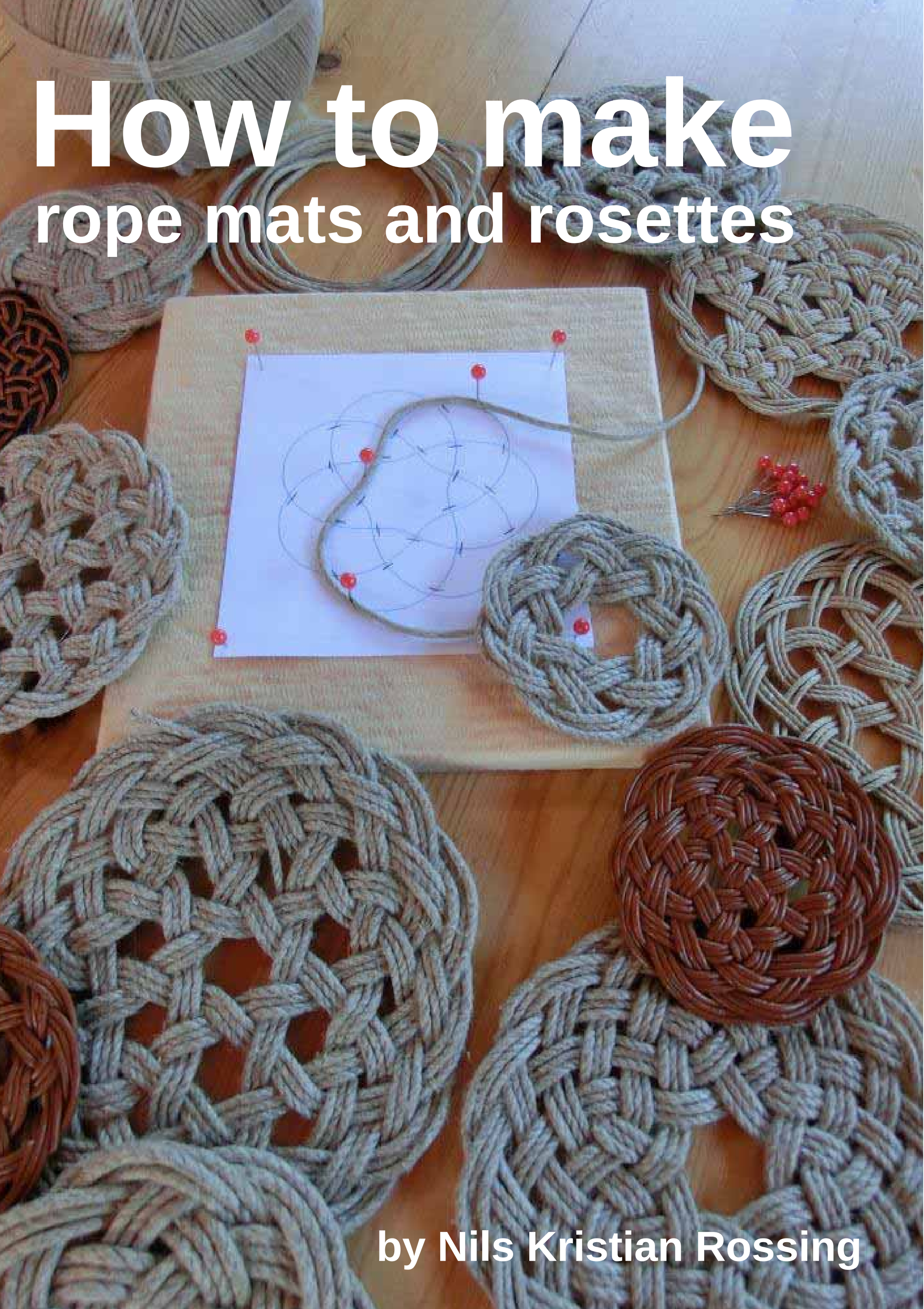
# How to make

## rope mats and rosettes



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*How to make rope mats and rosettes*

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# HOW TO MAKE ROPE MATS AND ROSETTES

by

NILS KR. ROSSING

# Content

# 1 Introduction

This booklet gives an introduction to how to use mathematics to synthesis rope mats and rosettes pattern. It also describes how to make the real thing with hemp rope. The booklet is made for the Bridges conference on Mathematics, Music, Art, Architecture and Culture at Towson University, Maryland US, 2012.

