The Braider

No.22 MAY 2000.

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A quarterly publication
for
the braiding artisan

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Solutions to the Questions in Issue No. 21

Question on pg. 478.

(1.) Let the Regular Knot p/b have an over–under coding and let's raise this knot to a bigger over–under coded Regular Knot P/B.

Say we give the over–under coded Regular Knot p/b an $(n+1)$ order enlargement[†] which results in an additional $2x$ half-cycles ($x = |\Delta b|_b + nb$ additional bights). In order to obtain an enlarged Regular Knot with an over–under coding, we must ensure that adjacent to each additional half-cycle there comes a further additional half-cycle. Hence in order to obtain an over–under coding for the enlarged Regular Knot P/B, we must give the over–under coded Regular Knot p/b an $(n+1)$ order enlargement followed by a first order enlargement of the same Enlargement Method. This is illustrated in Fig. 415 (Method I at the left of the thick vertical line, and Method II at the right of the thick vertical line). We see that the parities of p and P are the same, and that the parities of b and B are the same, hence the p/b parity-pair is an invariant.

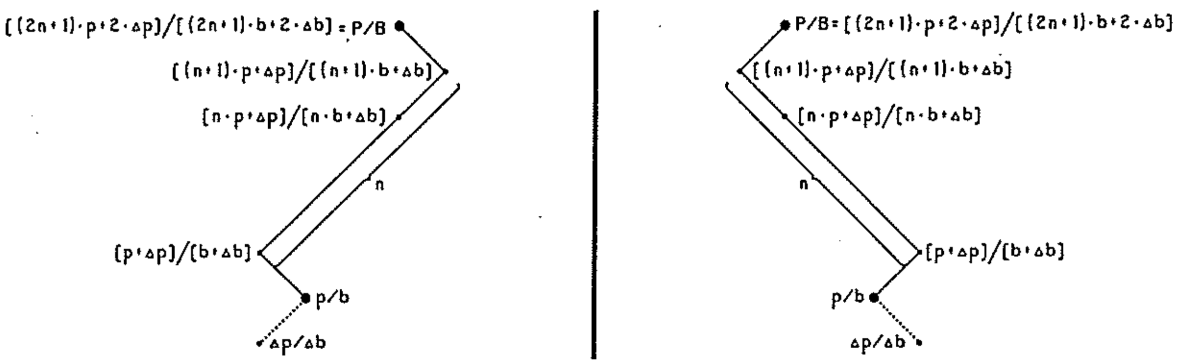


Fig. 415 — An $(n+1)$ order followed by a first order enlargement, both same Method.

A special case arises when $n = 0$ for the $(n+1)$ order enlargement. In this case we have two successive first order enlagements. By repeating this process a number of times we obtain the situation depicted in Fig. 416. Also here is of course the p/b parity-pair an invariant.

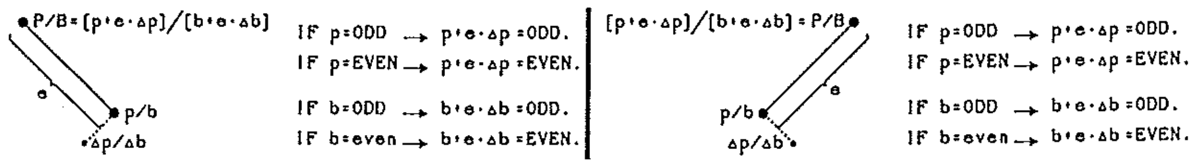


Fig. 416 — An even number (e) fundamental enlargements.

(2.) *Method I:* $p \cdot \Delta b = b \cdot \Delta p + 1$.

$$|\Delta b|_b = \frac{b \cdot |\Delta p|_p + 1}{p},$$
$$|\Delta p|_p = \frac{p \cdot |\Delta b|_b - 1}{b},$$

[†] See *The Braider*, Issue No. 6, pg. 109.

$$\Delta b = \frac{b(|\Delta p|_p + n \cdot p) + 1}{p} = |\Delta b|_b + n \cdot b\,,$$

$$\Delta p = \frac{p(|\Delta b|_b + n \cdot b) - 1}{b} = |\Delta p|_p + n \cdot p\,,$$

$$p^* = p + \Delta p = (n+1)p + |\Delta p|_p\,,$$

$$b^* = b + \Delta b = (n+1)b + |\Delta b|_b\,,$$

$$\Delta b^* = \Delta b = |\Delta b|_b + n \cdot b\,,$$

$$\text{since } |\Delta b|_b + n \cdot b < b^* \quad \rightarrow \quad |\Delta b|_b + n \cdot b = |\Delta b^*|_{b^*}\,,$$

$$\begin{aligned}
\Delta p^* &= \frac{p^* \cdot \Delta b^* - 1}{b^*} \\
&= \frac{p^*(|\Delta b|_b + n \cdot b) - 1}{|\Delta b|_b + (n+1)b} \\
&= \frac{\{(n+1)p + |\Delta p|_p\}(|\Delta b|_b + n \cdot b) - 1}{|\Delta b|_b + (n+1)b} \\
&= \frac{|\Delta b|_b \cdot n \cdot p + |\Delta b|_b \cdot p + |\Delta p|_p \cdot |\Delta b|_b + (n+1)n \cdot p \cdot b + |\Delta p|_p \cdot n \cdot b - 1}{|\Delta b|_b + (n+1)b}\,,
\end{aligned}$$

with $p \cdot |\Delta b|_b = b \cdot |\Delta p|_p + 1 \quad \rightarrow$

$$\begin{aligned}
\Delta p^* &= \frac{|\Delta b|_b \cdot n \cdot p + |\Delta p|_p \cdot b + |\Delta p|_p \cdot |\Delta b|_b + (n+1)n \cdot p \cdot b + |\Delta p|_p \cdot n \cdot b}{|\Delta b|_b + (n+1)b} \\
&= \frac{\{|\Delta b|_b + (n+1)b\}(|\Delta p|_p + n \cdot p)}{|\Delta b|_b + (n+1)b} = |\Delta p|_p + n \cdot p\,,
\end{aligned}$$

$$\text{since } |\Delta p|_p + n \cdot p < p^* \rightarrow \quad \Delta p^* = |\Delta p|_p + n \cdot p = \Delta p = |\Delta p^*|_{p^*}\,.$$

Method II: $p \cdot \Delta b = b \cdot \Delta p - 1\,.$

$$|\Delta b|_b = \frac{b \cdot |\Delta p|_p - 1}{p}\,,$$

$$|\Delta p|_p = \frac{p \cdot |\Delta b|_b + 1}{b}\,,$$

$$\Delta b = \frac{b(|\Delta p|_p + n \cdot p) - 1}{p} = |\Delta b|_b + n \cdot b\,,$$

$$\Delta p = \frac{p(|\Delta b|_b + n \cdot b) + 1}{b} = |\Delta p|_p + n \cdot p\,,$$

$$p^* = p + \Delta p = (n+1)p + |\Delta p|_p\,,$$

$$b^* = b + \Delta b = (n+1)b + |\Delta b|_b\,,$$

$$\Delta b^* = \Delta b = |\Delta b|_b + n \cdot b\,,$$

$$\text{since } |\Delta b|_b + n \cdot b < b^* \quad \rightarrow \quad |\Delta b|_b + n \cdot b = |\Delta b^*|_{b^*}\,,$$

$$\Delta p^* = \frac{p^* \cdot \Delta b^* + 1}{b^*}$$
$$= \frac{p^*(|\Delta b|_b + n \cdot b) + 1}{|\Delta b|_b + (n+1)b}$$
$$= \frac{\{(n+1)p + |\Delta p|_p\}(|\Delta b|_b + n \cdot b) + 1}{|\Delta b|_b + (n+1)b}$$
$$= \frac{|\Delta b|_b \cdot n \cdot p + |\Delta b|_b \cdot p + |\Delta p|_p \cdot |\Delta b|_b + (n+1)n \cdot p \cdot b + |\Delta p|_p \cdot n \cdot b + 1}{|\Delta b|_b + (n+1)b},$$

with $p \cdot |\Delta b|_b = b \cdot |\Delta p|_p - 1 \;\rightarrow$

$$\Delta p^* = \frac{|\Delta b|_b \cdot n \cdot p + |\Delta p|_p \cdot b + |\Delta p|_p \cdot |\Delta b|_b + (n+1)n \cdot p \cdot b + |\Delta p|_p \cdot n \cdot b}{|\Delta b|_b + (n+1)b}$$
$$= \frac{\{|\Delta b|_b + (n+1)b\}(|\Delta p|_p + n \cdot p)}{|\Delta b|_b + (n+1)b} = |\Delta p|_p + n \cdot p,$$

since $|\Delta p|_p + n \cdot p < p^* \;\rightarrow\quad \Delta p^* = |\Delta p|_p + n \cdot p = \Delta p = |\Delta p^*|_{p^*}$.

Thus if $\Delta b^* = \Delta b$, then also $\Delta p^* = \Delta p$. Hence when raising an over–under coded Regular Knot to a bigger over–under coded Regular Knot the p/b parity-pair is an invariant. Furthermore, when the first stage in the raising process is an $(n+1)$ order enlargement, then the second stage in the raising process must be a first order enlargement of the same Method. The special case in which the first stage is a first order enlargement $(n = 0)$ is of course contained in the general case.

Question on pg. 479.

An area which would complicate the 'classification' of over–under coded Regular Knots is, for example, in **appliquè**. Here a one-string Regular Cylindrical Braid does not necessarily remain a one-string braid (see *The Braider*, Issue No. 15, pp. 344 – 346).

Regular Cylindrical Bihelix Braids
with an
Interbraided Overlay

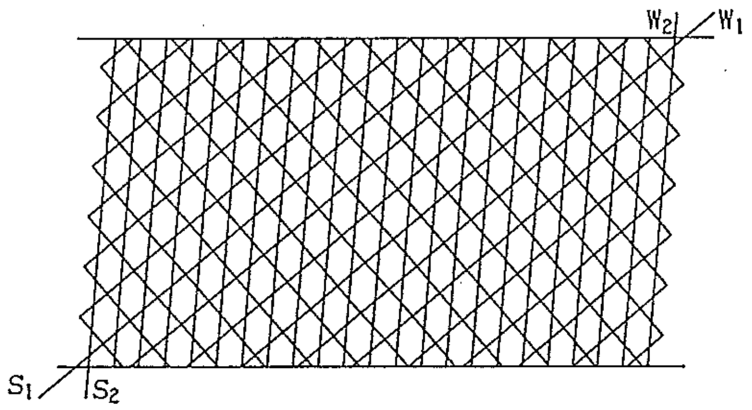


Fig. 417 — A string-run example of a Regular Cylindrical Bihelix Braid with an Interbraided Overlay.

The [7, 6] Prime Regular Cylindrical Bihelix Braid† $S_1 \longrightarrow W_1$ depicted in Fig. 417 has $N^* = 144$ crossings in its string-run. Its equivalent extended Premature Regular Cylindrical Braid has $b = 7$ and $p = 24$ with $\delta = 6$, hence is the $p/b = 25/7$ Premature Regular Cylindrical Knot. The Interbraided Overlay is the string-run $S_2 \longrightarrow W_2$.

