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A quarterly publication
for
the braiding artisan

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Nested Cylindrical Braids

Nested Cylindrical Braids are cylindrical braids in which at least on one of its two **bight-edges** not all the bights are on one **bight-boundary** (see Fig. 348).

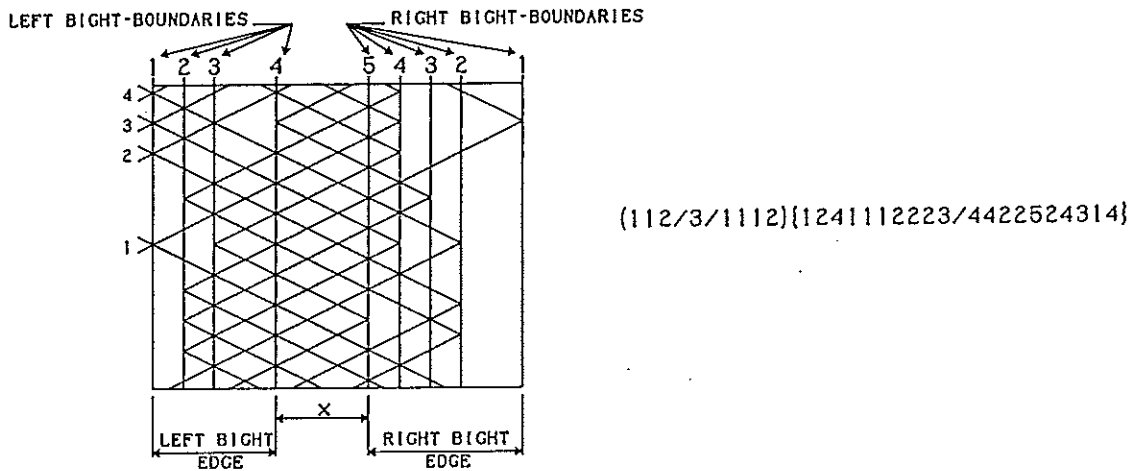


Fig. 348 — A Nested Cylindrical Braid.

The string-run of the Nested Cylindrical Braid in Fig. 348 is specified by :
 $(112/3/1112)\{1241112223/4422524314\}10$

In this specification $(112/3/1112)$ gives the position of the bight-boundaries relative to each other in column-distances from left to right.

The two sets of numerals in $\{1241112223/4422524314\}$ specify the positions of the bight-points in respectively a **period** at the left bight-edge and in a **period** at the right bight-edge. The final number (10) indicates the total number of bights (B_{total}) at each of the two bight-edges.

Note that in this braid there is only one period on each bight-edge since the number of numerals in 1241112223 and in 4422524314 is equal to B_{total} .

The first numeral in 1241112223 (hence 1) and the first numeral in 4422524314 (hence 4) indicate that there is a lower-left to upper-right half-cycle which runs from the left bight-boundary 1 to the right bight-boundary 4. This half-cycle is the **reference half-cycle** for the **right-hand bight-points** (the right reference half-cycle RRHC).

The left set of numerals (hence 1241112223) specify in a left period the left bight-boundaries of the left bight-points of the RRHC and the sequential adjacent parallel half-cycles in cyclic fashion above it. Similarly, the right set of numerals (hence 4422524314) specify in a right period the right bight-boundaries of the right bight-points of the RRHC and the sequential adjacent parallel half-cycles in cyclic fashion above it.

With reference to the RRHC, each numeral in the right sequence set 4422524314 is given a **ranking-number** j as subscript, starting with 1 for the RRHC. Hence we obtain for the right period the sequence $4_1 4_2 2_3 2_4 5_5 2_6 4_7 3_8 1_9 4_{10}$.

Each numeral in the left sequence set 1241112223 also receives a **ranking-number** j as subscript. These ranking-numbers j are calculated in the following way.

Say the n^{th} numeral in the left sequence set is k . Then the ranking-number j associated with this numeral is equal to :

$|n + \text{the sum of the first } (k - 1) \text{ numerals of the left bight-boundary position specification}|_{A_l}$,
 where A_l is the number of bights in a left period.

Consequently for the numerals of the left sequence set 1241112223 we obtain with the left bight-boundary position specification 112 and $A_l = 10$ the following ranking-numbers j :

- $n = 1$ for numeral $k = 1$, hence its ranking-number $j = |1 + 0|_{10} = 1$.
- $n = 2$ for numeral $k = 2$, hence its ranking-number $j = |2 + 1|_{10} = 3$.
- $n = 3$ for numeral $k = 4$, hence its ranking-number $j = |3 + (1 + 1 + 2)|_{10} = 7$.
- $n = 4$ for numeral $k = 1$, hence its ranking-number $j = |4 + 0|_{10} = 4$.
- $n = 5$ for numeral $k = 1$, hence its ranking-number $j = |5 + 0|_{10} = 5$.
- $n = 6$ for numeral $k = 1$, hence its ranking-number $j = |6 + 0|_{10} = 6$.
- $n = 7$ for numeral $k = 2$, hence its ranking-number $j = |7 + 1|_{10} = 8$.
- $n = 8$ for numeral $k = 2$, hence its ranking-number $j = |8 + 1|_{10} = 9$.
- $n = 9$ for numeral $k = 2$, hence its ranking-number $j = |9 + 1|_{10} = 0 = 10$.
- $n = 10$ for numeral $k = 3$, hence its ranking-number $j = |10 + (1 + 1)|_{10} = 2$.

Hence we obtain for the left period the sequence $1_1 2_3 4_7 1_4 1_5 1_6 2_8 2_9 2_{10} 3_2$.

The half-cycle from lower-right to upper-left whose left-hand bight-point has the ranking-number 1 is the **reference half-cycle** for the **left-hand bight-points** (the left reference half-cycle LRHC).

The **left nesting-number** A_l is the number of bights in a period at the left bight-edge, and the **right nesting-number** A_r is the number of bights in a period at the right bight-edge.

A **first return string-run** is the shortest consecutive half-cycle sequence to a half-cycle which is identical to the first half-cycle in the sequence. A first return string-run can readily be calculated after its first half-cycle has been chosen.

Let in a first return string-run the **first** half-cycle run from lower-left to upper-right and let its left bight-point position be specified by l_{1j_l} , where l_1 is its left bight-boundary number, and j_l its ranking-number; similarly let its right bight-point position be specified by r_{1j_r} , where r_1 is its right bight-boundary number, and j_r its ranking-number. Furthermore, let a general cycle with its associated bight-points, starting from the left bight-edge, be indicated as shown at the left in Fig. 349, and let a general cycle with its associated bight-points, starting from the right bight-edge, be indicated as shown at the right in Fig. 349.



Fig. 349 — A general left and right cycle.

The bight-boundary number l_{i+1} is the left bight-boundary number which is associated with the left ranking-number j'_l . Similarly the bight-boundary number r_{i+1} is the right bight-boundary number which is associated with the right ranking-number j'_r .

Let \mathcal{K}_l be the total number of left bight-boundaries, and let \mathcal{K}_r be the total number of right bight-boundaries.

$$j'_l = |j_l + \Delta_{l_i} + x + \Delta_{r_i}|_{A_l}.$$

$$j'_r = |j_r + \Delta_{r_i} + x + \Delta_{l_{i+1}}|_{A_r}.$$

Δ_{l_i} = sum of the last $(\mathcal{K}_l - l_i)$ numerals in left bight-boundary position specification sequence.

$\Delta_{l_{i+1}}$ = sum of the last $(\mathcal{K}_l - l_{i+1})$ numerals in left bight-boundary position specification sequence.

Δ_{r_i} = sum of the first $(\mathcal{K}_r - r_i)$ numerals in right bight-boundary position specification sequence.

B_l^* = number of periods at left bight-edge.

B_r^* = number of periods at right bight-edge.

$$B_{total} = A_l B_l^* = A_r B_r^* = A^{**} B^{**}.$$

$$d = \text{g.c.d.}(A_l, A_r).$$

$$A^{**} = \frac{B_{total}}{B^{**}} = \frac{A_l \cdot A_r}{d}.$$

$$B^{**} = \frac{B_{total} \cdot d}{A_l \cdot A_r} = \frac{B_l^* \cdot d}{A_r} = \frac{B_r^* \cdot d}{A_l}.$$

α = number of bights in first-return string-run.

$$P_{component} = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}}.$$

$$P_{total} = \sum P_{component}.$$

number of components } = number of first-return string-runs.

number of sub-components } = g.c.d. $(P_{component}, B^{**})$.

in a component }

total number of essential strings } = \sum sub-components.

For the string-run in Fig. 348 we obtain:

$$A_l = 10.$$

$$A_r = 10.$$

$$d = \text{g.c.d.}(A_l, A_r) = \text{g.c.d.}(10, 10) = 10.$$

$$A^{**} = \frac{A_l \cdot A_r}{d} = \frac{10 \cdot 10}{10} = 10.$$

$$B_l^* = \text{number of periods at left bight-edge} = 1.$$

$$B_r^* = \text{number of periods at right bight-edge} = 1.$$

$$B_{total} = A_l B_l^* = A_r B_r^* = A^{**} B^{**} = 10 \cdot 1 = 10.$$

$$B^{**} = \frac{B_{total}}{A^{**}} = 1.$$

$$\mathcal{K}_l = 4.$$

$$\mathcal{K}_r = 5.$$

Hence:

$$\begin{array}{ll} \Delta_{l_i} = 4 & \text{for } l_i = 1. \\ \Delta_{l_i} = 3 & \text{for } l_i = 2. \\ \Delta_{l_i} = 2 & \text{for } l_i = 3. \\ \Delta_{l_i} = 0 & \text{for } l_i = 4. \end{array} \qquad \begin{array}{ll} \Delta_{r_i} = 5 & \text{for } r_i = 1. \\ \Delta_{r_i} = 3 & \text{for } r_i = 2. \\ \Delta_{r_i} = 2 & \text{for } r_i = 3. \\ \Delta_{r_i} = 1 & \text{for } r_i = 4. \\ \Delta_{r_i} = 0 & \text{for } r_i = 5. \end{array}$$

Each lower-left to upper-right half-cycle type in a Nested Cylindrical Braid occurs only once in all the first-return string-runs of such a braid. The lower-left to upper-right half-cycle types can be read from the Nested Cylindrical Braid specification $(112/3/1112)\{1_1 2_3 4_7 1_4 1_5 1_6 2_8 2_9 2_{10} 3_2/4_1 4_2 2_3 2_4 5_5 2_6 4_7 3_8 1_9 4_{10}\}10$:

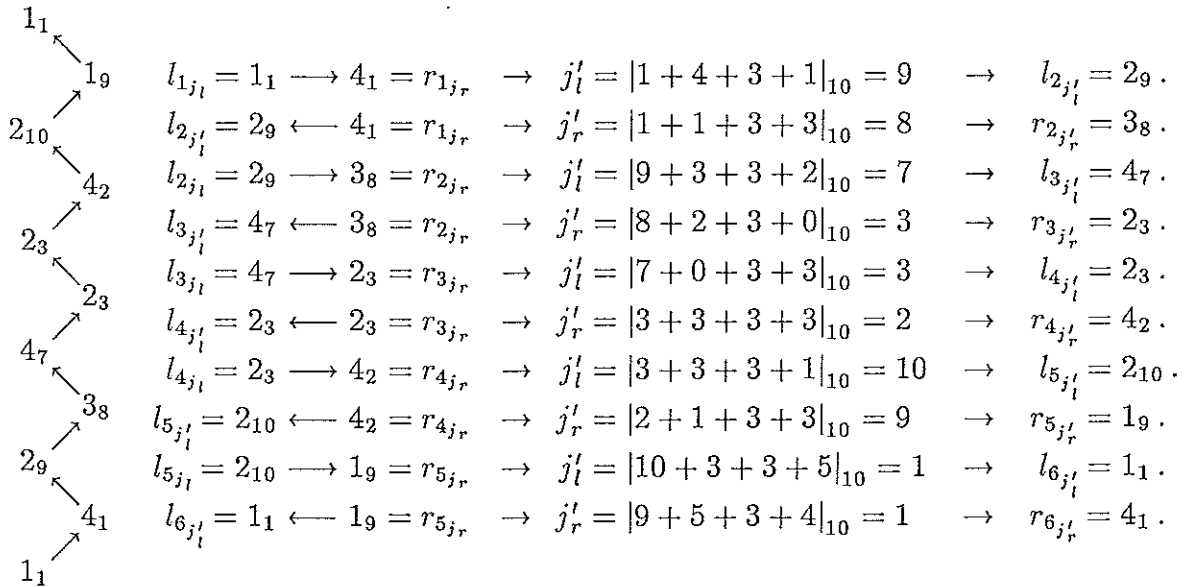
$$\begin{array}{ll} 1_1 \longrightarrow 4_1 & 1_6 \longrightarrow 2_6 \\ 2_3 \longrightarrow 4_2 & 2_8 \longrightarrow 4_7 \\ 4_7 \longrightarrow 2_3 & 2_9 \longrightarrow 3_8 \\ 1_4 \longrightarrow 2_4 & 2_{10} \longrightarrow 1_9 \\ 1_5 \longrightarrow 5_5 & 3_2 \longrightarrow 4_{10} \end{array}$$

Anyone of these listed types may be taken as the first lower-left to upper-right half-cycle in the 1st first-return string-run, but normally we take the first listed one. Every lower-left to upper-right half-cycle encountered in this 1st first-return string-run gets deleted from the type-list.

