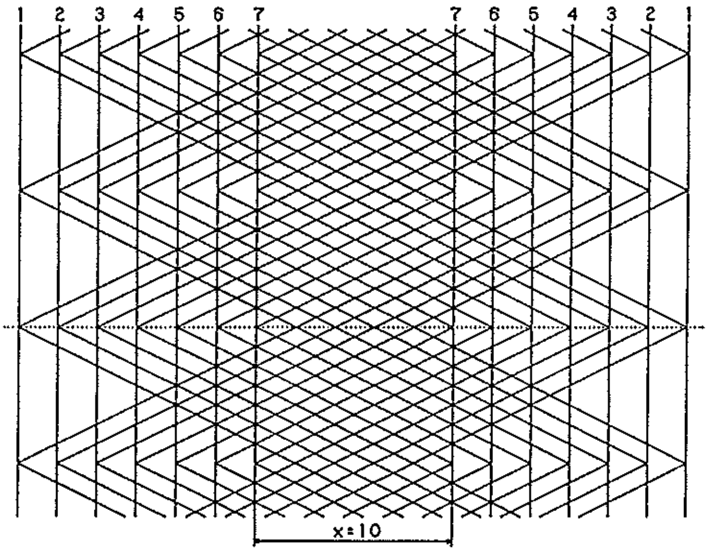


Fig. 439 — $x = 10$ and $A = 7$.

† l.c.m.(A_l, A_r) stands for the lowest common multiple of A_l and A_r.
Whole numbers are the numbers $0, 1, 2, 3, \cdots$.

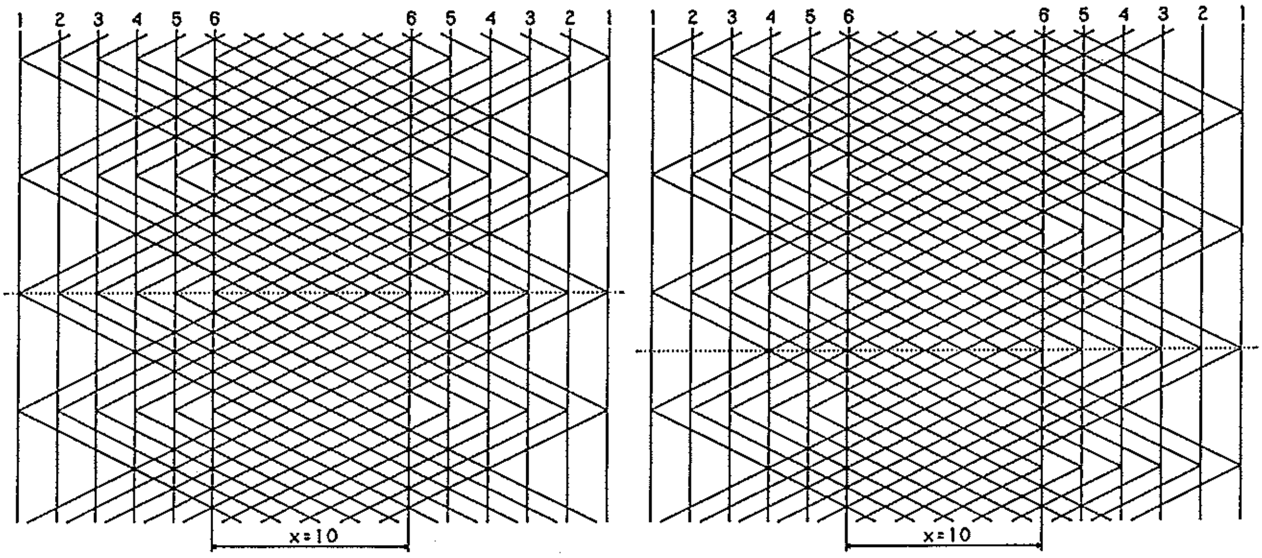


Fig. 440 — $x = 10$ and $A = 6$.

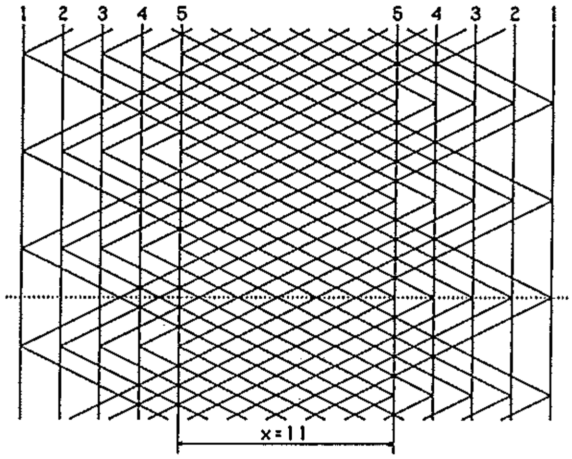


Fig. 441 — $x = 11$ and $A = 5$.

Since $\alpha = 1$ for the Standard and Semi-Standard Regular Nested Cylindrical Braids, we obtain $P_c = 4 + \frac{x-2(l_i+r_i)}{A}$.

When $r_1 = A$, hence when $x = 2 + nA$, where n is a whole number, we have the special case in which all A Components have the same number of parts $P_c = n + 2$. The associated lower-left to upper-right half-cycle types are tabulated in Fig. 442.

$l_i \longrightarrow r_i$	$l_i + r_i$
$1 \longrightarrow A$	$A+1$
$A \longrightarrow 1$	$A+1$
$A-1 \longrightarrow 2$	$A+1$
$A-2 \longrightarrow 3$	$A+1$
$\vdots$	$\vdots$
$4 \longrightarrow A-3$	$A+1$
$3 \longrightarrow A-2$	$A+1$
$2 \longrightarrow A-1$	$A+1$

Fig. 442 — The lower-left to upper-right half-cycle types when $r_1 = A$.

When n = odd, the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights, and when n = even, the apex positions of the right-hand nests of bights line exactly up with the apex positions of the left-hand nests of bights.

When $r_1 = k$, where $1 \le k < A$, hence where with $x = 2k + 2 + (2n-1)A$, where n is a whole number, the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights, and where with $x = 2k + 2 + 2nA$, where n is a whole number, the apex positions of the right-hand nests of bights line exactly up with the apex positions of the left-hand nests of bights, there are two different P_c values:

$(A-k)$ Components with $P_c = 2+\frac{x-2(k+1)}{A}$, and k Components with $P_c' = 4+\frac{x-2(k+1)}{A}$.

The associated lower-left to upper-right half-cycle types are tabulated in Fig. 443.

$l_i \longrightarrow r_i$	$l_i + r_i$
$1 \longrightarrow k$	$k+1$
$A \longrightarrow k+1$	$A+k+1$
$A-1 \longrightarrow k+2$	$A+k+1$
$A-2 \longrightarrow k+3$	$A+k+1$
$\vdots$	$\vdots$
$A-(z-1) \longrightarrow k+z$	$A+k+1$
$\vdots$	$\vdots$
$k+2 \longrightarrow A-1$	$A+k+1$
$k+1 \longrightarrow A$	$A+k+1$
$k \longrightarrow 1$	$k+1$
$k-1 \longrightarrow 2$	$k+1$
$k-2 \longrightarrow 3$	$k+1$
$\vdots$	$\vdots$
$4 \longrightarrow k-3$	$k+1$
$3 \longrightarrow k-2$	$k+1$
$2 \longrightarrow k-1$	$k+1$

Fig. 443 — The lower-left to upper-right half-cycle types when $r_1 = k$.

When the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights: $x = 2k + 2 + (2n-1)A$ with $P_c = 1 + 2n$ for $(A-k)$ Components and $P_c' = 3 + 2n$ for k Components, where n is a whole number. Thus $n = \frac{x+A-2k-2}{2A}$, and the components have an **odd** number of parts which differ by 2 for the two Component types.

When the apex positions of the right-hand nests of bights line up with the apex positions of the left-hand nests of bights: $x = 2k + 2 + 2nA$ with $P_c = 2 + 2n$ for $(A-k)$ Components and $P_c' = 4 + 2n$ for k Components, where n is a whole number. Thus $n = \frac{x-2k-2}{2A}$, and the components have an **even** number of parts which differ by 2 for the two Component types.