In Fig. 418 are indicated the first 17 single string Prime Regular Cylindrical Bihelix Braids (the Prime Regular Cylindrical Bihelix Knots). Those whose equivalent Premature Regular Cylindrical Braid has $\delta = 0$ or $\delta = n = 6$, hence which are equivalent Premature Regular Cylindrical Knots are generally the quickest and easiest to braid. Neglecting the trivial $p = 1$ with $\delta = 0$, their p-values with $\delta = 0$ can readily be found with the formulae:

$$\left.\begin{array}{l} p = 2kn + 1 \\ p = 2kn + 2 \end{array}\right\} \quad \text{where } k = 1, 2, 3, \cdots; \quad p \text{ and } b = n + 1 \text{ to be coprime.}$$

★★ *Show how these formulae can readily be derived.*

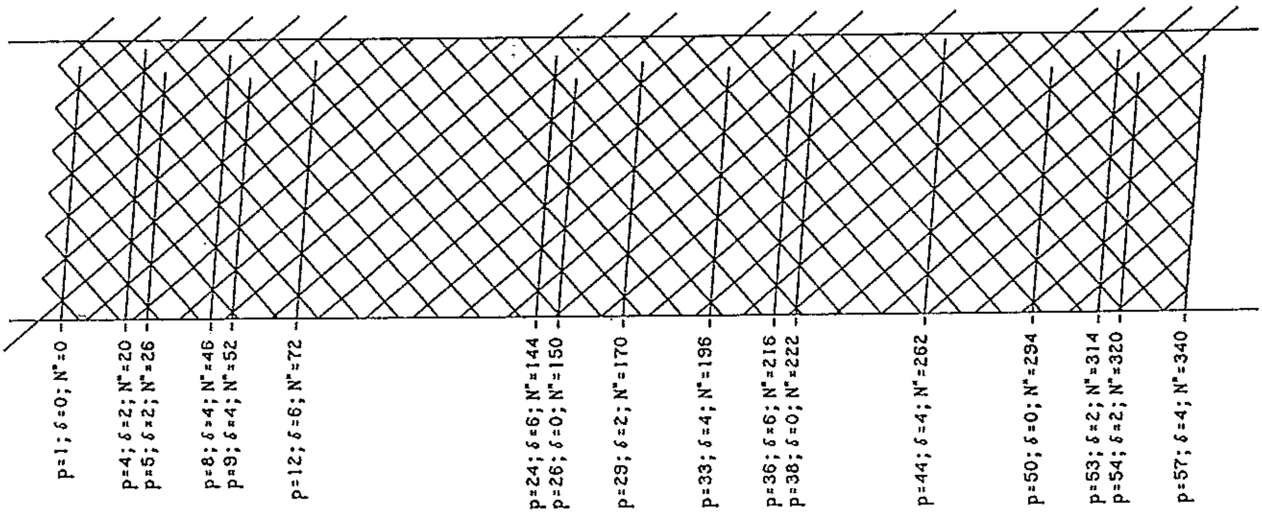


Fig. 418 — Prime Regular Cylindrical Bihelix Knots in the [7, 6] Bihelix Round Braid.

Note that a Premature Regular Cylindrical Braid with $p = j$ and $\delta = n$ is identical to the Premature Regular Cylindrical Braid with $p = j + 1$ and $\delta = 0$.

The string-run diagrams of the first ten [7, 6] Prime Regular Cylindrical Bihelix Knots are depicted in Fig. 419, while the table in Fig. 420 gives the particulars of the first 84 Prime Regular Cylindrical Bihelix Braids. Note that the entry $p = 13$, $\delta = 6$

† Refer to *The Braider*, Issue No. 17, pp. 378–392, and Issue No. 18, pp. 393–401.

also occurs as entry $p = 14$, $\delta = 0$; the entry $p = 24$, $\delta = 6$ also occurs as entry $p = 25$, $\delta = 0$; the entry $p = 36$, $\delta = 6$ also occurs as entry $p = 37$, $\delta = 0$; the entry $p = 48$, $\delta = 6$ also occurs as entry $p = 49$, $\delta = 0$; the entry $p = 60$, $\delta = 6$ also occurs as entry $p = 61$, $\delta = 0$; the entry $p = 72$, $\delta = 6$ also occurs as entry $p = 73$, $\delta = 0$.

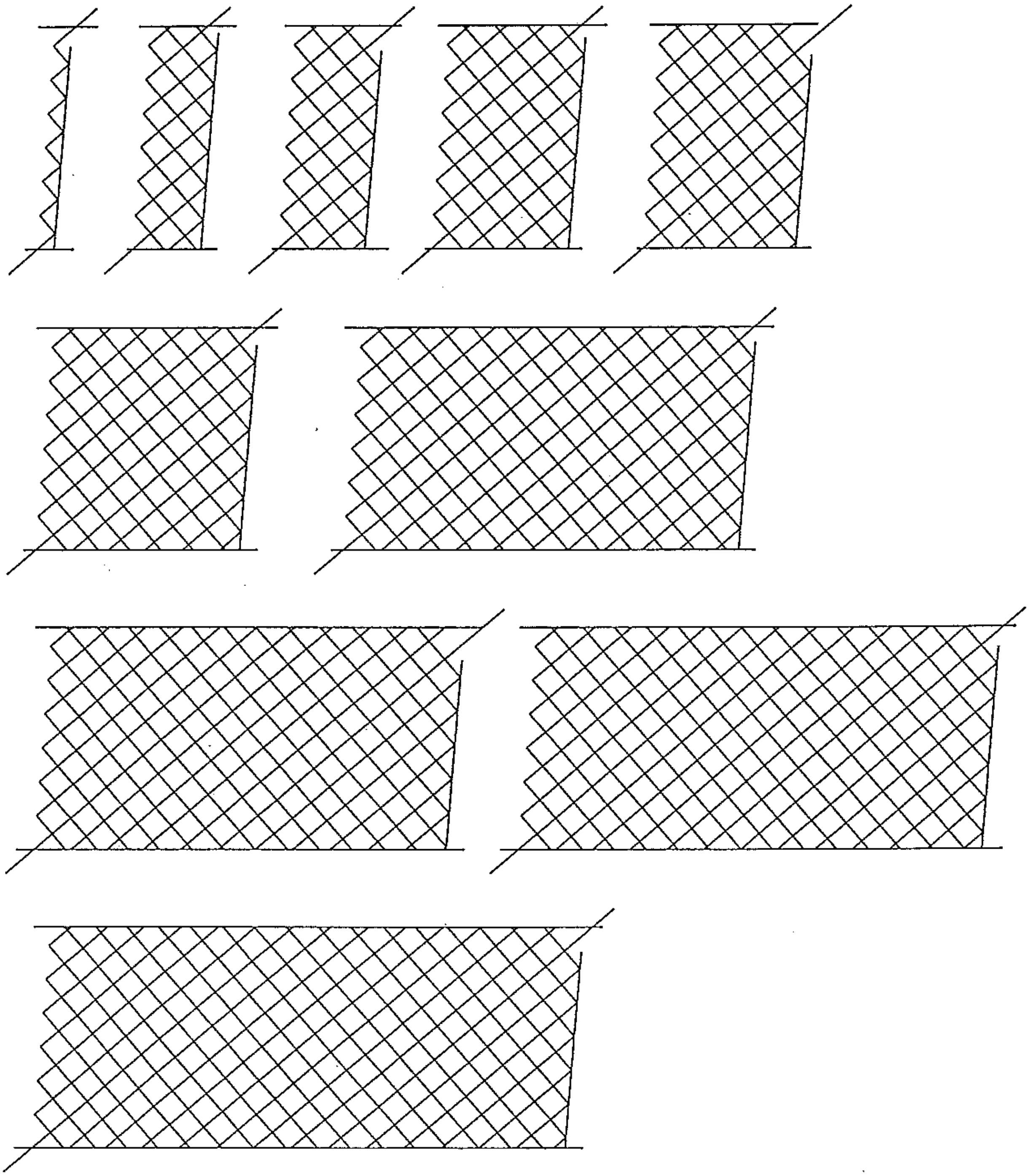


Fig. 419 — The first ten Prime Regular Cylindrical Bihelix Knots.

When we superimpose a **Matthew Walker** coding on the string-run of a Prime Regular Cylindrical Bihelix Knot, the braiding of any Prime Regular Cylindrical Bihelix Knot by means of its equivalent extended Premature Regular Cylindrical Knot becomes of course a very simple matter. Next we can then interbraid the Matthew Walker coded Prime Regular Cylindrical Bihelix Knot with an under–over coded overlay. An example is shown in Fig. 421. In this example, the Matthew Walker coded Prime Regular

Cylindrical Bihelix Knot (S_1) has $N^* = 144$ crossings and is equivalent to a Matthew Walker coded Premature Regular Knot with $p = 25$ ($\delta = 0$). It is interbraided with the under–over coded overlay (S_2) (see also Fig. 422).

PRIME REGULAR CYLINDRICAL BIHELIX BRAIDS NUMBER OF CROSSINGS N*	PREMATURE REGULAR CYLINDRICAL BRAIDS b=7		
	p	δ	NUMBER OF ESSENTIAL STRINGS
0	1	0	1
7	2	1	2
13	3	1	2
20	4	2	1
26	5	2	1
33	6	3	4
39	7	3	4
46	8	4	1
52	9	4	1
59	10	5	2
65	11	5	2
72	12	6	1
78	13	6	7
78	14	0	7
79	14	1	6
85	15	1	2
92	16	2	3
98	17	2	3
105	18	3	2
111	19	3	2
118	20	4	5
124	21	4	3
131	22	5	2
137	23	5	2
144	24	6	1
144	25	0	1
150	26	0	1
157	27	1	2
163	28	1	6
164	28	2	5
170	29	2	1
177	30	3	2
183	31	3	4
190	32	4	3
196	33	4	1
203	34	5	6
209	35	5	2
216	36	6	1
216	37	0	1
222	38	0	1
229	39	1	2
235	40	1	2
242	41	2	3
248	42	2	5
249	42	3	4
255	43	3	2
262	44	4	1
268	45	4	3
275	46	5	2
281	47	5	2
288	48	6	7
288	49	0	7
294	50	0	1
301	51	1	2
307	52	1	2
314	53	2	1
320	54	2	1
327	55	3	4
333	56	3	4
334	56	4	3
340	57	4	1
347	58	5	2
353	59	5	2
360	60	6	1
360	61	0	1
366	62	0	1
373	63	1	6
379	64	1	2
386	65	2	3
392	66	2	3
399	67	3	2
405	68	3	2
412	69	4	5
418	70	4	3
419	70	5	2
425	71	5	2
432	72	6	1
432	73	0	1
438	74	0	1
445	75	1	2
451	76	1	2
458	77	2	5
464	78	2	1
471	79	3	2
477	80	3	4
484	81	4	3
490	82	4	1
497	83	5	6
503	84	5	2
504	85	0	1

Fig. 420 — The first 84 Prime Regular Cylindrical Bihelix Braids.

The interbraided overlay can of course be braided with the string of the Prime Regular Cylindrical Bihelix Knot; the overall braid consists then of one string. This has been depicted in Fig. 423, the end W_1 can finally be worked away over W_2. The equivalent $p/b = 25/7$ Matthew Walker coded Premature Regular Knot with its interbraided under–over coded overlay is shown in Fig. 425.

A distance of $2w$ (twice the string width) is maintained between the adjacent strands as shown in Fig. 424.

This braid can make, for example, an attractive covering knot for a quirt handle.

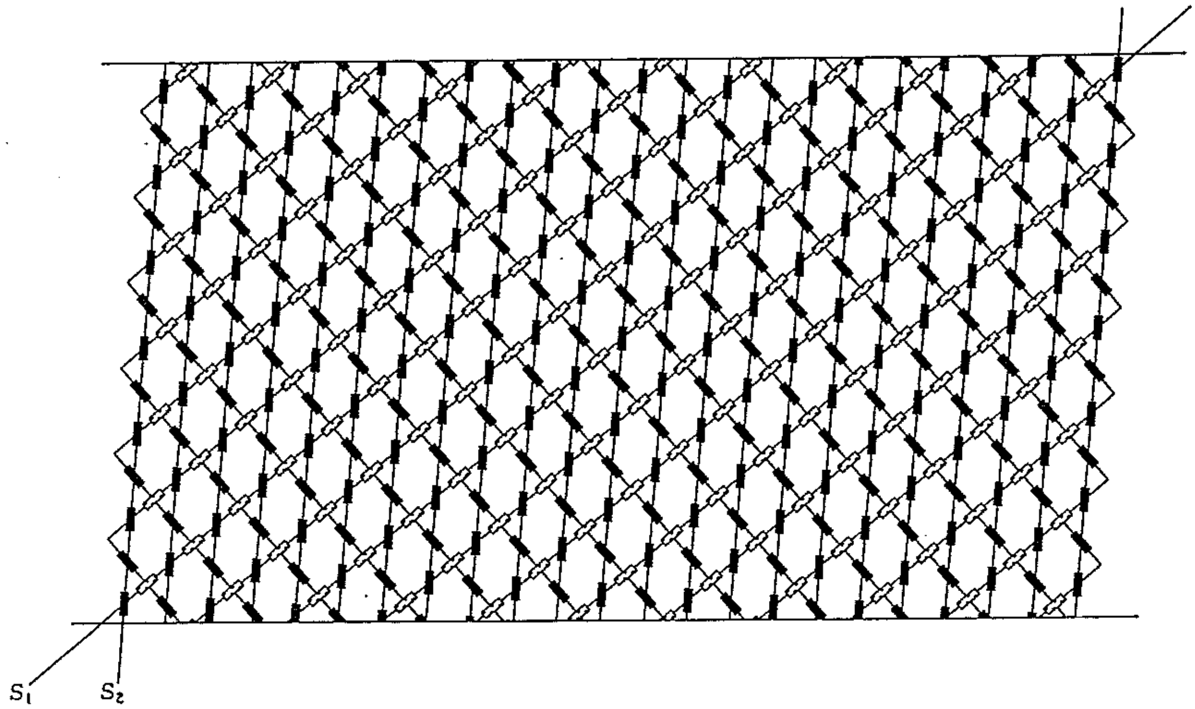


Fig. 421 — A Matthew Walker coded [7, 6] Prime Regular Cylindrical Bihelix Knot with an Interbraided under–over coded Overlay.