Anyone of the remaining types in the type-list may be taken as the first lower-left to upper-right half-cycle in the 2nd first-return string-run, and again every lower-left to upper-right half-cycle encountered in this 2nd first-return string-run gets deleted from the remaining type-list.

This process is carried on till all the lower-left to upper-right half-cycle types have been deleted from the type-list.

For the first-return string-runs we thus obtain:



$$P_c = \frac{\alpha \cdot x + \sum (\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{5 \cdot 3 + (4 + 3 + 0 + 3 + 3) + (1 + 2 + 3 + 1 + 5)}{10} = 4.$$

$$\text{g.c.d.}(P_c, B^{**}) = \text{g.c.d.}(4, 1) = 1.$$

$$\begin{array}{l}
 \begin{array}{l} 1_4 \\ \swarrow \searrow \\ 2_4 \\ \swarrow \searrow \\ 1_4 \end{array} \quad \begin{array}{l} l_{1j_l} = 1_4 \longrightarrow 2_4 = r_{1j_r} \rightarrow j'_l = |4 + 4 + 3 + 3|_{10} = 4 \rightarrow l_{2j'_l} = 1_4. \\ l_{2j'_l} = 1_4 \longleftarrow 2_4 = r_{1j_r} \rightarrow j'_r = |4 + 3 + 3 + 4|_{10} = 4 \rightarrow r_{2j'_r} = 2_4. \end{array}
 \end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 3 + (4) + (3)}{10} = 1.$$

$$\text{g.c.d.}(P_c, B^{**}) = \text{g.c.d.}(1, 1) = 1.$$

$$\begin{array}{l}
 \begin{array}{l} 1_5 \\ \swarrow \searrow \\ 4_7 \\ \swarrow \searrow \\ 2_8 \\ \swarrow \searrow \\ 4_{10} \\ \swarrow \searrow \\ 3_2 \\ \swarrow \searrow \\ 5_5 \\ \swarrow \searrow \\ 1_5 \end{array} \quad \begin{array}{l} l_{1j_l} = 1_5 \longrightarrow 5_5 = r_{1j_r} \rightarrow j'_l = |5 + 4 + 3 + 0|_{10} = 2 \rightarrow l_{2j'_l} = 3_2. \\ l_{2j'_l} = 3_2 \longleftarrow 5_5 = r_{1j_r} \rightarrow j'_r = |5 + 0 + 3 + 2|_{10} = 10 \rightarrow r_{2j'_r} = 4_{10}. \\ l_{2j_l} = 3_2 \longrightarrow 4_{10} = r_{2j_r} \rightarrow j'_l = |2 + 2 + 3 + 1|_{10} = 8 \rightarrow l_{3j'_l} = 2_8. \\ l_{3j'_l} = 2_8 \longleftarrow 4_{10} = r_{2j_r} \rightarrow j'_r = |10 + 1 + 3 + 3|_{10} = 7 \rightarrow r_{3j'_r} = 4_7. \\ l_{3j_l} = 2_8 \longrightarrow 4_7 = r_{3j_r} \rightarrow j'_l = |8 + 3 + 3 + 1|_{10} = 5 \rightarrow l_{4j'_l} = 1_5. \\ l_{4j'_l} = 1_5 \longleftarrow 4_7 = r_{3j_r} \rightarrow j'_r = |7 + 1 + 3 + 4|_{10} = 5 \rightarrow r_{4j'_r} = 5_5. \end{array}
 \end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{3 \cdot 3 + (4 + 2 + 3) + (0 + 1 + 1)}{10} = 2.$$

$$\text{g.c.d.}(P_c, B^{**}) = \text{g.c.d.}(2, 1) = 1.$$

$$\begin{array}{l}
 \begin{array}{l} 1_6 \\ \swarrow \searrow \\ 2_6 \\ \swarrow \searrow \\ 1_6 \end{array} \quad \begin{array}{l} l_{1j_l} = 1_6 \longrightarrow 2_6 = r_{1j_r} \rightarrow j'_l = |6 + 4 + 3 + 3|_{10} = 6 \rightarrow l_{2j'_l} = 1_6. \\ l_{2j'_l} = 1_6 \longleftarrow 2_6 = r_{1j_r} \rightarrow j'_r = |6 + 3 + 3 + 4|_{10} = 6 \rightarrow r_{2j'_r} = 2_6. \end{array}
 \end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 3 + (4) + (3)}{10} = 1.$$

$$\text{g.c.d.}(P_c, B^{**}) = \text{g.c.d.}(1, 1) = 1.$$

Hence:

$$P_{total} = \sum P_{component} = 4 + 1 + 1 + 2 = 8.$$

$$\left. \begin{array}{l} \text{number of} \\ \text{components} \end{array} \right\} = \text{number of first-return string-runs} = 4.$$

$$\left. \begin{array}{l} \text{total number of} \\ \text{essential strings} \end{array} \right\} = \sum \text{sub-components} = 1 + 1 + 1 + 1 = 4.$$

In general a Nested Cylindrical Braid will have different nesting-numbers A_l and A_r , and in general $B^{**} \neq 1$. A string-run example of such a Nested Cylindrical Braid for which $A_l = 4$ and $A_r = 6$ with $B_{total} = 24$ is depicted in Fig. 350.

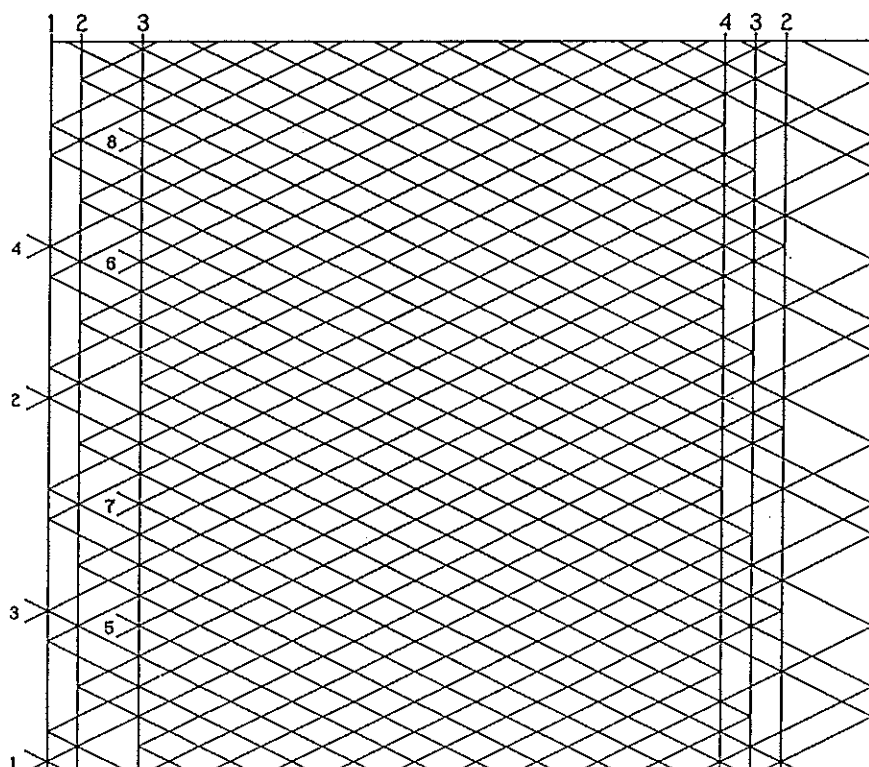


Fig. 350 — A Nested Cylindrical Braid with $A_l = 4$, $A_r = 6$, $x = 19$ and $B_{total} = 24$.

The specification of this braid is $(12/19/113)\{1123/113142\}24$, or with the ranking-numbers as subscripts attached to the bight-boundary numbers:

$$12/19/113)\{1_1 1_2 2_4 3_3 / 1_1 1_2 3_3 1_4 4_5 2_6\}24.$$

For the string-run we obtain:

$$A_l = 4.$$

$$A_r = 6.$$

$$d = \text{g.c.d.}(A_l, A_r) = \text{g.c.d.}(4, 6) = 2.$$

$$A^{**} = \frac{A_l \cdot A_r}{d} = \frac{4 \cdot 6}{2} = 12.$$

$$B_l^* = \text{number of periods at left bight-edge} = 6.$$

$$B_r^* = \text{number of periods at right bight-edge} = 4.$$

$$B_{total} = A_l B_l^* = A_r B_r^* = A^{**} B^{**} = 12 \cdot 2 = 24.$$

$$B^{**} = \frac{B_{total}}{A^{**}} = 2.$$

$$\mathcal{K}_l = 3.$$

$$\mathcal{K}_r = 4.$$

Hence:

$$\Delta_{l_i} = 3 \quad \text{for } l_i = 1.$$

$$\Delta_{l_i} = 2 \quad \text{for } l_i = 2.$$

$$\Delta_{l_i} = 0 \quad \text{for } l_i = 3.$$

$$\Delta_{r_i} = 5 \quad \text{for } r_i = 1.$$

$$\Delta_{r_i} = 2 \quad \text{for } r_i = 2.$$

$$\Delta_{r_i} = 1 \quad \text{for } r_i = 3.$$

$$\Delta_{r_i} = 0 \quad \text{for } r_i = 4.$$

The lower-left to upper-right half-cycle types can be read from the Nested Cylindrical Braid specification $(12/19/113)\{1_1 1_2 2_4 3_3 / 1_1 1_2 3_3 1_4 4_5 2_6\} 2_4$:

$$\begin{array}{lll} 1_1 \longrightarrow 1_1 & 1_1 \longrightarrow 4_5 & 1_1 \longrightarrow 3_3 \\ 1_2 \longrightarrow 1_2 & 1_2 \longrightarrow 2_6 & 1_2 \longrightarrow 1_4 \\ 2_4 \longrightarrow 3_3 & 2_4 \longrightarrow 1_1 & 2_4 \longrightarrow 4_5 \\ 3_3 \longrightarrow 1_4 & 3_3 \longrightarrow 1_2 & 3_3 \longrightarrow 2_6 \end{array}$$

For the first-return string-runs we thus obtain:

$1_1 \swarrow$	1_4	$l_{1j_l} = 1_1 \longrightarrow 1_1 = r_{1j_r} \rightarrow j'_l = 1 + 3 + 19 + 5 _4 = 4 \rightarrow l_{2j'_l} = 2_4.$
$1_2 \swarrow$	1_1	$l_{2j'_l} = 2_4 \longleftarrow 1_1 = r_{1j_r} \rightarrow j'_r = 1 + 5 + 19 + 2 _6 = 3 \rightarrow r_{2j'_r} = 3_3.$
$2_4 \swarrow$	3_3	$l_{2j_l} = 2_4 \longrightarrow 3_3 = r_{2j_r} \rightarrow j'_l = 4 + 2 + 19 + 1 _4 = 2 \rightarrow l_{3j'_l} = 1_2.$
$1_1 \swarrow$	4_5	$l_{3j'_l} = 1_2 \longleftarrow 3_3 = r_{2j_r} \rightarrow j'_r = 3 + 1 + 19 + 3 _6 = 2 \rightarrow r_{3j'_r} = 1_2.$
$2_4 \swarrow$	2_6	$l_{3j_l} = 1_2 \longrightarrow 1_2 = r_{3j_r} \rightarrow j'_l = 2 + 3 + 19 + 5 _4 = 1 \rightarrow l_{4j'_l} = 1_1.$
$1_1 \swarrow$	3_3	$l_{4j'_l} = 1_1 \longleftarrow 1_2 = r_{3j_r} \rightarrow j'_r = 2 + 5 + 19 + 3 _6 = 5 \rightarrow r_{4j'_r} = 4_5.$
$2_4 \swarrow$	4_5	$l_{4j_l} = 1_1 \longrightarrow 4_5 = r_{4j_r} \rightarrow j'_l = 1 + 3 + 19 + 0 _4 = 3 \rightarrow l_{5j'_l} = 3_3.$
$3_3 \swarrow$	2_6	$l_{5j'_l} = 3_3 \longleftarrow 4_5 = r_{4j_r} \rightarrow j'_r = 5 + 0 + 19 + 0 _6 = 6 \rightarrow r_{5j'_r} = 2_6.$
$1_1 \swarrow$	4_5	$l_{5j_l} = 3_3 \longrightarrow 2_6 = r_{5j_r} \rightarrow j'_l = 3 + 0 + 19 + 2 _4 = 4 \rightarrow l_{6j'_l} = 2_4.$
$2_4 \swarrow$	1_1	$l_{6j'_l} = 2_4 \longleftarrow 2_6 = r_{5j_r} \rightarrow j'_r = 6 + 2 + 19 + 2 _6 = 5 \rightarrow r_{6j'_r} = 4_5.$
$3_3 \swarrow$	1_2	$l_{6j_l} = 2_4 \longrightarrow 4_5 = r_{6j_r} \rightarrow j'_l = 4 + 2 + 19 + 0 _4 = 1 \rightarrow l_{7j'_l} = 1_1.$
$1_1 \swarrow$	3_3	$l_{7j'_l} = 1_1 \longleftarrow 4_5 = r_{6j_r} \rightarrow j'_r = 5 + 0 + 19 + 3 _6 = 3 \rightarrow r_{7j'_r} = 3_3.$
$2_4 \swarrow$	1_1	$l_{7j_l} = 1_1 \longrightarrow 3_3 = r_{7j_r} \rightarrow j'_l = 1 + 3 + 19 + 1 _4 = 4 \rightarrow l_{8j'_l} = 2_4.$
$3_3 \swarrow$	1_2	$l_{8j'_l} = 2_4 \longleftarrow 3_3 = r_{7j_r} \rightarrow j'_r = 3 + 1 + 19 + 2 _6 = 1 \rightarrow r_{8j'_r} = 1_1.$
$1_1 \swarrow$	3_3	$l_{8j_l} = 2_4 \longrightarrow 1_1 = r_{8j_r} \rightarrow j'_l = 4 + 2 + 19 + 5 _4 = 2 \rightarrow l_{9j'_l} = 1_2.$
$2_4 \swarrow$	1_1	$l_{9j'_l} = 1_2 \longleftarrow 1_1 = r_{8j_r} \rightarrow j'_r = 1 + 5 + 19 + 3 _6 = 4 \rightarrow r_{9j'_r} = 1_4.$
$3_3 \swarrow$	1_1	$l_{9j_l} = 1_2 \longrightarrow 1_4 = r_{9j_r} \rightarrow j'_l = 2 + 3 + 19 + 5 _4 = 1 \rightarrow l_{10j'_l} = 1_1.$
$1_1 \swarrow$	1_1	$l_{10j'_l} = 1_1 \longleftarrow 1_4 = r_{9j_r} \rightarrow j'_r = 4 + 5 + 19 + 3 _6 = 1 \rightarrow r_{10j'_r} = 1_1.$

$$P_c = \frac{\alpha \cdot x + \sum (\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{9 \cdot 19 + (3+2+3+3+0+2+3+2+3) + (5+1+5+0+2+0+1+5+5)}{12} = 18.$$

$$\text{g.c.d.}(P_c, B^{**}) = \text{g.c.d.}(18, 2) = 2.$$

$1_2 \swarrow$	2_6	$l_{1j_l} = 1_2 \longrightarrow 2_6 = r_{1j_r} \rightarrow j'_l = 2 + 3 + 19 + 2 _4 = 2 \rightarrow l_{2j'_l} = 1_2.$
$1_2 \swarrow$	1_2	$l_{2j'_l} = 1_2 \longleftarrow 2_6 = r_{1j_r} \rightarrow j'_r = 6 + 2 + 19 + 3 _6 = 6 \rightarrow r_{2j'_r} = 2_6.$

$$P_c = \frac{\alpha \cdot x + \sum (\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 19 + (3) + (2)}{12} = 2.$$

$$\text{g.c.d.}(P_c, B^{**}) = \text{g.c.d.}(2, 2) = 2.$$

$$\begin{array}{l}
 \begin{array}{c} 3_3 \\ \swarrow \\ 1_4 \\ \searrow \\ 3_3 \end{array} \quad \begin{array}{l} l_{1j_l} = 3_3 \longrightarrow 1_4 = r_{1j_r} \longrightarrow j'_l = |3 + 0 + 19 + 5|_4 = 3 \longrightarrow l_{2j'_l} = 3_3. \\ l_{2j'_l} = 3_3 \longleftarrow 1_4 = r_{1j_r} \longrightarrow j'_r = |4 + 5 + 19 + 0|_6 = 4 \longrightarrow r_{2j'_r} = 1_4. \end{array}
 \end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 19 + (0) + (5)}{12} = 2.$$

$$\text{g.c.d.}(P_c, B^{**}) = \text{g.c.d.}(2, 2) = 2.$$

$$\begin{array}{l}
 \begin{array}{c} 3_3 \\ \swarrow \\ 1_2 \\ \searrow \\ 3_3 \end{array} \quad \begin{array}{l} l_{1j_l} = 3_3 \longrightarrow 1_2 = r_{1j_r} \longrightarrow j'_l = |3 + 0 + 19 + 5|_4 = 3 \longrightarrow l_{2j'_l} = 3_3. \\ l_{2j'_l} = 3_3 \longleftarrow 1_2 = r_{1j_r} \longrightarrow j'_r = |2 + 5 + 19 + 0|_6 = 2 \longrightarrow r_{2j'_r} = 1_2. \end{array}
 \end{array}$$

$$P_c = \frac{\alpha \cdot x + \sum(\Delta_{l_i} + \Delta_{r_i})}{A^{**}} = \frac{1 \cdot 19 + (0) + (5)}{12} = 2.$$

$$\text{g.c.d.}(P_c, B^{**}) = \text{g.c.d.}(2, 2) = 2.$$

Hence :

$$P_{total} = \sum P_{component} = 18 + 2 + 2 + 2 = 24.$$

$$\left. \begin{array}{l} \text{number of} \\ \text{components} \end{array} \right\} = \text{number of first-return string-runs} = 4.$$

$$\left. \begin{array}{l} \text{total number of} \\ \text{essential strings} \end{array} \right\} = \sum \text{sub-components} = 2 + 2 + 2 + 2 = 8.$$

★ Calculate the first-return string-runs, the P_c -values and the number of essential strings required for the Nested Cylindrical Braids with the following string-run specifications:

(1). $(22/5/22)\{123/312\}12.$

(2). $(2/7/22)\{12/312\}12.$

(3). $(3/6/22)\{112/312\}12.$

Give some different specifications for the above braids.

Regular Cylindrical Bihelix Braids

The first thing we want to know about an $[n + 1, n]$ Prime Regular Cylindrical Bihelix Braid is the number of essential strings required. In the two previous Issues of *The Braider* we have seen how an $[n + 1, n]$ Prime Regular Cylindrical Bihelix Braid can be represented by an Extended Premature Regular Cylindrical Braid which is a Premature Regular Cylindrical Braid to which the necessary additional intersection-points have been added on its right-hand bight-boundary. Furthermore we have seen that these additional intersection-points are associated with the number of bights which were placed

to the right and adjacent to the right-hand bight-boundary of the Premature Regular Cylindrical Braid. We can thus calculate the number of essential strings required after we know the p and b values ($b = n + 1$) of the Regular Cylindrical Braid concerned and the number of bights adjacent to the right-hand bight-boundary of its premature form. In the previous Issue of *The Braider* on pg. 396 we denoted this number of bights adjacent to its right-hand bight-boundary by δ . The value of δ may be found from the string-run diagram which can be constructed in accordance with the procedures set out in the previous Issue of *The Braider* on pg. 397, or the value of δ may be calculated in accordance with the procedures set out on pg. 400.

The left-hand 'bight'-point of the Premature Regular Cylindrical Braid which has the index-number $I_{0_l} = 0$, contains the Standing-end of the string to which the Working-end of this string does **not** return. The Standing-end half-cycle associated with this left-hand 'bight'-point (hence the half-cycle, from lower-left to upper-right, which leaves the left index-number $I_{0_l} = 0$) runs to the right-hand bight-boundary of the Premature Regular Cylindrical Braid where it enters index-number $I_{0_{r_i}} = 0$. The succeeding half-cycle (which runs from lower-right to upper-left) leaves or passes through index-number $I_{0_{r_o}}$, and enters index-number I_{1_l} on the left-hand bight-boundary. Note that any two successive half-cycles enter and leave the same bight-point on the left-hand bight-boundary (hence the incoming index-number and the outgoing index-number have the same value), while on the right-hand bight-boundary of the Premature Regular Cylindrical Braid any two successive half-cycles do not necessarily enter and leave the same point. Consequently on the right-hand bight-boundary of the Premature Regular Cylindrical Braid, index-number $I_{x_{r_i}}$ associated with an incoming half-cycle and index-number $I_{x_{r_o}}$ associated with its succeeding outgoing half-cycle do not necessarily have the same value (see Fig. 351).

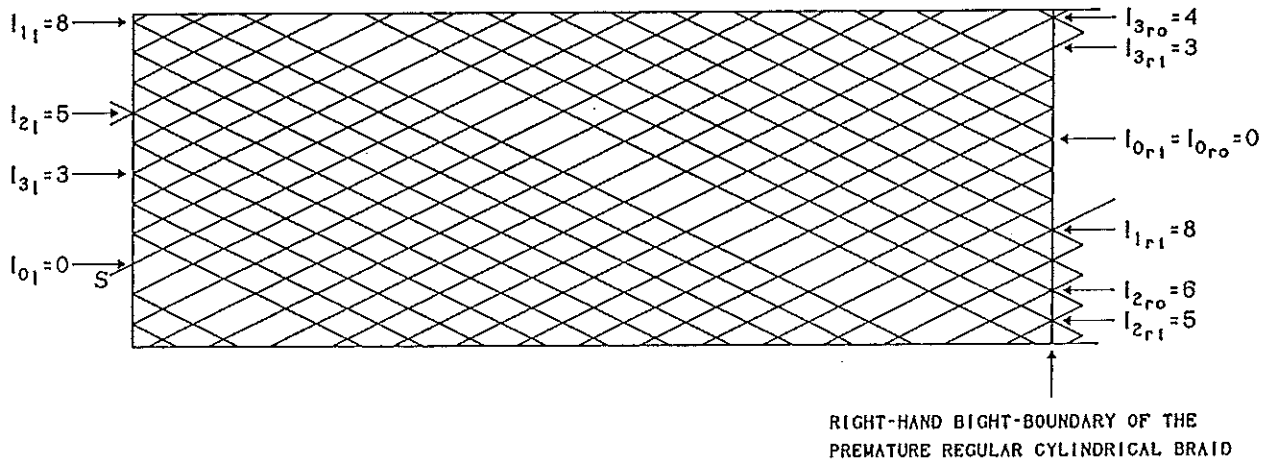


Fig. 351 — The string-run of an Extended Premature Regular Cylindrical Braid.