Example 2:

Let $x = 10$ and $A = 7$. Then: (see also Fig. 439 on pg. 519)

Since x = even and A = odd, the apex positions of the right-hand nests of bights line up with the apex positions of the left-hand nests of bights (see pg. 514).

$$r_i = r_1 = k = \left|\frac{x}{2} - 1\right|_A = \left|\frac{10}{2} - 1\right|_7 = |4|_7 = 4\,.$$

$$n = \frac{x - 2k - 2}{2A} = \frac{10 - 2\times 4 - 2}{2\times 7} = \frac{0}{14} = 0\,.$$

$$P_c = 2 + 2n = 2\,,\quad \text{number of Component} = A - k = 7 - 4 = 3\,.$$

$$P_c' = 4 + 2n = 4\,,\quad \text{number of Component} = k = 4\,.$$

$$P_{total} = 3\times 2 + 4\times 4 = 22 = x + 2A - 2\,.$$

Example 3:

Let $x = 10$ and $A = 6$. Then: (see also Fig. 440 on pg. 520)

Since $x =$ even and $A =$ even, the apex positions of the right-hand nests of bights line up with the apex positions of the left-hand nests of bights when: (see pg. 514)

$$r_i = r_1 = k = \left|\frac{x}{2} - 1\right|_A = \left|\frac{10}{2} - 1\right|_6 = |4|_6 = 4\,.$$

Then :

$$n = \frac{x - 2k - 2}{2A} = \frac{10 - 2\times 4 - 2}{2\times 6} = \frac{0}{12} = 0\,.$$

$$P_c = 2 + 2n = 2\,,\quad \text{number of Component} = A - k = 6 - 4 = 2\,.$$

$$P_c' = 4 + 2n = 4\,,\quad \text{number of Component} = k = 4\,.$$

$$P_{total} = 2\times 2 + 4\times 4 = 20 = x + 2A - 2\,.$$

The apex positions of the right-hand nests of bights fall midway between the apex positions of the left-hand nests of bights when: (see pg. 514)

$$r_i = r_1 = k = \left|\frac{x + A}{2} - 1\right|_A = \left|\frac{10 + 6}{2} - 1\right|_6 = |7|_6 = 1\,.$$

Then :

$$n = \frac{x + A - 2k - 2}{2A} = \frac{10 + 6 - 2\times 1 - 2}{2\times 6} = \frac{12}{12} = 1\,.$$

$$P_c = 1 + 2n = 3\,,\quad \text{number of Component} = A - k = 6 - 1 = 5\,.$$

$$P_c' = 3 + 2n = 5\,,\quad \text{number of Component} = k = 1\,.$$

$$P_{total} = 5\times 3 + 1\times 5 = 20 = x + 2A - 2\,.$$

Example 4:

Let $x = 11$ and $A = 5$. Then: (see also Fig. 441 on pg. 520)

Since $x =$ odd and $A =$ odd, the apex positions of the right-hand nests of bights fall midway between the apex positions of the left-hand nests of bights (see pg. 514).

$$r_i = r_1 = k = \left|\frac{x + A}{2} - 1\right|_A = \left|\frac{11 + 5}{2} - 1\right|_5 = |7|_5 = 2\,.$$

$$n = \frac{x + A - 2k - 2}{2A} = \frac{11 + 5 - 2\times 2 - 2}{2\times 5} = \frac{10}{10} = 1\,.$$

$$P_c = 1 + 2n = 3\,,\quad \text{number of Component} = A - k = 5 - 2 = 3\,.$$

$$P_c' = 3 + 2n = 5\,,\quad \text{number of Component} = k = 2\,.$$

$$P_{total} = 3\times 3 + 2\times 5 = 19 = x + 2A - 2\,.$$

Instead of starting with values for x and A, we can also start with values for P_{total} and A and then calculate the value of x from $P_{total} = x + 2A - 2$, hence with the formula $x = P_{total} + 2 - 2A$. However, in practice it is generally more convenient to start with values for x and A, since x measures the cylindrical length of the braid and A is a measure for its domed ends.

As we already mentioned on pg. 514, the well-known **Standard Herringbone Pineapple Knots** and the **Semi-Standard Herringbone Pineapple Knots** have a string-run in which the apex positions of the right-hand nests of bights fall exactly midway between the apex positions of the left-hand nests of bights. The essential coding for these knots is depicted in Fig. 444.

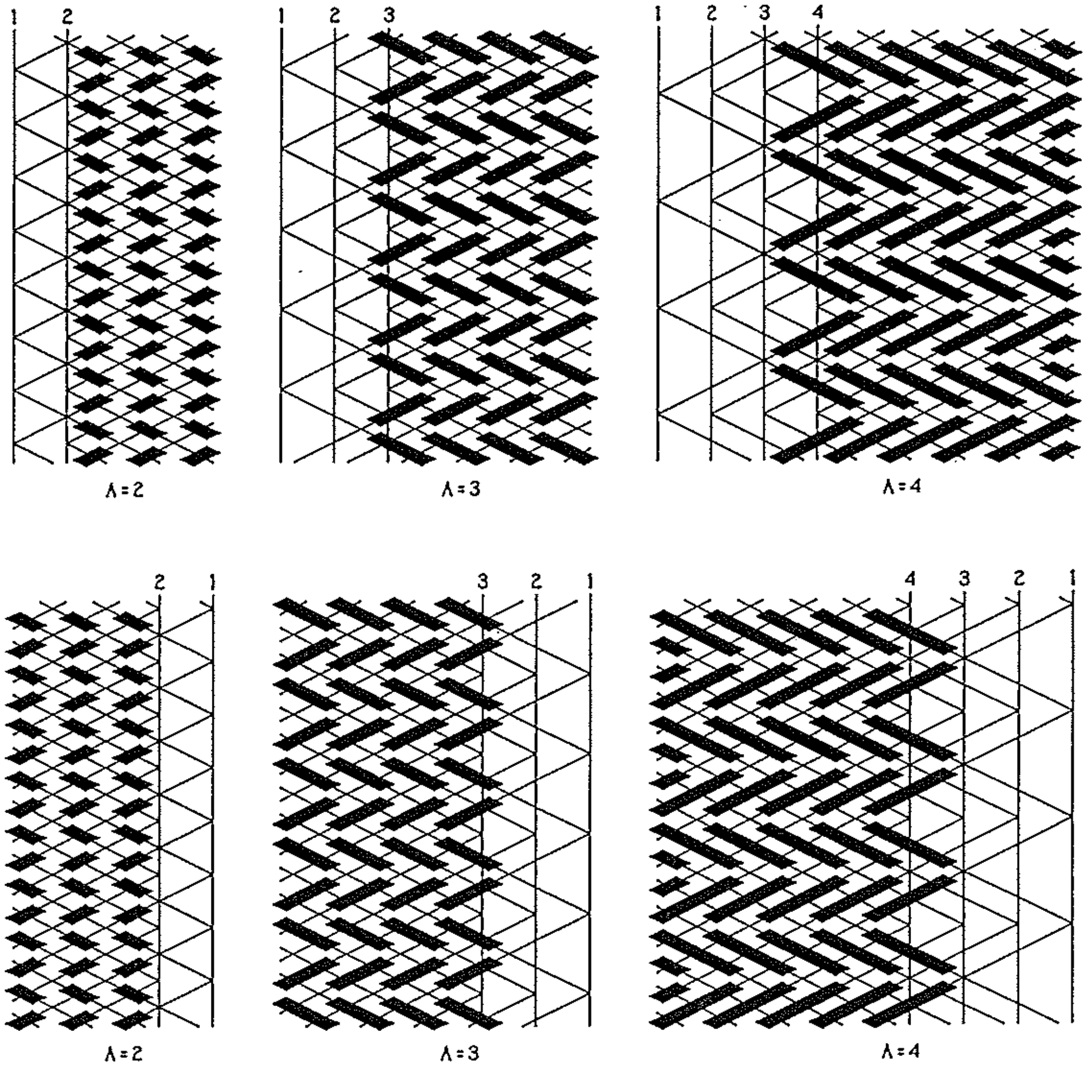


Fig. 444 — The essential coding.

The upper row of diagrams shows the essential coding at the left-hand bight-edge, while the lower row of diagrams shows the essential coding at the right-hand bight-edge. The rows which have not yet received a coding may be coded so as to obtain either the upper two rows of diagrams or the lower two rows of diagrams in Fig. 445. However, when the braiding-material is a flat string with opposite face pairs identical in size, shape, texture and colour, it should be noted that the upper two rows of diagrams are identical to the lower two rows of diagrams (turn one set of diagrams through an angle of 180° about an axis perpendicular to the paper in order to obtain the other set).