Mathematically there is a close connection between pendulum drawing and rope mats. This article describes a method to analyse traditional rope mats and rosettes. By sampling the rope track one finds out that the rope follows a periodic function both in x- and y-directions. Having the two periodic curves, it is possible to do a Fourier decomposition in both directions. By doing this the Fourier components for the two curves are determined, or, as we may call it: The two-dimensional spectrum for the rosette or mat. The Fourier components for the rope work may now be used to synthesize the mat with a two-dimensional curve drawing software like Matlab or Winplot. By changing the Fourier components' frequency and amplitude, it is possible to make new variants of the mats and rosettes within the same category.

This booklet and future supplements can be downloaded from:
http://www.ntnu.no/skolelab/fagartikler

# 2 The Old Art of Rope Work and Fourier Decomposition

## 2.1 Introduction

A sailing-ship cuts through the sea. There is a moderate breeze and it is time for a rest. A sailor is sitting on a barrel working with a piece of rope. The coarse fingers of the sailor makes the finest shapes with the simple rope. After a while, a work of art takes shape in his hands and a *rope rosette* is placed on the deck.

A pendulum is suspended from the ceiling. The heavy weight is carried by two cords attached to the ceiling. The pen, which is fastened to the weight, is following the movement perfectly and strokes gently across the sheet of paper which is stretched out on the floor. The pen makes arcs of the finest sort on the paper. The movements are slow and steady. A lonely spectator sitting by the side of the pendulum is spellbound by the curves becoming visible on the paper. After some time the pendulum settles down and the curve is tied up in a single point at the centre of the drawing. A beautiful *pendulum drawing* lays on the floor.

It is late in the evening, an enthusiastic scientist is sitting by his computer meditating on a mathematical problem. An irregular time function emerges on the screen. He presses a couple of keys on the keyboard and the computer starts analysing the signal. After a few seconds the time function is transformed into bars in a diagram and suddenly some of the inner secrets of the function are revealed. The computer screen shows, in sharp contrast to the dark background, a *signal spectrum*.

Are there any common denominators between these three so very different topics?

The next sections describes both my process of discovering the connection between pendulum drawing and rope works, and how my work as a researcher in communication technology inspired me to develop a method of analysing rosettes.

## 2.2 Pendulum drawing

By suspending the cables of a pendulum from two points in the ceiling and then leading the cables together into one point between the points of suspension and the weight, it is possible to make the pendulum oscillate with two different frequencies in two directions perpendicular to each other.

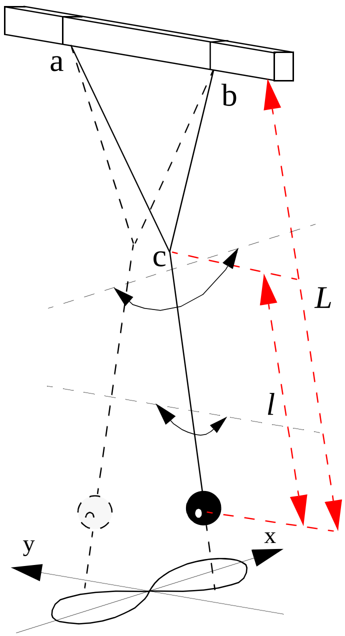



*Figure 1 Simple two-point pendulum.*

Figur 1 shows how the two-point pendulum works in principle. When the square root of the ratio between the long part, *L*, and the short part, *l*, of the pendulum, is 2, the pendulum weight describes a figure of eight, if the phase between the two oscillations is suitable. Such a pendulum was described by Dr. ***Nathanael Bowditch*** (1773–1838) [4] and Prof. ***James Dean*** (1776–1849) in the beginning of the 19$^{th}$ century. Prof. Dean used the two-point pendulum to illustrate the apparent motion of the earth, observed from the moon [3].