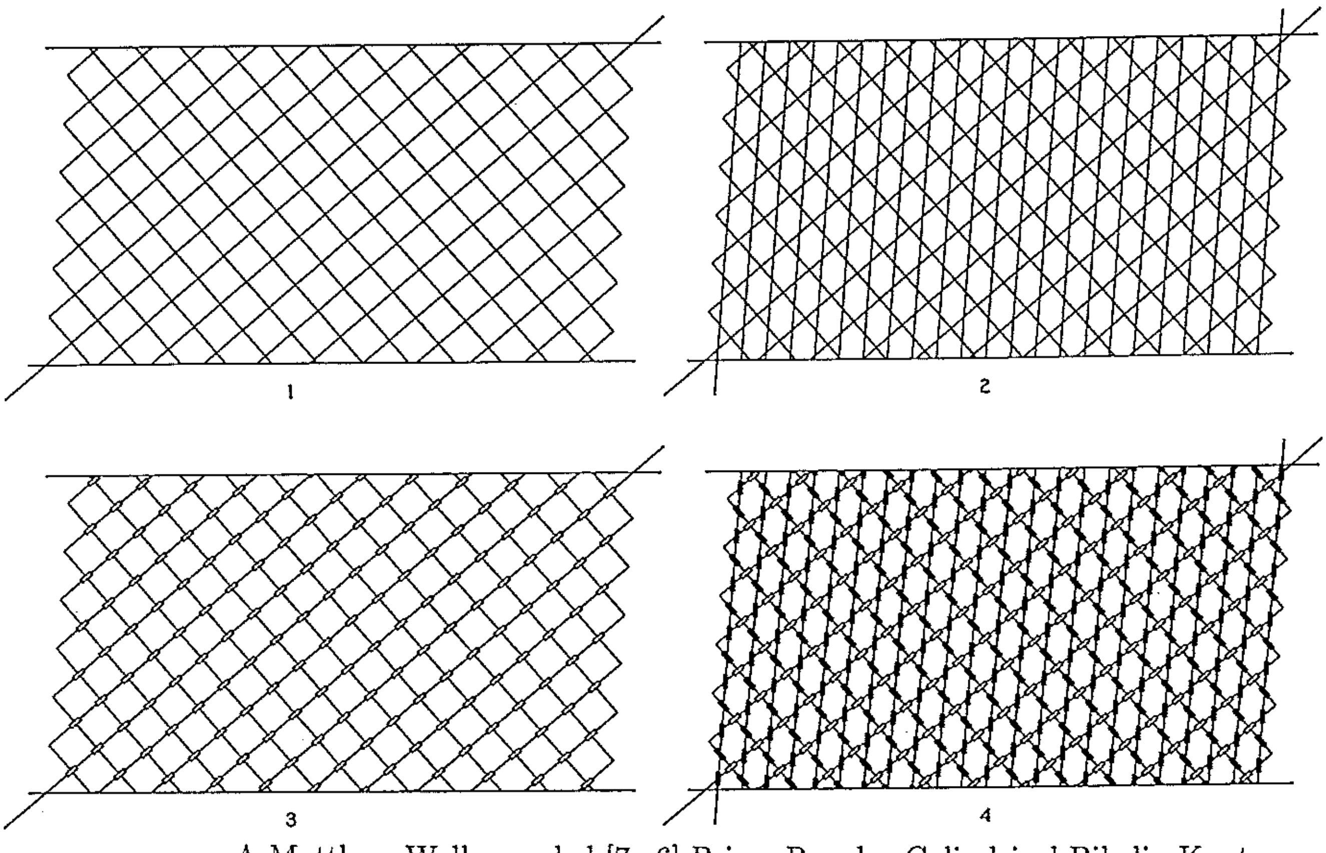


Fig. 422 — A Matthew Walker coded [7, 6] Prime Regular Cylindrical Bihelix Knot with an Interbraided under–over coded Overlay.

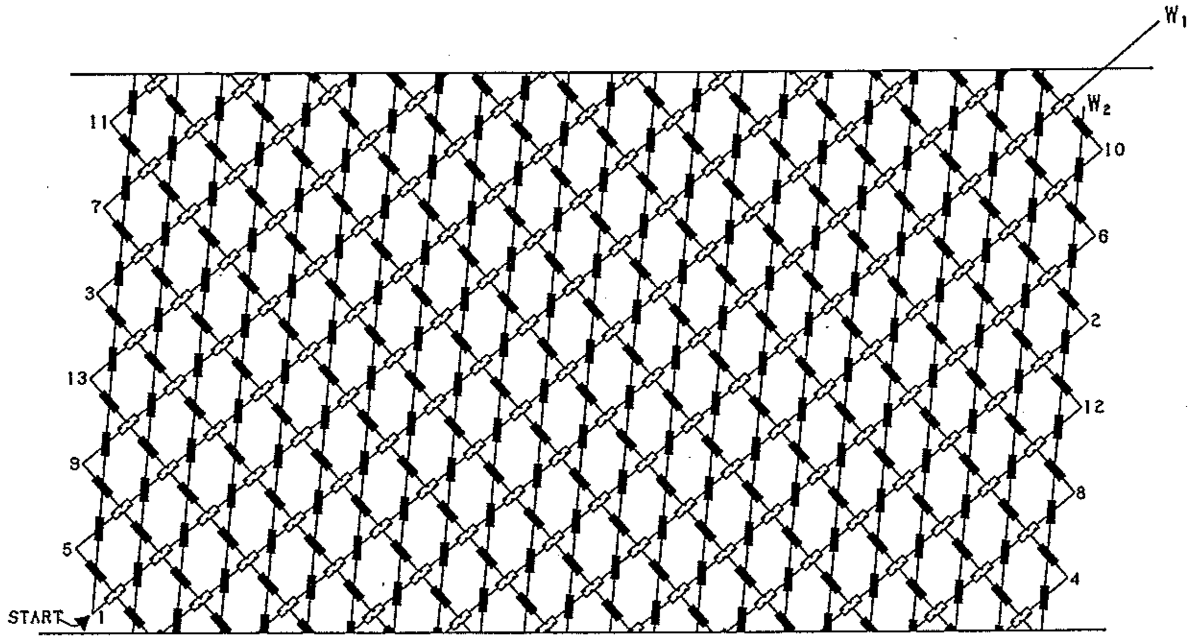


Fig. 423 — The single string version of the braid in Fig. 421.

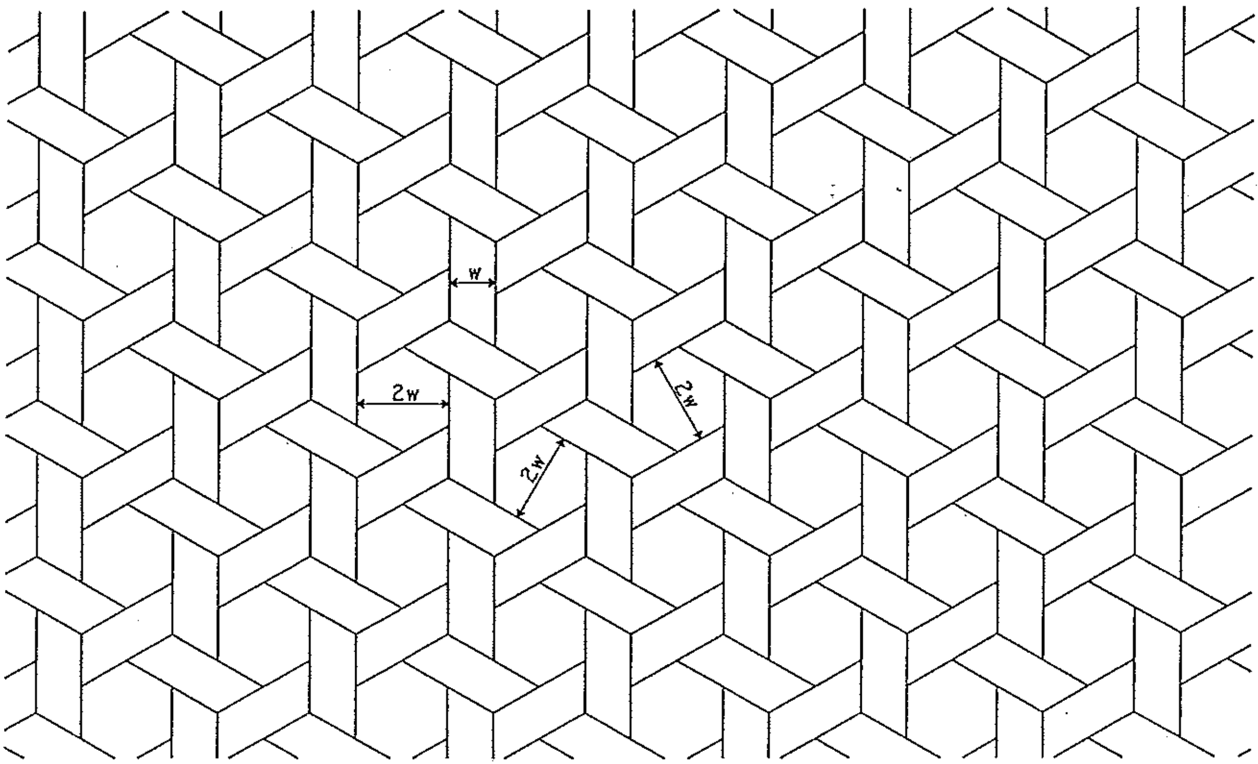
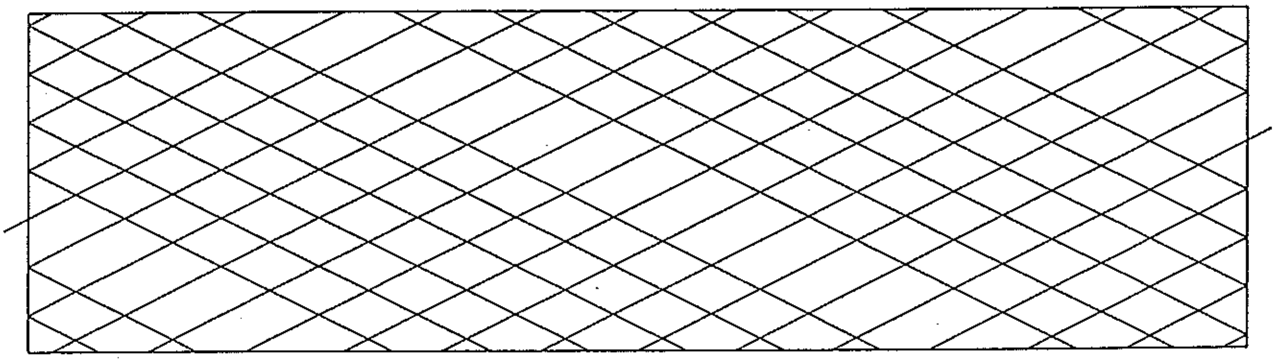
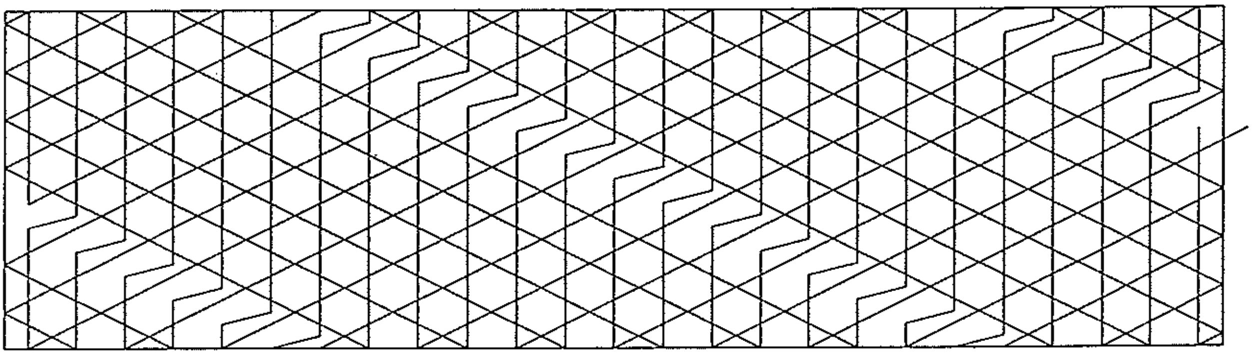