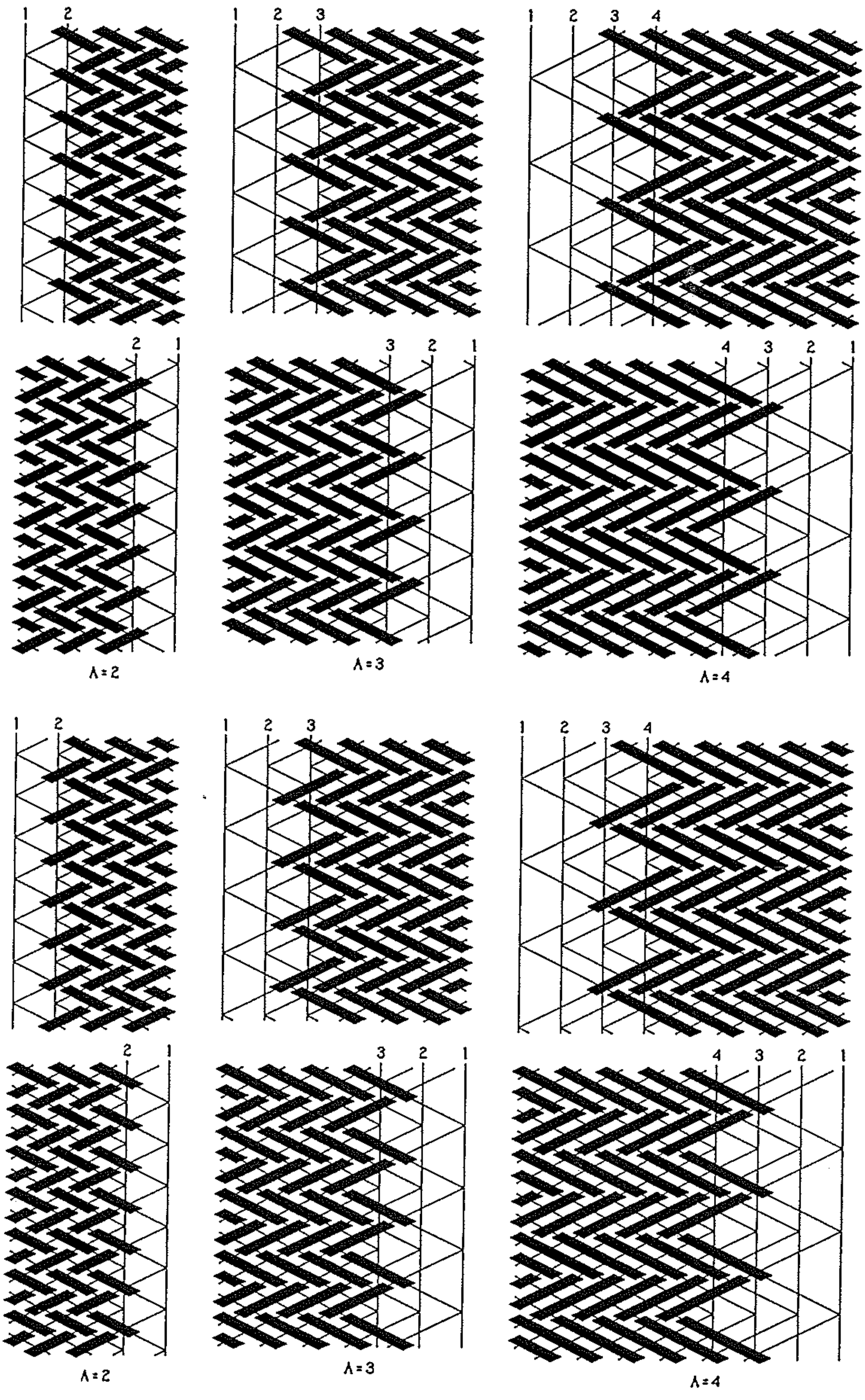


Fig. 445 — The Herringbone-coding.

Fig. 446 shows in relation to the left-hand and right-hand bight-edges once again for the Standard and Semi-Standard Herringbone Pineapple Knots the two identical herringbone coding types. Note that although the coding at the right-hand and left-hand bight-edges are each others mirror image (lateral inversions), they are not lateral equivalent.

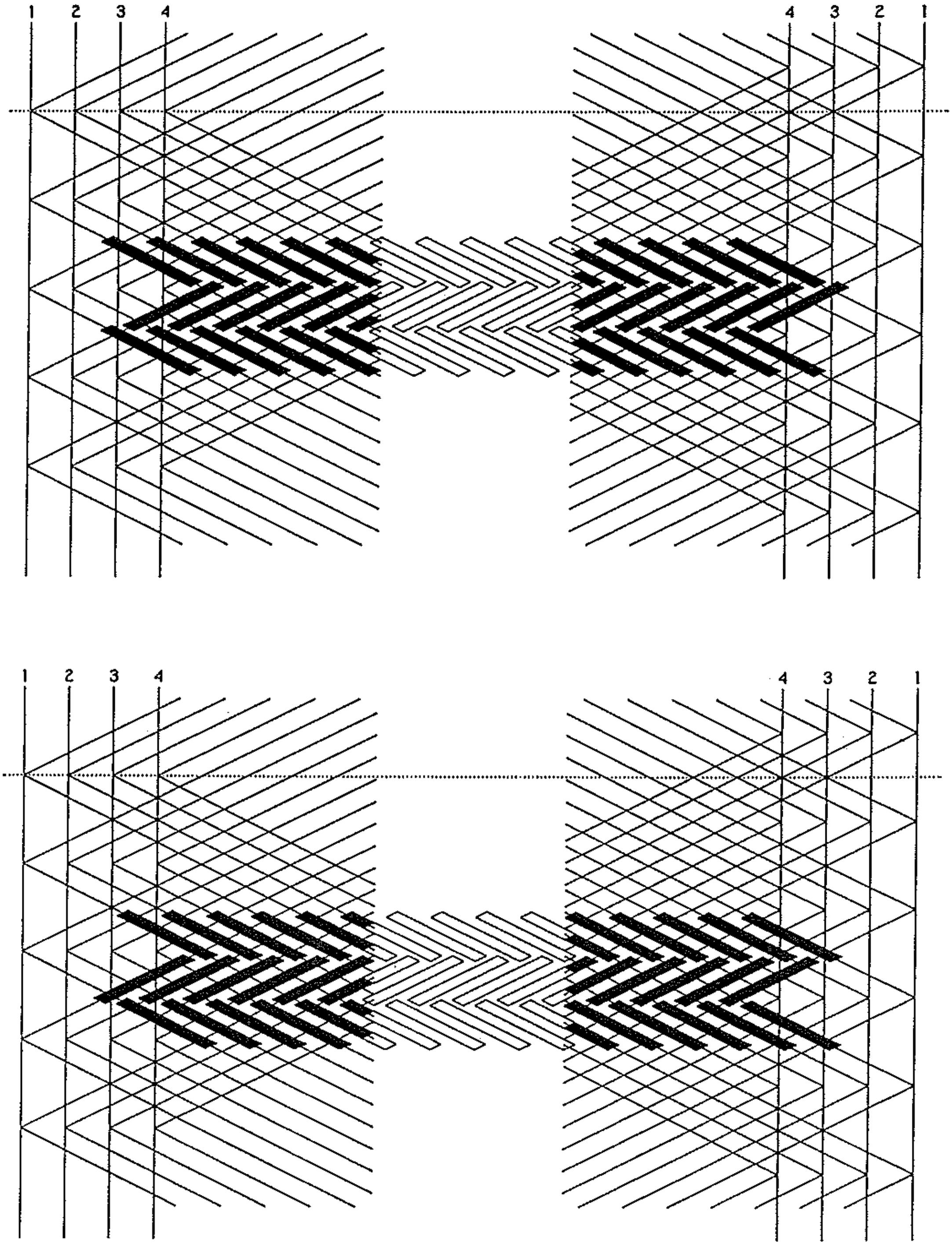


Fig. 446 — The Standard and Semi-Standard Herringbone Pineapple Knots.

Since there is thus only one general herringbone coding-form for nesting-number A, it is sufficient to consider the herringbone coding-form depicted in the diagrams of the upper two rows in Fig. 445, hence the herringbone coding-form depicted in the upper diagram of Fig. 446.