For the Example in Fig. 351 we obtain the following:

$$\begin{aligned}
 n + 1 &= 11, \\
 n &= 10, \\
 p &= 30, \\
 m &= \left\lfloor \frac{pn}{2n^2 + 2n + 1} \right\rfloor = \left\lfloor \frac{300}{221} \right\rfloor = 1, \\
 c &= |m|_n = |1|_{10} = 1,
 \end{aligned}$$

$$M = |-p|_{2n+2} = |-30|_{22} = 14,$$

$$T = \left| 2m + \left\lfloor \frac{m}{n} \right\rfloor \right|_{2n+2} = \left| 2 \times 1 + \left\lfloor \frac{1}{10} \right\rfloor \right|_{22} = |2 + 0|_{22} = 2,$$

$$z = p - m(2n + 2) - \left\lfloor \frac{m}{n} \right\rfloor = 30 - 1(22) - \left\lfloor \frac{1}{10} \right\rfloor = 30 - 22 - 1 = 7,$$

$$r = |-m|_n = |-1|_{10} = 9.$$

Since $r > \left\lfloor \frac{z-1}{2} \right\rfloor$ we obtain :

$$z^* = z + 1 = 7 + 1 = 8,$$

$$r^* = \left\lfloor \frac{z}{2} \right\rfloor = \left\lfloor \frac{7}{2} \right\rfloor = 4,$$

$$\delta = n - (r - r^*) = 10 - (9 - 4) = 5.$$

The missing half-cycle in the Premature Regular Cylindrical Braid is the lower-right to upper-left half-cycle which would begin at the right-hand bight-boundary in index-number $|-p|_{n+1} = |-30|_{11} = 3$. Hence the lower-left to upper-right half-cycle which passes at the right-hand bight-boundary of the Premature Regular Cylindrical Braid through index-number $I_{j_{r_i}} = 3$ is followed by the lower-right to upper-left half-cycle which passes at that same boundary through index-number $I_{j_{r_o}} = 4$. Similarly for the adjacent $\delta - 1 = 5 - 1 = 4$ left to right and right to left half-cycles:

left to right (in) through $I_{j_{r_i}} = 4$ followed by right to left (out) through $I_{j_{r_o}} = 5$,
left to right (in) through $I_{j_{r_i}} = 5$ followed by right to left (out) through $I_{j_{r_o}} = 6$,
left to right (in) through $I_{j_{r_i}} = 6$ followed by right to left (out) through $I_{j_{r_o}} = 7$,
left to right (in) through $I_{j_{r_i}} = 7$ followed by right to left (out) through $I_{j_{r_o}} = 8$.

The lower-left to upper-right half-cycle which passes through $I_{j_{r_i}} = 8$ exits the braid to the right.

Thus we obtain:

1 st half-cycle :		$I_{0_{l_o}} = 0 \longrightarrow I_{0_{r_i}} = 0$
2 nd half-cycle :	$I_{1_{l_i}} = I_{0_{r_o}} + p _{n+1} _{n+1} = 0 + 8 _{11} = 8 \longleftarrow I_{0_{r_o}} = 0$	
3 rd half-cycle :		$I_{1_{l_o}} = 8 \longrightarrow I_{1_{r_i}} = 8$
4 th half-cycle :		$I_{2_{l_o}} = 5 \longrightarrow I_{2_{r_i}} = 5$
5 th half-cycle :	$I_{3_{l_i}} = I_{2_{r_o}} + p _{n+1} _{n+1} = 6 + 8 _{11} = 3 \longleftarrow I_{2_{r_o}} = 6$	
6 th half-cycle :		$I_{3_{l_o}} = 3 \longrightarrow I_{3_{r_i}} = 3$
7 th half-cycle :	$I_{4_{l_i}} = I_{3_{r_o}} + p _{n+1} _{n+1} = 4 + 8 _{11} = 1 \longleftarrow I_{3_{r_o}} = 4$	
8 th half-cycle :		$I_{4_{l_o}} = 1 \longrightarrow I_{4_{r_i}} = 1$
9 th half-cycle :	$I_{5_{l_i}} = I_{4_{r_o}} + p _{n+1} _{n+1} = 1 + 8 _{11} = 9 \longleftarrow I_{4_{r_o}} = 1$	
10 th half-cycle :		$I_{5_{l_o}} = 9 \longrightarrow I_{5_{r_i}} = 9$
11 th half-cycle :	$I_{6_{l_i}} = I_{5_{r_o}} + p _{n+1} _{n+1} = 9 + 8 _{11} = 6 \longleftarrow I_{5_{r_o}} = 9$	
12 th half-cycle :		$I_{6_{l_o}} = 6 \longrightarrow I_{6_{r_i}} = 6$
13 th half-cycle :	$I_{7_{l_i}} = I_{6_{r_o}} + p _{n+1} _{n+1} = 7 + 8 _{11} = 4 \longleftarrow I_{6_{r_o}} = 7$	
14 th half-cycle :		$I_{7_{l_o}} = 4 \longrightarrow I_{7_{r_i}} = 4$
15 th half-cycle :	$I_{8_{l_i}} = I_{7_{r_o}} + p _{n+1} _{n+1} = 5 + 8 _{11} = 2 \longleftarrow I_{7_{r_o}} = 5$	

$$\begin{aligned}
 16^{th} \text{ half-cycle} & : & I_{8_{l_o}} = 2 & \longrightarrow & I_{8_{r_i}} = 2 \\
 17^{th} \text{ half-cycle} & : & I_{9_{i_i}} = |I_{8_{r_o}} + |p|_{n+1}|_{n+1} = |2 + 8|_{11} = 10 & \longleftarrow & I_{8_{r_o}} = 2 \\
 18^{th} \text{ half-cycle} & : & & & I_{9_{l_o}} = 10 & \longrightarrow & I_{9_{r_i}} = 10 \\
 19^{th} \text{ half-cycle} & : & I_{10_{i_i}} = |I_{9_{r_o}} + |p|_{n+1}|_{n+1} = |10 + 8|_{11} = 7 & \longleftarrow & I_{9_{r_o}} = 10 \\
 20^{th} \text{ half-cycle} & : & & & I_{10_{l_o}} = 7 & \longrightarrow & I_{10_{r_i}} = 7 \\
 21^{st} \text{ half-cycle} & : & I_{11_{i_i}} = |I_{10_{r_o}} + |p|_{n+1}|_{n+1} = |8 + 8|_{11} = 5 & \longleftarrow & I_{10_{r_o}} = 8
 \end{aligned}$$

The Standing-end of the second string may of course be placed in any left-hand bight-point which is not occupied by the first string.

Not only do we now know that we require two strings for this braid, but we also know the number of half-cycles in each of the two interbraided braids, furthermore we know how these two strings are interbraided.

The above displayed calculation procedure can of course be set out in a much more compact and hence simpler manner as shown below:

$$\delta = 5; |-30|_{11} = 3; I_{j,r_i} = 3, 4, 5, 6, 7; |30|_{11} = 8.$$

0, 0	5, 6
8	3, 4
-----	1, 1
3 half-cycles	9, 9
	6, 7
	4, 5
	2, 2
	10, 10
	7, 8

	18 half-cycles

The total number of half-cycles is equal to $2n + 1 = 21$.

The coding which is superimposed on the string-run is generally a row-coding (see for example *The Braider*, No. 17, Figs. 329 & 331). Although in the grid-diagram of the equivalent Extended Premature Regular Cylindrical Braid such a row-coding is not a true row-coding, it can however be treated as such by using the appropriate adjustments in the calculation procedures for the determination of the half-cycle algorithms.

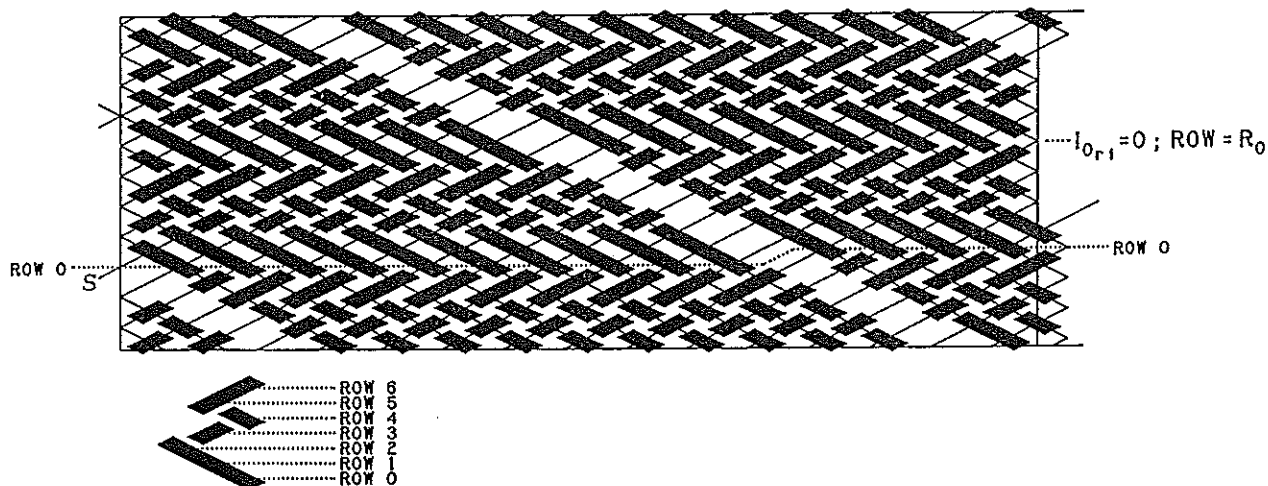


Fig. 352 — The string-run of Fig. 351 with a superimposed 'row'-coding.

Since the number of 'rows' is equal to $2n + 1$, we can obtain with a suitable selection for n a 'row'-coding with a periodic pattern. In our Example $2n + 1 = 21 = 3 \times 7$, consequently for a repeating coding-pattern we can have a seven 'row' periodic coding-pattern which repeats three times or we can have a three 'row' periodic coding-pattern which repeats seven times. In Fig. 352 we have selected the first option, hence the 'row'-coding period $\pi = 7$ which repeats three times; the 'row'-coding period is shown under the grid-diagram.

The coding of row 0 is the coding on the dotted line which passes on the left-hand bight-boundary through the Standing-end point S (hence index-number $I_{0,0} = 0$), and which passes on the right-hand bight-boundary of the Premature Regular Cylindrical Braid through the point which lies $\left(|p|_{2n+2} - \left\lfloor \frac{p}{2n+2} \right\rfloor \right)$ rows below the intersection-point of this boundary with the Standing-end half-cycle, hence $\left(|p|_{2n+2} - \left\lfloor \frac{p}{2n+2} \right\rfloor \right)$ rows below index-number $I_{0,r} = 0$.

The intersection-rows associated with the half-cycle to be laid down may be calculated as follows:

For a lower-left to upper-right half-cycle:

$$R = \left| 2I_{j_{l_0}} + C_l - \left\lfloor \frac{I_{j_{l_0}} + C_l}{n+1} \right\rfloor \right|_{\pi},$$

where C_l is the intersection-column number from the left-hand bight-boundary of the Premature Regular Cylindrical Braid.

For a lower-right to upper-left half-cycle:

$$R = \left| p - \left\lfloor \frac{p}{n+1} \right\rfloor + 2I_{j_{r_0}} + C_r - \lambda \right|_{\pi},$$

where C_r is the intersection-column number from the right-hand bight-boundary of the Premature Regular Cylindrical Braid,

$\lambda = 0$ for $|-p|_{n+1} = 0$,

$\lambda = 0$ for $I_{j_{r_0}} < |-p|_{n+1}$,

$\lambda = 1$ for $I_{j_{r_0}} > |-p|_{n+1}$.

The C_l and C_r values are calculated with the following formulae:

$C_l = \left| I_{j_{li_{\text{previous}}}} - I_{j_{li_{\text{new}}}} \right|_{n+1} + t(n+1)$ where $t = 0, 1, 2, 3, \dots$ and $0 < C_l \leq p$ when $C_l = p$ is a valid intersection, or $0 < C_l < p$ when $C_l = p$ is not a valid intersection.

$C_r = \left| I_{j_{ri_{\text{previous}}}} - I_{j_{ri_{\text{new}}}} \right|_{n+1} + t(n+1)$ where $t = 0, 1, 2, 3, \dots$ and $0 \leq C_r < p$ when $C_r = 0$ is a valid intersection, or $0 < C_r < p$ when $C_r = 0$ is not a valid intersection.

