THE BRAIDER

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A quarterly publication for the braiding artisan

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A Braid Name Change

Let's kick this new century off by changing the name of a braid that is known by at least two rather ambiguous names. It is the Regular Cylindrical Braid[†] of **one** string with an **over–under** weaving pattern throughout. Initially we defined this braid as a **Turk's Head Knot**[‡] Since the name *Turk's Head* is used by many people for a large number of single and multi string cylindrical braids, it is not a good name for a specific braidform. Later we adopted the name **Casa Knot**, solely for the shortness of the name. This name was extensively used by Tom Hall, for in the purely pragmatic world of braiding this braidform formed, and still forms, the main basis of interwoven braids.[††] It is therefore not surprising that in most publications far too much emphasis is placed on these so-called 'Casa Knots' in the case of interwoven knots.

It should be remembered that for interbraided knots, one knotform is not any more important than any other knotform.

The by us previously adopted names *'Turk's Head Knot'* and *'Casa Knot'* may cause a further problem in that an **over–under coding** throughout a braid not only may be seen as a *'Turk's Head' coding* or *'Casa' coding*, but also as a **column-coding** and **row-coding**. However, an **over–under coding** throughout a braid does in general **not** imply that the braid has a column-coding or a row-coding! The braid in Fig. 387 has an *over–under coding* throughout, its coding is a *column-coding*, but it is **not** a *row-coding* (see arrowed rows).

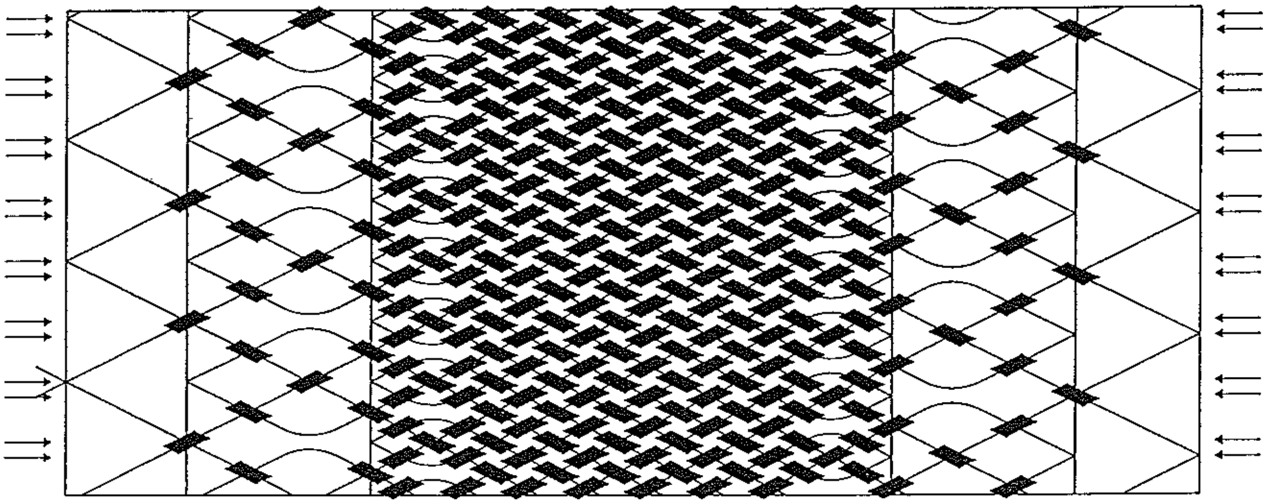


Fig. 387 — Over–under coding throughout; a column-coding, but not a row-coding.

The braid in Fig. 388 has also an *over–under coding* throughout, its coding is a *row-coding*, but it is **not** a *column-coding* (see arrowed columns).

In order to overcome the ambiguity associated with the names *'Turk's Head'* and *'Casa'*, we shall in future denote a braid which has an *over–under coding* throughout, as

† For the definition of **Regular Cylindrical Braid**, see *The Braider*, Issue No. 2, pg. 30, and Issue No. 1, pg. 6.

‡ Refer to *Braiding — Regular Knots; pg. 16.*

†† Casa is the Spanish word for: house, home, building. In the purely pragmatic world of braiding, a 'Casa Knot' forms the foundation knot of nearly all interwoven knots, and in that world many interwoven knots do consist of interbraided 'Casa Knots' — in interwoven knots, 'Casa Knots' house other knots, or are used in building them.

an **over–under coded** braid. Although the name is somewhat long, there is at least no ambiguity. For example, in future we shall call a 'Turk's Head Knot' or 'Casa Knot' (a single string Regular Cylindrical Braid with an over–under coding throughout) an **over–under coded Regular Cylindrical Knot**. A multi-string Regular Cylindrical Braid with a Casa-coding we shall in future call an **over-under coded Regular Cylindrical Braid**. In this context, the term **Braid** will be used as a general term, hence one or more strings may be required in the construction, while the term **Knot** will indicate that only one string is required in the construction.

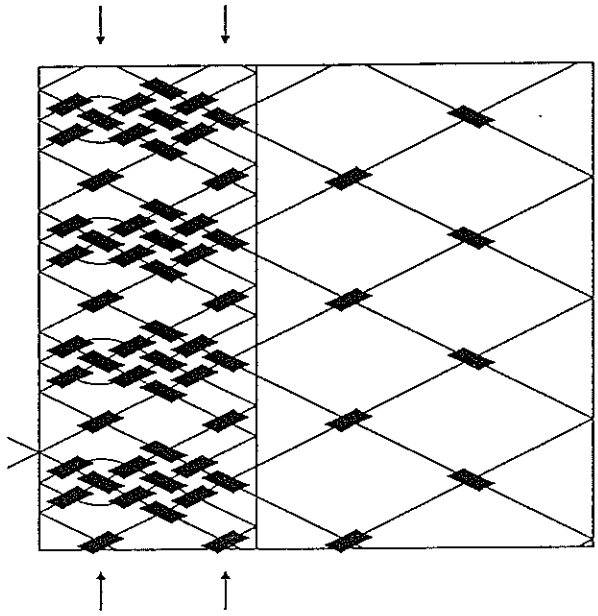


Fig. 388 — Over–under coding throughout; a row-coding, but not a column-coding.

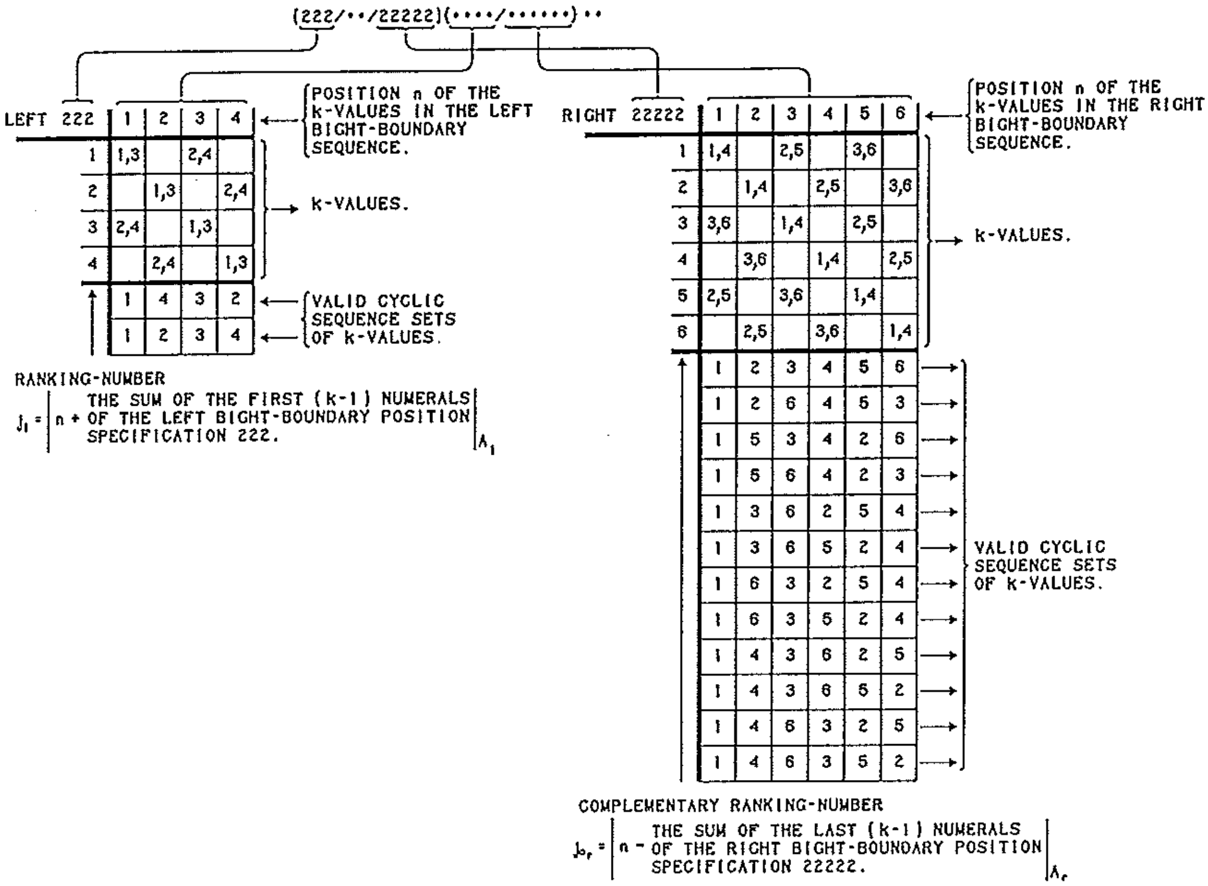
Nested Cylindrical Braids

In the previous Issue of *The Braider*, No. 20, pp. 437-441 we have seen that a set number $(\eta_l \cdot \eta_r \cdot d)$ of possible braid-types is associated with a left and right bight-boundary position specification pair. Under these possible braid-types there are η_l left bight-boundary arrangements and η_r right bight-boundary arrangements.

One of the most often encountered left and right bight-boundary position specifications is $222\cdots$ with $\mathcal{K}_l = A_l$ and $\mathcal{K}_r = A_r$ (the well-known 'asymmetric Pineapple knots' have such a left and right bight-boundary position specification with $\mathcal{K}_l = A_l$ and $\mathcal{K}_r = A_r$). Hence let's have a look at an example of such a left and right bight-boundary specification and see which left and right bight-boundary arrangements are associated with them.

Let the left bight-boundary specification be 222 and the right bight-boundary specification be 22222. Hence $\mathcal{K}_l = 4 = A_l$ and $\mathcal{K}_r = 6 = A_r$. The calculation of the left and right valid cyclic sequence sets of k-values is shown in Fig. 389. These calculations show that there are $\eta_l = 2$ valid left cyclic sequence sets of k-values and $\eta_r = 12$ valid right cyclic sequence sets of k-values. Since the g.c.d. $(A_l , A_r) =$ g.c.d. $(4 , 6) = 2 = d$, the number of possible braid-types is $\eta_l \cdot \eta_r \cdot d = 2 \cdot 12 \cdot 2 = 48$.

The $\eta_l = 2$ valid left cyclic sequence sets of k-values give us the 2 left bight-boundary arrangements shown in Fig. 390, while the 12 right bight-boundary arrangements associated with the $\eta_r = 12$ valid right cyclic sequence sets of k-values are depicted in Figs. 391 & 392.



	1	2	3	4
1	1,3		2,4	
2		1,3		2,4
3	2,4		1,3	
4		2,4		1,3
	1	4	3	2
	1	2	3	4

$j_l = \left| n + \right.$ THE SUM OF THE FIRST (k-1) NUMERALS OF THE LEFT BIGHT-BOUNDARY POSITION SPECIFICATION 222. $\left.\right|_{\Lambda_l}$

	1	2	3	4	5	6
1	1,4		2,5		3,6	
2		1,4		2,5		3,6
3	3,6		1,4		2,5	
4		3,6		1,4		2,5
5	2,5		3,6		1,4	
6		2,5		3,6		1,4
	1	2	3	4	5	6
	1	2	6	4	5	3
	1	5	3	4	2	6
	1	5	6	4	2	3
	1	3	6	2	5	4
	1	3	6	5	2	4
	1	6	3	2	5	4
	1	6	3	5	2	4
	1	4	3	6	2	5
	1	4	3	6	5	2
	1	4	6	3	2	5
	1	4	6	3	5	2

$j_{r} = \left| n - \right.$ THE SUM OF THE LAST (k-1) NUMERALS OF THE RIGHT BIGHT-BOUNDARY POSITION SPECIFICATION 22222. $\left.\right|_{\Lambda_r}$

Fig. 389 — The calculation of the left and right valid cyclic sequence sets of k-values.

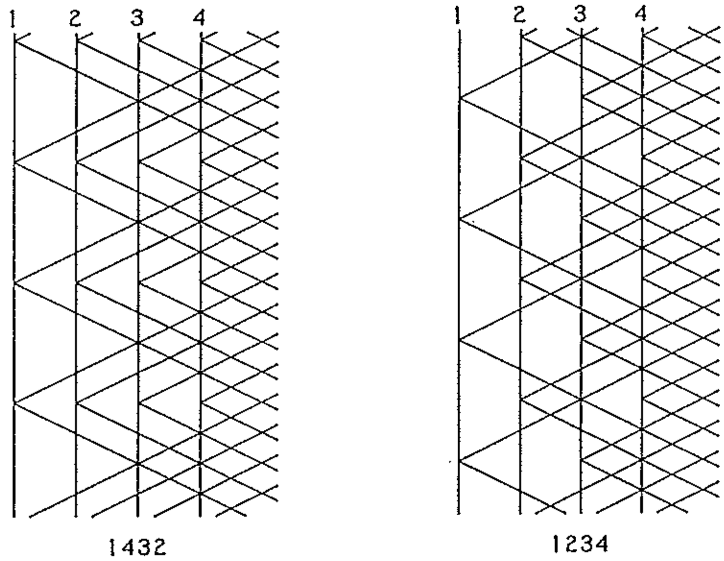
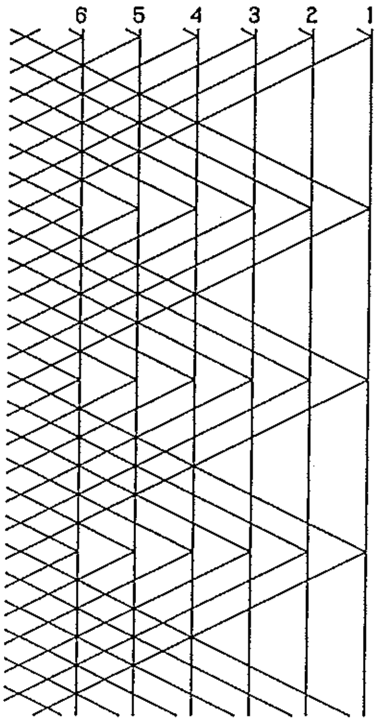


Fig. 390 — The $\eta_l = 2$ possible left bight-boundary arrangements.