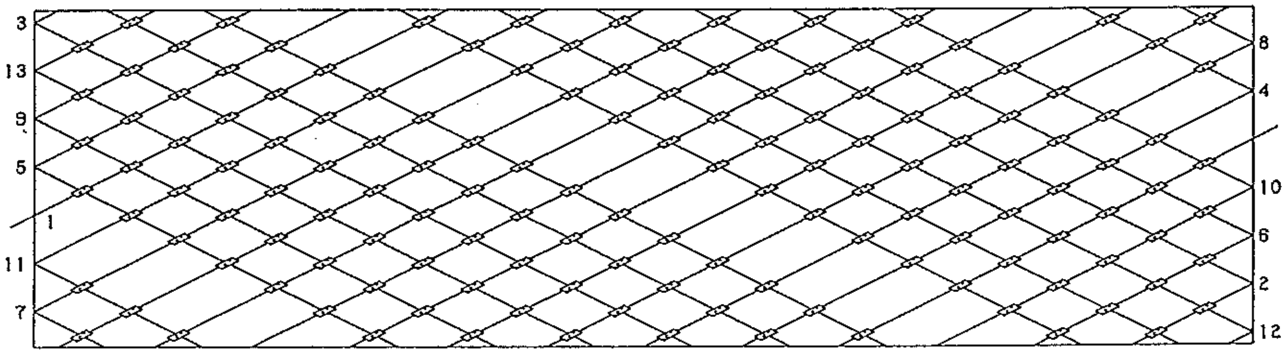
Fig. 424 — A section of the braid in detail.



1



2



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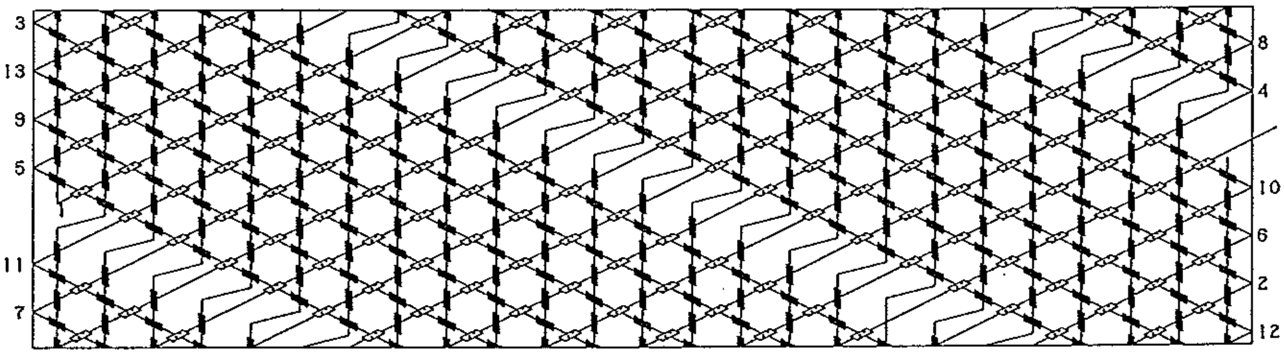


4

Fig. 425 — The equivalent $p/b = 25/7$ Matthew Walker coded Premature Regular Knot with its Interbraided under–over coded Overlay.

When thin lace is used for braiding the $[n{+}1\,,\, n]$, and $[n\,,\, n{+}1]$, Regular Cylindrical Bihelix Braids ($\sigma = 1$) with interbraided under–over coded Overlay, a useful formula is:

$$2n + 1 = \frac{0.73C}{w}\,,$$

where C is the circumference of the object to be covered. For the $[n+1\,,\, n]$, or the $[n\,,\, n+1]$, Regular Cylindrical Bihelix Braid ($\sigma = 1$), the value of w can then be made such that $2n + 1$ is very near an odd positive integer.

Fig. 426 depicts an example of a Matthew Walker coded 2-string Regular Cylindrical Bihelix Braid with an Interbraided under–over coded Overlay.

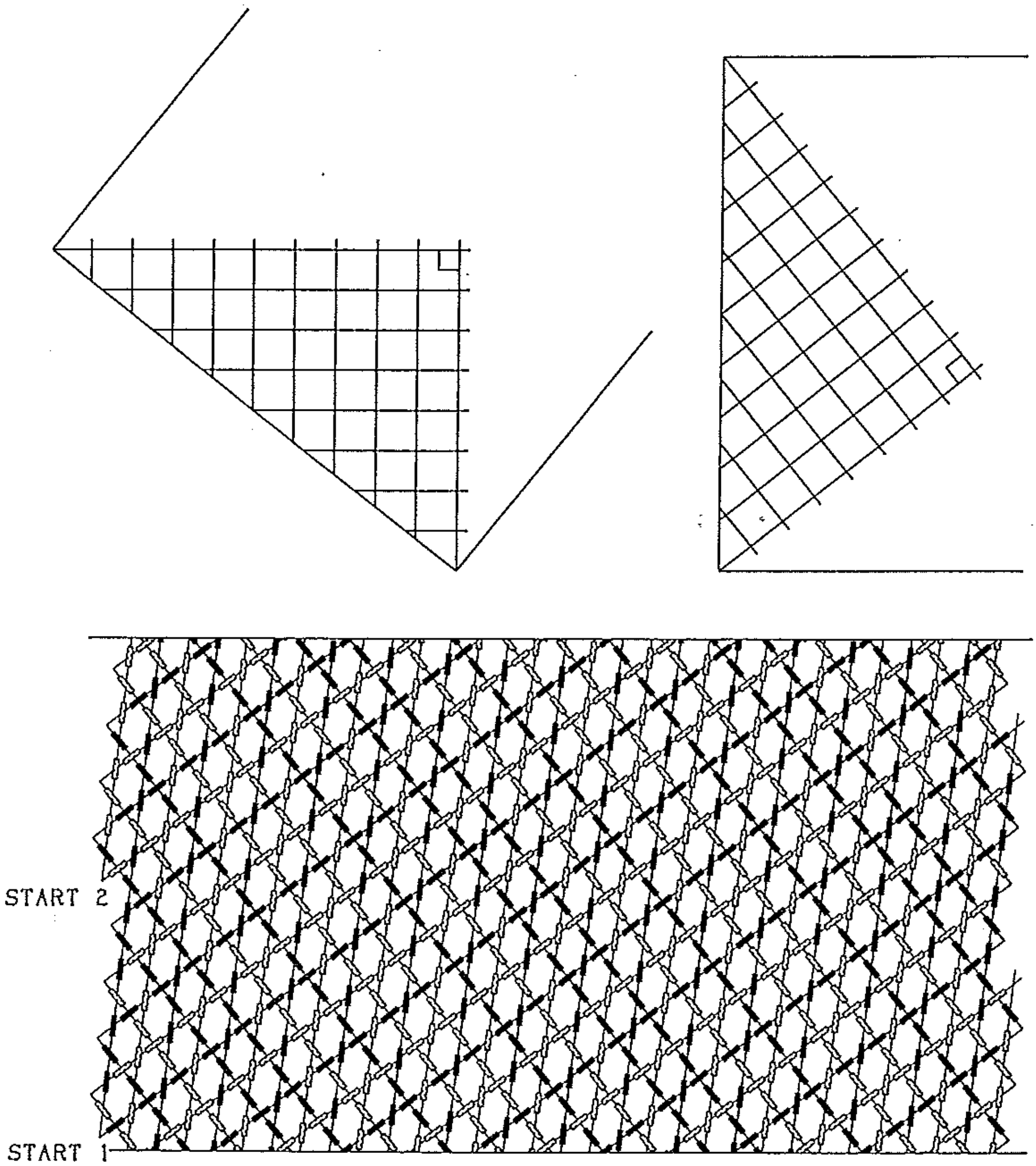


Fig. 426 — A Matthew Walker coded two string [10 , 8] Regular Cylindrical Bihelix Braid ($\sigma = 2$) with Interbraided under–over coded Overlay.

Since for the same braiding material the distortion of the hexagons depends on the slant of the bight-boundary lines, this distortion is for an $[n^* + 2, n^*]$ Regular Cylindrical Bihelix Braid the same as for an $[n + 1, n]$ Regular Cylindrical Bihelix Braid when $n^* = 2n$. Thus for bigger diameter braids, the Matthew Walker coded $[n^* + 2, n^*]$ Regular Cylindrical Bihelix Braids with an interbraided under–over coded overlay provide us with an opportunity to create some interesting colour patterns.

Nested Cylindrical Braids

In the previous Issue of *The Braider*, No. 21, pp. 466 – 470 we have discussed how a right-hand valid sequence set is obtained from its corresponding left-hand valid sequence set. We also mentioned that the most often encountered left and right bight-boundary position specifications were $222\cdots$ with $\mathcal{K}_l = A_l$ and $\mathcal{K}_r = A_r$. Since in practice we generally limit A_l and/or A_r to 7, we have shown for $2 \leq A_l \leq 7$ in Figs. 427 – 432 the left valid cyclic sequence sets of k-values and their associated left bight-boundary arrangements.

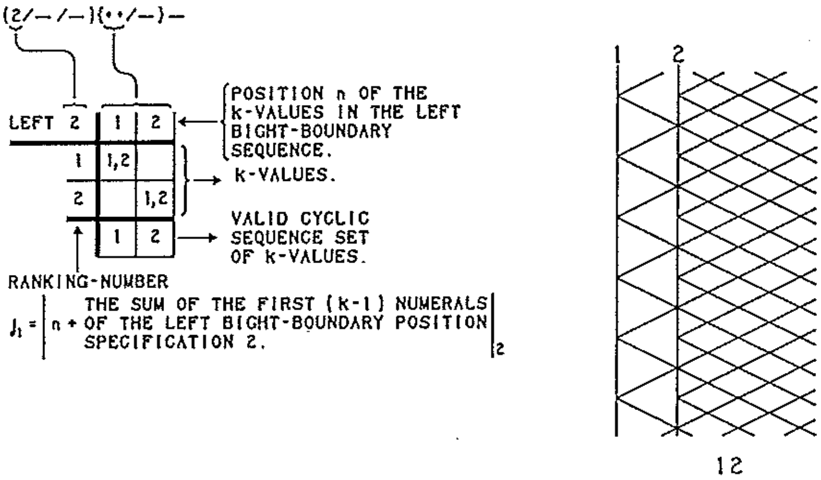


Fig. 427 — Left bight-boundary specification 2 with $\mathcal{K}_l = A_l = 2$.

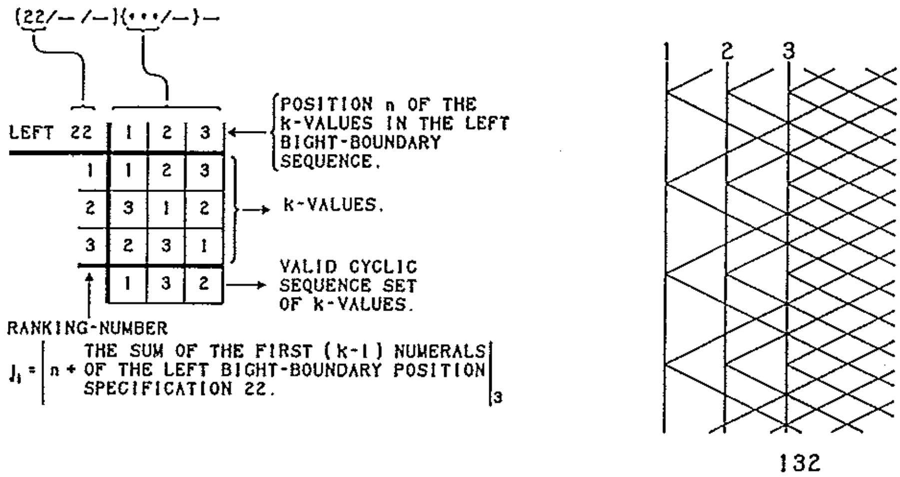


Fig. 428 — Left bight-boundary specification 22 with $\mathcal{K}_l = A_l = 3$.