A calculation example in full for a left to right and a right to left half-cycle is shown below.

For the 12th half-cycle in our Example we thus obtain:

$$I_{j_{l_0}} = I_{6_{l_0}} = 6; C_l = \left| I_{j_{li_{\text{previous}}}} - 6 \right|_{11} + t(11) \text{ with } I_{j_{li_{\text{previous}}}} = 8, 3, 1, 9, 6.$$

$$C_l = |8 - 6|_{11} + 11t = 2 + 11t = 2, 13, 24.$$

$$C_l = |3 - 6|_{11} + 11t = 8 + 11t = 8, 19, 30.$$

$$C_l = |1 - 6|_{11} + 11t = 6 + 11t = 6, 17, 28.$$

$$C_l = |9 - 6|_{11} + 11t = 3 + 11t = 3, 14, 25.$$

$$C_l = |6 - 6|_{11} + 11t = 0 + 11t = 11, 22.$$

Hence $C_l = 2, 3, 6, 8, 11; 13, 14, 17, 19, 22; 24, 25, 28, 30$.

$\lfloor \frac{6+C_l}{11} \rfloor = 1, 2, 3$ for $C_l = 5, 16, 27$; $R = \lfloor 12 + C_l - \lfloor \frac{6+C_l}{11} \rfloor \rfloor_7 = \lfloor 5 + C_l - \lfloor \frac{6+C_l}{11} \rfloor \rfloor_7$.
 $C_l = 2, 3 \mid 6, 8, 11, 13, 14 \mid 17, 19, 22, 24, 25 \mid 28, 30$.

$R = 0, 1; 3, 5, 1, 3, 4; 6, 1, 4, 6, 0; 2, 4$. Hence the sequence is:

Left to Right: $2u - 2o - u - o - u - o - 2u - o - 3u$.

For the 15th half-cycle in our Example we thus obtain:

$I_{j_{r_o}} = I_{7_{r_o}} = 5$; $C_r = \left| I_{j_{ri_{previous}}} - 5 \right|_{11} + t(11)$ with $I_{j_{ri_{previous}}} = 0, 8, 5, 3, 1, 9, 6, 4$.

$C_r = \left| 0 - 5 \right|_{11} + 11t = 6 + 11t = 6, 17, 28$.

$C_r = \left| 8 - 5 \right|_{11} + 11t = 3 + 11t = 3, 14, 25$.

$C_r = \left| 5 - 5 \right|_{11} + 11t = 0 + 11t = 0, 11, 22$.

$C_r = \left| 3 - 5 \right|_{11} + 11t = 9 + 11t = 9, 20$.

$C_r = \left| 1 - 5 \right|_{11} + 11t = 7 + 11t = 7, 18, 29$.

$C_r = \left| 9 - 5 \right|_{11} + 11t = 4 + 11t = 4, 15, 26$.

$C_r = \left| 6 - 5 \right|_{11} + 11t = 1 + 11t = 1, 12, 23$.

$C_r = \left| 4 - 5 \right|_{11} + 11t = 10 + 11t = 10, 21$.

Hence $C_r = 0, 1, 3, 4, 6, 7, 9, 10; 11, 12, 14, 15, 17, 18, 20, 21; 22, 23, 25, 26, 28, 29$.

$\lambda = 1$; $R = \lfloor 28 + 10 + C_r - \lambda \rfloor_7 = \lfloor 2 + C_r \rfloor_7$.

$C_r = 0, 1, 3, 4, 6, 7, 9, 10; 11, 12, 14, 15, 17, 18, 20, 21; 22, 23, 25, 26, 28, 29$.

$R = 2, 3, 5, 6, 1, 2, 4, 5, 6, 0, 2, 3, 5, 6, 1, 2, 3, 4, 6, 0, 2, 3$. Hence the sequence is:

Right to Left: $o - 3u - 3o - 2u - 2o - 3u - 2o - u - o - u - 2o - u$.

The calculation results for the half-cycle algorithms of this Prime Regular Cylindrical Bihelix Braid (Fig. 352) are as follows:

Half-cycle 1:

Left \rightarrow Right: Free run.

Half-cycle 2: $I_{j_{r_o}} = 0$; $I_{j_{ri_{previous}}} = 0$; $\lambda = 0$; $R = \lfloor C_r \rfloor_7$.

$C_r = 11; 22$.

$R = 4, 1$.

Right \rightarrow Left: $2o$.

Half-cycle 3: $I_{j_{l_o}} = 8$; $I_{j_{li_{previous}}} = 8$; $\lfloor \frac{I_{j_{l_o}} + C_l}{11} \rfloor = 1, 2, 3$ for $C_l = 3, 14, 25$;

$R = \lfloor 2 + C_l - \lfloor \frac{8+C_l}{11} \rfloor \rfloor_7$.

$C_l = 11 \mid 22$.

$R = 5; 1$.

Left \rightarrow Right: $o - u$. String exits braid.

Half-cycle 4: $I_{j_{l_o}} = 5$; $I_{j_{li_{previous}}} = 8$; $\lfloor \frac{I_{j_{l_o}} + C_l}{11} \rfloor = 1, 2, 3$ for $C_l = 6, 17, 28$;

$R = \lfloor 3 + C_l - \lfloor \frac{5+C_l}{11} \rfloor \rfloor_7$. New string.

$C_l = 3 \mid 14 \mid 25$.

$R = 6; 2; 5$.

Left \rightarrow Right: $o - u - o$.

Half-cycle 5: $I_{j_{r_o}} = 6$; $I_{j_{ri_{previous}}} = 0, 8, 5$; $\lambda = 1$; $R = \lfloor 4 + C_r \rfloor_7$.

$C_r = 2, 5, 10; 13, 16, 21; 24, 27$.

$R = 6, 2, 0, 3, 6, 4, 0, 3$.

Right \rightarrow Left: $u - 2o - 2u - 2o - u$.

Half-cycle 6: $I_{j_{i_0}} = 3$; $I_{j_{i_{i_{\text{previous}}}}} = 8, 3$; $\left\lfloor \frac{I_{j_{i_0}} + C_l}{11} \right\rfloor = 1, 2, 3$ for $C_l = 8, 19, 30$;

$$R = \left| 6 + C_l - \left\lfloor \frac{3 + C_l}{11} \right\rfloor \right|_7.$$

$$C_l = 5 \mid 11, 16 \mid 22, 27.$$

$$R = 4; 2, 0; 5, 3.$$

$$\text{Left} \rightarrow \text{Right: } 3u - 2o.$$

Half-cycle 7: $I_{j_{r_0}} = 4$; $I_{j_{r_{i_{\text{previous}}}}} = 0, 8, 5, 3$; $\lambda = 1$; $R = |C_r|_7$.

$$C_r = 1, 4, 7, 10; 12, 15, 18, 21; 23, 26, 29.$$

$$R = 1, 4, 0, 3, 5, 1, 4, 0, 2, 5, 1.$$

$$\text{Right} \rightarrow \text{Left: } 3o - 2u - 4o - u - o.$$

Half-cycle 8: $I_{j_{i_0}} = 1$; $I_{j_{i_{i_{\text{previous}}}}} = 8, 3, 1$; $\left\lfloor \frac{I_{j_{i_0}} + C_l}{11} \right\rfloor = 1, 2$ for $C_l = 10, 21$;

$$R = \left| 2 + C_l - \left\lfloor \frac{1 + C_l}{11} \right\rfloor \right|_7.$$

$$C_l = 2, 7 \mid 11, 13, 18 \mid 22, 24, 29.$$

$$R = 4, 2; 5, 0, 5; 1, 3, 1.$$

$$\text{Left} \rightarrow \text{Right: } 2u - o - u - o - u - o - u.$$

Half-cycle 9: $I_{j_{r_0}} = 1$; $I_{j_{r_{i_{\text{previous}}}}} = 0, 8, 5, 3, 1$; $\lambda = 0$; $R = |2 + C_r|_7$.

$$C_r = 2, 4, 7, 10, 11; 13, 15, 18, 21, 22; 24, 26, 29.$$

$$R = 4, 6, 2, 5, 6, 1, 3, 6, 2, 3, 5, 0, 3.$$

$$\text{Right} \rightarrow \text{Left: } o - u - o - 2u - o - 2u - o - 2u - o - u.$$

Half-cycle 10: $I_{j_{i_0}} = 9$; $I_{j_{i_{i_{\text{previous}}}}} = 8, 3, 1, 9$; $\left\lfloor \frac{I_{j_{i_0}} + C_l}{11} \right\rfloor = 1, 2, 3$ for $C_l = 2, 13, 24$;

$$R = \left| 4 + C_l - \left\lfloor \frac{9 + C_l}{11} \right\rfloor \right|_7.$$

$$C_l = 3, 5, 10, 11 \mid 14, 16, 21, 22 \mid 25, 27.$$

$$R = 6, 1, 6, 0; 2, 4, 2, 3; 5, 0.$$

$$\text{Left} \rightarrow \text{Right: } o - u - o - 4u - 2o - u.$$

Half-cycle 11: $I_{j_{r_0}} = 9$; $I_{j_{r_{i_{\text{previous}}}}} = 0, 8, 5, 3, 1, 9$; $\lambda = 1$; $R = |3 + C_r|_7$.

$$C_r = 2, 3, 5, 7, 10, 11; 13, 14, 16, 18, 21, 22; 24, 25, 27, 29.$$

$$R = 5, 6, 1, 3, 6, 0, 2, 3, 5, 0, 3, 4, 6, 0, 2, 4.$$

$$\text{Right} \rightarrow \text{Left: } 2u - o - 2u - 2o - 2u - o - u - o - u - 3o.$$

Half-cycle 12: $I_{j_{i_0}} = 6$; $I_{j_{i_{i_{\text{previous}}}}} = 8, 3, 1, 9, 6$; $\left\lfloor \frac{I_{j_{i_0}} + C_l}{11} \right\rfloor = 1, 2, 3$ for $C_l = 5, 16, 27$;

$$R = \left| 5 + C_l - \left\lfloor \frac{6 + C_l}{11} \right\rfloor \right|_7.$$

$$C_l = 2, 3 \mid 6, 8, 11, 13, 14 \mid 17, 19, 22, 24, 25 \mid 28, 30.$$

$$R = 0, 1; 3, 5, 1, 3, 4; 6, 1, 4, 6, 0; 2, 4.$$

$$\text{Left} \rightarrow \text{Right: } 2u - 2o - u - o - u - o - 2u - o - 3u.$$

Half-cycle 13: $I_{j_{r_0}} = 7$; $I_{j_{r_{i_{\text{previous}}}}} = 0, 8, 5, 3, 1, 9, 6$; $\lambda = 1$; $R = |6 + C_r|_7$.

$$C_r = 1, 2, 4, 5, 7, 9, 10; 12, 13, 15, 16, 18, 20, 21; 23, 24, 26, 27, 29.$$

$$R = 0, 1, 3, 4, 6, 1, 2, 4, 5, 0, 1, 3, 5, 6, 1, 2, 4, 5, 0.$$

$$\text{Right} \rightarrow \text{Left: } 2o - u - o - u - 3o - u - 2o - 3u - 3o - u - o.$$

Half-cycle 14: $I_{j_{i_0}} = 4$; $I_{j_{i_{i_{\text{previous}}}}} = 8, 3, 1, 9, 6, 4$; $\left\lfloor \frac{I_{j_{i_0}} + C_l}{11} \right\rfloor = 1, 2, 3$ for

$$C_l = 7, 18, 29; R = \left| 1 + C_l - \left\lfloor \frac{4 + C_l}{11} \right\rfloor \right|_7.$$

$$C_l = 2, 4, 5 \mid 8, 10, 11, 13, 15, 16 \mid 19, 21, 22, 24, 26, 27 \mid 30.$$

$$R = 3, 5, 6; 1, 3, 4, 6, 1, 2; 4, 6, 0, 2, 4, 5; 0.$$

$$\text{Left} \rightarrow \text{Right: } 3o - u - o - u - o - 3u - o - 3u - o - u.$$