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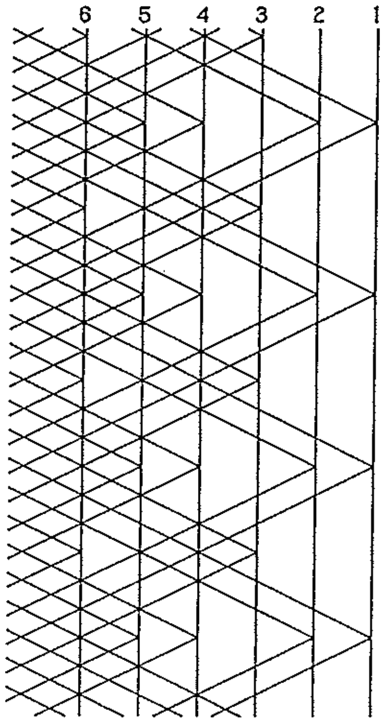

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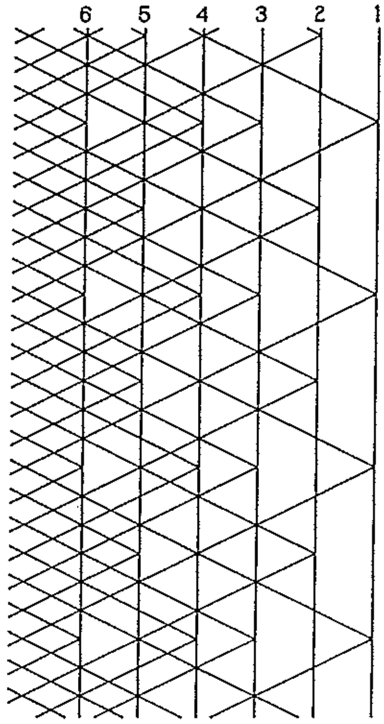

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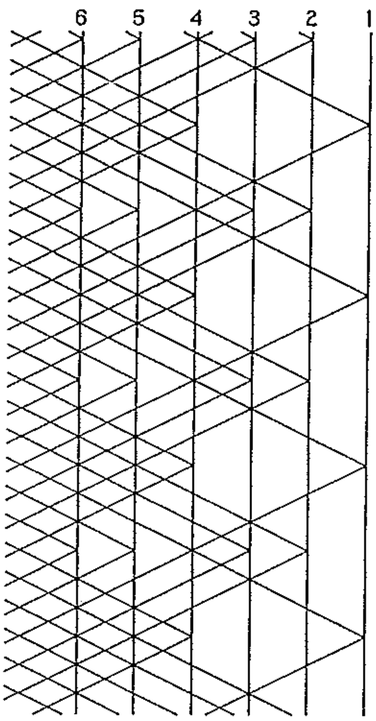

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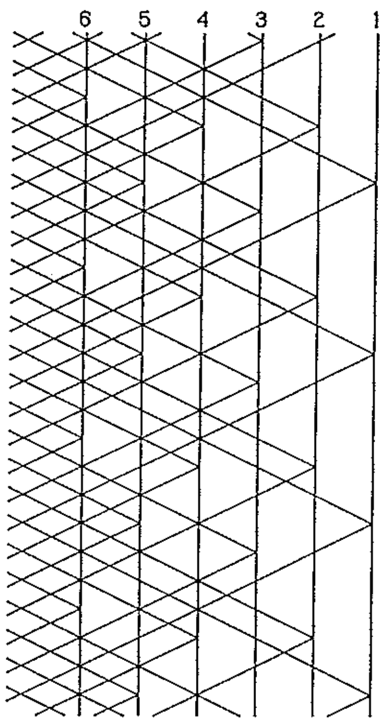

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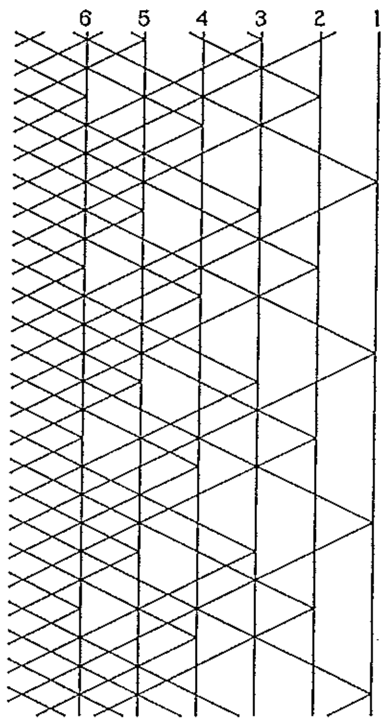

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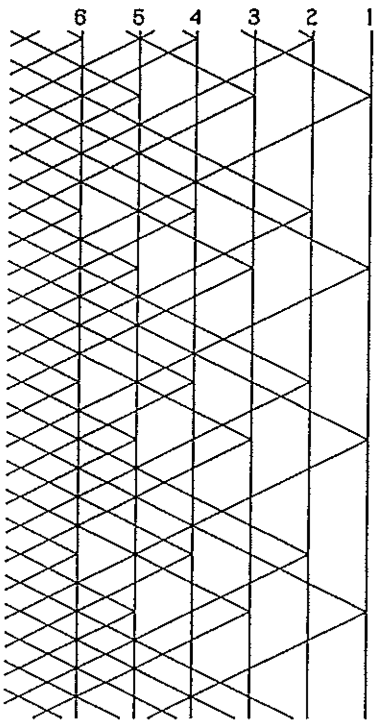
Fig. 391 — Together with Fig. 392: the $\eta_r = 12$ possible right bight-boundary arrangements.



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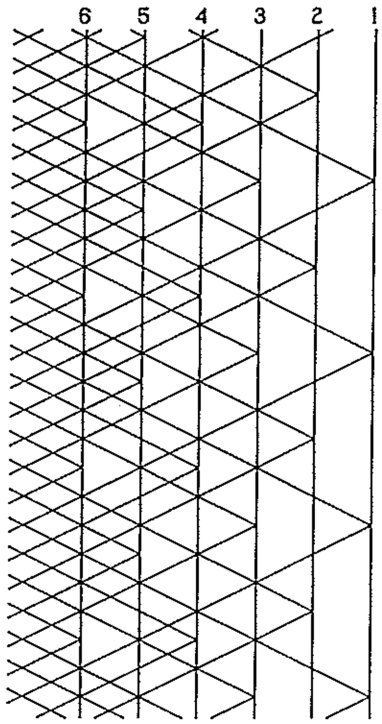

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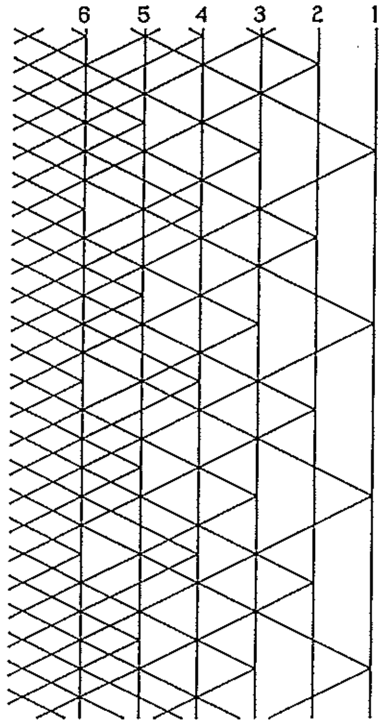

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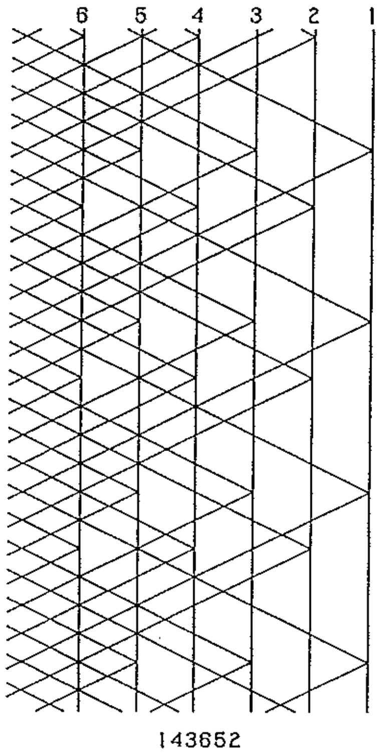

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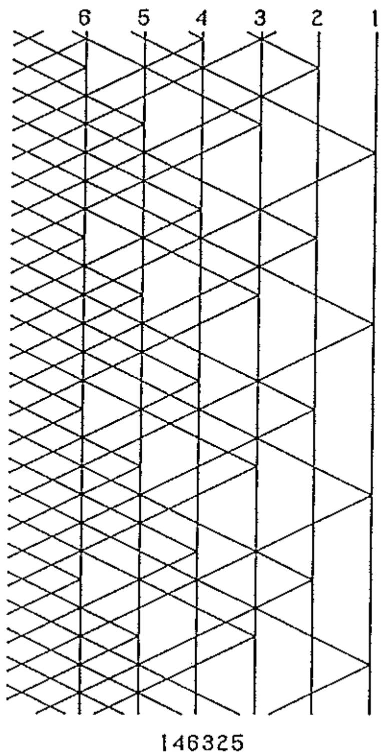

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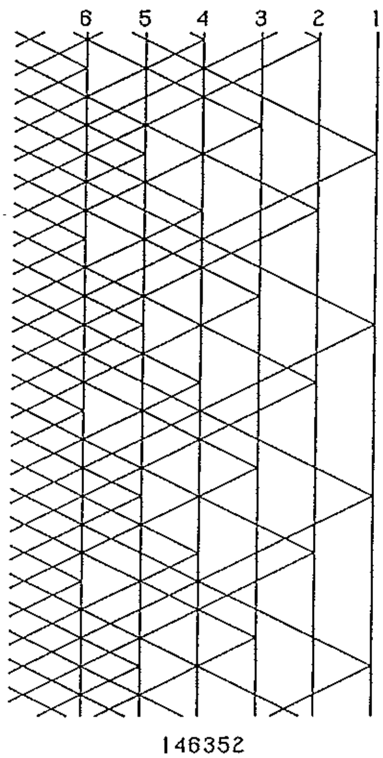

146352

Fig. 392 — Together with Fig. 391: the $\eta_r = 12$ possible right bight-boundary arrangements.

The 'asymmetric Pineapple knots' with $A_l = 4$ and $A_r = 6$, hence with the bight-boundary position specification (222/ ··· /22222), have the string-run specifications:

(1). (222/ ··· /22222){1432/123456} ··· .
(2). (222/ ··· /22222){1432/234561} ··· .

Although in the foregoing we calculated the left valid sequence sets and the right valid sequence sets in their respective ways, it is more convenient to obtain the right valid sequence sets from its corresponding left valid sequence sets. By rotating the string-run diagram through an angle of 180 deg so that the right bight-edge becomes the left bight-edge (and the left bight-edge becomes the right bight-edge) we can calculate for this new left bight-edge the left valid sequence sets. The corresponding right valid sequence sets (those associated with the original right bight-edge) are then the left valid sequence sets read in reverse cyclic order. Let's illustrate this with an example.

In Fig. 362 (see *The Braider*, Issue No. 20, pg. 439) we have shown the calculation of the right valid sequence sets (those associated with the right bight-boundary position specification 113 and the right nesting-number $A_r = 6$). The corresponding left bight-boundary position specification becomes then 311 (read the right bight-boundary position specification in reverse order) with the corresponding left nesting-number $A_l = 6$. The calculation of the associated left valid sequence sets is shown in Fig. 393. Note that the right valid sequence sets associated with the right bight-boundary position specification 113 and the right nesting-number $A_r = 6$, are readily obtained from the left valid sequence sets, associated with the left bight-boundary position specification 311 and the left nesting-number $A_l = 6$, by reading these sequence sets in reverse cyclic order.

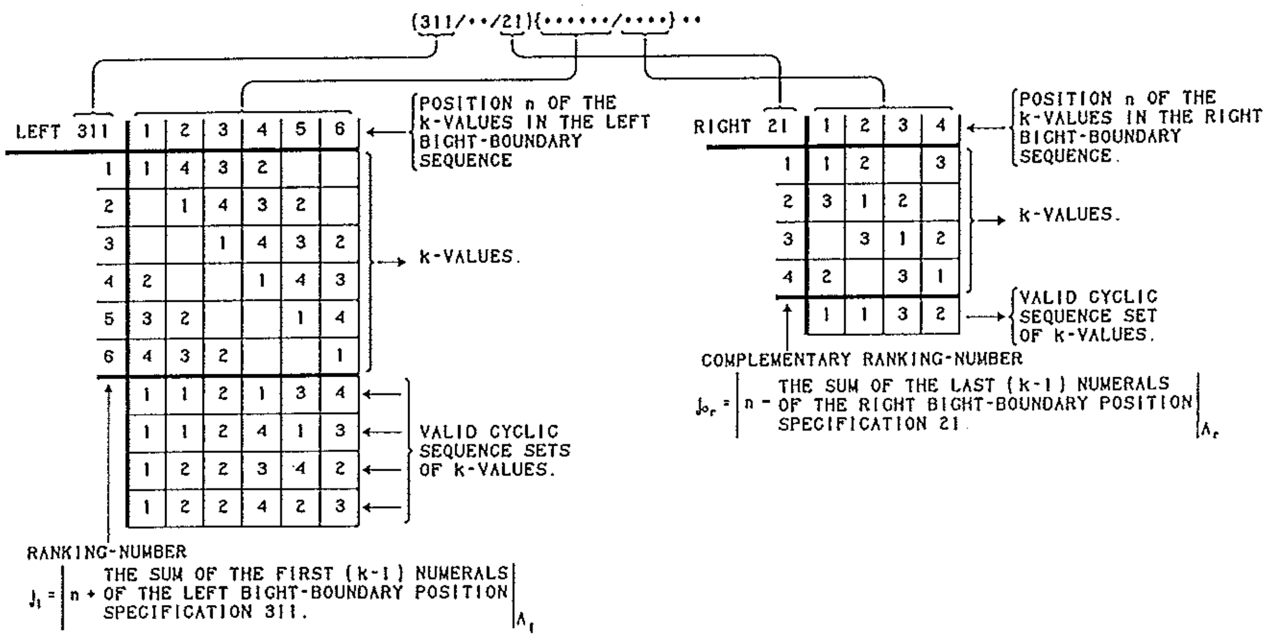

Fig. 393 — The string-run specification (311/ * */21){* * * * * * / * * * *} * *.