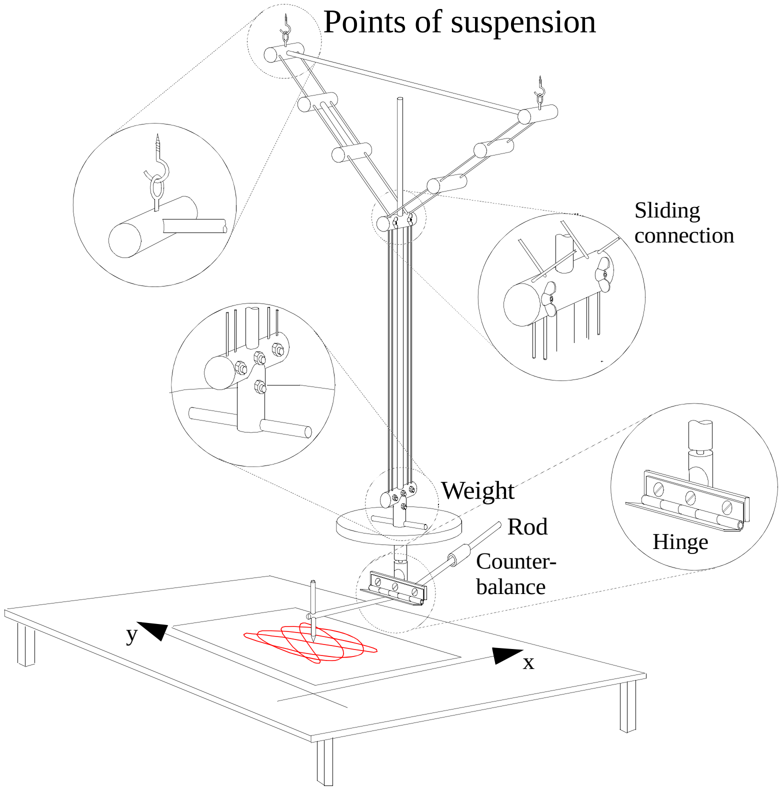



Figure 2 . *Two-point pendulum drawer.*

Figur 2 shows a two-point pendulum *drawer* first described by ***Hubert Airy*** (1838–1903) in *Nature* 1971 [6]. By using a sliding connection between the two wires, it was possible to change the frequency ratio between the two oscillations, and a great variety of figures could be created on the paper. The pen is attached to a rod fastened to a hinge so the pen can tip up and down and keep a constant pressure on the paper. The pressure may be adjusted by the counterbalance. These figures are also called *Lissajous (or Bowditch) curves* after the French physicist ***Jules A. Lissajous*** (1822–80) [5].

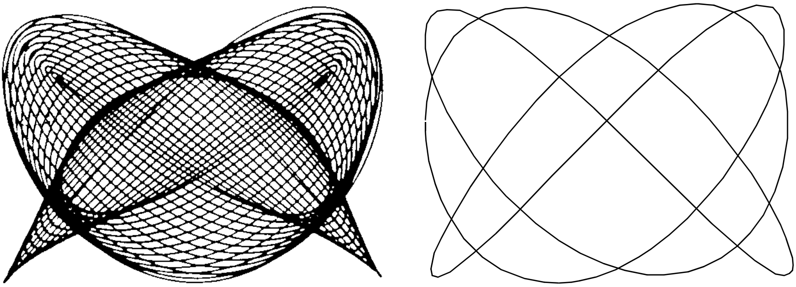


Figure 3 *Examples of Lissajous curves drawn by the pendulum.*

## 2.3 Mats of rope

The Danish cordage artist and writer *Kaj Lund*[1] has for a number of years collected and recreated old rope mats and knots. Some of them can be traced back to the time of the Vikings (800–1100AD) [7]. One of the most simple models is the *Rectangular mat (or Expansion mat)* shown in figure 4. Comparing figure 3 and 4 one may see that the Rectangular mat and the Lissajous curves have almost the exact same shape.