- Half-cycle 15:** $I_{j_{ro}} = 5$; $I_{j_{ri_{previous}}} = 0, 8, 5, 3, 1, 9, 6, 4$; $\lambda = 1$; $R = |2 + C_r|_7$.
 $C_r = 0, 1, 3, 4, 6, 7, 9, 10, 11, 12, 14, 15, 17, 18, 20, 21, 22, 23, 25, 26, 28, 29$.
 $R = 2, 3, 5, 6, 1, 2, 4, 5, 6, 0, 2, 3, 5, 6, 1, 2, 3, 4, 6, 0, 2, 3$.
 Right \rightarrow Left: $o - 3u - 3o - 2u - 2o - 3u - 2o - u - o - u - 2o - u$.
- Half-cycle 16:** $I_{j_{io}} = 2$; $I_{j_{ii_{previous}}} = 8, 3, 1, 9, 6, 4, 2$; $\left\lfloor \frac{I_{j_{io}} + C_l}{11} \right\rfloor = 1, 2$ for $C_l = 9, 20$;
 $R = |4 + C_l - \left\lfloor \frac{2 + C_l}{11} \right\rfloor|_7$.
 $C_l = 1, 2, 4, 6, 7 | 10, 11, 12, 13, 15, 17, 18 | 21, 22, 23, 24, 26, 28, 29$.
 $R = 5, 6, 1, 3, 4; 6, 0, 1, 2, 4, 6, 0; 2, 3, 4, 5, 0, 2, 3$.
 Left \rightarrow Right: $2o - u - o - u - o - 4u - o - 2u - o - u - o - 2u - o$.
- Half-cycle 17:** $I_{j_{ro}} = 2$; $I_{j_{ri_{previous}}} = 0, 8, 5, 3, 1, 9, 6, 4, 2$; $\lambda = 0$; $R = |4 + C_r|_7$.
 $C_r = 1, 2, 3, 4, 6, 7, 9, 10, 11; 12, 13, 14, 15, 17, 18, 20, 21, 22; 23, 24, 25, 26, 28, 29$.
 $R = 5, 6, 0, 1, 3, 4, 6, 0, 1, 2, 3, 4, 5, 0, 1, 3, 4, 5, 6, 0, 1, 2, 4, 5$.
 Right \rightarrow Left: $2u - 2o - u - o - u - 3o - u - o - u - 2o - u - o - 2u - 4o - u$.
- Half-cycle 18:** $I_{j_{io}} = 10$; $I_{j_{ii_{previous}}} = 8, 3, 1, 9, 6, 4, 2, 10$; $\left\lfloor \frac{I_{j_{io}} + C_l}{11} \right\rfloor = 1, 2, 3$ for
 $C_l = 1, 12, 23$; $R = |6 + C_l - \left\lfloor \frac{10 + C_l}{11} \right\rfloor|_7$.
 $C_l = |2, 3, 4, 5, 7, 9, 10, 11 | 13, 14, 15, 16, 18, 20, 21, 22 | 24, 25, 26, 27, 29$.
 $R = 0, 1, 2, 3, 5, 0, 1, 2; 3, 4, 5, 6, 1, 3, 4, 5; 6, 0, 1, 2, 4$.
 Left \rightarrow Right: $3u - 2o - 3u - o - u - 2o - u - o - u - 2o - 4u$.
- Half-cycle 19:** $I_{j_{ro}} = 10$; $I_{j_{ri_{previous}}} = 0, 8, 5, 3, 1, 9, 6, 4, 2, 10$; $\lambda = 1$; $R = |5 + C_r|_7$.
 $C_r = 1, 2, 3, 4, 5, 6, 7, 9, 10, 11; 12, 13, 14, 15, 16, 17, 18, 20, 21, 22;$
 $23, 24, 25, 26, 27, 28, 29$.
 $R = 6, 0, 1, 2, 3, 4, 5, 0, 1, 2, 3, 4, 5, 6, 0, 1, 2, 3, 4, 5, 6$.
 Right \rightarrow Left: $u - 3o - u - o - u - 3o - u - o - 2u - 4o - 2u - 3o - u - o - 2u$.
- Half-cycle 20:** $I_{j_{io}} = 7$; $I_{j_{ii_{previous}}} = 8, 3, 1, 9, 6, 4, 2, 10, 7$; $\left\lfloor \frac{I_{j_{io}} + C_l}{11} \right\rfloor = 1, 2, 3$ for
 $C_l = 4, 15, 26$; $R = |C_l - \left\lfloor \frac{7 + C_l}{11} \right\rfloor|_7$.
 $C_l = 1, 2, 3 | 5, 6, 7, 8, 10, 11, 12, 13, 14 | 16, 17, 18, 19, 21, 22, 23, 24, 25 | 27, 28, 29, 30$.
 $R = 1, 2, 3; 4, 5, 6, 0, 2, 3, 4, 5, 6; 0, 1, 2, 3, 5, 6, 0, 1, 2; 3, 4, 5, 6$.
 Left \rightarrow Right: $2u - o - u - 2o - 2u - o - u - 2o - 3u - 3o - 3u - o - u - 2o$.
- Half-cycle 21:** $I_{j_{ro}} = 8$; $I_{j_{ri_{previous}}} = 0, 8, 5, 3, 1, 9, 6, 4, 2, 10, 7$; $\lambda = 1$; $R = |1 + C_r|_7$.
 $C_r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21;$
 $22, 23, 24, 25, 26, 27, 28, 29$.
 $R = 1, 2, 3, 4, 5, 6, 0, 1, 2, 3, 4, 5, 6, 0, 1, 2, 3, 4, 5, 6, 0, 1, 2$.
 Right \rightarrow Left: $2o - u - o - 2u - 3o - u - o - 2u - 3o - u - o - 2u - 3o - u - o - 2u - 3o$.

Although the calculations are simple, there are many to be performed with the consequence that errors may go undetected. It is therefore much more convenient to let a computer do this work, and especially for colour-patterns the computer can then readily draw the actual grid-diagram of the Prime Regular Cylindrical Bihelix Braid.

Integrated Braids

In this issue we shall discuss the Gaucho Button Knots as integrated braids. Although the Gaucho Knot Button described by Bruce Grant in the *Encyclopedia of*

Rawhide and Leather Braiding (Fig. 353 depicts its grid-diagram) can be regarded as an integrated braid with respect to the loop which goes over into the four strings of the braid, the resulting button however can in general not be regarded as being a satisfactory one. The reason being that due to the diameter of the Gaucho Knot it will in general not be possible to obtain a sufficiently firm button.

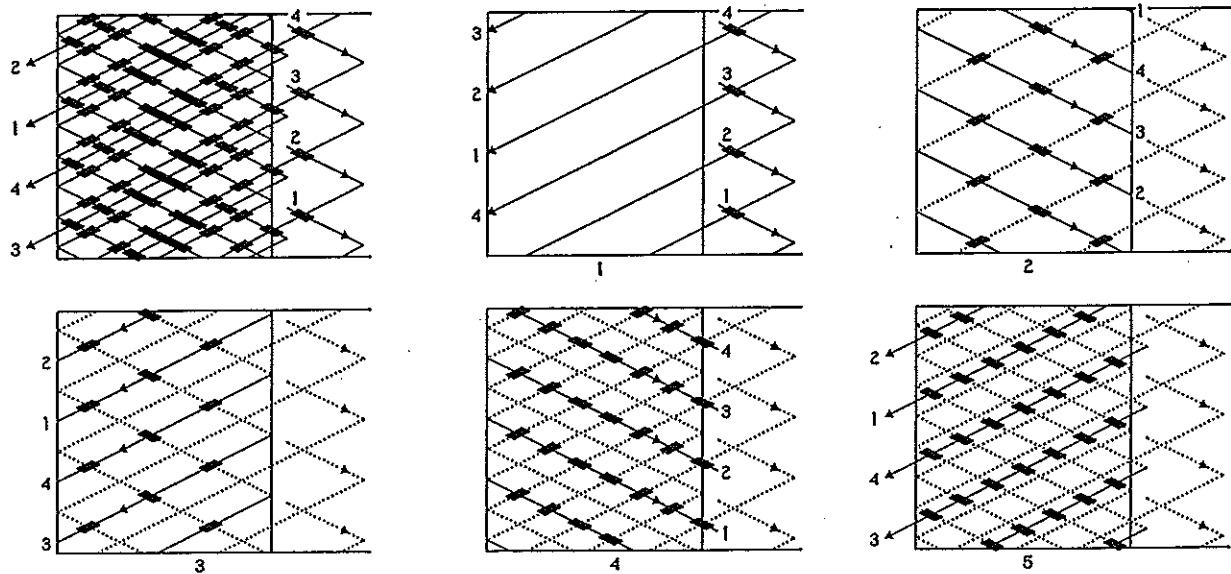


Fig. 353 — The Gaucho Knot Button as described by Bruce Grant in the *Encyclopedia of Rawhide and Leather Braiding*.

In order to improve this button sufficiently, we must place a foundation knot under it. The construction of such a Gaucho Button Knot as an integrated braid at the end of a four string round braid has been depicted in Fig. 354.

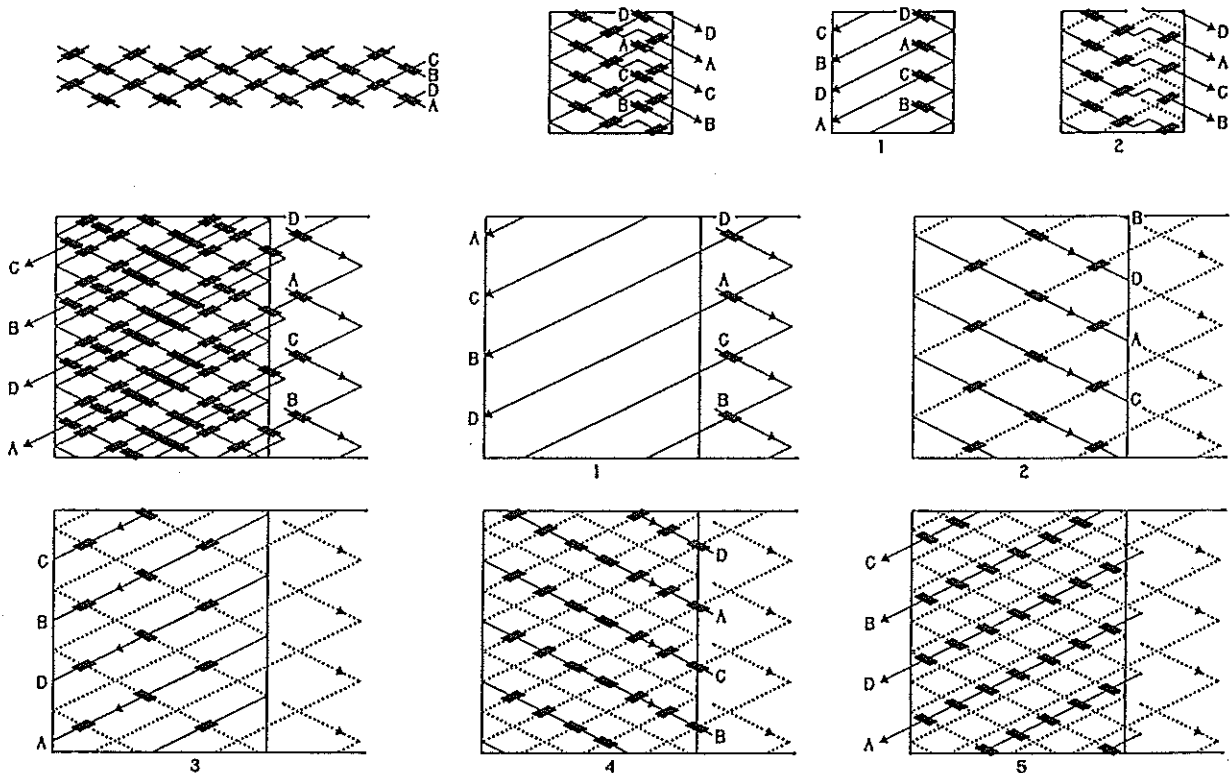


Fig. 354 — A simple integrated Gaucho Button Knot.

In most applications a bigger or bulkier button is often required. This can readily be accomplished by omitting the last braiding step 5 in Fig. 354 and continuing the braiding process after step 4 by braiding a further integrated Gaucho-coded knot over the braid so far obtained. Examples of such further integrated Gaucho-coded knots are shown in Figs. 357-361.