Similarly, the right valid sequence set associated with the right bight-boundary position specification 21 and the right nesting number $A_r = 4$, may be obtained from the left valid sequence set, associated with the left bight-boundary position specification 12 and the left nesting-number $A_l = 4$, by reading this sequence set in reverse cyclic order.

THE BRAIDER'S NOTEBOOK

A few years ago we read in a publication on braiding the statement: *"Almost all of the knots braiders use are tied from two bight Casa Knots."*. For certain 'braiders' this may well be true, but it is also true that such 'braiders' cannot really be regarded to be braiders. Thus, with regards the above mentioned statement, it will be instructive to examine the **over–under coded Regular Knots** (Casa Knots) in some greater detail.

Let's first see how various people might 'classify' the **over–under coded Regular Knots**, braided from a flat string which has for each opposite face pair identical faces in size, shape, texture and colour.

Some people might think that there are only two types, respectively depicted in Fig. 394 and Fig. 395, of **over–under coded Regular Knots** for the above specified string conditions.

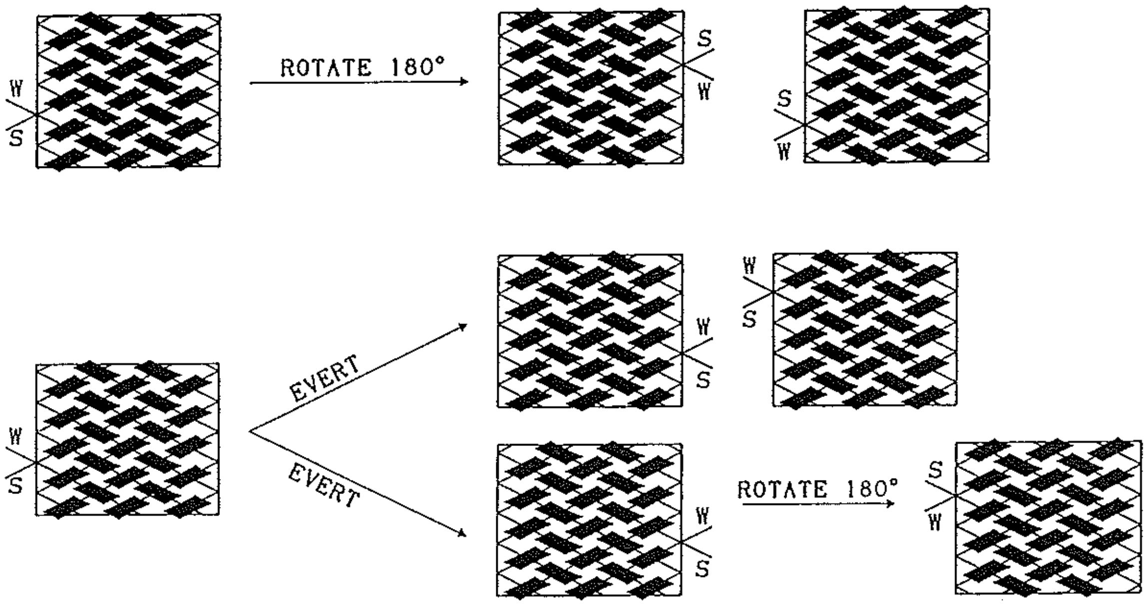

Fig. 394 — The type with an **even** number of parts.

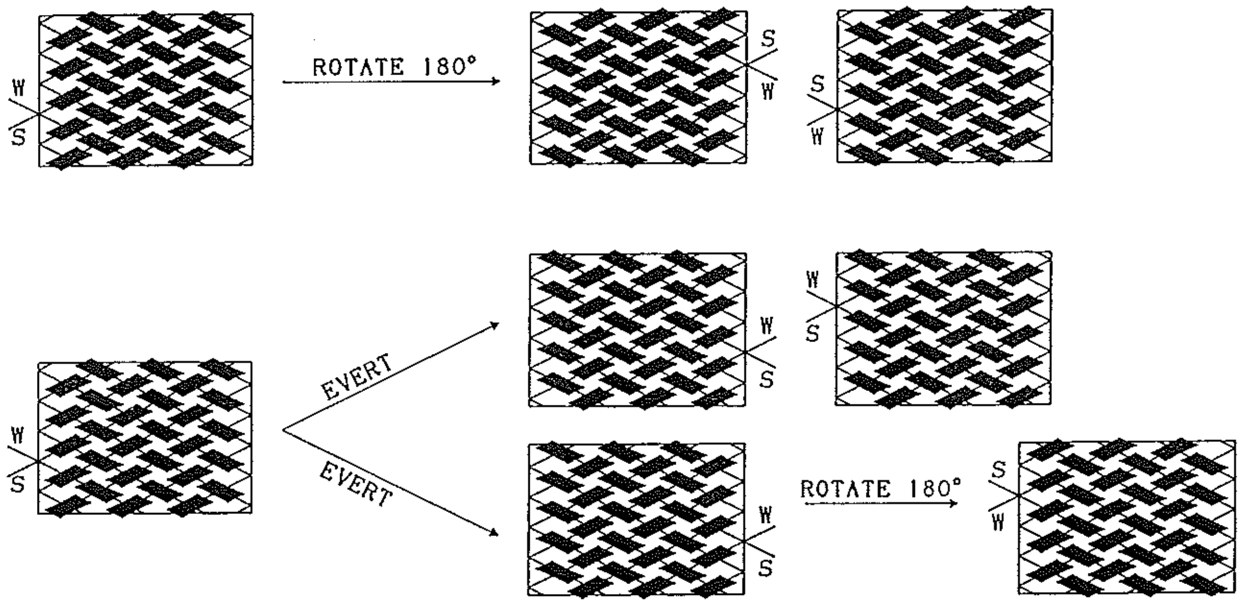

Fig. 395 — The type with an **odd** number of parts.

g.c.d. (parts , bights) = 1 , an **even** number of parts can only have an **odd** number of bights). Hence we can consider four types: two with respectively the complementary weaving-patterns, an **even** number of parts and an **odd** number of bights; and two with respectively an **odd** or an **even** number of bights and an **odd** number of parts. These four types are then illustrated in respectively Figs. 394 , 396 , 395 & 397.

Note that by only taking into account the possible parity relationships between the parts and the bights, one would also have obtained three types: one type for an **even** number of parts with an **odd** number of bights and two types for an **odd** number of parts with respectively an **odd** or **even** number of bights. This three-type 'classification' is of course quite different to the previous three-type 'classification' outlined before on pg. 472.

Let's consider the three possible parity-pairs associated with the parts/bights numbers:

$$p/b = \text{even}/\text{odd} = e/o\,,$$
$$p/b = \text{odd}/\text{odd} = o/o\,,$$
$$p/b = \text{odd}/\text{even} = o/e\,.$$

Each of these three parity-pairs occupies **four** different positions in the RKT (Regular Knot Tree). For the raising of the associated over–under coded Regular Knots to bigger over–under coded Regular Knots, each position is associated with **six** enlargement types, two of these are respectively an **even** multiple of first order enlargements in accordance with Enlargement Method I and an **even** multiple of first order enlargements in accordance with Enlargement Method II, while four of these are each a combination of a higher order (**odd** or **even**) enlargement followed by a first order enlargement both of the same Enlargement Method. The paths in the RKT of these six enlargement types are depicted in Fig. 398, where left of the thick vertical line are depicted the three types associated with Enlargement Method I, and where right of the thick vertical line are depicted the three types associated with Enlargement Method II (for $n =$ **odd** a higher **even** order, and for $n =$ **even** a higher **odd** order).

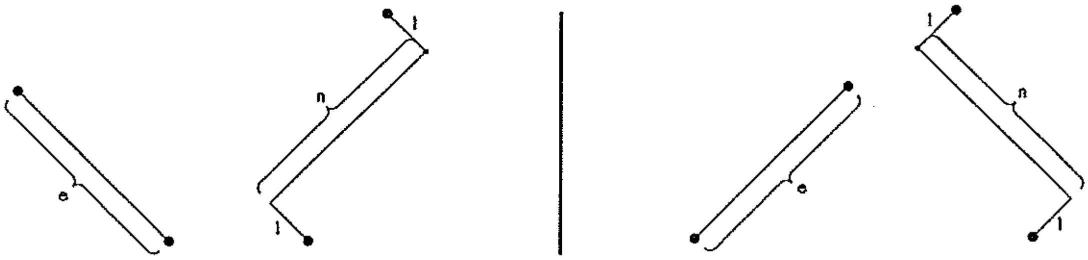

Fig. 398 — The **six** enlargement types.

Thus for each of the three parity-pairs $p/b = e/o$, $p/b = o/o$ & $p/b = o/e$ we obtain, with the four positions of the parity-pair in the RKT, **twenty four** enlargement types. With the four positions of each parity-pair in the RKT we can 'classify' the over–under coded Regular Knots into $4 \times 4 = 16$ types with each type having 6 raising procedures.

The 24 enlargement types associated with the parity-pair $p/b = e/o$ are depicted in Fig. 399; the 24 enlargement types associated with the parity-pair $p/b = o/o$ are depicted in Fig. 400; and The 24 enlargement types associated with the parity-pair $p/b = o/e$ are depicted in Fig. 401.

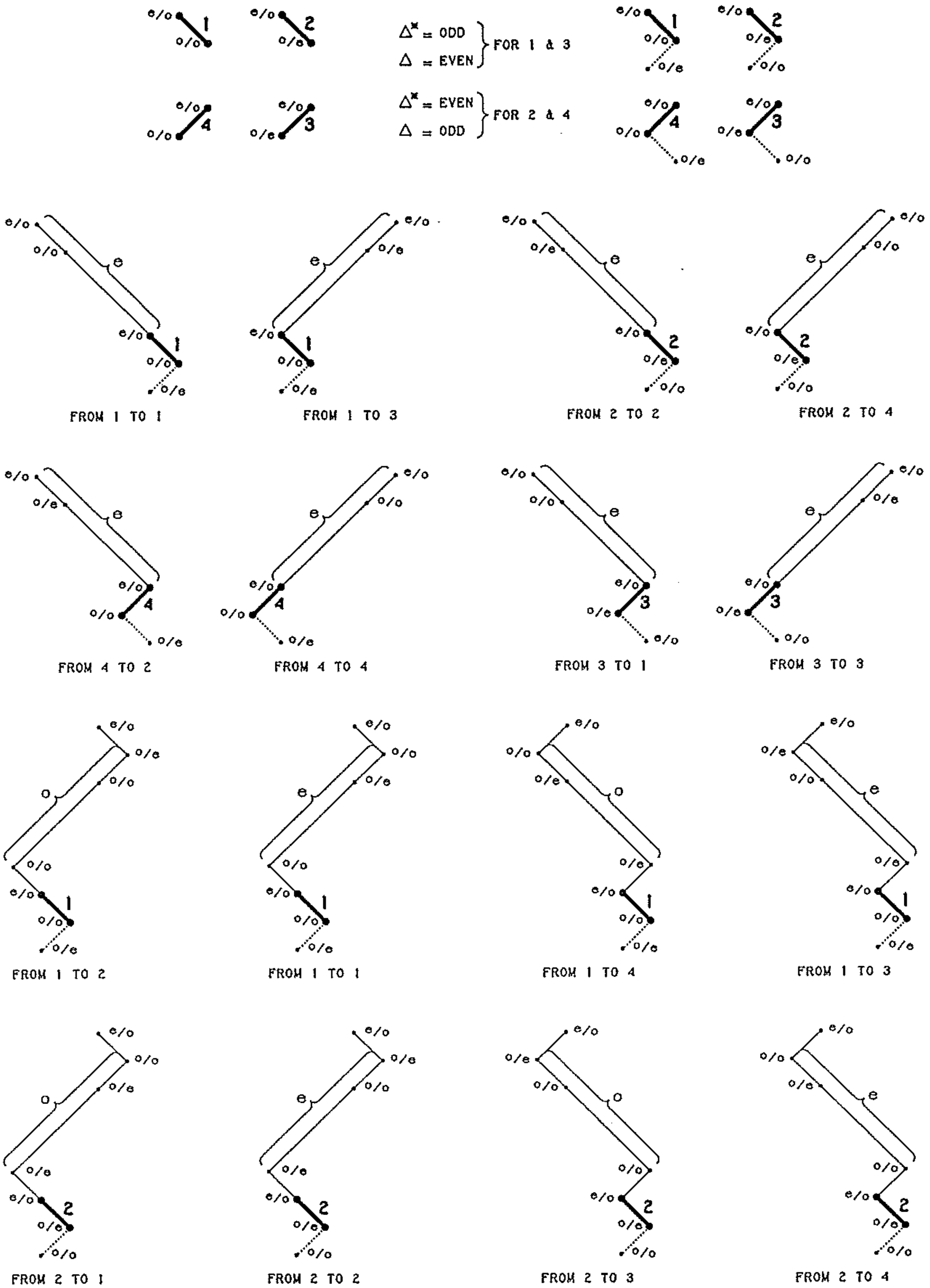

Fig. 399 — $p/b = e/o$, and its 24 enlargement types.