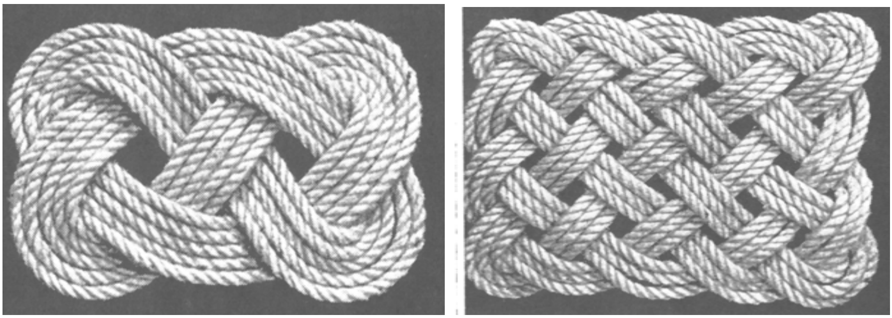


Figure 4 *Rectangular mats.*

## 2.4 The mathematics of Lissajous curves

Lissajous curves can easily be described mathematically by two sine-shaped functions

1. *Kaj Lund* published several books on rope work from 1969 to 1979 (Borgen – Danish publisher).

which are oriented perpendicular to each other. This is shown in figure 5.

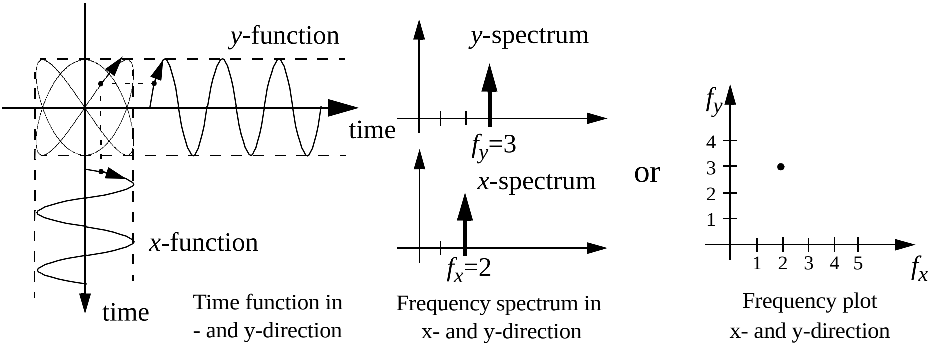



Figure 5 *Lissajous curves can easily be described mathematically by two sinusoidal functions.*

Mathematically the Lissajous figure may be expressed by two equations:

$$x = A_x \cos(2\pi f_x t) \tag{6}$$

$$y = A_y \sin(2\pi f_y t) \tag{7}$$

Different Lissajous curves can be obtained by changing the ratio between the frequencies, $f_x$ and $f_y$ (figure 5) of the two sine-shaped functions. The shape of the Lissajous curves may be changed by the amplitudes $A_x$ and $A_y$.

It is also possible to represent Lissajous curves with two spectra, one for the *x*- and one for the *y*-function.

## 2.5 Rectangular mats

We have observed that the Rectangular mat and the Lissajous curves are almost exactly the same. For this reason we may represent the Rectangular mat with the same mathematical expression as we did for the Lissajous curves. Figur 8 shows some examples of simulated Rectangular mats. The shape of the mat is, as for the Lissajous curves, mainly determined by the ratio between the two frequencies, $f_x$ and $f_y$.