The shape of the final button depends on the lengths of the successive integrated knots. With the integrated overall foundation up to and including step 4 of Fig. 354 the shape of the button will be as follows: with the final integrated knots of Fig. 357 & 358, the shape of the button will be as depicted by the left-most drawing in Fig. 355; with the final integrated knot of Fig. 359, the shape of the button will be as depicted by the second drawing from the left in Fig. 355; and with the final integrated knots of Figs. 360 & 361, the shape of the button will be as depicted by the third drawing from the left in Fig. 355.

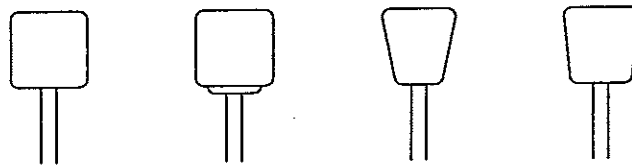


Fig. 355 — The various shapes of the integrated Gaucho-coded Button Knots.

With a longer integrated overall foundation as depicted in Fig. 356 and a final integrated Gaucho-coded knot as in Figs. 360 & 361, the shape of the button will be as depicted by the right-most drawing in Fig. 355.

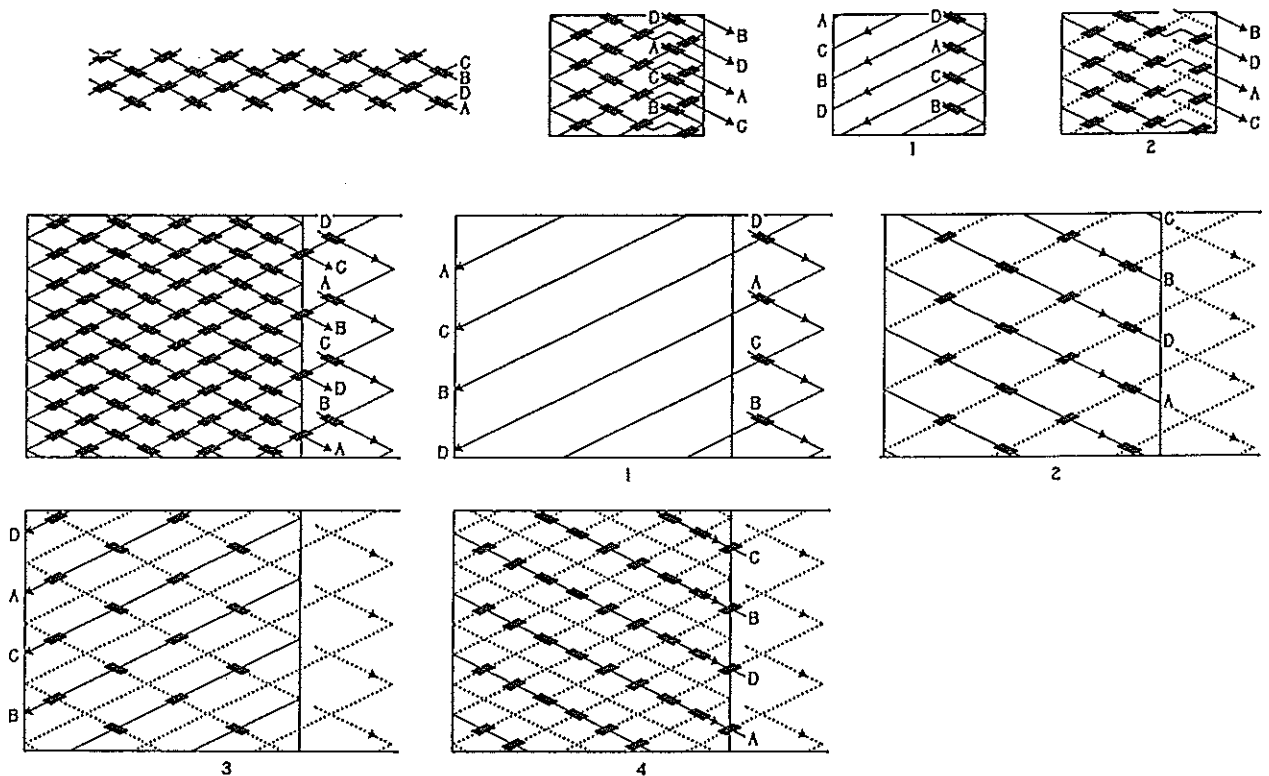


Fig. 356 — A longer integrated foundation.

We like to remind the reader that our depicted examples are intended for the string-dimensions of the inexpensive fabric reinforced "Dura Flex" lace. For other string-dimensions the sizes of the various integrated knots have to be adjusted accordingly.

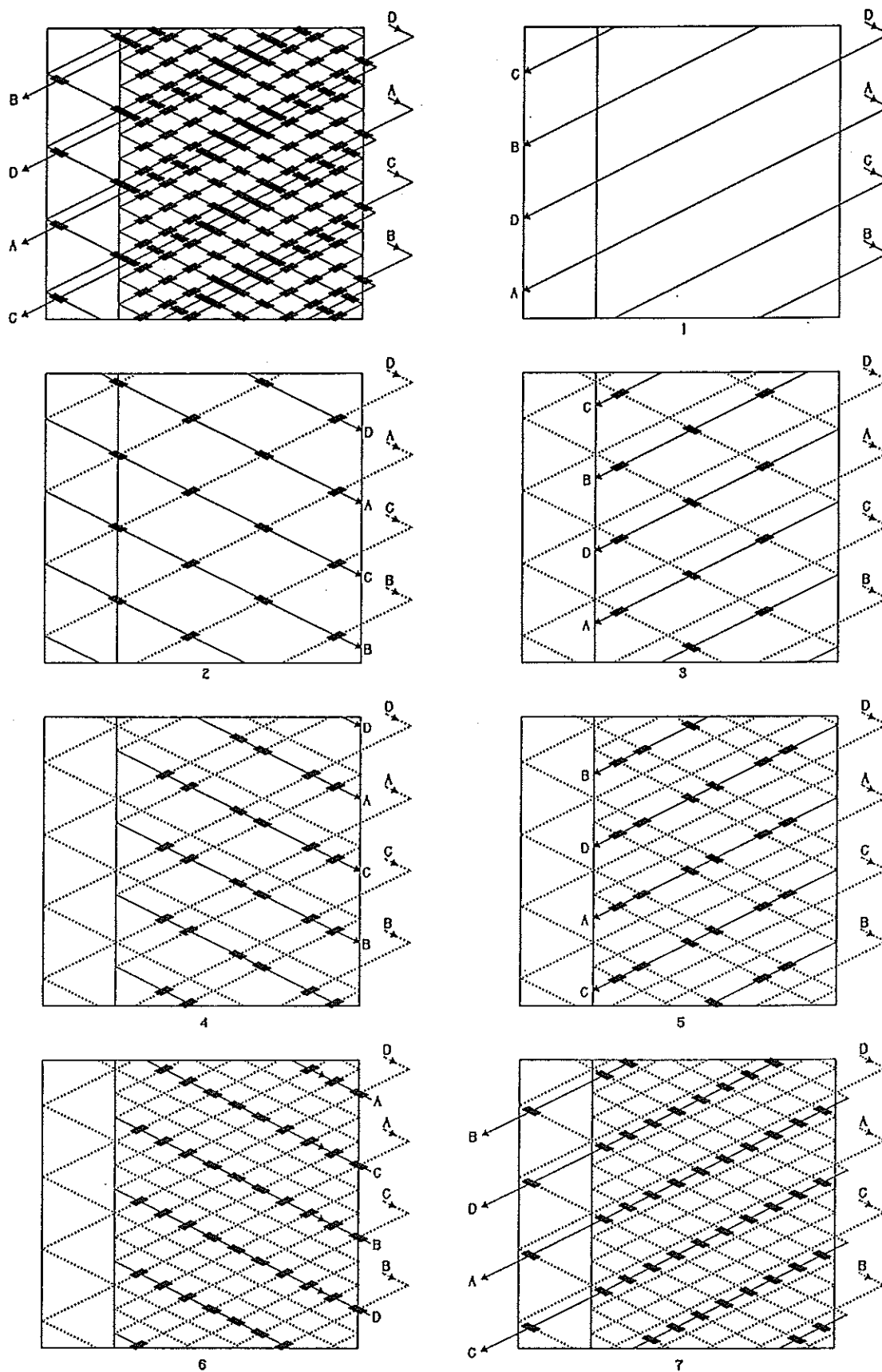


Fig. 357 — A final Gauchocoded integrated knot for the Gaucho Button Knot.

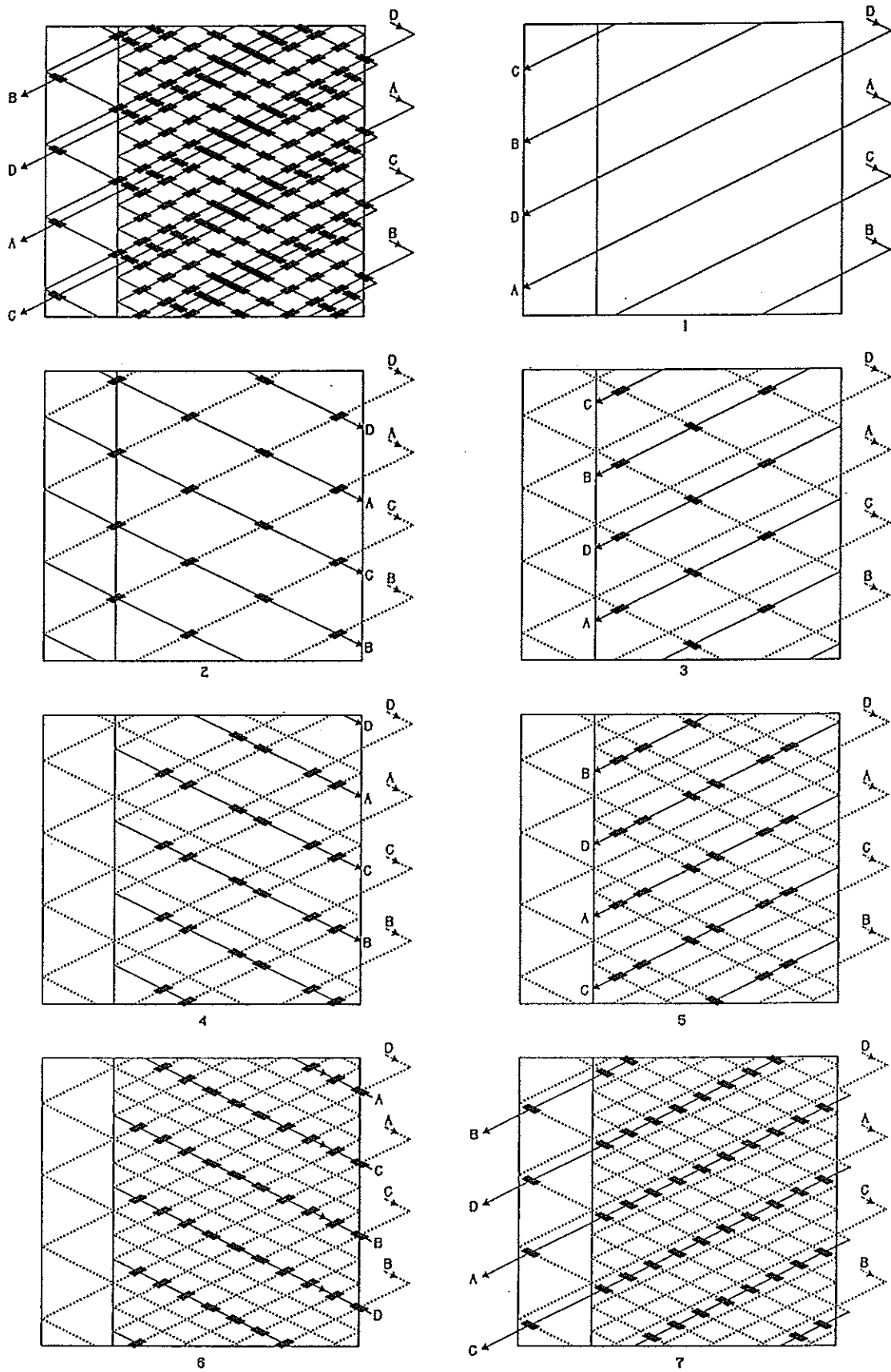


Fig. 358 — A final Gauchocoded integrated knot for the Gaucho Button Knot.