A Standard and a Semi-Standard A-pass Herringbone Pineapple Knot respectively consists of A interbraided over–under coded Regular and Semi-Regular Knots. When $l_1 = 1$ and $r_1 = A$, the interbraided knots are identical with the same odd number of parts each. When $l_1 = 1$ and $r_1 = k < A$, the interbraided knots have an odd number of parts each: $(A-k)$ with $p = (2m-1)$ parts each and k with $p = (2m+1)$ parts each, where m is a natural number.

A lower-left to upper-right half-cycle running from l_i to r_i has the coding-sequence:

$$(l_i)u - Ao - Au - \cdots - Au - Ao - (r_i - 1)u\,,$$

in which each set of the p sets of crossings, except the last set $(r_i - 1)u$, has one crossing belonging to the string-run of the knot between the bight-boundaries l_i and r_i.

A lower-right to upper-left half-cycle running from r_i to l_i has the coding-sequence:

$$(r_i)u - Ao - Au - \cdots - Au - Ao - (l_i - 1)u\,,$$

in which each set of the p sets of crossings, except the last set $(l_i - 1)u$, has one crossing belonging to the string-run of the knot between the bight-boundaries l_i and r_i.

Hence the reference half-cycle sequences, consequently the first lower-left to upper-right half-cycle sequence, are

from lower-left l_i to upper-right r_i :

$$(l_i - 1)u - (A-1)o - (A-1)u - \cdots - (A-1)u - (A-1)o - (r_i - 1)u\,.$$

from lower-right r_i to upper-left l_i :

$$(r_i - 1)u - (A-1)o - (A-1)u - \cdots - (A-1)u - (A-1)o - (l_i - 1)u\,.$$

It will now be evident that for the Standard and Semi-Standard Herringbone Pineapple Knots we can, for the determination of the half-cycle algorithms of an interbraided over–under coded Regular respectively Semi-Regular Knot, employ, with a small modification, their algorithm diagrams.

The general form of such an algorithm diagram for an interbraided over–under coded Regular Knot between the bight-boundaries l_i and r_i is as follows:

$$\begin{array}{ccccccccc}
\longrightarrow & l_i - 1 & A-1 & A-1 & \ldots & A-1 & A-1 & A-1 & r_i - 1 \\
0 & \star & \star & \star & \ldots & \star & \star & \star & \\
/ & \backslash & / & \backslash & \ldots & / & \backslash & / & \backslash \\
 & \star & \star & \star & \ldots & \star & \star & \star & 0 \\
l_i - 1 & A-1 & A-1 & A-1 & \ldots & A-1 & A-1 & r_i - 1 & \longleftarrow
\end{array}$$

The positions of the stars are occupied by the i-values of the complementary bight-number scheme associated with the knot to be interbraided. The value above an upper star, or below a lower star, increases by 1 when its associated i-value is applicable to the half-cycle concerned. The last entry for the lower-left to upper-right half-cycles, $(r_i - 1)u$, remains the same for all of them. Similarly for all the lower-right to upper-left half-cycles, the last entry $(l_i - 1)u$ remains the same.

Example 5:

Let $A = 5$; $B^* = 4$; $x = 23$. Then for a Standard or Semi-Standard Herringbone Pineapple Knot :

For $l_1 = 1 \quad \longrightarrow \quad r_1 = \left|\dfrac{x+A}{2} - 1\right|_A = \left|\dfrac{23+5}{2} - 1\right|_5 = 3 = k\,.$

$$n = \frac{x + A - 2k - 2}{2A} = \frac{23 + 5 - 2 \times 3 - 2}{2 \times 5} = 2\,,\text{hence :}$$

$P_c = 1 + 2n = 1 + 2 \times 2 = 5$ for $A - k = 5 - 3 = 2$ Components.

$P'_c = 3 + 2n = 3 + 2 \times 2 = 7$ for $k = 3$ Components.

Since:

$$\text{g.c.d.}\,(P_c, B^*) = \text{g.c.d.}\,(5,4) = 1 \text{ and}$$
$$\text{g.c.d.}\,(P'_c, B^*) = \text{g.c.d.}\,(7,4) = 1\,,$$

the string-run is that of a Standard Herringbone Pineapple Knot (one essential string for each Component). The string-run diagram and the grid-diagram are depicted in Fig. 447.

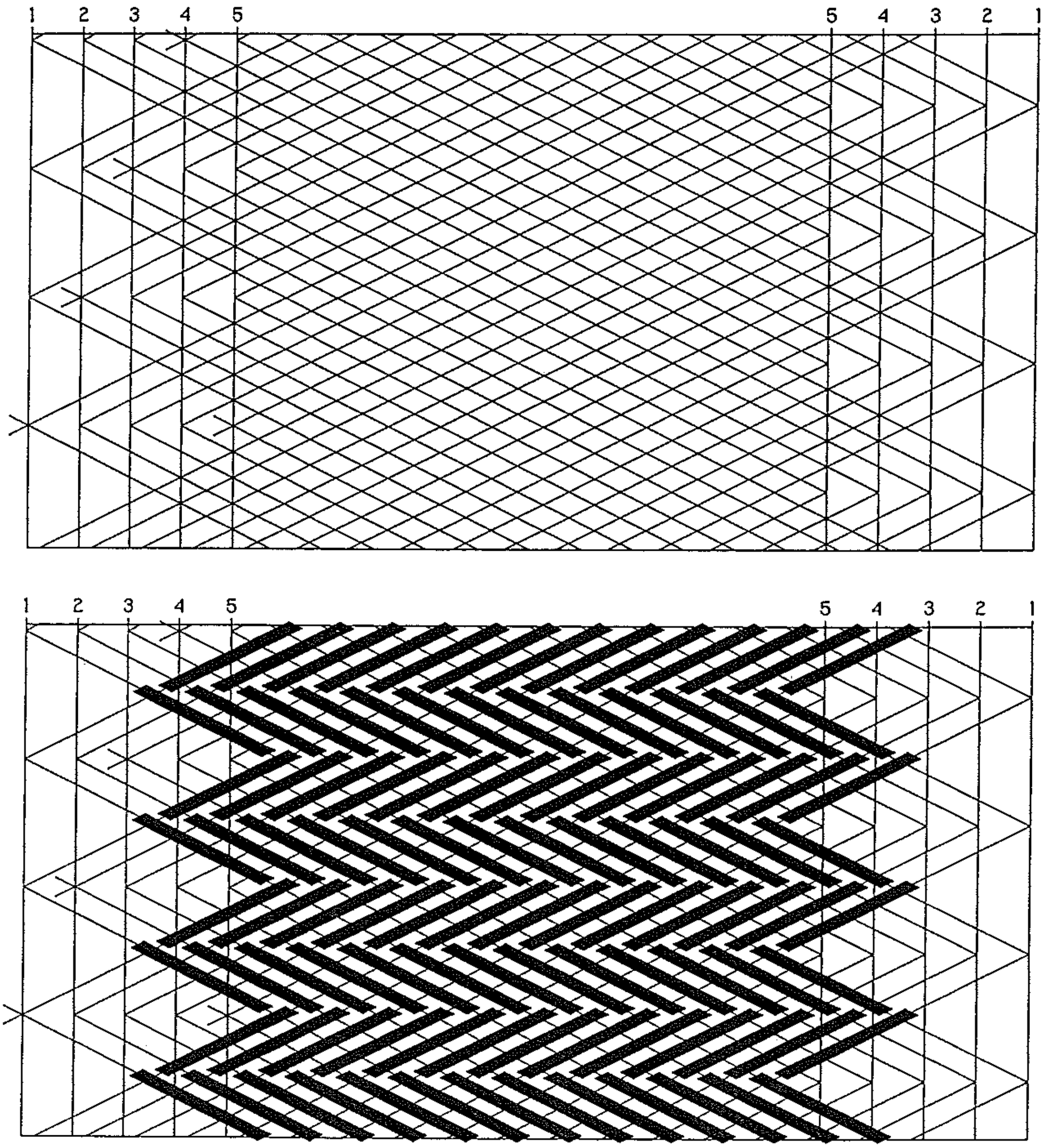


Fig. 447 — A 5-pass Standard Herringbone Pineapple Knot with $x = 23$ and $B^* = 4$.

This knot can be braided in 120 different ways†. We shall discuss two of these 120 different ways, randomly selected, in this example.