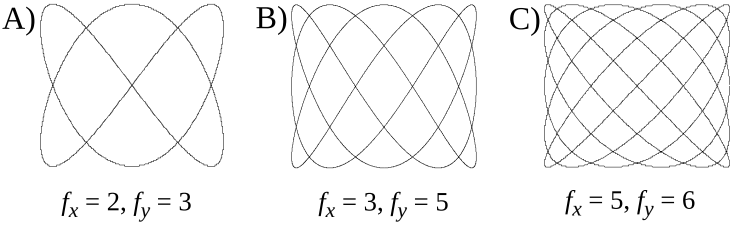



Figure 8 *Examples of simulated Rectangular mats.*

However, the Rectangular mat is one of the most simple mats. A natural question is: Is it possible to find equations describing more complex mats and rosettes. Let us take a look at the Turk's Head [9].

## 2.6 Turk's Head rosette

The sign of *The International Guild of Knot Tyers* is a Four-Bight Turk's Head rosette (figure 9). A "bight" is a knot tier's term for a loop.

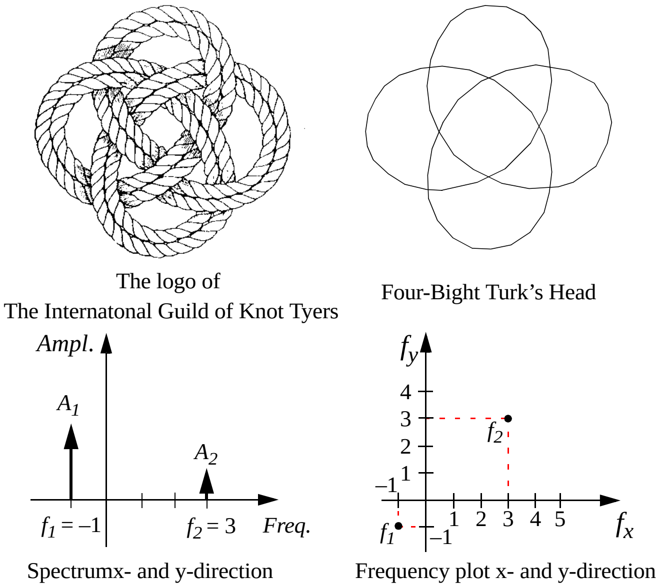



Figure 9 *Four Bight Turk's Head.*

## How to make rope mats and rosettes

Curves like the Four-Bight Turk's Head are known from spirograph curves, ornamental turning [10] and Guilloché patterns [2]. By studying the spirograph it is quite obvious that simple spirograph curves appear from two rotating vectors, which can be expressed mathematically by equation (10) and (11):

$$x = A_{x1}\cos(2\pi f_{x1}t)+A_{x2}\cos(2\pi f_{x2}t) = A_1\cos(2\pi f_1 t)+A_2\cos(2\pi f_2 t) \tag{10}$$

$$y = A_{y1}\sin(2\pi f_{y1}t)+A_{y2}\sin(2\pi f_{y2}t) = A_1\sin(2\pi f_1 t)+A_2\sin(2\pi f_2 t) \tag{11}$$

As we see, the expression contains two frequency components in x- and y-direction, $f_{x1}$, $f_{x2}$ and $f_{1y}$, $f_{2y}$. Due to symmetry the components are equal in both x- and y-direction ($f_{x1} = f_{y1} = f_1$ and $f_{x2} = f_{y2} = f_2$ and $A_{x1} = A_{y1} = A_1$ and $A_{x2} = A_{y2} = A_2$). By manipulating these components, it is possible to make more complex Turk's Head's rosettes.

The distance between the two frequency components reflects the number of bights (figure 12). By drawing a line from the centre to the periphery, the value of the positive frequency ($f_2$) defines the number of strands crossing the line. By changing the distance between and location of the frequency components, we can get several variants of the Turk's Head rosette as shown in figure 12.