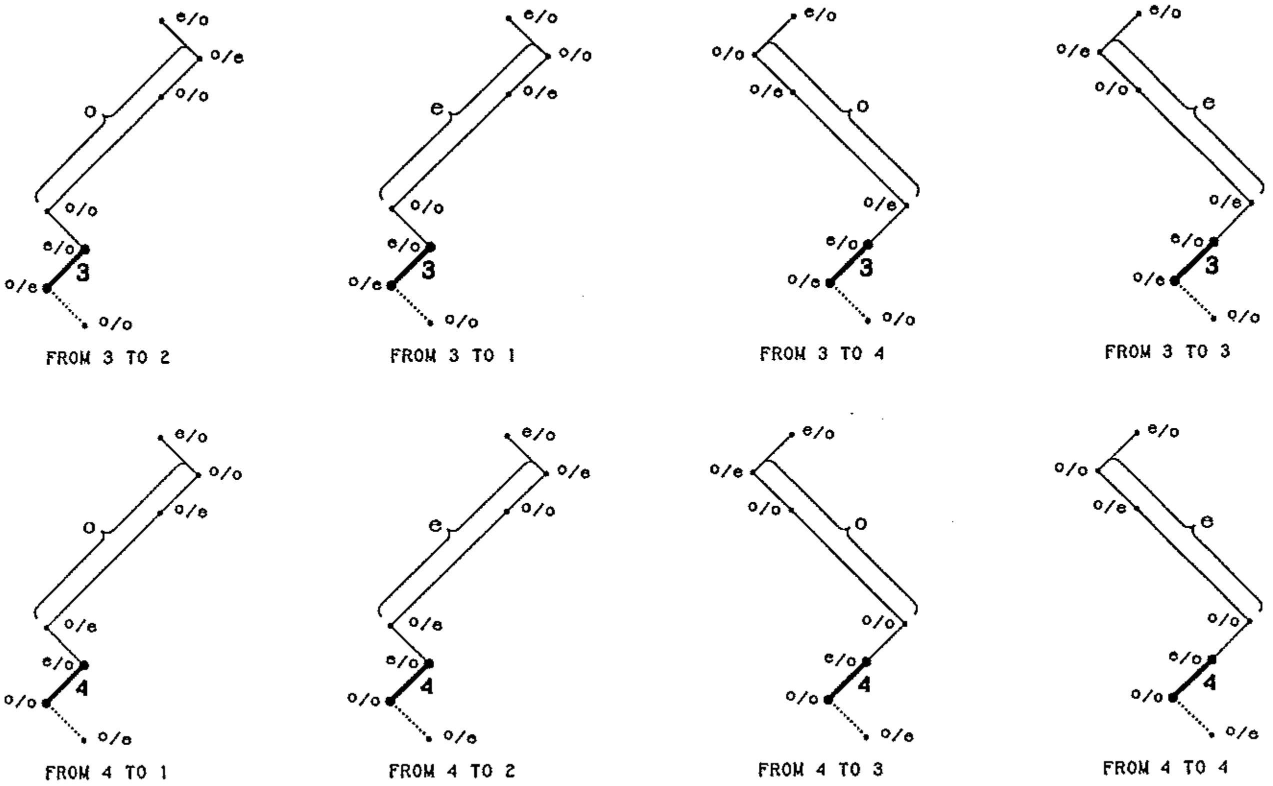

Fig. 399 continued — $p/b = e/o$, and its 24 enlargement types.

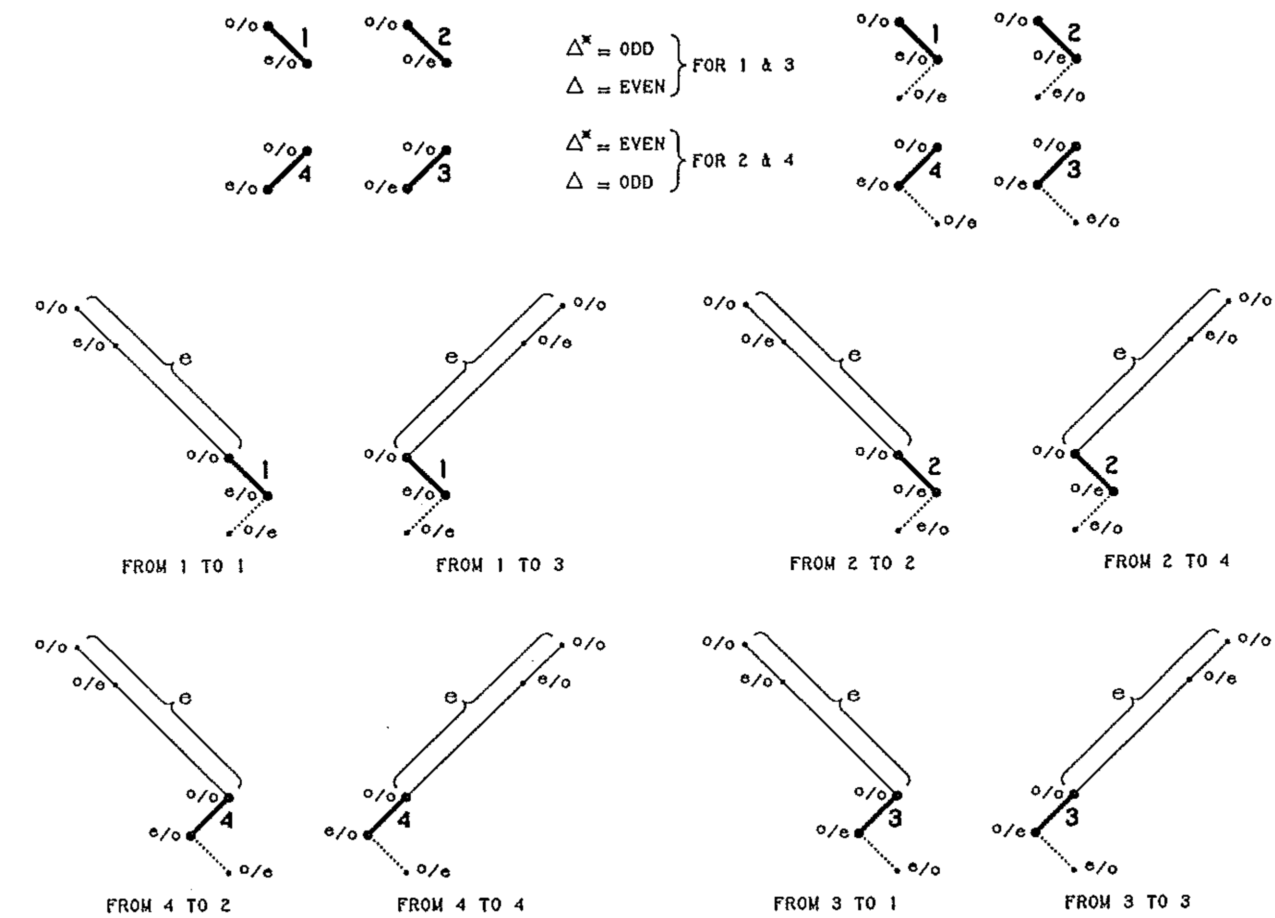

Fig. 400 — $p/b = o/o$, and its 24 enlargement types.

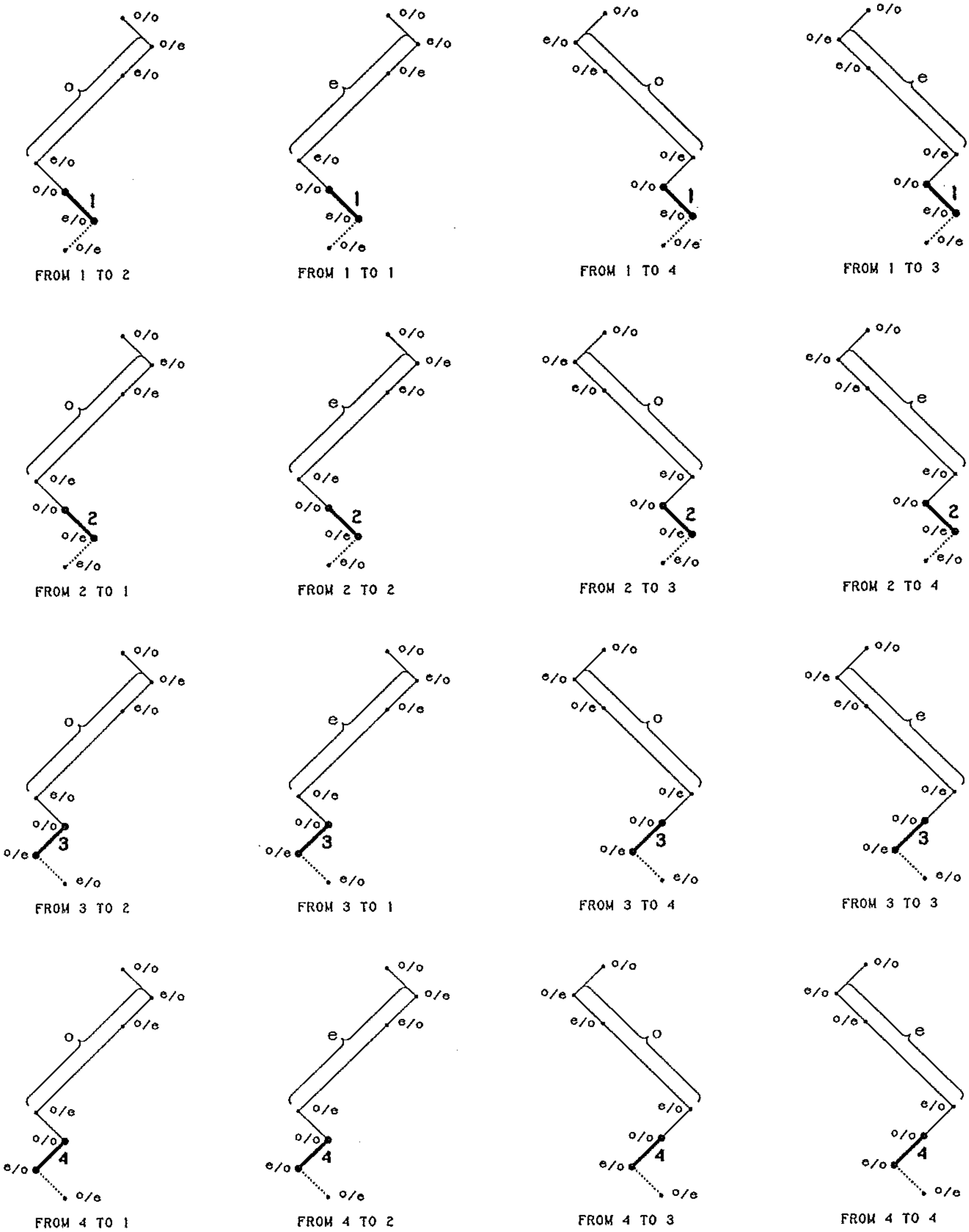

Fig. 400 continued — $p/b = o/o$, and its 24 enlargement types.

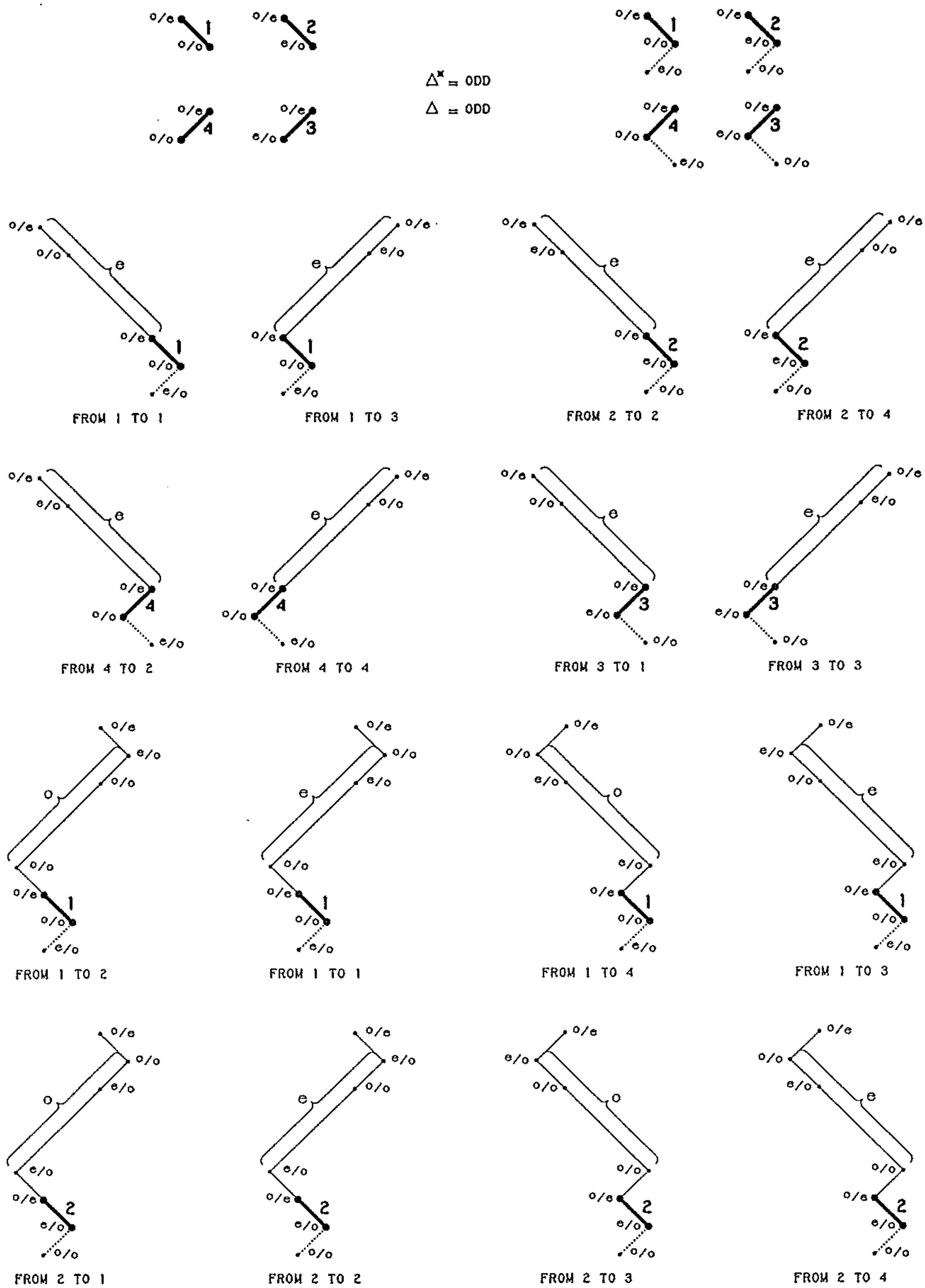

Fig. 401 — $p/b = o/e$, and its 24 enlargement types.