† An A-pass Standard Herringbone Pineapple Knot can be braided in $A!$ different ways. $A!$, pronounced as **A factorial**, stands for $1 \times 2 \times 3 \times 4 \times \cdots \times (A-2) \times (A-1) \times A$.

(1). Say that we first braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 1 and its right bight-boundary 3. For this knot $p/b = 7/4$, with $A = 1$ and $l_i = 1$; $r_i = 1$. Its algorithm diagram is thus:

$$\begin{array}{cccccccc}
\longrightarrow & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 3 & 0 & 1 & 2 & \\
/ & \backslash & / & \backslash & / & \backslash & / & \backslash \\
 & 2 & 1 & 0 & 3 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \longleftarrow
\end{array}$$

Its half-cycle algorithms are therefore:

half-cycle 1		:	$L \longrightarrow R$	Free Run.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	o.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	o.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - o - u$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$u - o - u$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$u - 2o - u - o$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$u - 2o - u - o$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$u - o - u - o - u - o$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 2 and its right bight-boundary 2. For this knot $p/b = 7/4$, with $A = 2$ and $l_i = 2$; $r_i = 1$. Its algorithm diagram is thus:

$$\begin{array}{cccccccc}
\longrightarrow & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 2 & 3 & 0 & 1 & 2 & \\
/ & \backslash & / & \backslash & / & \backslash & / & \backslash \\
 & 2 & 1 & 0 & 3 & 2 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & \longleftarrow
\end{array}$$

Its half-cycle algorithms are therefore:

half-cycle 1		:	$L \longrightarrow R$	$u - o - u - o - u - o$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$o - u - 2o - u - o - u$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$u - o - u - 2o - u - o$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - o - u - 2o - 2u - o - u$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$2u - o - u - 2o - 2u - o$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$u - 2o - u - 2o - 2u - 2o - u$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$2u - 2o - u - 2o - 2u - 2o$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$u - 2o - 2u - 2o - 2u - 2o - u$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 3 and its right bight-boundary 1. For this knot $p/b = 7/4$, with $A = 3$ and $l_i = 3$; $r_i = 1$. Its algorithm diagram is thus:

$$\begin{array}{cccccccc}
\longrightarrow & 2 & 2 & 2 & 2 & 2 & 2 & 0 \\
0 & 1 & 2 & 3 & 0 & 1 & 2 & \\
/ & \backslash & / & \backslash & / & \backslash & / & \backslash \\
 & 2 & 1 & 0 & 3 & 2 & 1 & 0 \\
2 & 2 & 2 & 2 & 2 & 2 & 0 & \longleftarrow
\end{array}$$

Its half-cycle algorithms are therefore:

half-cycle 1		:	$L \longrightarrow R$	$2u - 2o - 2u - 2o - 2u - 2o$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$2o - 2u - 3o - 2u - 2o - 2u$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$2u - 2o - 2u - 3o - 2u - 2o$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - 2o - 2u - 3o - 3u - 2o - 2u$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$3u - 2o - 2u - 3o - 3u - 2o$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$u - 3o - 2u - 3o - 3u - 3o - 2u$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$3u - 3o - 2u - 3o - 3u - 3o$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$u - 3o - 3u - 3o - 3u - 3o - 2u$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 4 and its right bight-boundary 5. For this knot $p/b = 5/4$, with $A = 4$ and $l_i = 4$; $r_i = 4$. Its algorithm diagram is thus:

```
--->  3  3  3  3    3
 0    3  2  1  0
 /    \  /  \  /    \
      0  1  2  3    0
 3    3  3  3  3  <---
```

Its half-cycle algorithms are therefore:

half-cycle 1		:	$L \longrightarrow R$	$3u - 3o - 3u - 3o - 3u$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$3u - 3o - 3u - 4o - 3u$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$3u - 3o - 3u - 4o - 3u$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$3u - 3o - 4u - 4o - 3u$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$3u - 3o - 4u - 4o - 3u$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$3u - 4o - 4u - 4o - 3u$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$3u - 4o - 4u - 4o - 3u$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$4u - 4o - 4u - 4o - 3u$.

Finally we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 5 and its right bight-boundary 4. For this knot $p/b = 5/4$, with $A = 5$ and $l_i = 5$; $r_i = 4$. Its algorithm diagram is thus:

```
--->  4  4  4  4    3
 0    3  2  1  0
 /    \  /  \  /    \
      0  1  2  3    0
 4    4  4  4  3  <---
```

Its half-cycle algorithms are therefore:

half-cycle 1		:	$L \longrightarrow R$	$4u - 4o - 4u - 4o - 3u$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$3u - 4o - 4u - 5o - 4u$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$4u - 4o - 4u - 5o - 3u$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$3u - 4o - 5u - 5o - 4u$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$4u - 4o - 5u - 5o - 3u$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$3u - 5o - 5u - 5o - 4u$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$4u - 5o - 5u - 5o - 3u$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$4u - 5o - 5u - 5o - 4u$.

The 5-pass Standard Herringbone Pineapple Knot has now been completed.

Instead of using the five algorithm diagrams, one for each interbraided over–under coded Regular Knot, we could have used a general algorithm diagram for the $p/b = 7/4$

over–under coded Regular Knots, and a general algorithm diagram for the $p/b = 5/4$ over–under coded Regular Knots. These general algorithm diagrams are respectively:

$\longrightarrow$	$l_i - 1$	$A-1$	$A-1$	$A-1$	$A-1$	$A-1$	$r_i - 1$
0	1	2	3	0	1	2	
/	\	/	\	/	\	/	\
	2	1	0	3	2	1	0
$l_i - 1$	$A-1$	$A-1$	$A-1$	$A-1$	$A-1$	$r_i - 1$	$\longleftarrow$

$\longrightarrow$	$l_i - 1$	$A-1$	$A-1$	$A-1$	$r_i - 1$
0	3	2	1	0	
/	\	/	\	/	\
	0	1	2	3	0
$l_i - 1$	$A-1$	$A-1$	$A-1$	$r_i - 1$	$\longleftarrow$

(2). Lets use these general algorithm diagrams, and say that we first braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 4 and its right bight-boundary 5. For this knot $p/b = 5/4$, with $A = 1$ and $l_i = 1$; $r_i = 1$. Hence we obtain the following half-cycle algorithms from its associated general algorithm diagram:

half-cycle 1		:	$L \longrightarrow R$	Free Run.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	o.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	o.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - o$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$u - o$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$o - u - o$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$o - u - o$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$u - o - u - o$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 2 and its right bight-boundary 2. For this knot $p/b = 7/4$, with $A = 2$ and $l_i = 1$; $r_i = 1$. Hence from its associated general algorithm diagram we obtain the following half-cycle algorithms:

half-cycle 1		:	$L \longrightarrow R$	$o - u - o - u - o$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$o - u - 2o - u - o$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$o - u - 2o - u - o$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - o - u - 2o - 2u - o$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$u - o - u - 2o - 2u - o$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$u - 2o - u - 2o - 2u - 2o$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$u - 2o - u - 2o - 2u - 2o$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$u - 2o - 2u - 2o - 2u - 2o$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 5 and its right bight-boundary 4. For this knot $p/b = 5/4$, with $A = 3$ and $l_i = 3$; $r_i = 2$. Hence we obtain the following half-cycle algorithms from its associated general algorithm diagram:

half-cycle 1		:	$L \longrightarrow R$	$2u - 2o - 2u - 2o - u$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$u - 2o - 2u - 3o - 2u$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$2u - 2o - 2u - 3o - u$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - 2o - 3u - 3o - 2u$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$2u - 2o - 3u - 3o - u$.

half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$u - 3o - 3u - 3o - 2u$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$2u - 3o - 3u - 3o - u$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$2u - 3o - 3u - 3o - 2u$.

Next we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 1 and its right bight-boundary 3. For this knot $p/b = 7/4$, with $A = 4$ and $l_i = 1$; $r_i = 2$. Hence from its associated general algorithm diagram we obtain the following half-cycle algorithms:

half-cycle 1		:	$L \longrightarrow R$	$3o - 3u - 3o - 3u - 3o - u$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$u - 3o - 3u - 4o - 3u - 3o$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$3o - 3u - 4o - 3u - 3o - u$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$2u - 3o - 3u - 4o - 4u - 3o$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$u - 3o - 3u - 4o - 4u - 3o - u$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$2u - 4o - 3u - 4o - 4u - 4o$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$u - 4o - 3u - 4o - 4u - 4o - u$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$2u - 4o - 4u - 4o - 4u - 4o$.