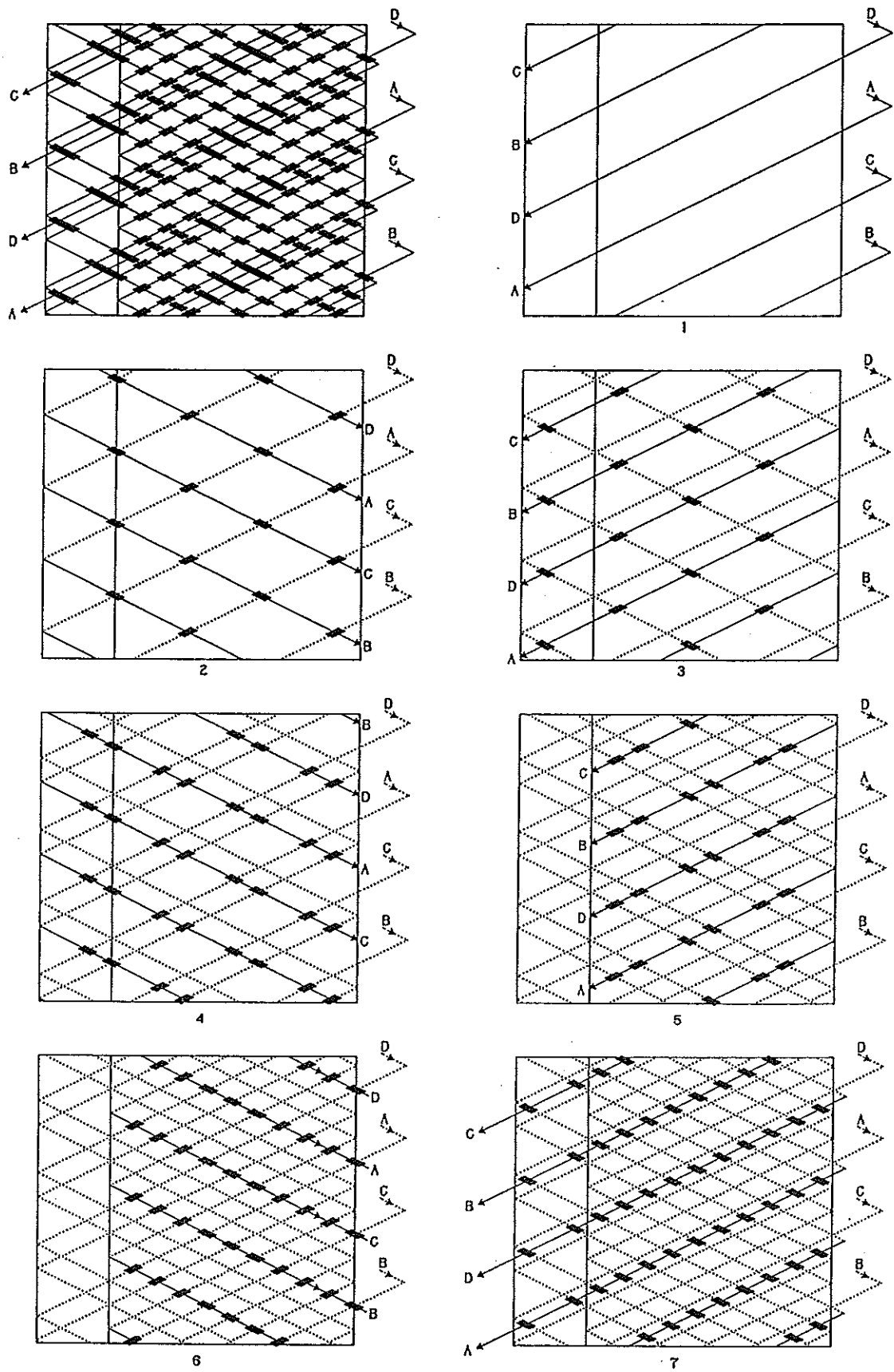


Fig. 359 — A final Gauchocoded integrated knot for the Gaucho Button Knot.

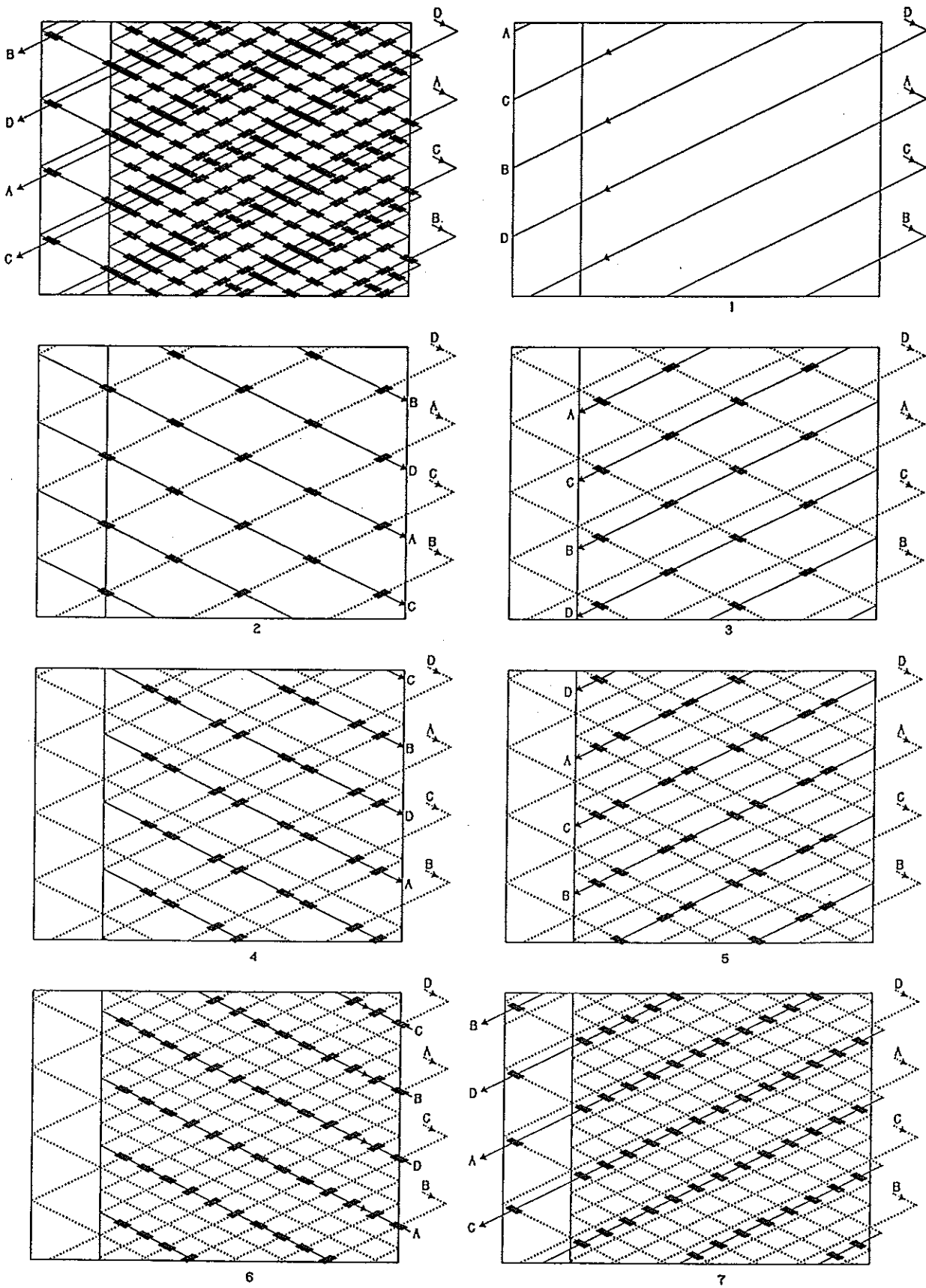


Fig. 360 — A final Gauchocoded integrated knot for the Gaucho Button Knot.

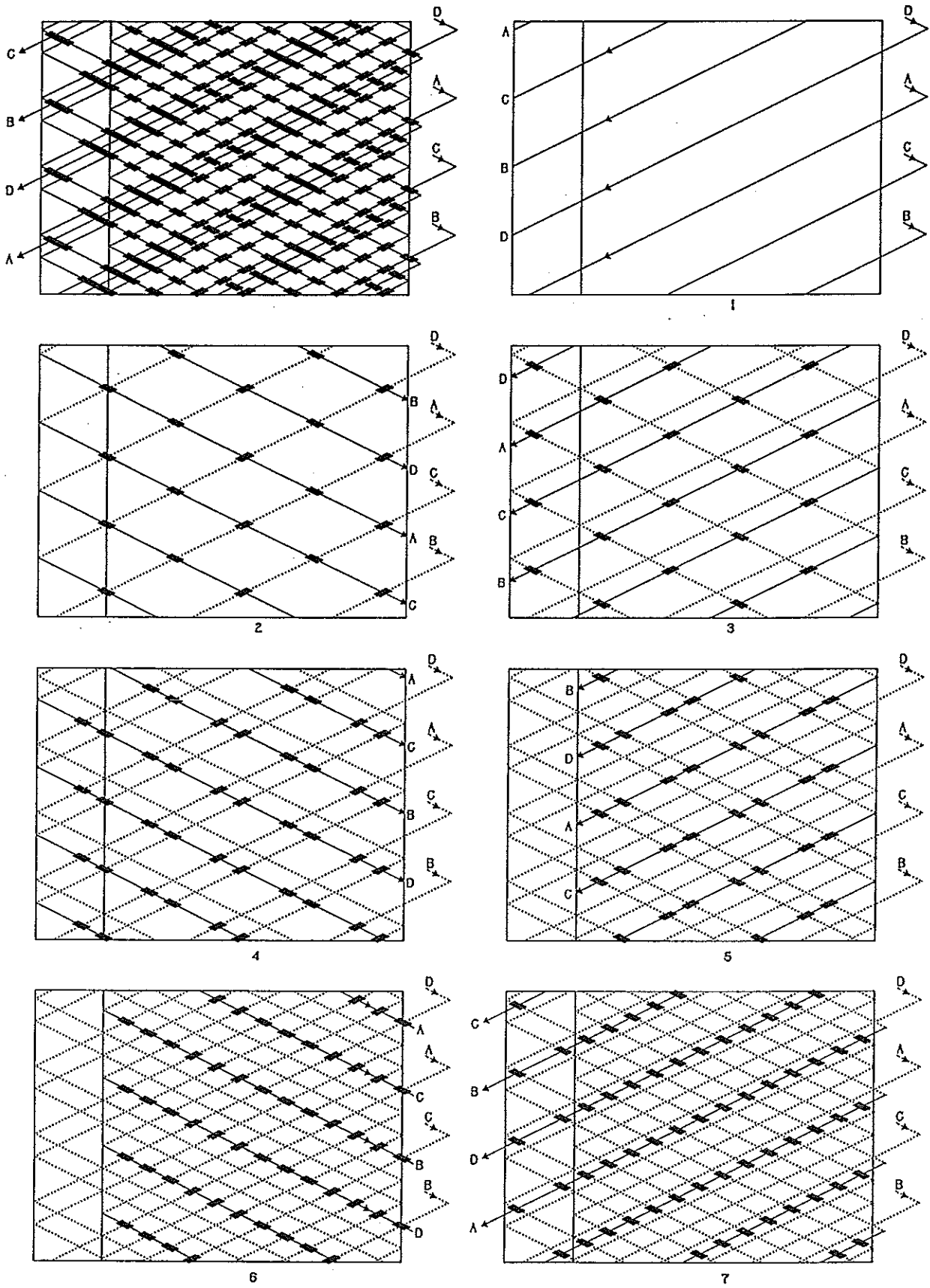


Fig. 361 — A final Gaucho-coded integrated knot for the Gaucho Button Knot.