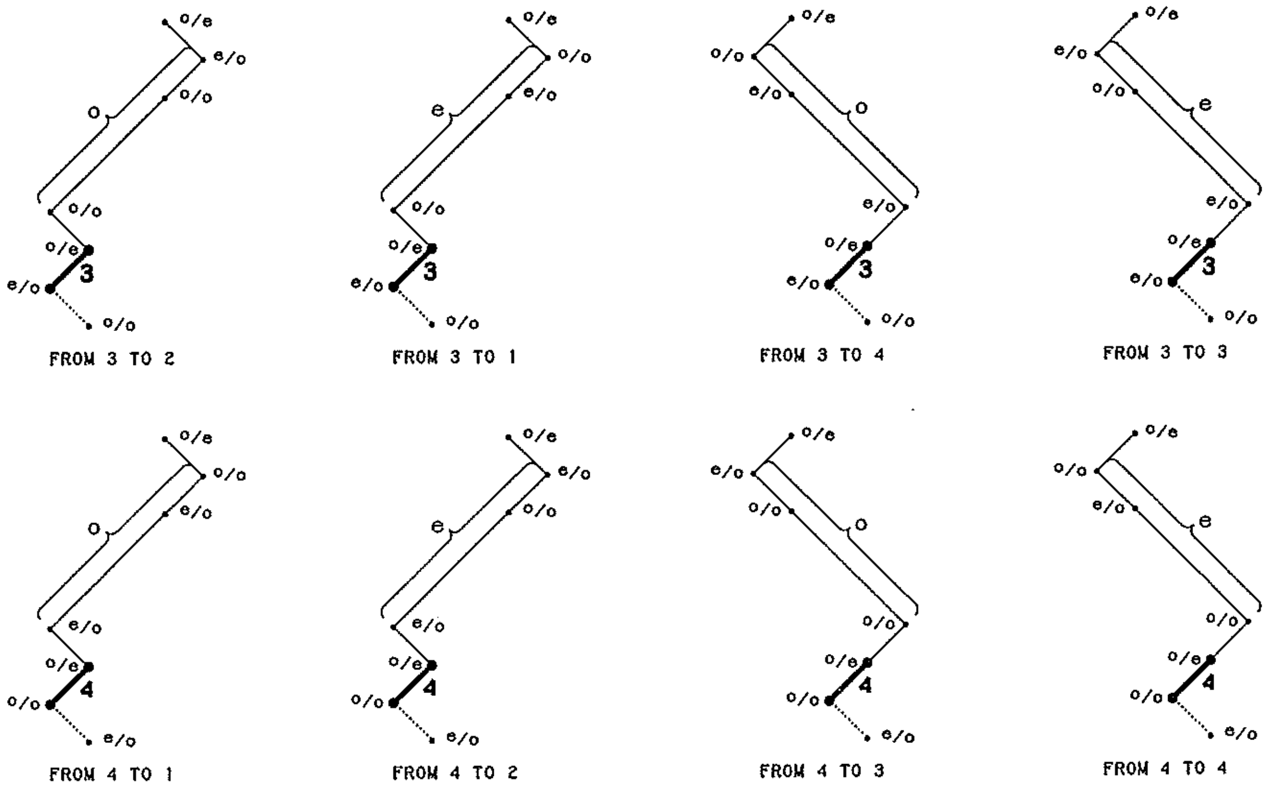

Fig. 401 continued — $p/b = o/e$, and its 24 enlargement types.

We note from these enlargement types that the parity-pair of a over–under coded Regular Knot is an **invariant**, hence when $p/b = e/o$, then any over–under coded Regular Knot raised from it also has $p/b = e/o$. Similarly, when $p/b = o/o$, then any over–under coded Regular Knot raised from it also has $p/b = o/o$, and when $p/b = o/e$, then any over–under coded Regular Knot raised from it also has $p/b = o/e$.

⋆⋆ *Prove in two much easier and quicker ways that a p/b parity-pair is an invariant in the enlargement from one over–under coded Regular Knot to another bigger over–under coded Regular Knot.*

Fig. 402 depicts the various relationships between p/b, the colour of the pattern (the weaving-pattern), the p/b parity-pairs, and the parities of Δ^* & Δ.

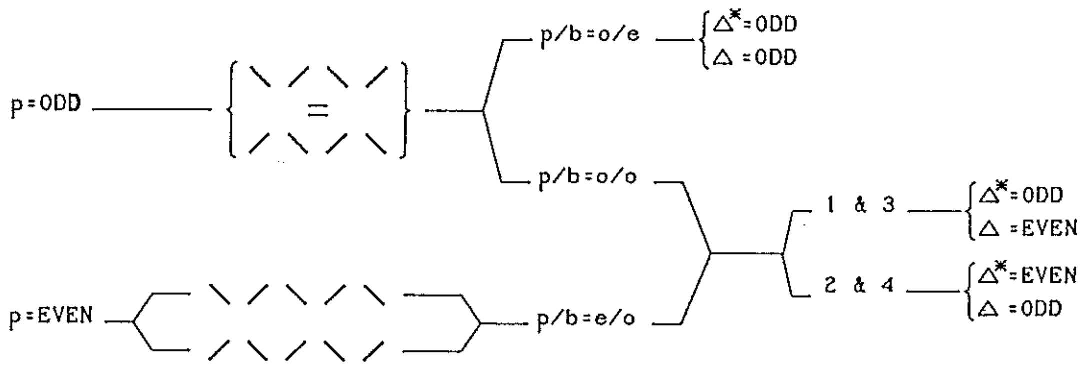

Fig. 402 — The relationships between the important parameters.

From the relationships depicted in Fig. 402 it follows that one must at least 'classify' the over–under coded Regular Knots into **seven** types, although a full 'classification'

within the over–under coded Regular Knots requires **sixteen** types as we have seen earlier.

Even the 'classification' of the simple over–under coded Regular Knots **within** their own restricted area of **over–under coded Regular Knots** is no doubt more extensive than most classification-proponents could imagine. If we would also try to cover in their 'classification' their properties **outside** their own, above indicated, restricted area, then the complications are enormous.

⋆⋆ *Name, as an example, an area which would complicate their 'classification' (hint: we have met this area in some previous Issues of The Braider).*

Let's now return to the statement mentioned in the beginning (see pg. 471). Since a p/b parity-pair is an invariant in the enlargement from one over–under coded Regular Knot (Casa Knot) to another bigger over–under coded Regular Knot (Casa Knot) and since by using over–under coded Regular Knots which almost all have been enlarged (raised) from 2-bight over–under coded Regular Knots, it follows that almost all over–under coded Regular Knots used by those so-called braiders have an **even** number of bights, hence with the p/b parity-pair o/e. Let's therefore see what proportion of over–under coded Regular Knots are apparently rarely, if ever, used by those people.

First we examine the distribution of each p/b parity-pair.

In Fig. 403 is depicted a general section of the RKT, starting near the bottom with a node which has a $p/b = o/o$ parity-pair. The uppermost two levels give the two cyclic p/b parity-pair sequences in the levels of the RKT. These two cyclic p/b parity-pair sequences in the levels of the RKT are diagrammetically depicted by the right-hand diagram. By reading the parity-pairs of a level from **left** to **right** we have the cyclic sequences:

$$\text{Odd levels: } o/e - e/o - o/o\,,$$
$$\text{Even levels: } o/o - e/o - o/e\,.$$

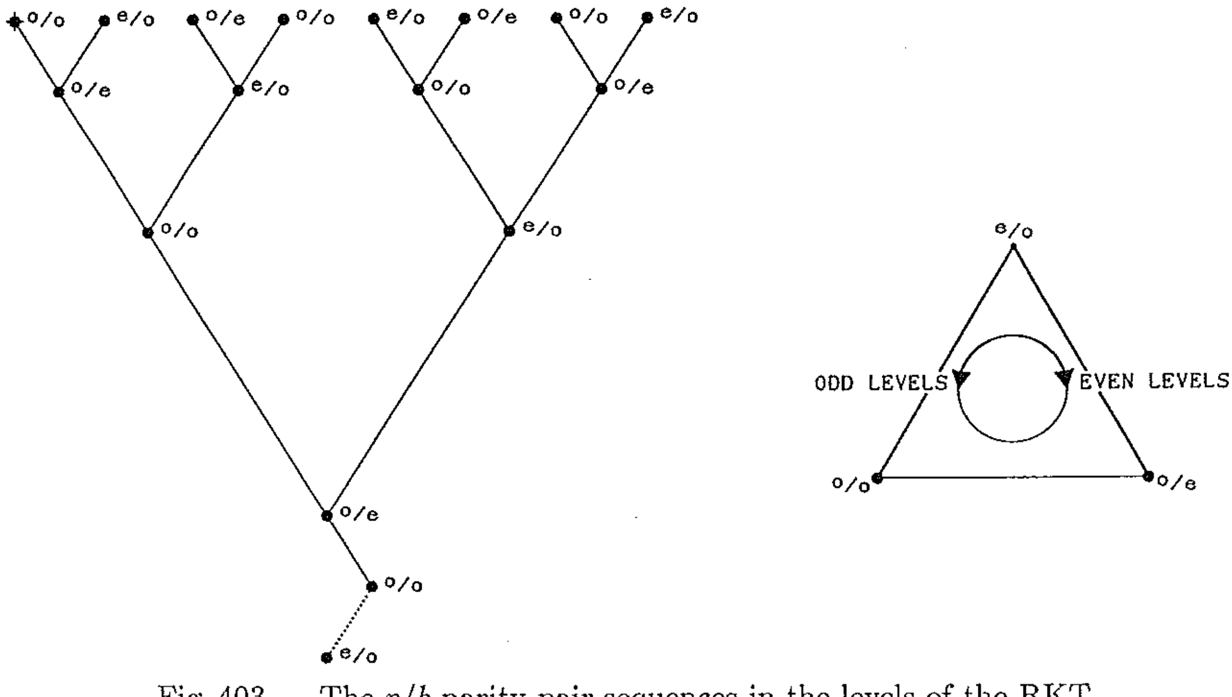

Fig. 403 — The p/b parity-pair sequences in the levels of the RKT.

The RKT up to and including level 7 has been shown in Fig. 404. In Fig. 405 the p/b-values in Fig. 404 have been replaced by their corresponding parity-pairs.

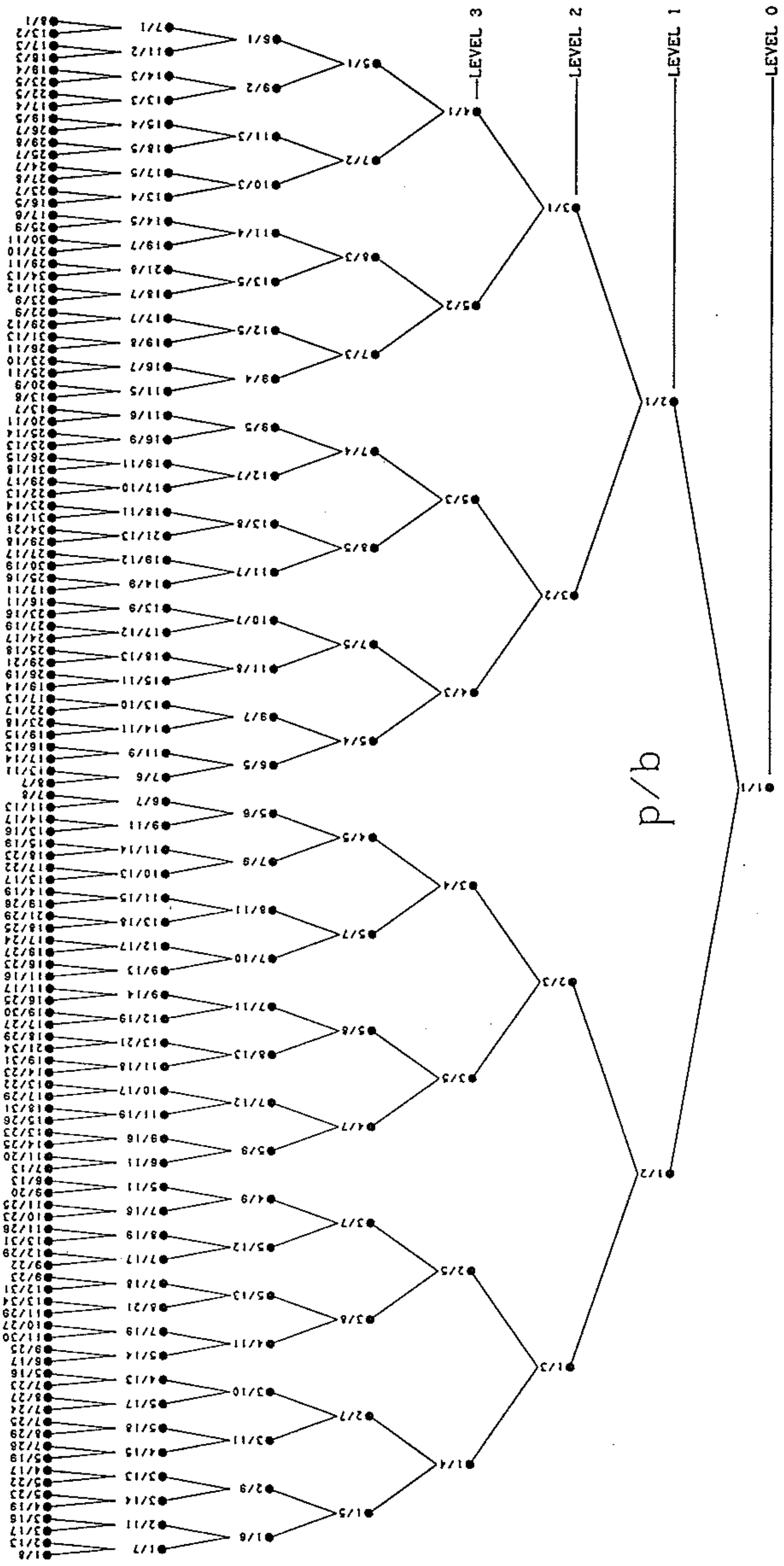

Fig. 404 — The RKT up to and including level 7.