Finally we braid in the $A = 5$ Standard Herringbone Pineapple Knot the over–under coded Regular Knot between its left bight-boundary 3 and its right bight-boundary 1. For this knot $p/b = 7/4$, with $A = 5$ and $l_i = 3$; $r_i = 1$. Hence from its associated general algorithm diagram we obtain the following half-cycle algorithms:

half-cycle 1		:	$L \longrightarrow R$	$2u - 4o - 4u - 4o - 4u - 4o$.
half-cycle 2	$i = 0$	:	$L \longleftarrow R$	$4o - 4u - 5o - 4u - 4o - 2u$.
half-cycle 3	$i = 0$	:	$L \longrightarrow R$	$2u - 4o - 4u - 5o - 4u - 4o$.
half-cycle 4	$i = 1$	:	$L \longleftarrow R$	$u - 4o - 4u - 5o - 5u - 4o - 2u$.
half-cycle 5	$i = 1$	:	$L \longrightarrow R$	$3u - 4o - 4u - 5o - 5u - 4o$.
half-cycle 6	$i = 2$	:	$L \longleftarrow R$	$u - 5o - 4u - 5o - 5u - 5o - 2u$.
half-cycle 7	$i = 2$	:	$L \longrightarrow R$	$3u - 5o - 4u - 5o - 5u - 5o$.
half-cycle 8	$i = 3$	:	$L \longleftarrow R$	$u - 5o - 5u - 5o - 5u - 5o - 2u$.

The 5-pass Standard Herringbone Pineapple Knot has now been completed.

From a general algorithm diagram for an interbraided over–under coded Regular Knot, we can compile its associated general half-cycle algorithm table. This has been done in *Book 4/1, Braiding — Standard Herringbone Pineapple Knots* for B^*-values ranging from 3 to 7 inclusive, paired with P_c-values ranging from 3 to 13 inclusive. Especially the novice might find using those tables easier.

A Grant Knot

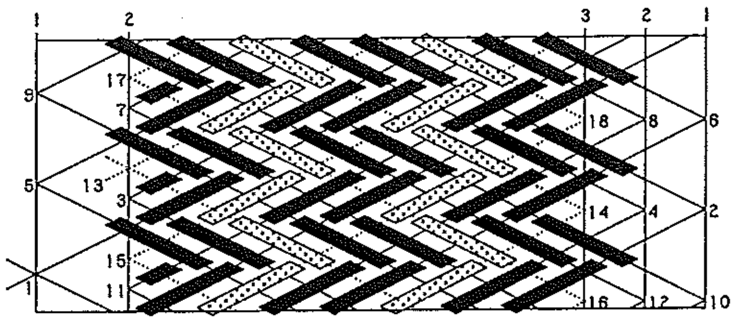


Fig. 448 — (3/15/22){122/123}9.

The Grant Knot in Fig. 448 is an interweave of a Perfect Pineapple Knot and an over–under coded Regular Knot. For the numerals of the left sequence set 122 we obtain with the left bight-boundary position specification 3 and $A_l = 3$ the following ranking-numbers j :†

$n = 1$ for numeral $k = 1$, hence its ranking-number $j = |1 + 0|_3 = 1$.
$n = 2$ for numeral $k = 2$, hence its ranking-number $j = |2 + 3|_3 = 2$.
$n = 3$ for numeral $k = 2$, hence its ranking-number $j = |3 + 3|_3 = 0 = 3$.

Hence with the ranking-numbers the Nested Cylindrical Braid specification becomes:

$$(3/15/22)\{1_1 2_2 2_3 / 1_1 2_2 3_3\}9 .$$

From this specification we read the lower-left to upper-right half-cycle types:

$$1_1 \longrightarrow 1_1$$
$$2_2 \longrightarrow 2_2$$
$$2_3 \longrightarrow 3_3$$

Furthermore with $\mathcal{K}_l = 2$ and $\mathcal{K}_r = 3$:

$\Delta_{l_i} = 3$ for $l_i = 1$. $\quad$ $\Delta_{r_i} = 4$ for $r_i = 1$.
$\Delta_{l_i} = 0$ for $l_i = 2$. $\quad$ $\Delta_{r_i} = 2$ for $r_i = 2$.
$\Delta_{r_i} = 0$ for $r_i = 3$.

For the first-return string-runs we thus obtain:

$$\begin{array}{l} 1_1 \\ \quad\nwarrow\!\searrow 2_2 \\ \quad\nearrow \\ 2_2 \\ \quad\nwarrow\!\searrow 1_1 \\ \quad\nearrow \\ 1_1 \end{array}
\qquad
\begin{array}{lllll}
l_{1_{j_l}} = 1_1 \longrightarrow 1_1 = r_{1_{j_r}} & \rightarrow & j'_l = |1+3+15+4|_3 = 2 & \rightarrow & l_{2_{j'_l}} = 2_2 . \\
l_{2_{j'_l}} = 2_2 \longleftarrow 1_1 = r_{1_{j_r}} & \rightarrow & j'_r = |1+4+15+0|_3 = 2 & \rightarrow & r_{2_{j'_r}} = 2_2 . \\
l_{2_{j_l}} = 2_2 \longrightarrow 2_2 = r_{2_{j_r}} & \rightarrow & j'_l = |2+0+15+2|_3 = 1 & \rightarrow & l_{3_{j'_l}} = 1_1 . \\
l_{3_{j'_l}} = 1_1 \longleftarrow 2_2 = r_{2_{j_r}} & \rightarrow & j'_r = |2+2+15+3|_3 = 1 & \rightarrow & r_{3_{j'_r}} = 1_1 .
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{2 \cdot 15 + (3+0) + (4+2)}{3} = 13 .$$

g.c.d. (P_c , B^{**}) = g.c.d. $(13 , 3) = 1$.

$$\begin{array}{l} 2_3 \\ \quad\nwarrow\!\searrow 3_3 \\ \quad\nearrow \\ 2_3 \end{array}
\qquad
\begin{array}{lllll}
l_{1_{j_l}} = 2_3 \longrightarrow 3_3 = r_{1_{j_r}} & \rightarrow & j'_l = |3+0+15+0|_3 = 3 & \rightarrow & l_{2_{j'_l}} = 2_3 . \\
l_{2_{j'_l}} = 2_3 \longleftarrow 3_3 = r_{1_{j_r}} & \rightarrow & j'_r = |3+0+15+0|_3 = 3 & \rightarrow & r_{2_{j'_r}} = 3_3 .
\end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 15 + (0) + (0)}{3} = 5 .$$

g.c.d. (P_c , B^{**}) = g.c.d. $(5 , 3) = 1$.

Hence:

$$P_{total} = \sum P_{component} = 13 + 5 = 18 .$$

† For the formulae used in the following calculations refer to *The Braider*, Issue No. 19, pp. 415–422.

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 2\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1 + 1 = 2\,.$$

1^{st} component; 1^{st} colour; between bight-boundaries 1 & 2 (left) and 1 & 2 (right).

1. $L \longrightarrow R$: Free Run.
2. $R \longrightarrow L$: $o - u$.
3. $L \longrightarrow R$: u.
4. $R \longrightarrow L$: $u - 2o$.
5. $L \longrightarrow R$: $u - 2o - u$.
6. $R \longrightarrow L$: $2u - 3o - u$.
7. $L \longrightarrow R$: $2o - 3u$.
8. $R \longrightarrow L$: $o - 2u - o - u - 2o$.
9. $L \longrightarrow R$: $o - 2u - o - u - 2o - u$.
10. $R \longrightarrow L$: $2o - 2u - o - 2u - 2o - u$.
11. $L \longrightarrow R$: $2u - 2o - u - 2o - 2u$.
12. $R \longrightarrow L$: $u - 2o - 2u - 2o - 2u - 2o$.

2^{nd} component; 2^{nd} colour; between bight-boundaries 2 (left) and 3 (right).

13. $L \longrightarrow R$: $2u - 2o - 2u - 2o - 2u$.
14. $R \longrightarrow L$: $2u - 2o - 2u - 3o - 2u$.
15. $L \longrightarrow R$: $2u - 2o - 2u - 3o - 2u$.
16. $R \longrightarrow L$: $2u - 3o - 2u - 3o - 3u$.
17. $L \longrightarrow R$: $2u - 3o - 2u - 3o - 3u$.
18. $R \longrightarrow L$: $2u - 3o - 3u - 3o - 3u$.