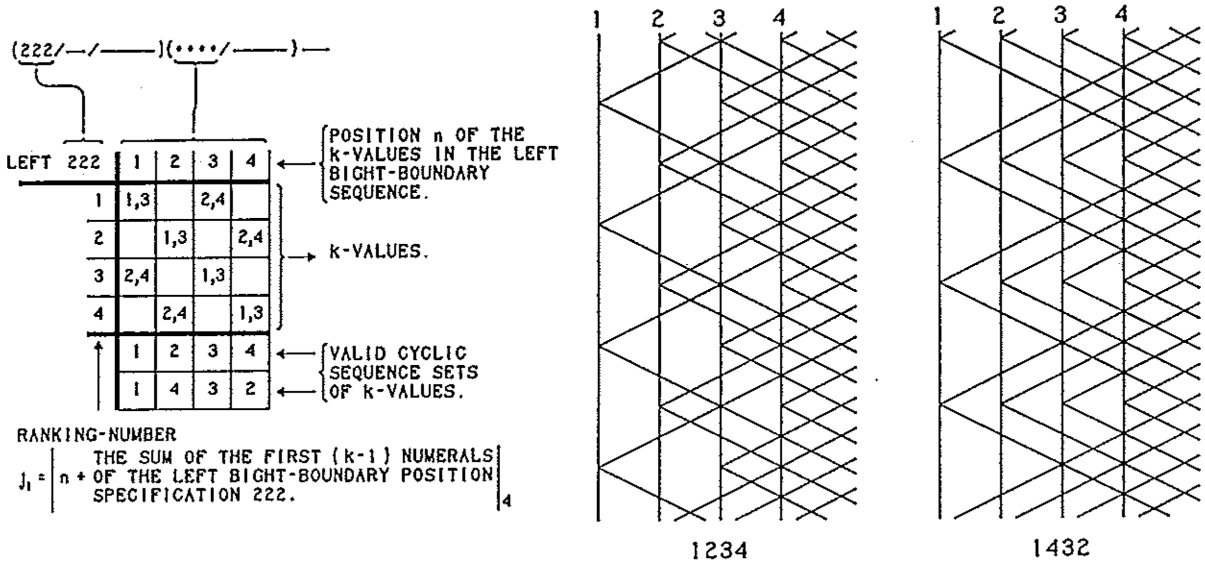


Fig. 429 — Left bight-boundary specification 222 with $\mathcal{K}_l = A_l = 4$.

The more commonly encountered Nested Cylindrical Braids which have the bight-boundary position specifications $222\cdots$ with $\mathcal{K}_l = A_l$ and $\mathcal{K}_r = A_r$ are those which have the left sequence set $1A_l(A_l-1)(A_l-2)(A_l-3)\cdots 432$ and the right sequence set $k|k+1|_{A_r}|k+2|_{A_r}|k+3|_{A_r}\cdots A_r 123\cdots|k-2|_{A_r}|k-1|_{A_r}$, where $1 \le k \le A_r$. They are the **Asymmetric Regular Nested Cylindrical Braids** when $A_r \neq A_l$, and they are the **Regular Nested Cylindrical Braids** when $A_r = A_l = A$. Thus the **Regular Nested Cylindrical Braids** have left sequence set $1A(A-1)(A-2)(A-3)\cdots 432$ and the right sequence set $k|k+1|_A|k+2|_A|k+3|_A\cdots A123\cdots|k-2|_A|k-1|_A$, where $1 \le k \le A$. We shall discuss first the Regular Nested Cylindrical Braids and then the Asymmetric Regular Nested Cylindrical Braids.

The Regular Nested Cylindrical Braids:

Since the bight-boundary position specification for both the left and right bight-edges are the same and equal to $222\cdots$ with $\mathcal{K}_l = A_l = \mathcal{K}_r = A_r = A$ the formulae on pg. 417 can be simplified considerably.

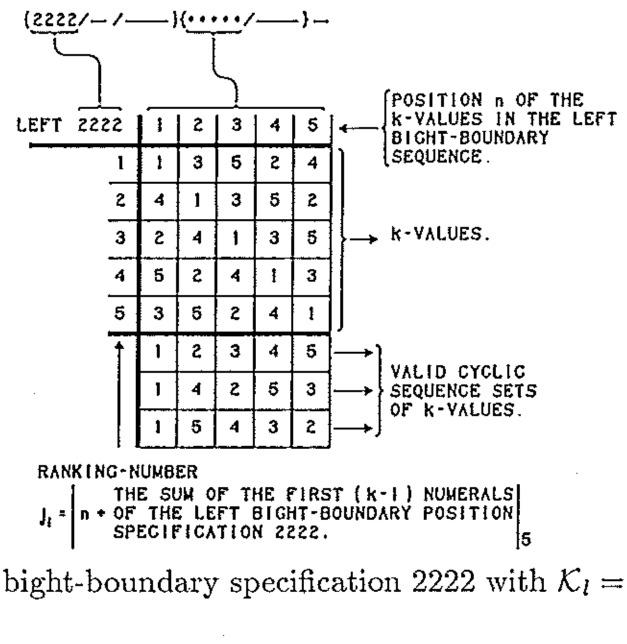
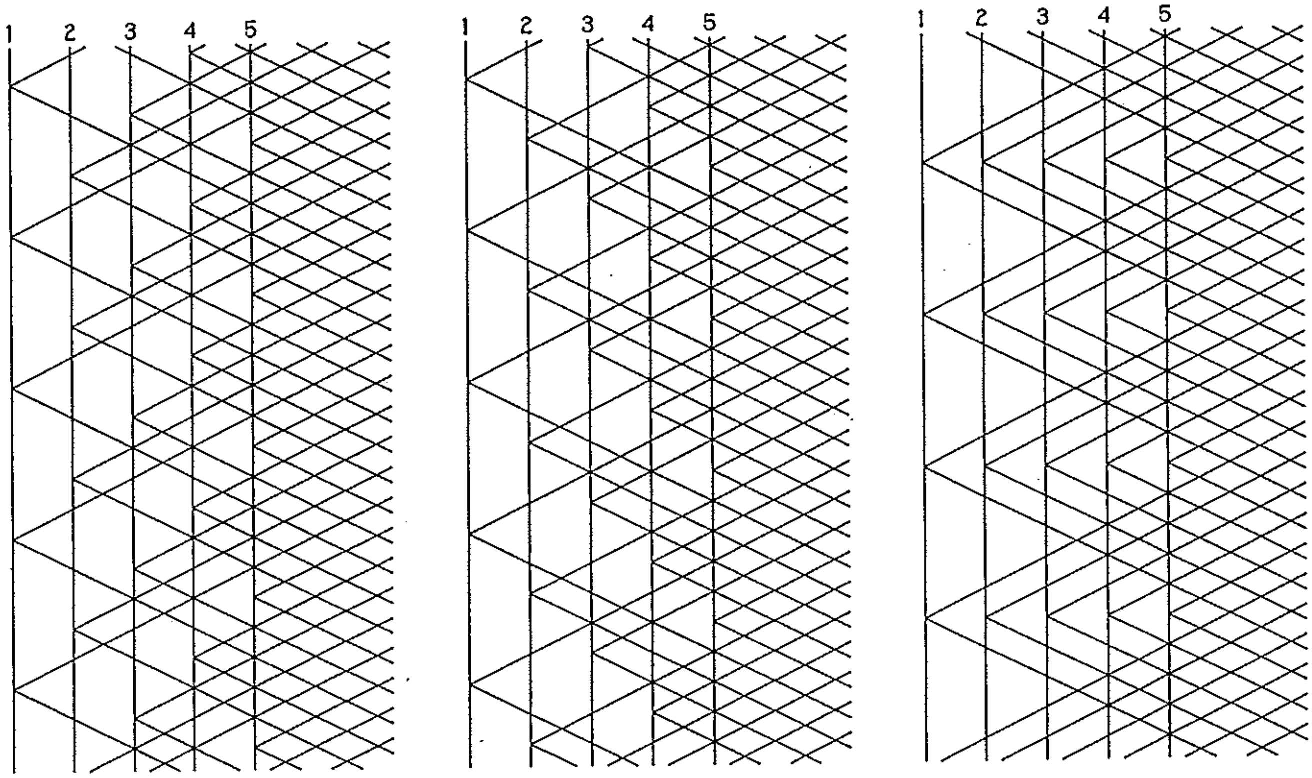


Fig. 430 — Left bight-boundary specification 2222 with $\mathcal{K}_l = A_l = 5$.



12345 14253 15432

Fig. 430 cont.

(22222/—/——){••••••/——}—

LEFT 22222	1	2	3	4	5	6
1	1,4		3,6		2,5	
2		1,4		3,6		2,5
3	2,5		1,4		3,6	
4		2,5		1,4		3,6
5	3,6		2,5		1,4	
6		3,6		2,5		1,4
	1	2	5	3	6	4
	1	2	5	6	3	4
	1	3	2	4	6	5
	1	3	5	4	6	2
	1	4	2	5	6	3
	1	4	2	5	3	6
	1	4	5	2	3	6
	1	4	5	2	6	3
	1	5	2	3	6	4
	1	5	2	6	3	4
	1	6	2	4	3	5
	1	6	5	4	3	2

POSITION n OF THE K-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE.

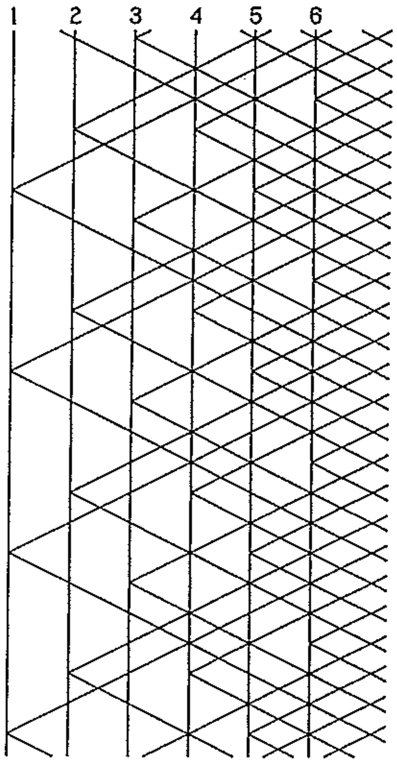
K-VALUES.

VALID CYCLIC SEQUENCE SETS OF K-VALUES.

RANKING-NUMBER

$J_l = \left| n + \right.$ THE SUM OF THE FIRST (K-1) NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 22222. $\left.\right|_6$

Fig. 431 — Left bight-boundary specification 22222 with $\mathcal{K}_l = A_l = 6$.