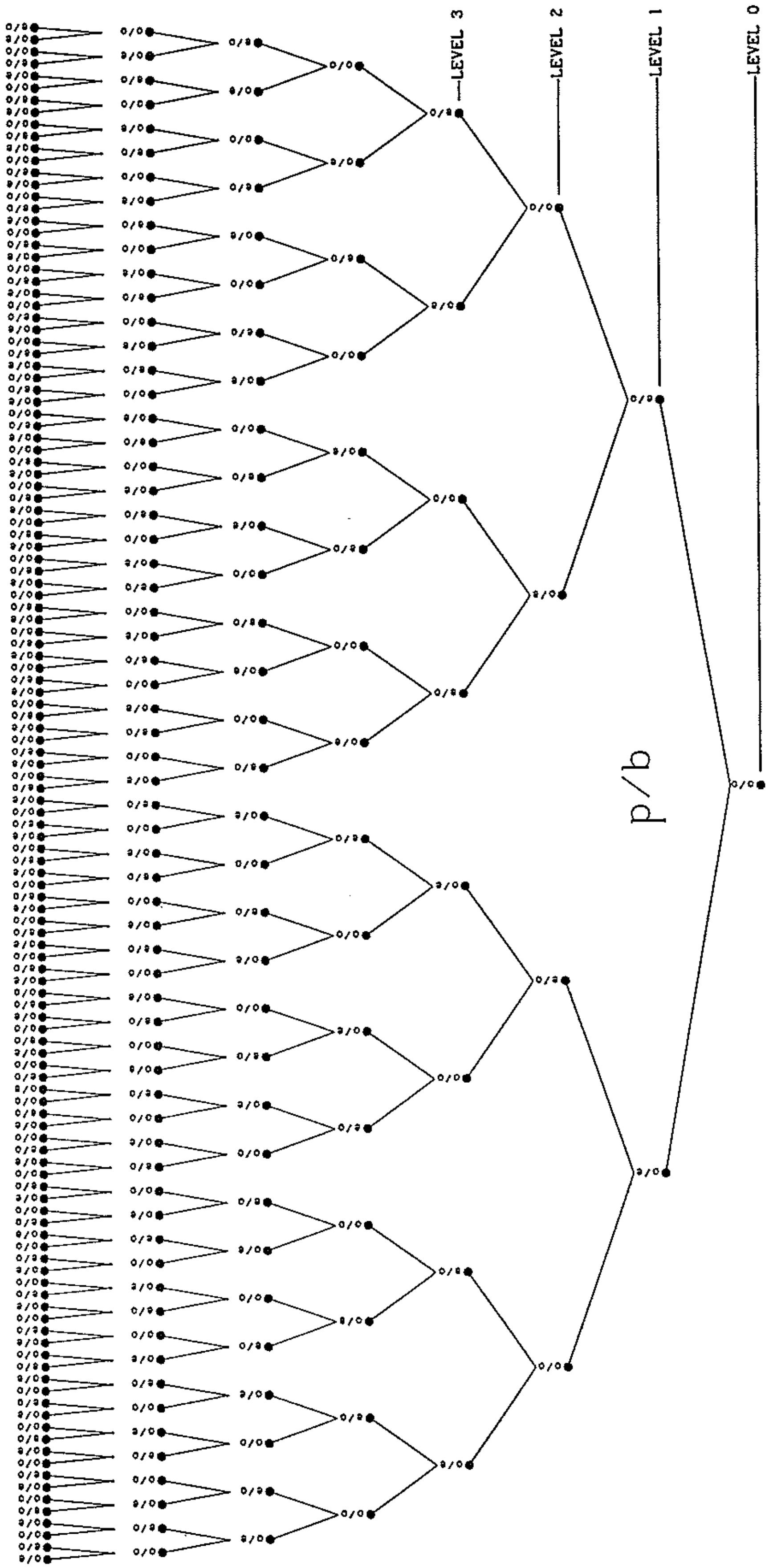

Fig. 405 — The p/b parity-pairs in the RKT up to and including level 7.

The level i in the RKT has 2^i nodes (parity-pairs), hence for $i = \mathbf{odd}$ we obtain:

$$\text{number of } o/e\text{'s is:} \quad \lfloor \tfrac{2^i-1}{3} \rfloor + 1 = \lceil \tfrac{2^i}{3} \rceil\,,$$
$$\text{number of } e/o\text{'s is:} \quad \lfloor \tfrac{2^i-2}{3} \rfloor + 1 = \lceil \tfrac{2^i}{3} \rceil\,,$$
$$\text{number of } o/o\text{'s is:} \quad \lfloor \tfrac{2^i-3}{3} \rfloor + 1 = \lfloor \tfrac{2^i}{3} \rfloor\,.$$

For $i = \mathbf{even}$ we obtain:

$$\text{number of } o/o\text{'s is:} \quad \lfloor \tfrac{2^i-1}{3} \rfloor + 1 = \lceil \tfrac{2^i}{3} \rceil\,,$$
$$\text{number of } e/o\text{'s is:} \quad \lfloor \tfrac{2^i-2}{3} \rfloor + 1 = \lfloor \tfrac{2^i}{3} \rfloor\,,$$
$$\text{number of } o/e\text{'s is:} \quad \lfloor \tfrac{2^i-3}{3} \rfloor + 1 = \lfloor \tfrac{2^i}{3} \rfloor\,.$$

Example:

For level 5:

$$\text{number of } o/e\text{'s is:} \quad \lfloor \tfrac{2^i-1}{3} \rfloor + 1 = \lfloor \tfrac{32-1}{3} \rfloor + 1 = 11\,,$$
$$\text{number of } e/o\text{'s is:} \quad \lfloor \tfrac{2^i-2}{3} \rfloor + 1 = \lfloor \tfrac{32-2}{3} \rfloor + 1 = 11\,,$$
$$\text{number of } o/o\text{'s is:} \quad \lfloor \tfrac{2^i-3}{3} \rfloor + 1 = \lfloor \tfrac{32-3}{3} \rfloor + 1 = 10\,.$$

For level 6:

$$\text{number of } o/o\text{'s is:} \quad \lfloor \tfrac{2^i-1}{3} \rfloor + 1 = \lfloor \tfrac{64-1}{3} \rfloor + 1 = 22\,,$$
$$\text{number of } e/o\text{'s is:} \quad \lfloor \tfrac{2^i-2}{3} \rfloor + 1 = \lfloor \tfrac{64-2}{3} \rfloor + 1 = 21\,,$$
$$\text{number of } o/e\text{'s is:} \quad \lfloor \tfrac{2^i-3}{3} \rfloor + 1 = \lfloor \tfrac{64-3}{3} \rfloor + 1 = 21\,.$$

Since the 1-part, the 2-parts column-coded Regular Knots and the 3-parts over–under coded Regular Knots as well as the 1-bight and the 2-bights over–under coded Regular Knots can, for the raising of over–under coded Regular Knots, be deducted from each level†, we obtain:

For $i = \mathbf{odd}$:

$$\text{number of } o/e\text{'s is:} \quad \lfloor \tfrac{2^i-1}{3} \rfloor - 2 = \lfloor \tfrac{2^i}{3} \rfloor - 2\,,$$
$$\text{number of } e/o\text{'s is:} \quad \lfloor \tfrac{2^i-2}{3} \rfloor - 1 = \lfloor \tfrac{2^i}{3} \rfloor - 1\,,$$
$$\text{number of } o/o\text{'s is:} \quad \lfloor \tfrac{2^i-3}{3} \rfloor = \lfloor \tfrac{2^i}{3} \rfloor - 1\,.$$

For $i = \mathbf{even}$:

$$\text{number of } o/o\text{'s is:} \quad \lfloor \tfrac{2^i-1}{3} \rfloor - 2 = \lfloor \tfrac{2^i}{3} \rfloor - 2\,,$$
$$\text{number of } e/o\text{'s is:} \quad \lfloor \tfrac{2^i-2}{3} \rfloor = \lfloor \tfrac{2^i}{3} \rfloor - 1\,,$$
$$\text{number of } o/e\text{'s is:} \quad \lfloor \tfrac{2^i-3}{3} \rfloor - 1 = \lfloor \tfrac{2^i}{3} \rfloor - 2\,.$$

It will be evident that just over 2/3 of the over–under coded Regular Knots will be out of bounds to those who only would use the over–under coded Regular Knots which can be enlarged from 2-bights over–under coded Regular Knots. It will thus be obvious that in general the statement: *"Almost all of the knots braiders use are tied from two bight Casa Knots."* is certainly incorrect.

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† The 3-parts over–under coded Regular Knots may be raised from the $p/b = 3/1$ and the $p/b = 3/2$ over–under coded Regular Knots as discussed in *The Braider*, Appendix 1996, pp. *iii* – *v*.

Integrated Braids

Sometimes we require long flat braids with a buttonhole or buttonholes at one end and a button Knot at the other end. An Example of the end with the buttonholes is shown in Fig. 406 for a six, respectively an eight part over–under coded Regular Flat Braid. This is the end at which braiding begins.

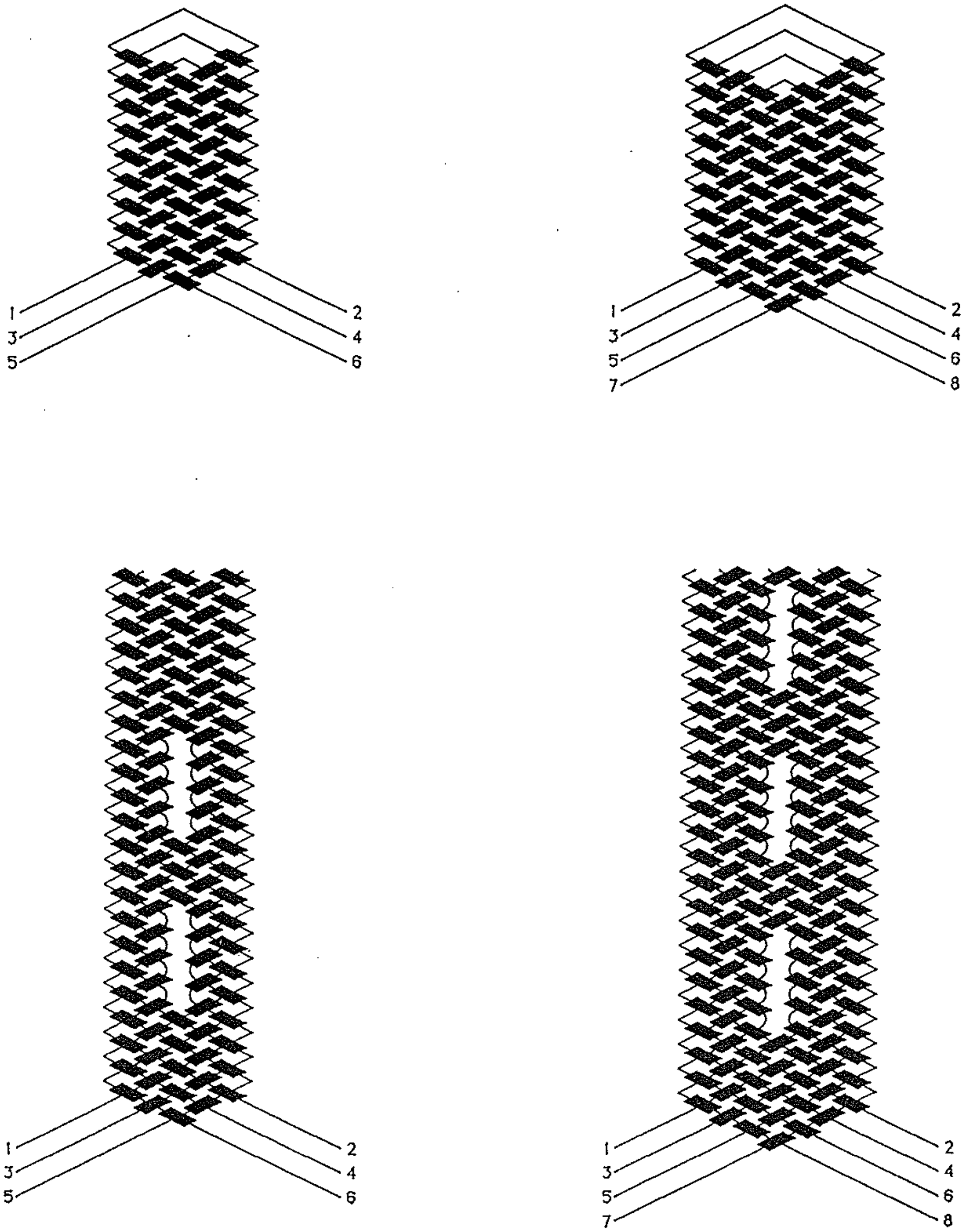

Fig. 406 — The start of the braid with the buttonholes.

At the other end of the flat braid (the end which will receive the button), the flat braid goes over into a round braid. For both the 6-part and the 8-part Regular Flat Braid, the round braid is a four-string over–under Round Braid. After braiding the round braid for a short distance, as indicated in Fig. 407, we braid over it the foundation knot of the button knot. In the case of the 6-part Regular Flat Braid, the foundation knot is constructed with the string-ends A and B, while in the case of the 8-part Regular Flat Braid, the foundation knot is constructed with the string-ends A, B, C and D.

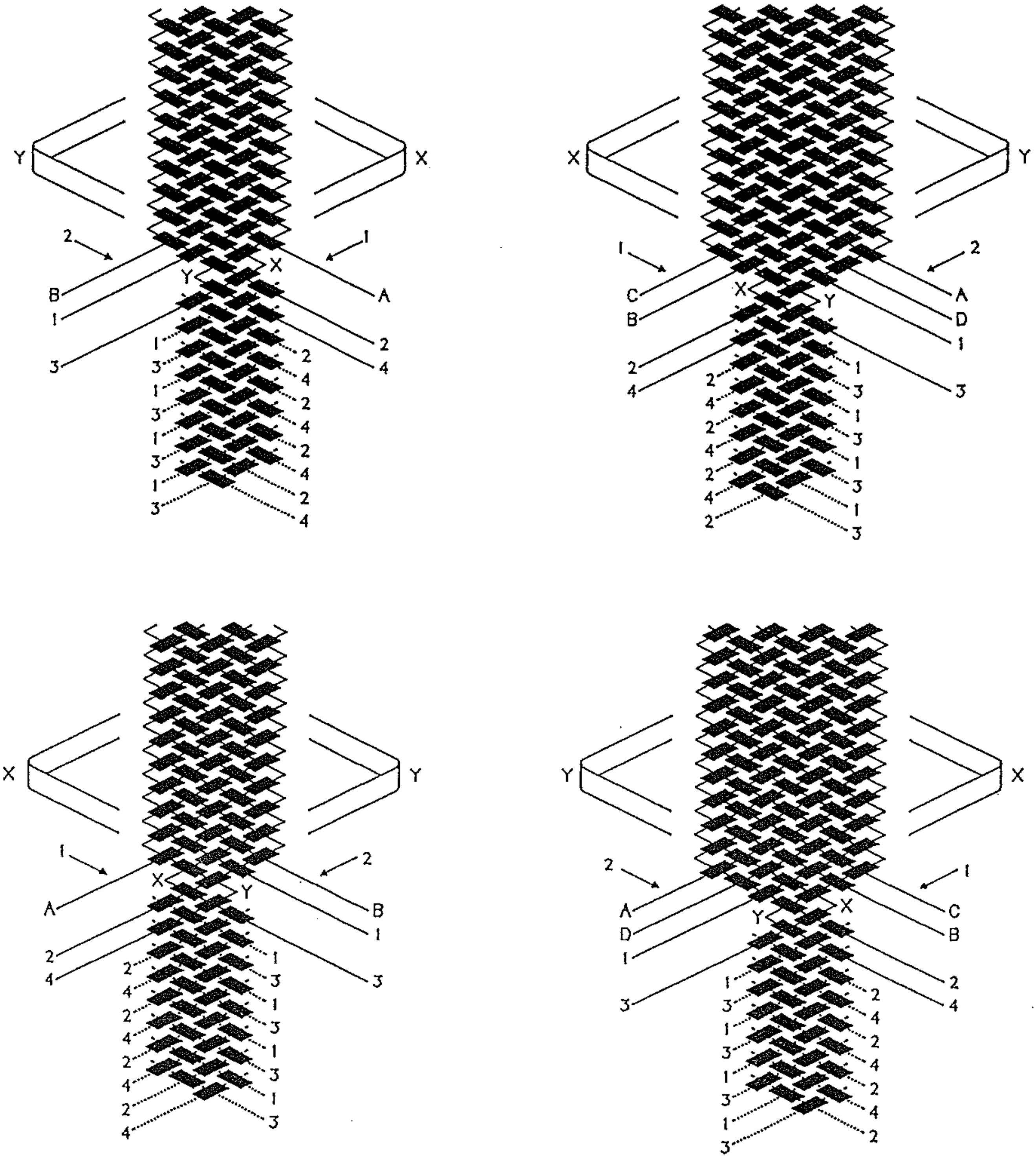

Fig. 407 — The transition from flat braid to round braid.