A Pineapple Knot

The Pineapple Knot in Fig. 449 is an interweave of two Perfect Pineapple Knots.

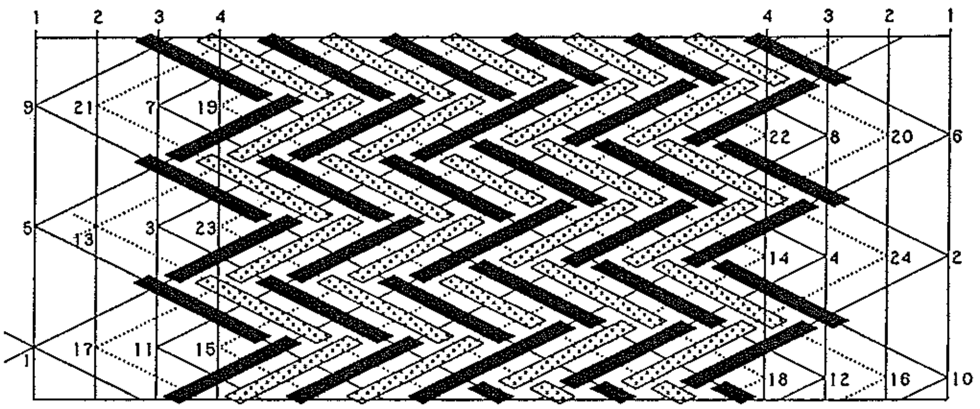


Fig. 449 — (222/18/222){1432/1234}12.

This is a Regular Nested Cylindrical Braid, and as we have seen in *The Braider*, Issue No. 22, pg. 510, we don't require the ranking-numbers. We can therefore use the simplified formulae on pg. 510.

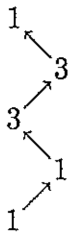
From the Regular Nested Cylindrical Braid specification we obtain:

$$
\begin{aligned}
A &= 4\,. \\
x &= 18\,. \\
d &= A = 4\,. \\
B_{total} &= AB^* = 12\,. \\
B^* &= \frac{B_{total}}{A} = \frac{12}{4} = 3\,. \\
\mathcal{K}_l &= \mathcal{K}_r = \mathcal{K} = A = 4\,.
\end{aligned}
$$

From the given Regular Nested Cylindrical Braid specification we can read the lower-left to upper-right half-cycle types:

$$
\begin{array}{lcl}
1 \longrightarrow 1 & \qquad & 3 \longrightarrow 3 \\
4 \longrightarrow 2 & \qquad & 2 \longrightarrow 4
\end{array}
$$

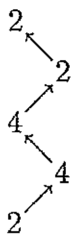
For the first-return string-runs we thus obtain:



$$
\begin{array}{lcl}
l_1 = 1 \longrightarrow 1 = r_1 & \rightarrow & l_2 = |1 + 18 - 2(1+1)|_4 = 3\,. \\
l_2 = 3 \longleftarrow 1 = r_1 & \rightarrow & r_2 = |1 + 18 - 2(1+3)|_4 = 3\,. \\
l_2 = 3 \longrightarrow 3 = r_2 & \rightarrow & l_3 = |3 + 18 - 2(3+3)|_4 = 1\,. \\
l_3 = 1 \longleftarrow 3 = r_2 & \rightarrow & r_3 = |3 + 18 - 2(3+1)|_4 = 1\,.
\end{array}
$$

$$P_c = 4\alpha + \frac{\alpha x - 2\sum(l_i + r_i)}{A} = 4 \cdot 2 + \frac{2 \cdot 18 - 2\{(1+3) + (1+3)\}}{4} = 13\,.$$

g.c.d. $(P_c\,,\, B^*) =$ g.c.d. $(13\,,\, 3) = 1\,.$



$$
\begin{array}{lcl}
l_1 = 2 \longrightarrow 4 = r_1 & \rightarrow & l_2 = |2 + 18 - 2(2+4)|_4 = 4\,. \\
l_2 = 4 \longleftarrow 4 = r_1 & \rightarrow & r_2 = |4 + 18 - 2(4+4)|_4 = 2\,. \\
l_2 = 4 \longrightarrow 2 = r_2 & \rightarrow & l_3 = |4 + 18 - 2(4+2)|_4 = 2\,. \\
l_3 = 2 \longleftarrow 2 = r_2 & \rightarrow & r_3 = |2 + 18 - 2(2+2)|_4 = 4\,.
\end{array}
$$

$$P_c = 4\alpha + \frac{\alpha x - 2\sum(l_i + r_i)}{A} = 4 \cdot 2 + \frac{2 \cdot 18 - 2\{(2+4) + (4+2)\}}{4} = 11\,.$$

g.c.d. $(P_c\,,\, B^*) =$ g.c.d. $(11\,,\, 3) = 1\,.$

Hence:

$$P_{total} = \sum P_{component} = 13 + 11 = 24\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 2\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 1 + 1 = 2\,.$$

1^{st} component; 1^{st} colour; between bight-boundaries 1 & 3 (left) and 1 & 3 (right).

1. $L \longrightarrow R$: Free Run.
2. $R \longrightarrow L$: $o - u$.
3. $L \longrightarrow R$: u.
4. $R \longrightarrow L$: $u - 2o$.
5. $L \longrightarrow R$: $u - 2o - u$.
6. $R \longrightarrow L$: $2u - 3o - u$.
7. $L \longrightarrow R$: $2o - 3u$.
8. $R \longrightarrow L$: $o - 2u - o - u - 2o$.
9. $L \longrightarrow R$: $o - 2u - o - u - 2o - u$.
10. $R \longrightarrow L$: $2o - 2u - o - 2u - 2o - u$.
11. $L \longrightarrow R$: $2u - 2o - u - 2o - 2u$.
12. $R \longrightarrow L$: $u - 2o - 2u - 2o - 2u - 2o$.

2^{nd} component; 2^{nd} colour; between bight-boundaries 2 & 4 (left) and 2 & 4 (right).

13. $L \longrightarrow R$: $u - 2o - 3u - 2o - 2u$.
14. $R \longrightarrow L$: $2u - 2o - u - 3o - 2u$.
15. $L \longrightarrow R$: $2u - 2o - 4u - 2o - u$.
16. $R \longrightarrow L$: $u - 3o - 2u - 2o - 3u - 2o - u$.
17. $L \longrightarrow R$: $u - 2o - 4u - 3o - 2u$.
18. $R \longrightarrow L$: $2u - 3o - 2u - 3o - 3u$.
19. $L \longrightarrow R$: $4u - 2o - 4u - 4o - u$.
20. $R \longrightarrow L$: $u - 4o - 3u - 2o - 4u - 3o - u$.
21. $L \longrightarrow R$: $2u - 3o - 4u - 4o - 3u$.
22. $R \longrightarrow L$: $3u - 3o - 3u - 4o - 3u$.
23. $L \longrightarrow R$: $4u - 4o - 4u - 4o - 2u$.
24. $R \longrightarrow L$: $u - 4o - 4u - 3o - 4u - 4o - u$.