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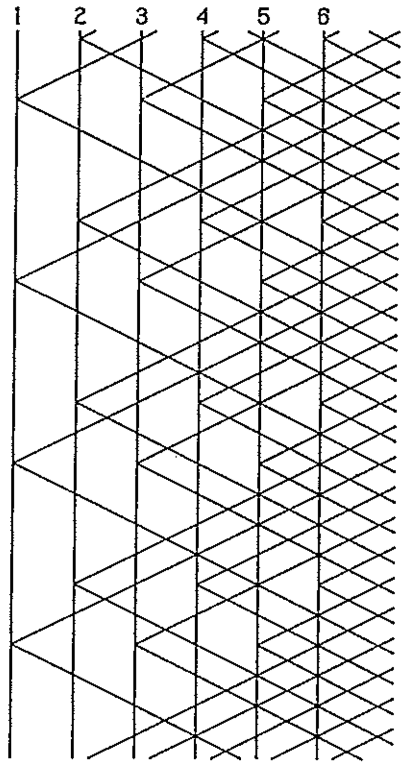

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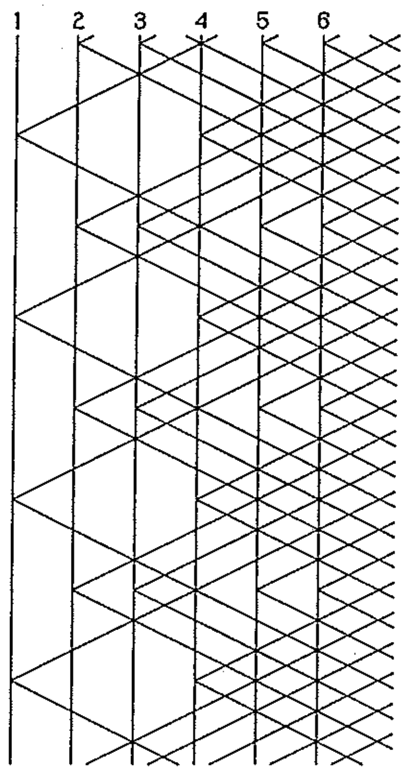
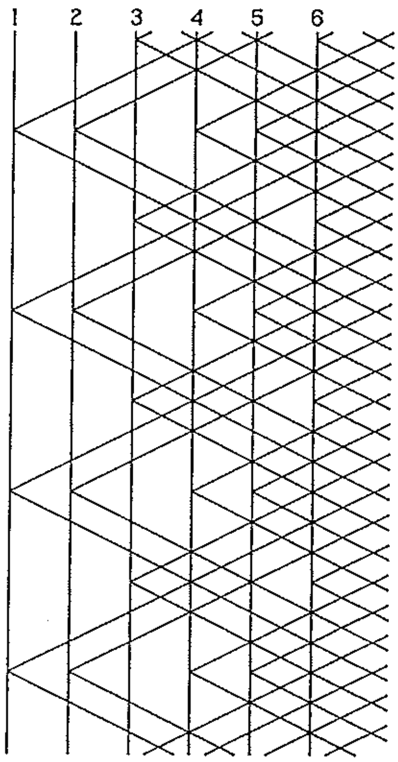
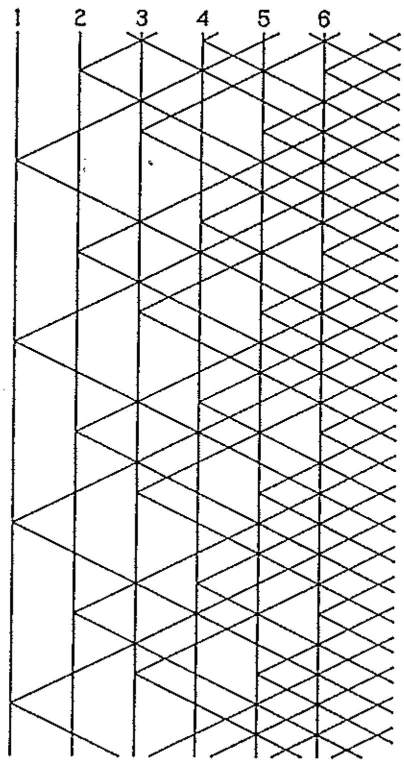


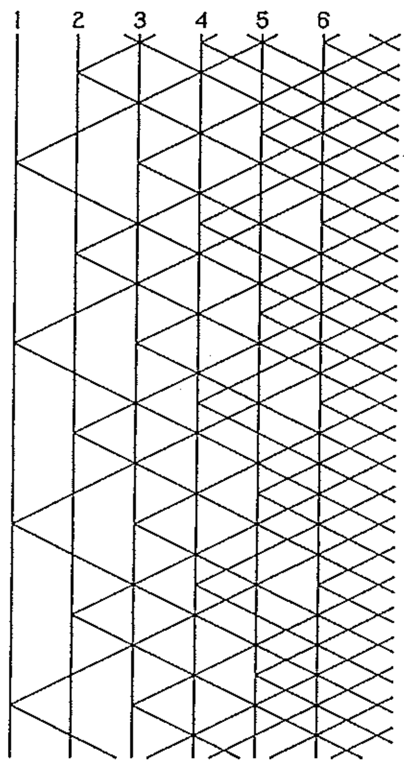
132465



135462

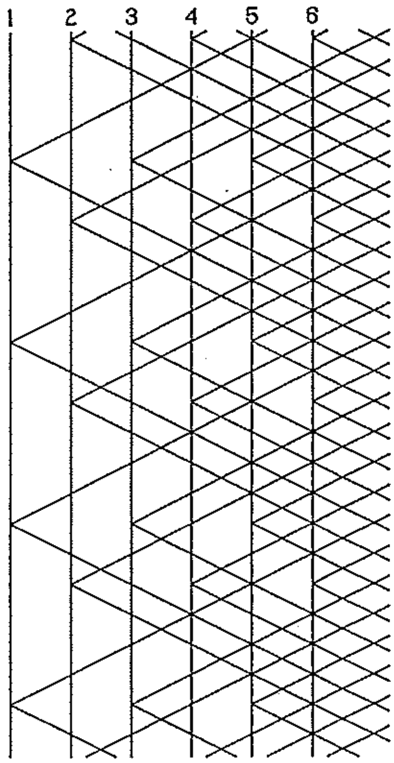


142563



142536

Fig. 431 cont.



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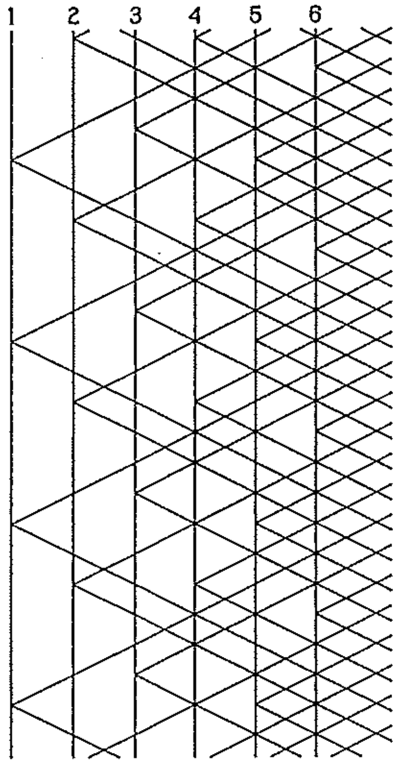
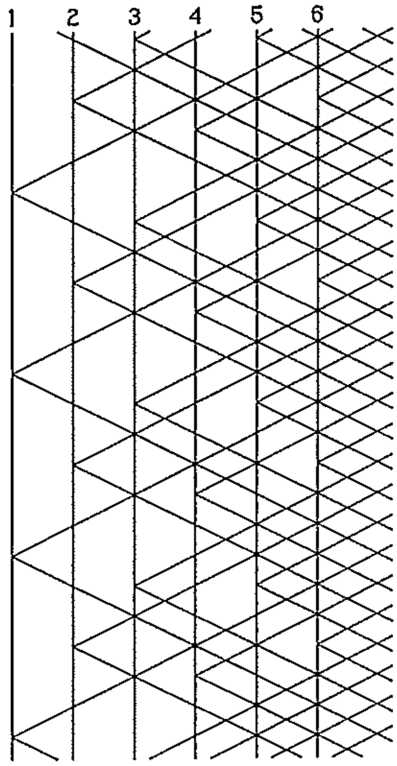
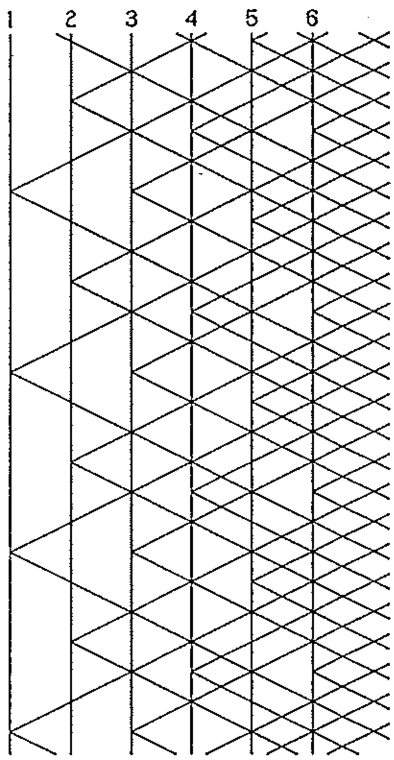


145263



152364



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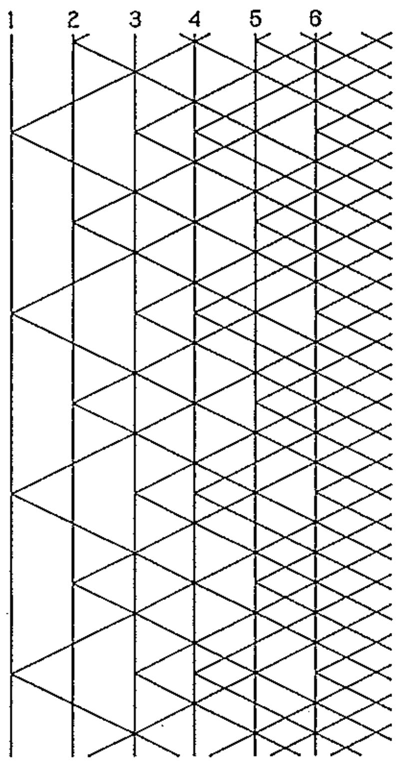
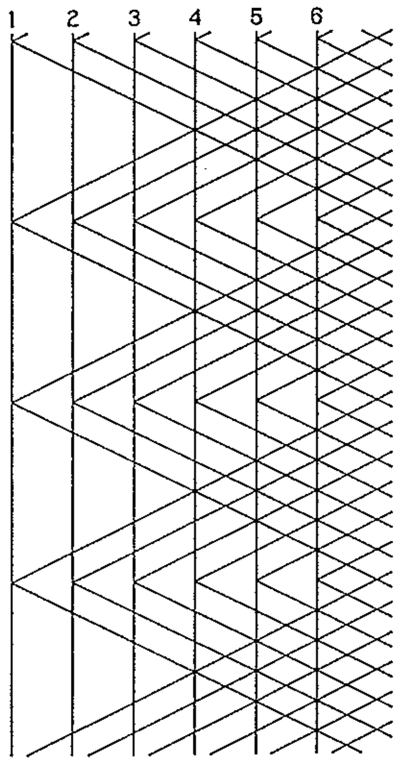


162435



165432

Fig. 431 cont.

The formulae (see pg. 417):

Δ_{l_i} = sum of the last $(\mathcal{K}_l - l_i)$ numerals in left bight-boundary position specification sequence.

$\Delta_{l_{i+1}}$ = sum of the last $(\mathcal{K}_l - l_{i+1})$ numerals in left bight-boundary position specification sequence.

Δ_{r_i} = sum of the first $(\mathcal{K}_r - r_i)$ numerals in right bight-boundary position specification sequence.

$$j_l' = |j_l + \Delta_{l_i} + x + \Delta_{r_i}|_{A_l} .$$

$$j_r' = |j_r + \Delta_{r_i} + x + \Delta_{l_{i+1}}|_{A_r} .$$

modify to:

$$\Delta_{l_i} = 2(A - l_i) .$$

$$\Delta_{l_{i+1}} = 2(A - l_{i+1}) .$$

$$\Delta_{r_i} = 2(A - r_i) .$$

$$j_l' = |j_l + x - 2(l_i + r_i)|_A .$$

$$j_r' = |j_r + x - 2(r_i + l_{i+1})|_A .$$

(222222/—/——)(•••••••/——)—

POSITION n OF THE k-VALUES IN THE LEFT BIGHT-BOUNDARY SEQUENCE.

LEFT 222222	1	2	3	4	5	6	7
1	1	4	7	3	6	2	5
2	5	1	4	7	3	6	2
3	2	5	1	4	7	3	6
4	6	2	5	1	4	7	3
5	3	6	2	5	1	4	7
6	7	3	6	2	5	1	4
7	4	7	3	6	2	5	1

→ k-VALUES

1	2	3	4	5	6	7
1	2	3	5	7	6	4
1	2	3	7	5	4	6
1	2	4	6	5	3	7
1	2	6	4	3	5	7
1	3	2	7	4	5	6
1	3	4	5	2	7	6
1	3	5	4	2	6	7
1	3	5	6	7	4	2
1	3	5	7	2	4	6
1	5	2	6	3	7	4
1	5	3	2	4	6	7
1	5	6	7	2	4	3
1	6	3	4	5	7	2
1	6	4	2	7	5	3
1	6	5	7	2	3	4
1	7	2	4	5	6	3
1	7	5	2	3	4	6
1	7	6	5	4	3	2

→ VALID CYCLIC SEQUENCE SETS OF k-VALUES.