For the upper 6-part Regular Flat Braid in Fig. 407, the construction of the foundation knot is shown in Fig. 408, while for the lower 6-part Regular Flat Braid in Fig. 407, the construction of the foundation knot is shown in Fig. 409. Thus for Fig. 408, turn string *A* in the upper left diagram of Fig. 407 to the left, and for Fig. 409, turn string *A* in the lower left diagram of Fig. 407 to the right.

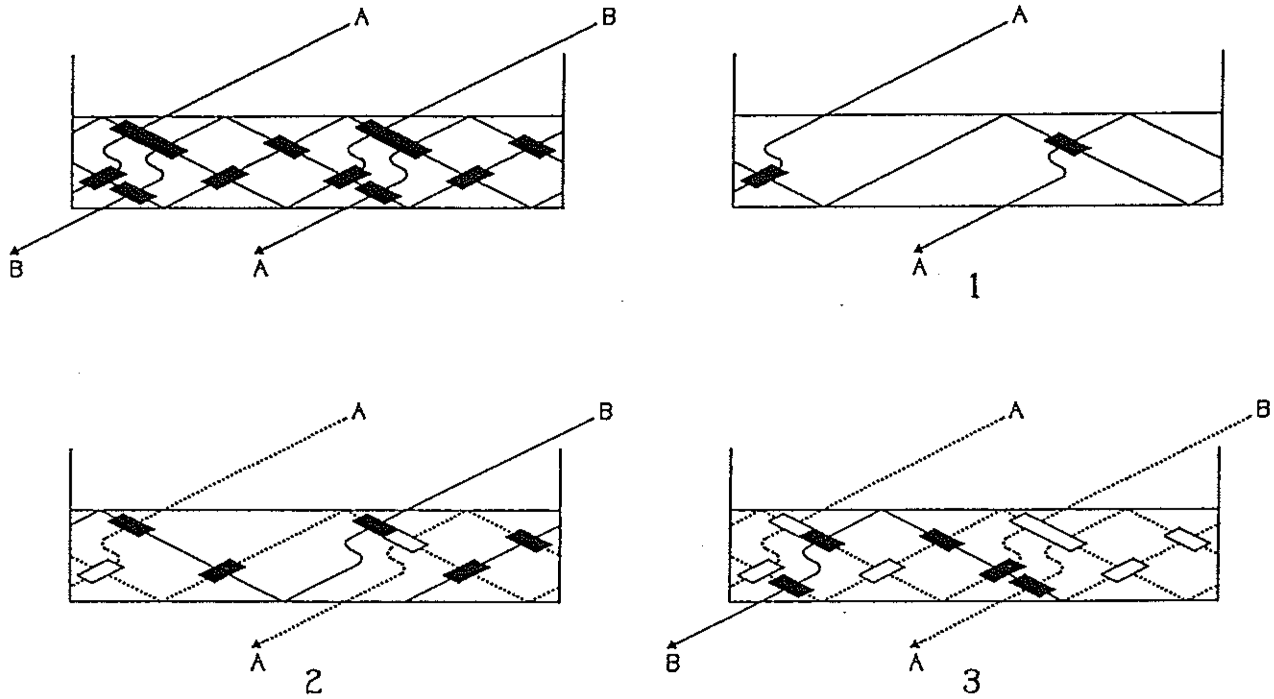

Fig. 408 — The foundation knot for the upper 6-part Regular Flat Braid in Fig. 407.

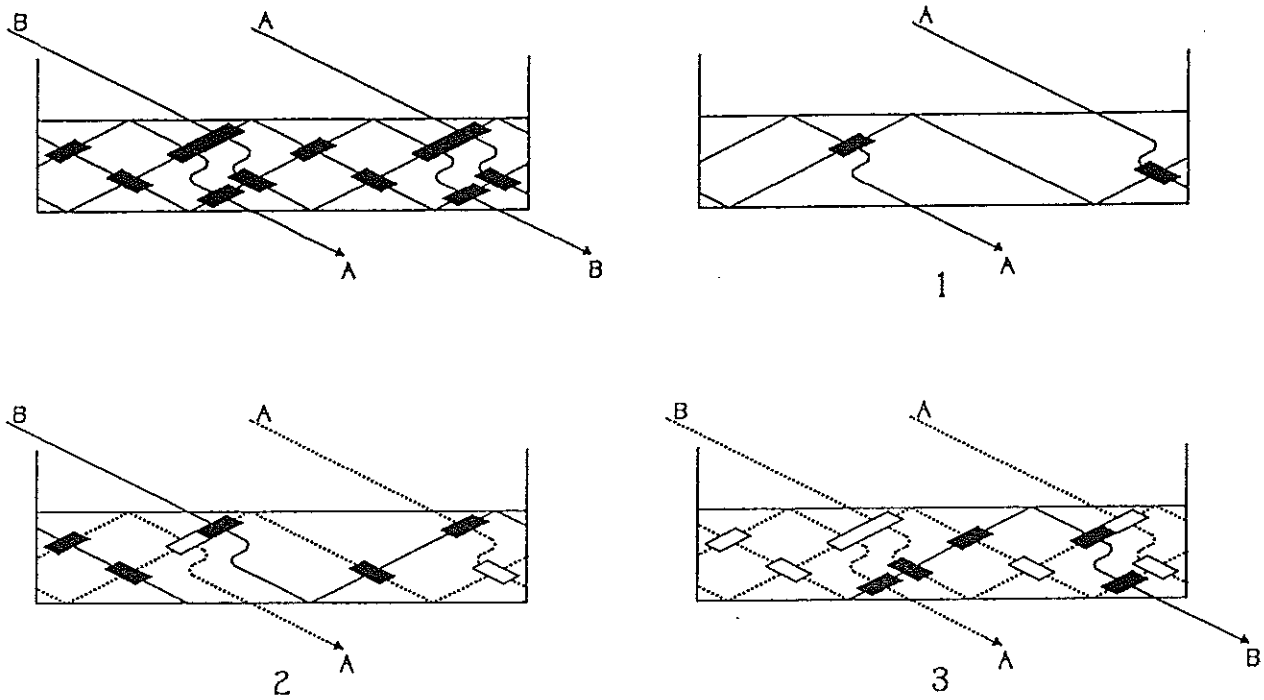

Fig. 409 — The foundation knot for the lower 6-part Regular Flat Braid in Fig. 407.

For the upper 8-part Regular Flat Braid in Fig. 407, the construction of the foundation knot is shown in Fig. 410. Here we turn string *A* to the left and string *B* to the right **over** string *A* to form crossing *V* on the front; then we turn string *C* to the right and string *D* to the left **under** string *C* to form crossing *W* on the back. After the foundation knot has been braided, turn *A* and *C* to lower left and turn *B* and *D* to upper right. Tie down with a constrictor knot around them and cut the strings flush with the upper and lower edge of the knot. Unbraid the four-string round braid as far as the foundation knot.

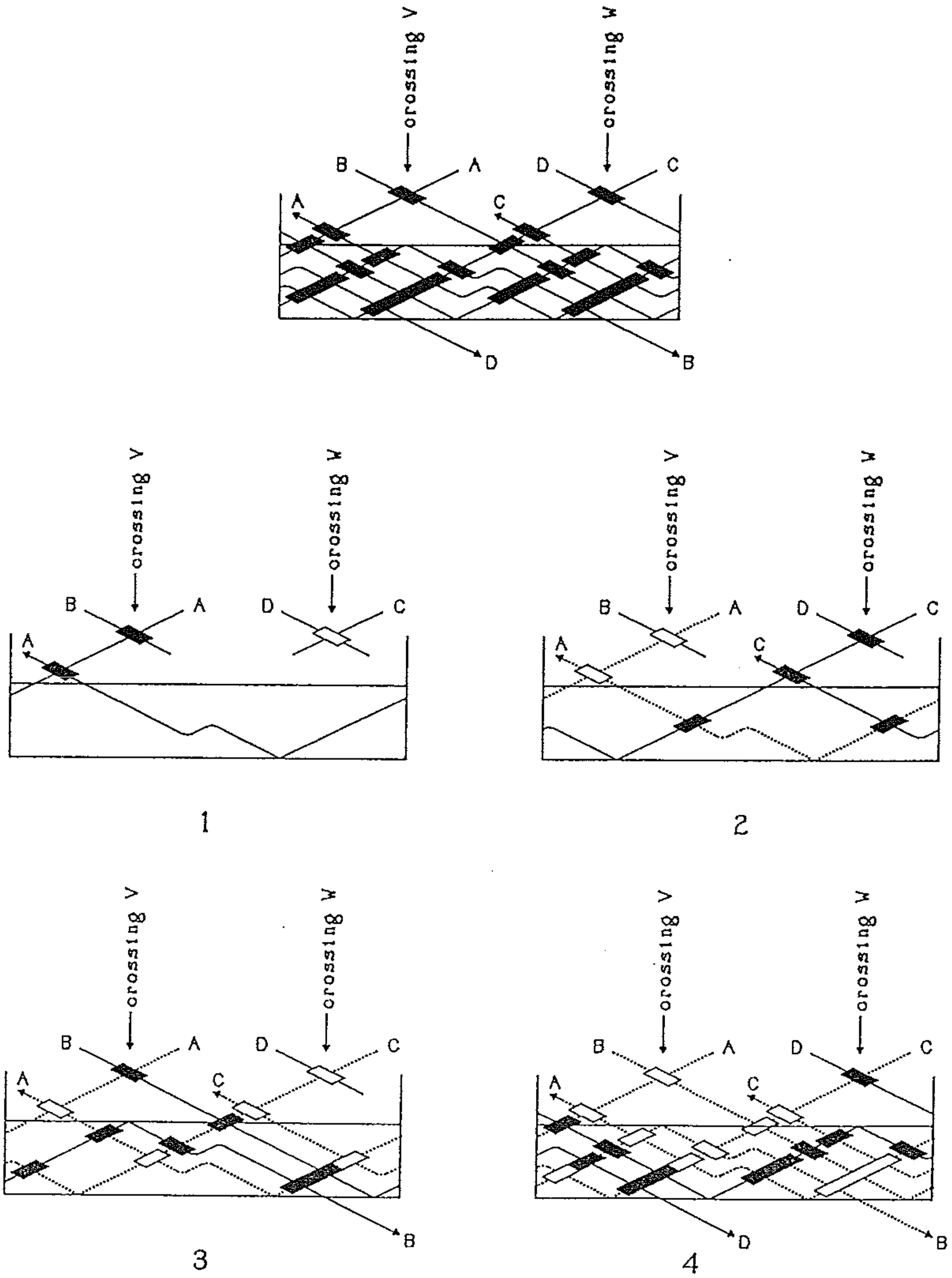

Fig. 410 — The foundation knot for the upper 8-part Regular Flat Braid in Fig. 407.

For the lower 8-part Regular Flat Braid in Fig. 407, the construction of the foundation knot is shown in Fig. 411. Here we turn string *A* to the right and string *B* to the left over string *A* to form crossing *V* on the front; then we turn string *C* to the left and string *D* to the right **under** string *C* to form crossing *W* on the back. After the foundation knot has been braided, turn *A* and *C* to lower right and turn *B* and *D* to upper left. Tie down with a constrictor knot around them and cut the strings flush with the upper and lower edge of the knot. Unbraid the four-string round braid as far as the foundation knot.