Some starts for a 6 thong Round Braid

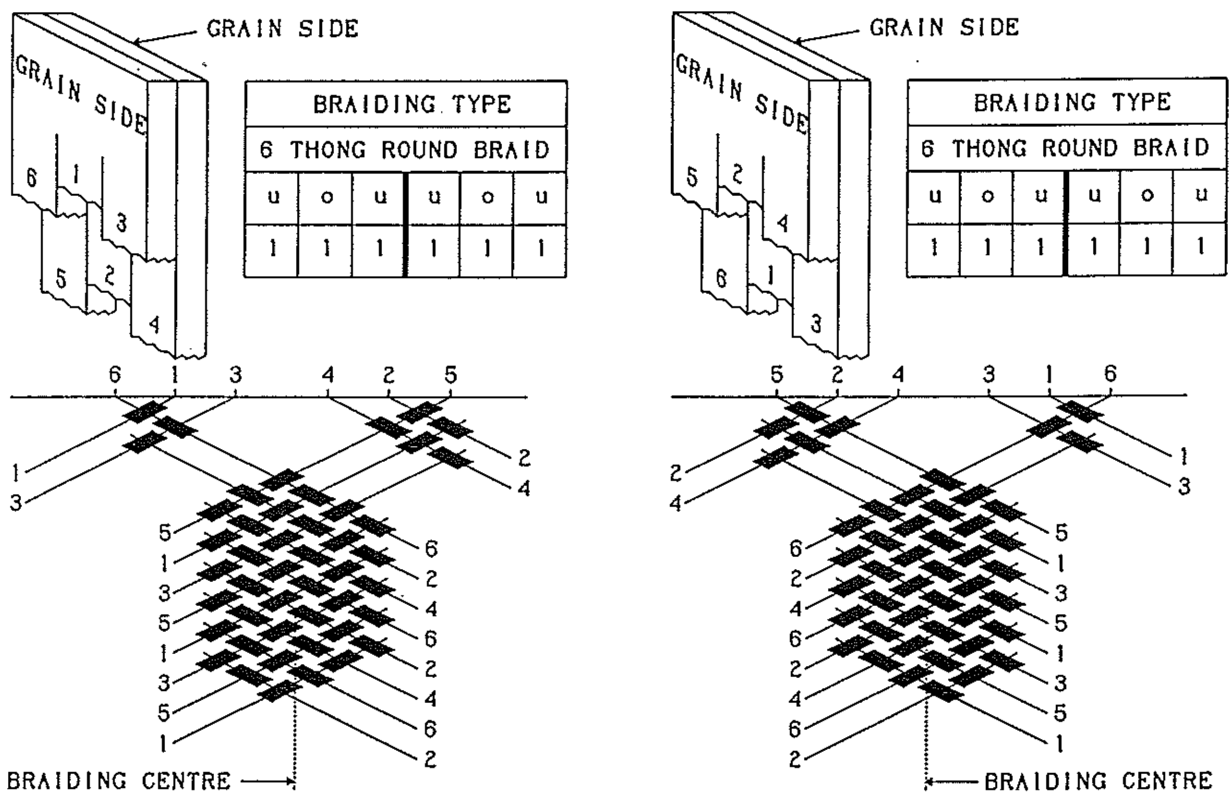


Fig. 450 — A start for a 6 thong Round Braid.

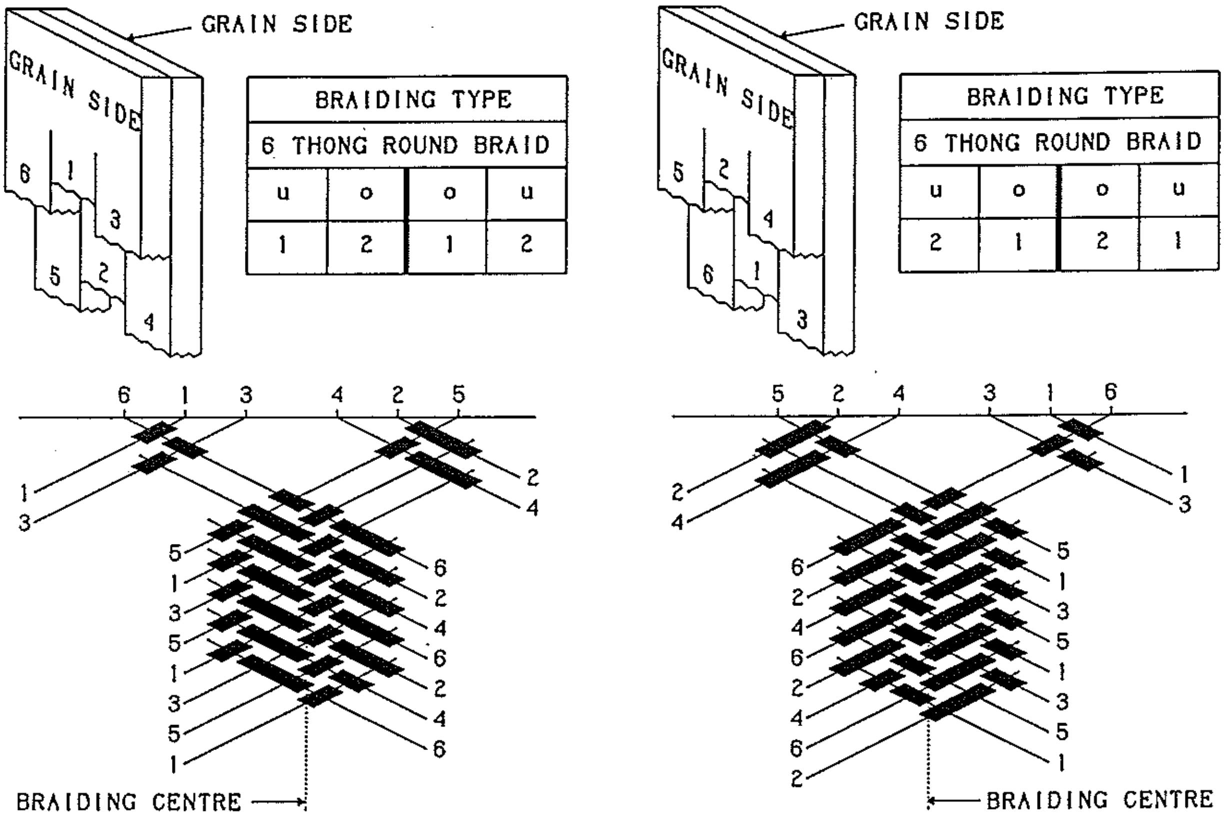


Fig. 451 — A start for a 6 thong Round Braid.

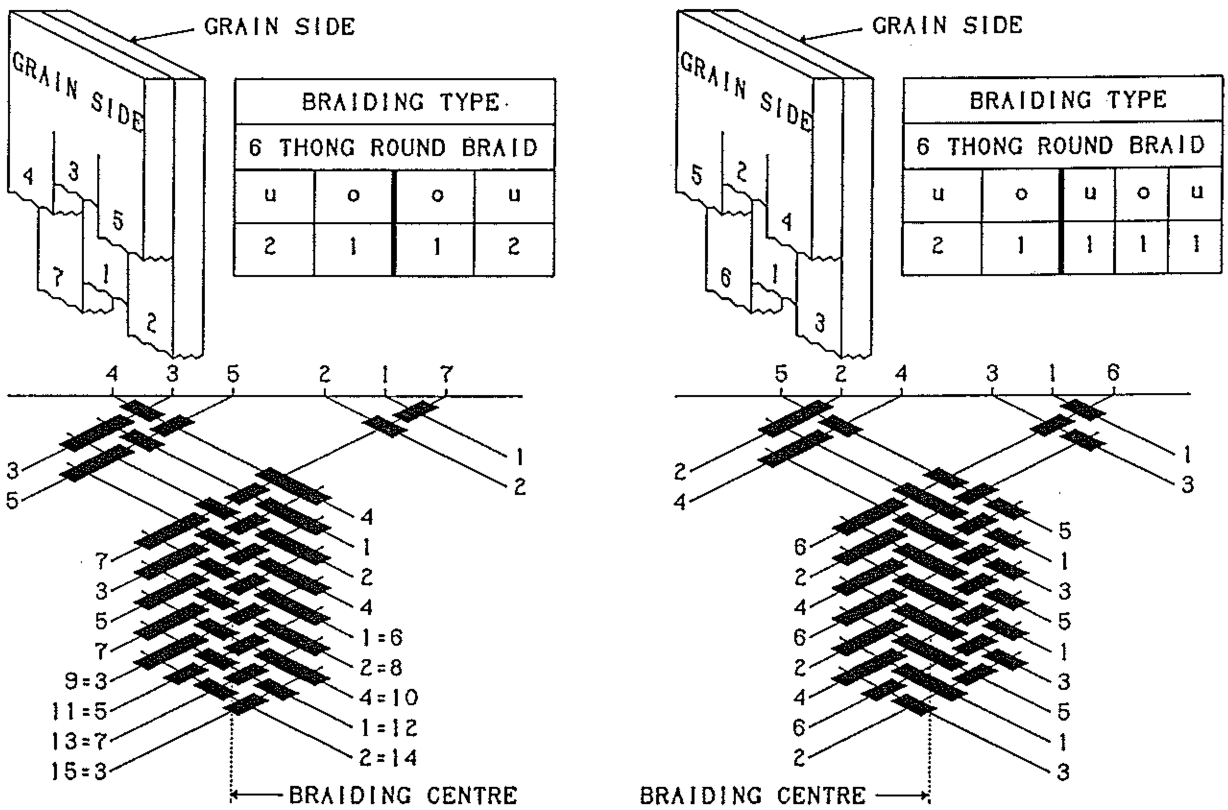


Fig. 452 — A start for a 6 thong Round Braid.