RANKING-NUMBER

$J_l = \left| n + \text{THE SUM OF THE FIRST } (k-1) \text{ NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 222222.} \right|_7$

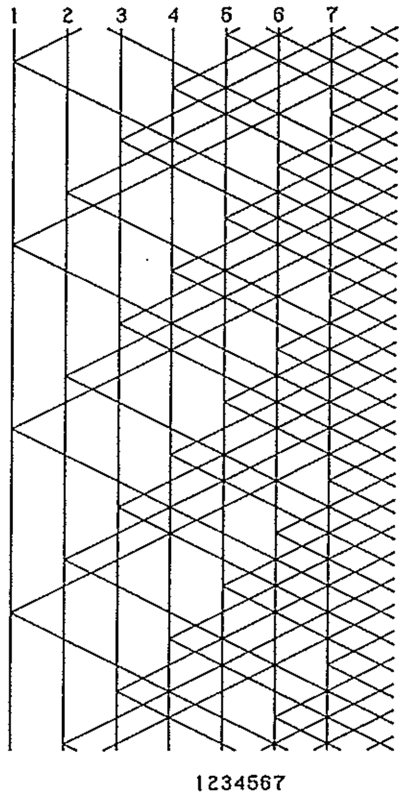


Fig. 432 — Left bight-boundary specification 222222 with $\mathcal{K}_l = A_l = 7$.

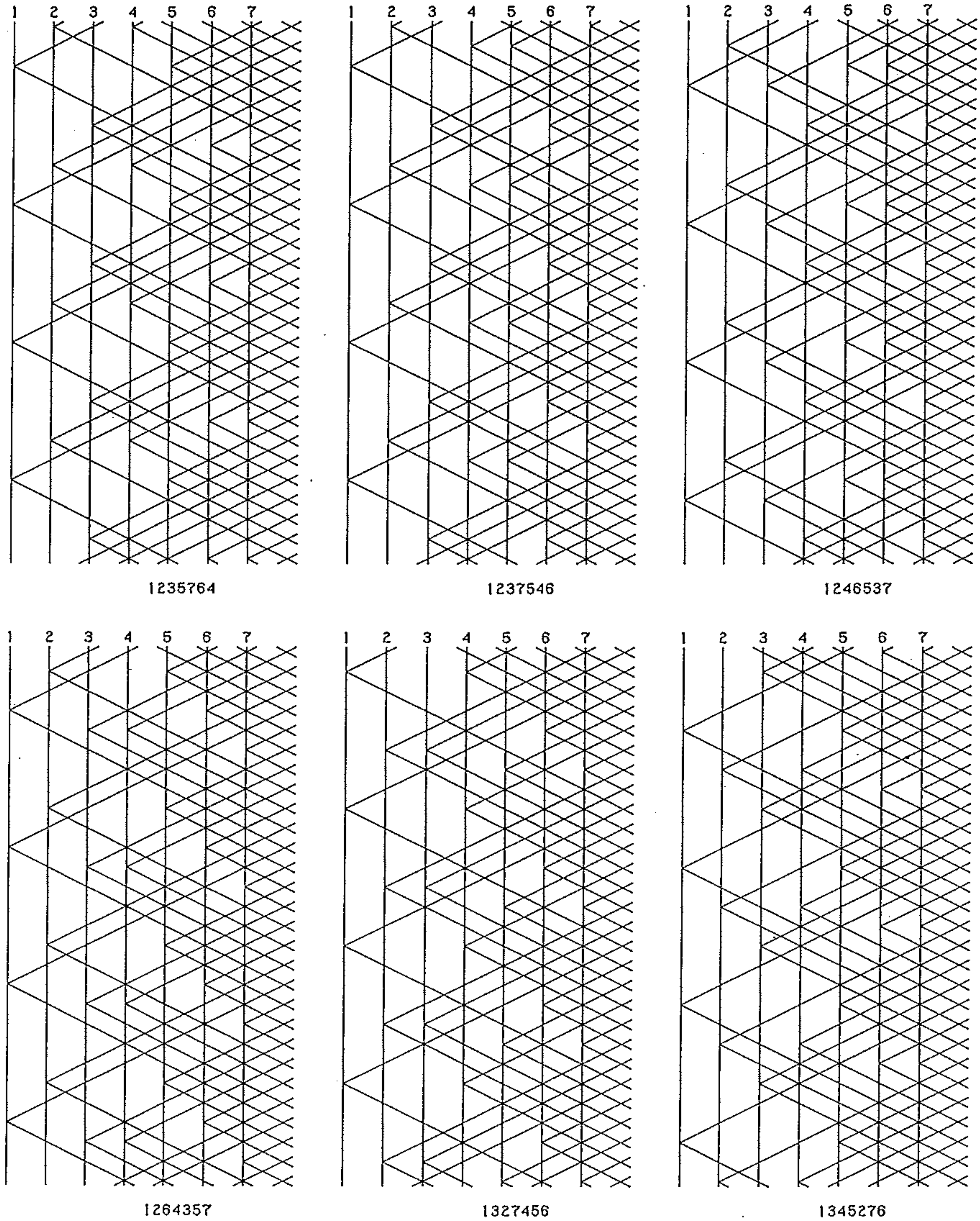
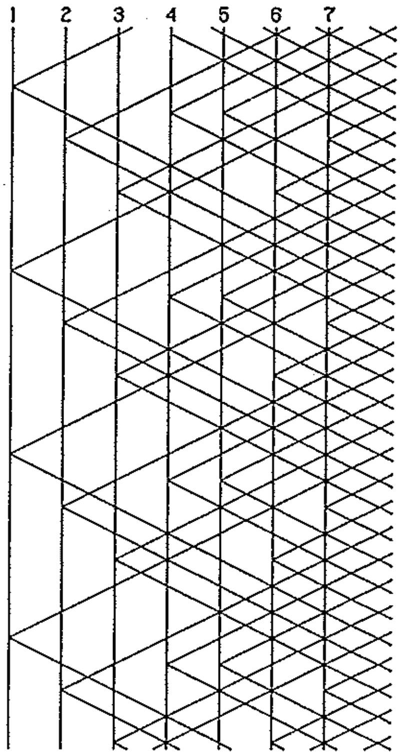
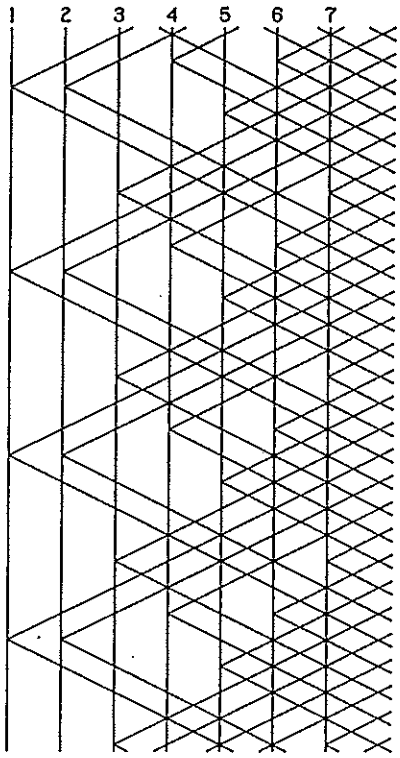


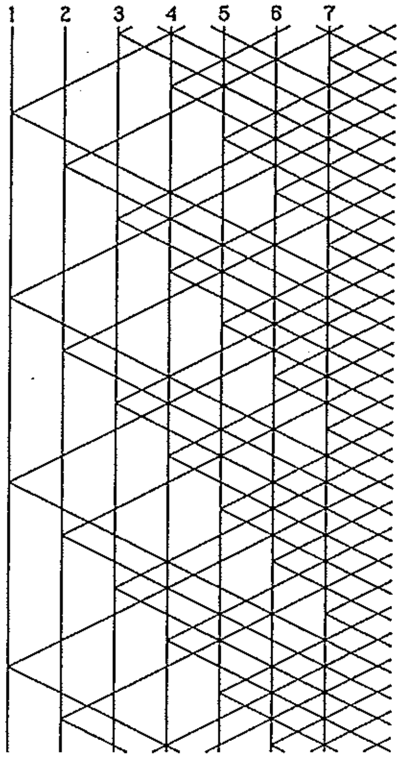
Fig. 432 cont.



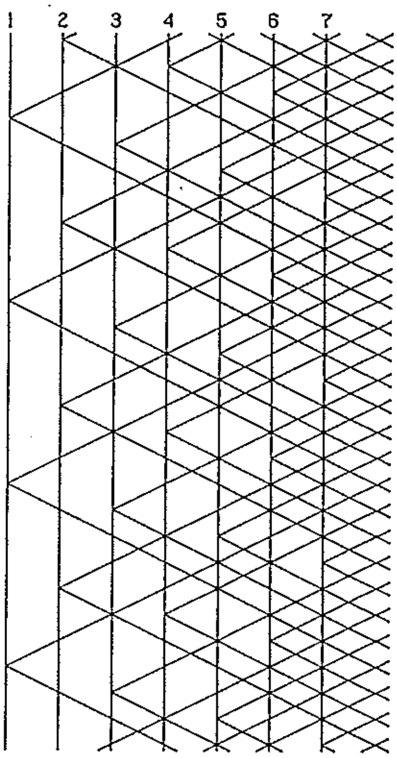
1354267



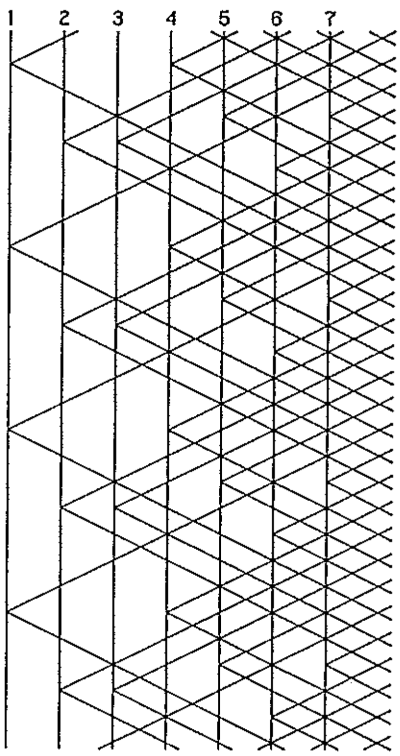
1356742



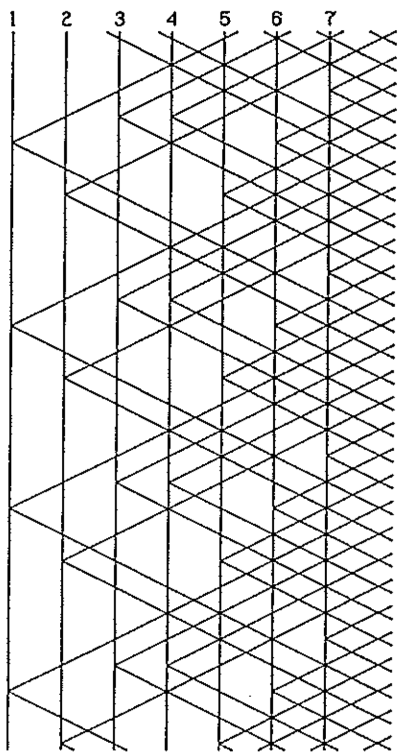
1357246



1526374



1532467



1567243

Fig. 432 cont.