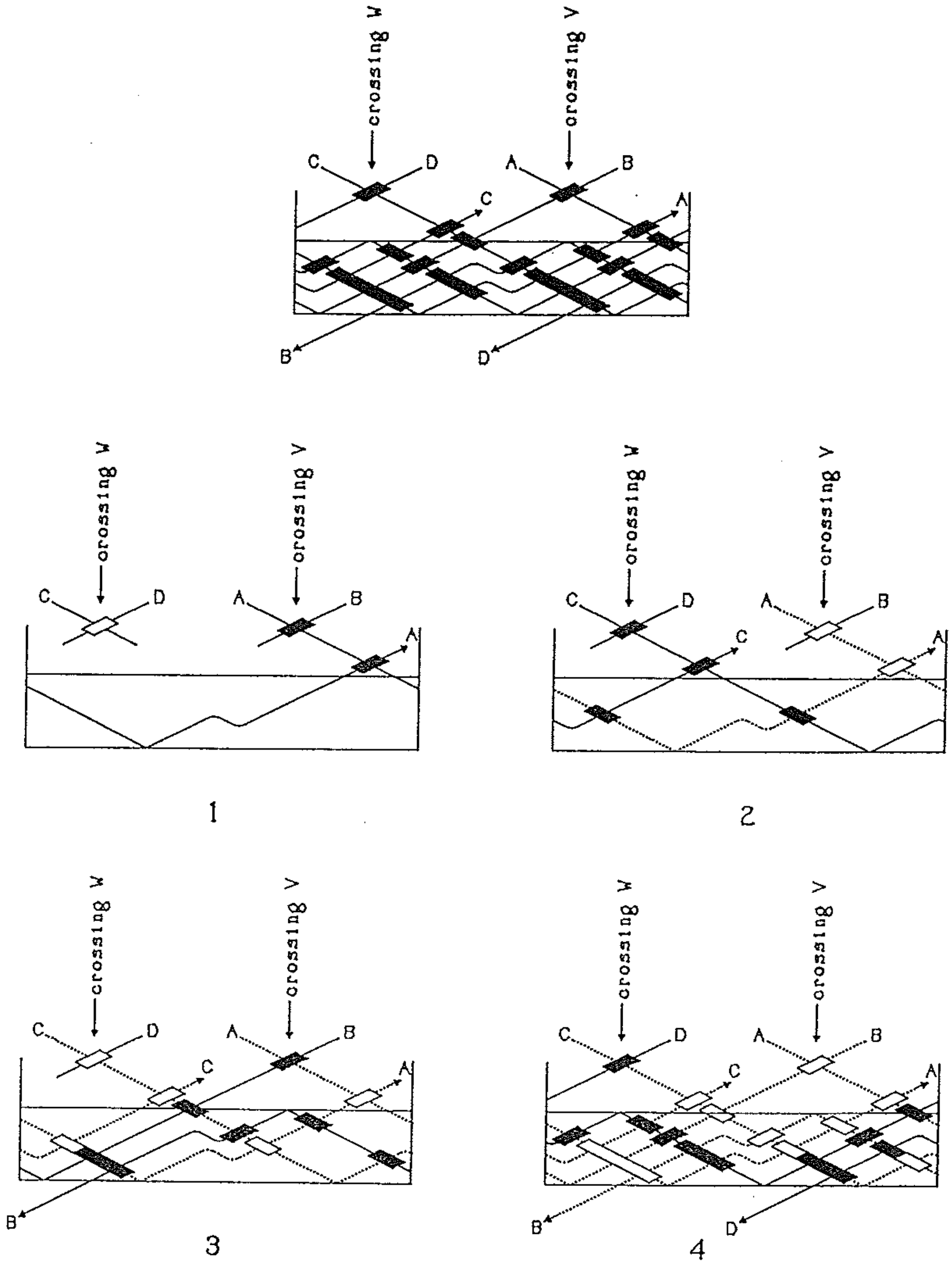

Fig. 411 — The foundation knot for the lower 8-part Regular Flat Braid in Fig. 407.

Fig. 412 shows an alternative braiding sequence for the foundation knot in Fig. 411.

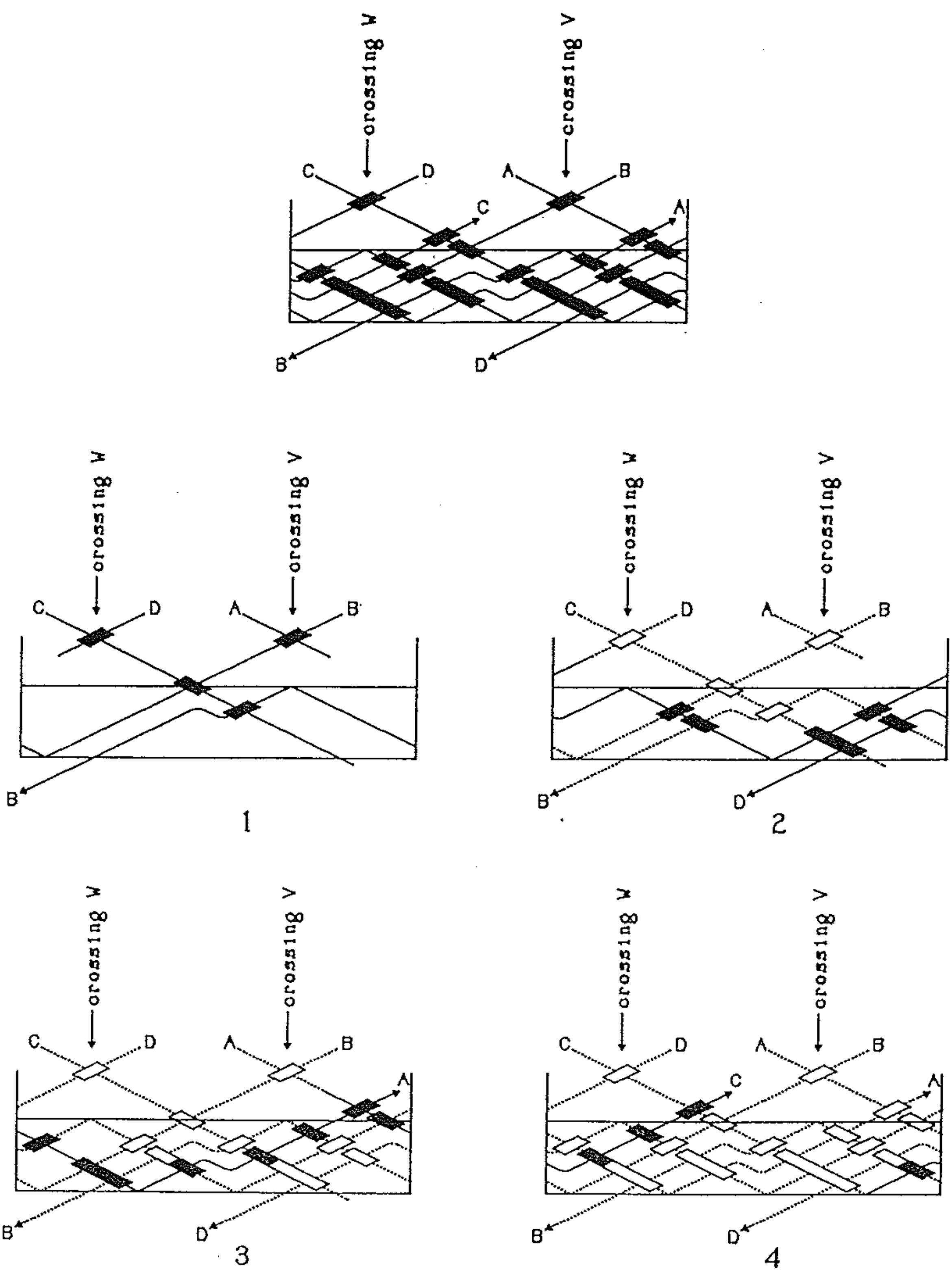

Fig. 412 — The foundation knot for the lower 8-part Regular Flat Braid in Fig. 407.

A similar alternative braiding sequence may be employed for the foundation knot in Fig. 410.

The final integrated Gaucho button knot, braided with the string-ends of the four-string round braid, is depicted in Fig. 413 and Fig. 414.

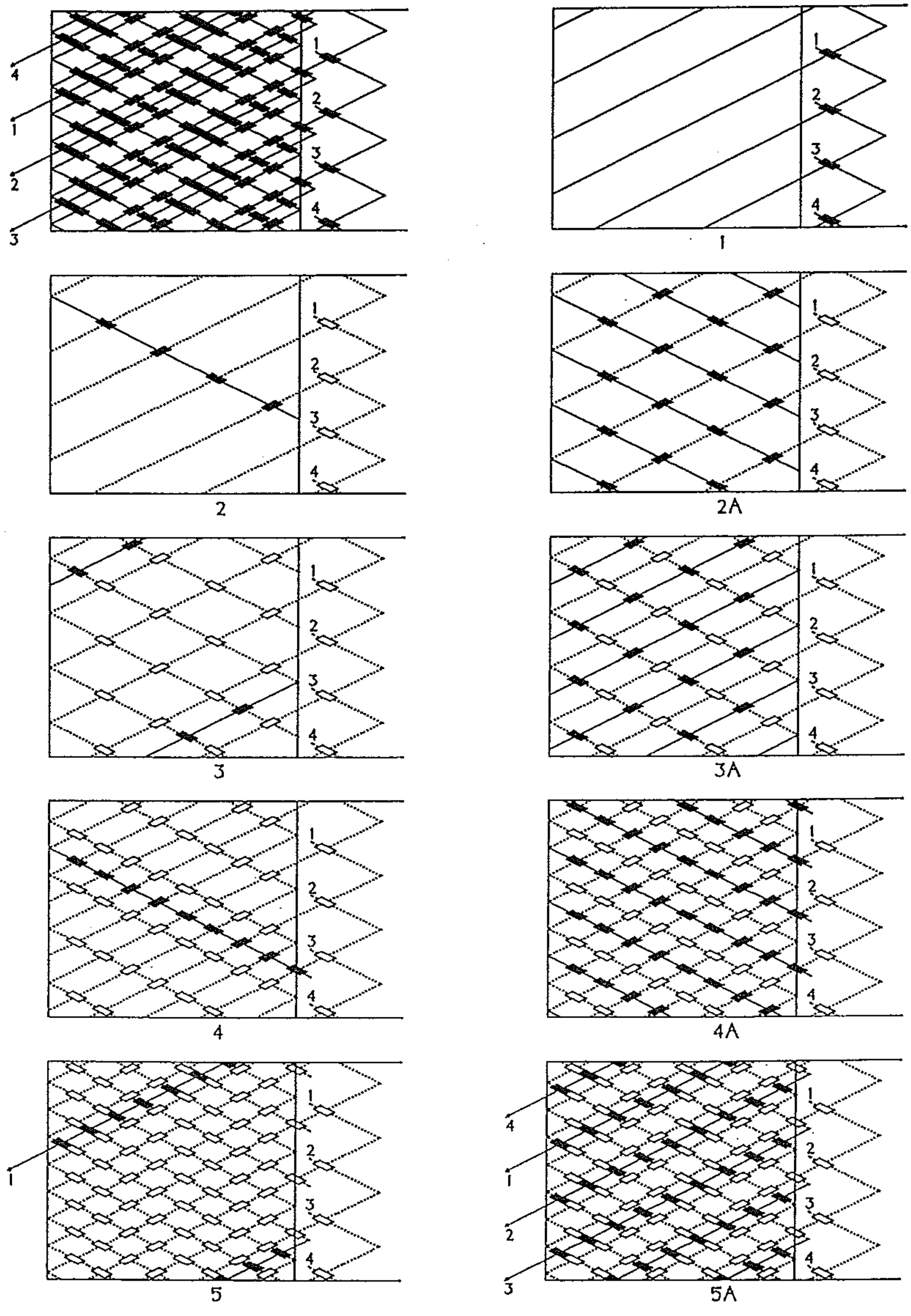

Fig. 413 — The final integrated Gaucho button knot.

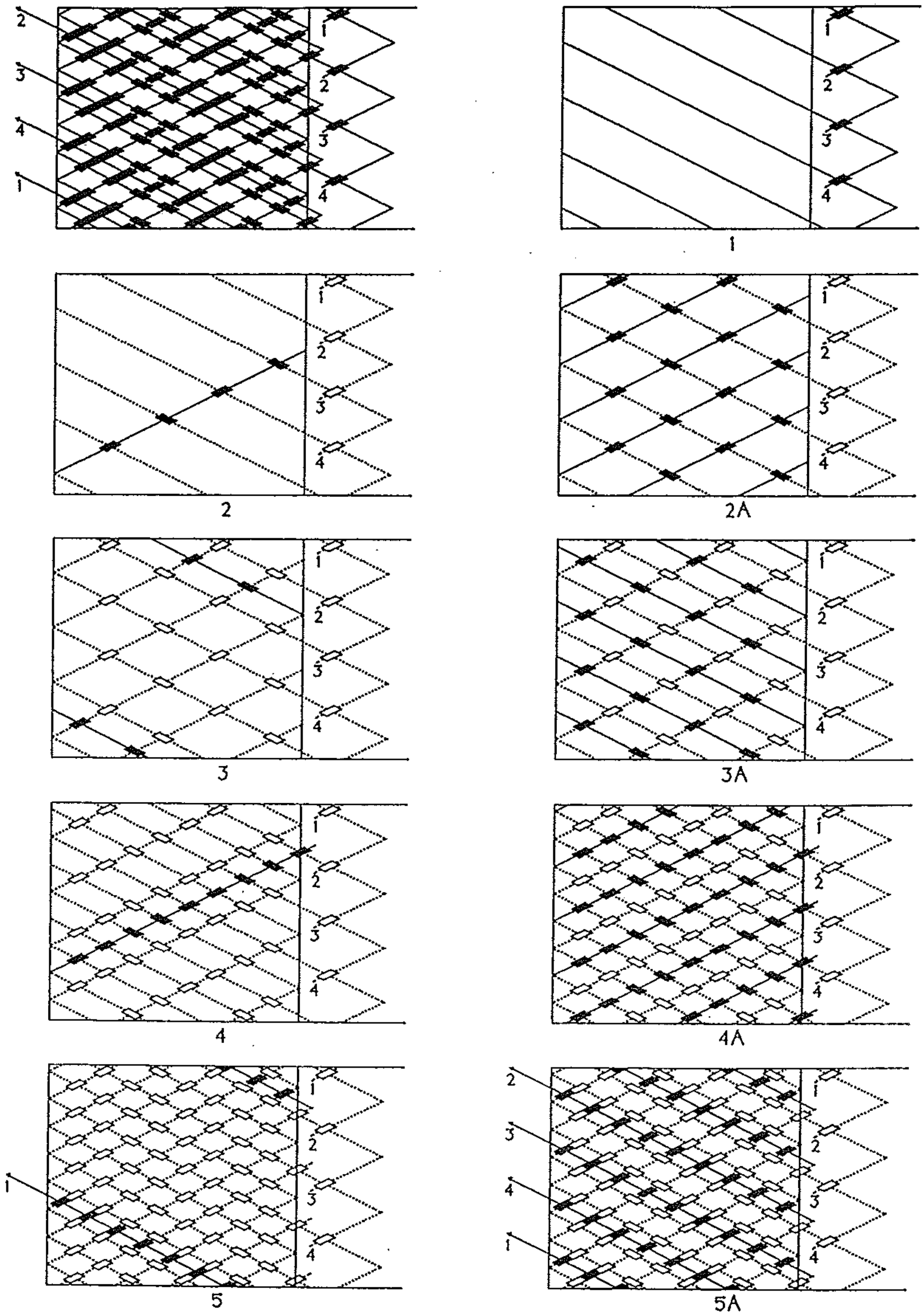

Fig. 414 — The final integrated Gaucho button knot.