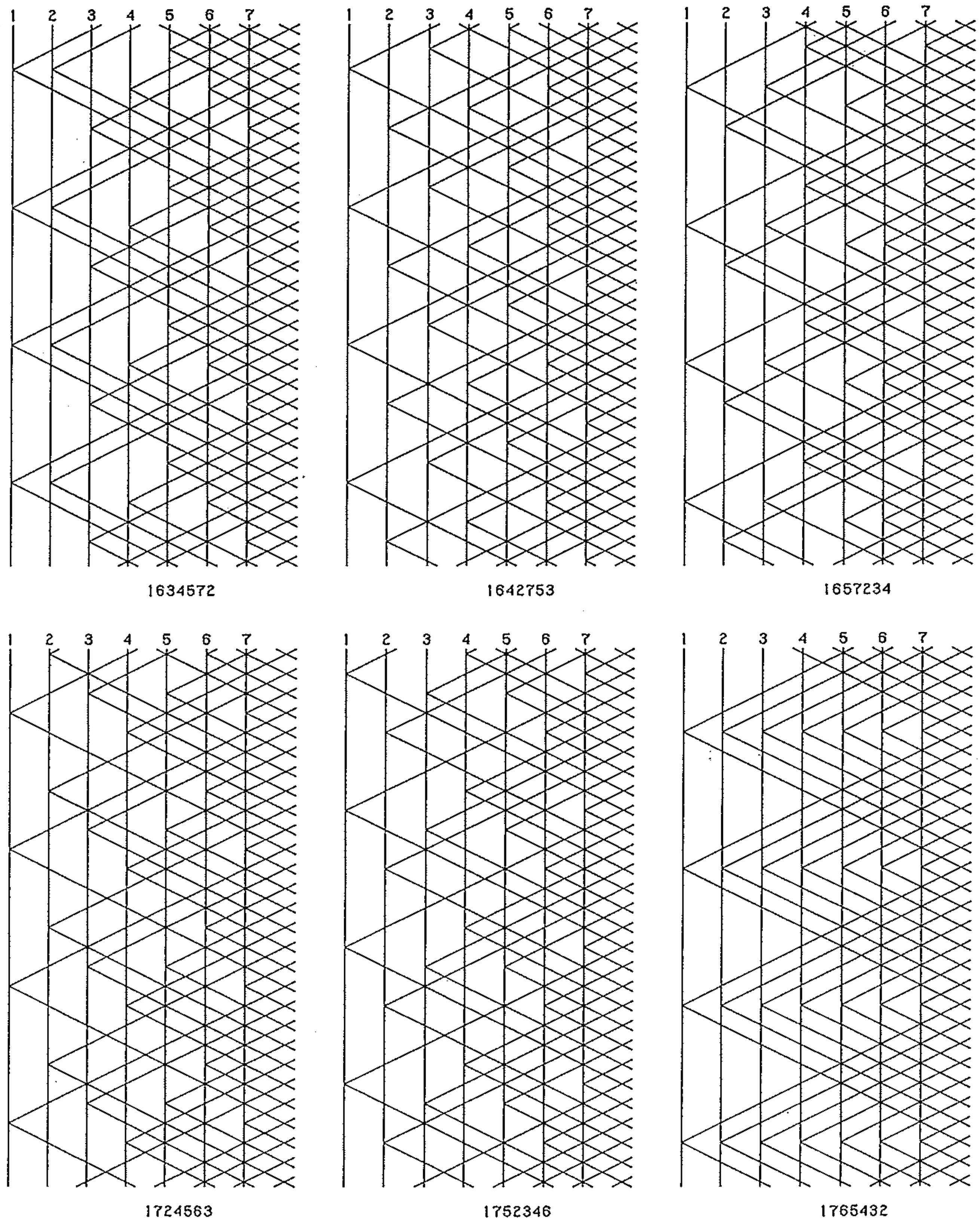

1 2 3 4 5 6 7
1634572
1 2 3 4 5 6 7
1642753
1 2 3 4 5 6 7
1657234
1 2 3 4 5 6 7
1724563
1 2 3 4 5 6 7
1752346
1 2 3 4 5 6 7
1765432

Fig. 432 cont.

Hence due to the left sequence set $1A(A-1)(A-2)(A-3)\cdots 432$ and the right sequence set $k|k+1|_A|k+2|_A|k+3|_A\cdots A123\cdots|k-2|_A|k-1|_A$, where $1 \le k \le A$ there is no need to carry the ranking-numbers, and consequently we obtain for a general left and right cycle (see Fig. 433):

$$l_{i+1} = |l_i + x - 2(l_i + r_i)|_A \, .$$
$$r_{i+1} = |r_i + x - 2(r_i + l_{i+1})|_A \, .$$

Fig. 433 — A general left and right cycle associated with the Regular Nested Cylindrical Braids.

The further formulae on pg. 417 modify to:

$$B_l^* = B_r^* = B^* \, .$$
$$B_{total} = A_l B_l^* = A_r B_r^* = A^{**}B^{**} = AB^* \, .$$
$$d = \text{g.c.d.}(A_l\,,\, A_r) = \text{g.c.d.}(A\,,\, A) = A \, .$$
$$A^{**} = A \, .$$
$$B^{**} = B^* \, .$$
$$\alpha = \text{number of bights in first-return string-run} \, .$$
$$P_{component} = 4\alpha + \frac{\alpha \cdot x - 2\sum(l_i + r_i)}{A} \, .$$
$$P_{total} = \sum P_{component} \, .$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} \, .$$

$$\left.\begin{array}{r}\text{number of}\\ \text{sub-components}\\ \text{in a component}\end{array}\right\} = \text{g.c.d.}(P_{component}\,, B^*) \, .$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} \, .$$

Example:

The Regular Nested Cylindrical Braid specification is:

(22222/8/22222){165432/561234}24.

The string-run of this braid is shown in Fig. 434.

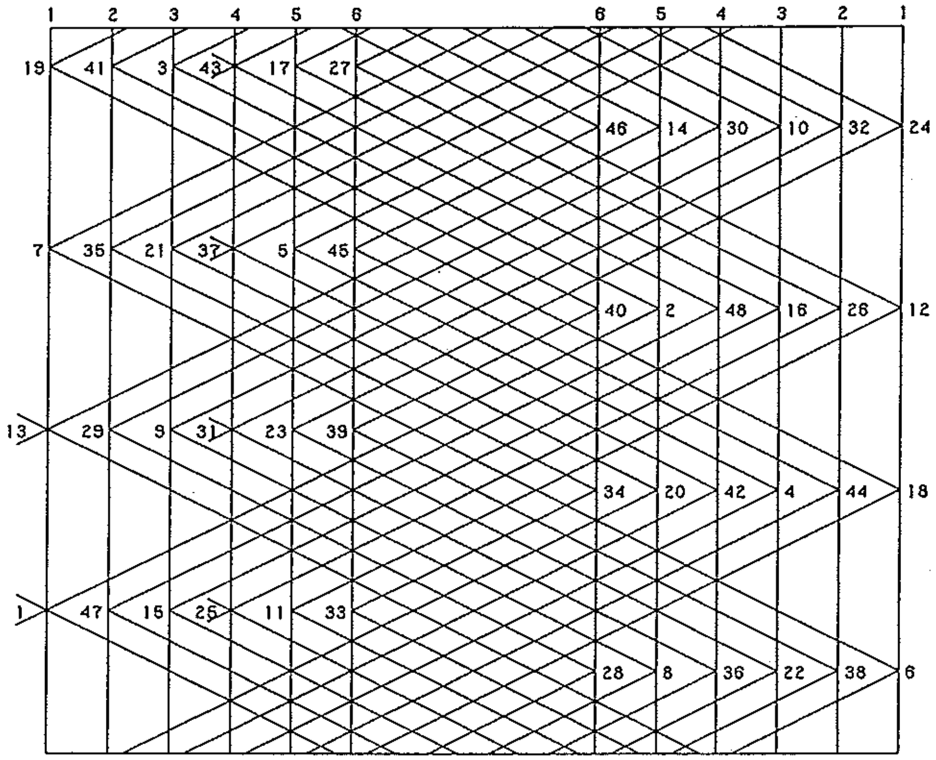


Fig. 434 — The string-run diagram of (22222/8/22222){165432/561234}24.

From the Regular Nested Cylindrical Braid specification we obtain:

$$A = 6\,.$$
$$x = 8\,.$$
$$d = A = 6\,.$$
$$B_{total} = AB^* = 24\,.$$
$$B^* = \frac{B_{total}}{A} = \frac{24}{6} = 4\,.$$
$$\mathcal{K}_l = \mathcal{K}_r = \mathcal{K} = A = 6\,.$$

From the given Regular Nested Cylindrical Braid specification we can read the lower-left to upper-right half-cycle types:

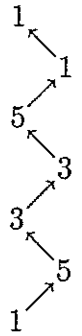
$1 \longrightarrow 5$	$4 \longrightarrow 2$
$6 \longrightarrow 6$	$3 \longrightarrow 3$
$5 \longrightarrow 1$	$2 \longrightarrow 4$

Anyone of these listed types may be taken as the first lower-left to upper-right half-cycle in the 1^{st} first-return string-run, but normally we take the first listed one. Every lower-left to upper-right half-cycle encountered in this 1^{st} first-return string-run gets deleted from the type-list.

Anyone of the remaining types in the type-list may be taken as the first lower-left to upper-right half-cycle in the 2^{nd} first-return string-run, and again every lower-left to upper-right half-cycle encountered in this 2^{nd} first-return string-run gets deleted from the remaining type-list.

This process is carried on till all the lower-left to upper-right half-cycle types have been deleted from the type-list.

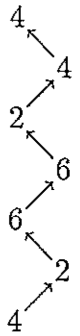
For the first-return string-runs we thus obtain:



$$\begin{array}{lll} l_1 = 1 \longrightarrow 5 = r_1 & \rightarrow & l_2 = |1+8-2(1+5)|_6 = 3\,. \\ l_2 = 3 \longleftarrow 5 = r_1 & \rightarrow & r_2 = |5+8-2(5+3)|_6 = 3\,. \\ l_2 = 3 \longrightarrow 3 = r_2 & \rightarrow & l_3 = |3+8-2(3+3)|_6 = 5\,. \\ l_3 = 5 \longleftarrow 3 = r_2 & \rightarrow & r_3 = |3+8-2(3+5)|_6 = 1\,. \\ l_3 = 5 \longrightarrow 1 = r_3 & \rightarrow & l_4 = |5+8-2(5+1)|_6 = 1\,. \\ l_4 = 1 \longleftarrow 1 = r_3 & \rightarrow & r_4 = |1+8-2(1+1)|_6 = 5\,. \end{array}$$

$$P_c = 4\alpha + \frac{\alpha x - 2\sum(l_i + r_i)}{A} = 4 \cdot 3 + \frac{3 \cdot 8 - 2\{(1+3+5)+(5+3+1)\}}{6} = 10\,.$$

g.c.d. $(P_c\,,\,B^*) =$ g.c.d. $(10\,,\,4) = 2\,.$



$$\begin{array}{lll} l_1 = 4 \longrightarrow 2 = r_1 & \rightarrow & l_2 = |4+8-2(4+2)|_6 = 6\,. \\ l_2 = 6 \longleftarrow 2 = r_1 & \rightarrow & r_2 = |2+8-2(2+6)|_6 = 6\,. \\ l_2 = 6 \longrightarrow 6 = r_2 & \rightarrow & l_3 = |6+8-2(6+6)|_6 = 2\,. \\ l_3 = 2 \longleftarrow 6 = r_2 & \rightarrow & r_3 = |6+8-2(6+2)|_6 = 4\,. \\ l_3 = 2 \longrightarrow 4 = r_3 & \rightarrow & l_4 = |2+8-2(2+4)|_6 = 4\,. \\ l_4 = 4 \longleftarrow 4 = r_3 & \rightarrow & r_4 = |4+8-2(4+4)|_6 = 2\,. \end{array}$$

$$P_c = 4\alpha + \frac{\alpha x - 2\sum(l_i + r_i)}{A} = 4 \cdot 3 + \frac{3 \cdot 8 - 2\{(4+6+2)+(2+6+4)\}}{6} = 8\,.$$

g.c.d. $(P_c\,,\,B^*) =$ g.c.d. $(8\,,\,4) = 4\,.$

Hence:

$$P_{total} = \sum P_{component} = 10 + 8 = 18\,.$$

$$\left.\begin{array}{r}\text{number of}\\ \text{components}\end{array}\right\} = \text{number of first-return string-runs} = 2\,.$$

$$\left.\begin{array}{r}\text{total number of}\\ \text{essential strings}\end{array}\right\} = \sum \text{sub-components} = 2 + 4 = 6\,.$$

⋆⋆ *The* **Standard Regular Nested Cylindrical Braids** *(g.c.d.* $(P_c\,,\,B^*) = 1$*), and the* **Semi-Standard Regular Nested Cylindrical Braids** *(g.c.d.* $(P_c\,,\,B^*) \neq 1$*) have a number of Components equal to* A*. For the Regular Nested Cylindrical Braids with their respective left and right sequence sets* $1A(A-1)(A-2)(A-3)\cdots 432$ *and* $k|k+1|_A|k+2|_A|k+3|_A \cdots A123\cdots|k-2|_A|k-1|_A$*, where* $1 \leq k \leq A$*, we know that when for the first half-cycle, starting at the left bight-boundary* $l_1 = 1$*, the right bight-boundary* r_1 *is known, the left and right bight-boundaries of all the left to right half-cycle types are known. How do we calculate* r_1 *for the Standard and Semi-Standard Regular Nested Cylindrical Braids, and what do we observe about their string-runs?*