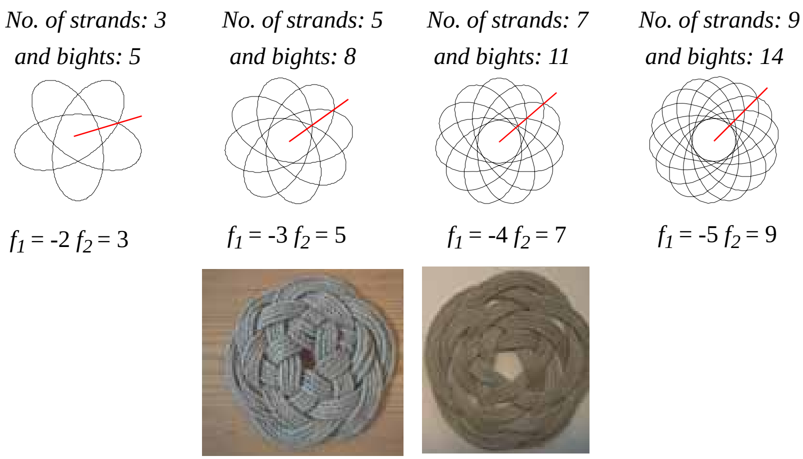



Figure 12 *Examples of simulated Turk's Head rosettes.*

Similar patterns can be achieved by a *twin pendulum* described by ***J. Goold*** et.al. [11].

Let us now go one step further and look at even more complex mats.

## 2.7 The Twisted rosette

Figur 13 shows examples of the Twisted rosette.The twists (pretzels) on the left mat overlap in the centre. The middle and the right ones do not overlap. To find the mathematical expression of these rosettes we may use Fourier decomposition.

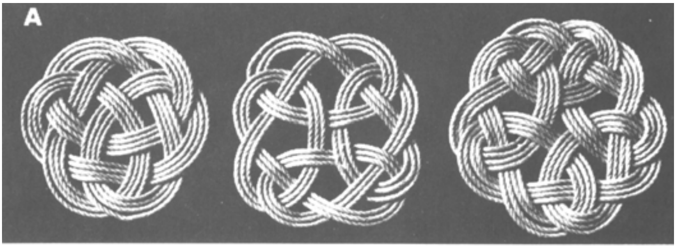
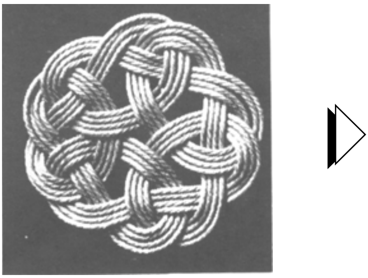
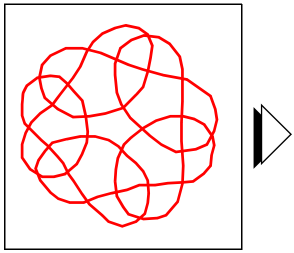



Figure 13 *Examples of Twisted rosettes [8].*

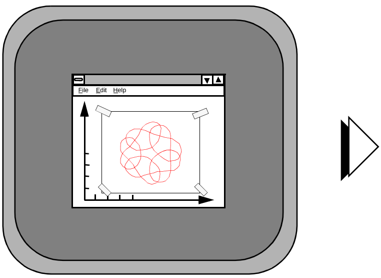
To do the analysis we transfer the rosette curve to transparent plastic which is taped onto the computer screen. The running Matlab program shows a coordinate diagram behind the plastic with the rosette drawing. The sampling is done by clicking the mouse cursor along the rosette curve, from the beginning to the end (figure 14).



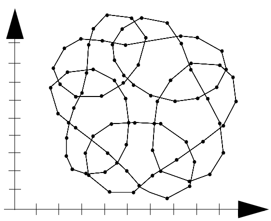
*Original rosette*



*Rosette copied to plastic*



*Plastic placed on the computer screen and sampled by the mouse*



*Sampled rosette*

Figure 14 *Sampling of the rosette.*

Next the curvature of the rosette is transferred to a set of coordinate points, from which the computer calculates the *x*- and *y*-function. The spectra of the two functions are found by Fourier decomposition. The rosette shown in figure 15 is made by five overlapping twists. From the *x*- and *y*-spectra we can see that the twisted rosette may be described by *three* frequency components. The distance between the frequency components is five and equal to the number of twists in the rosette.

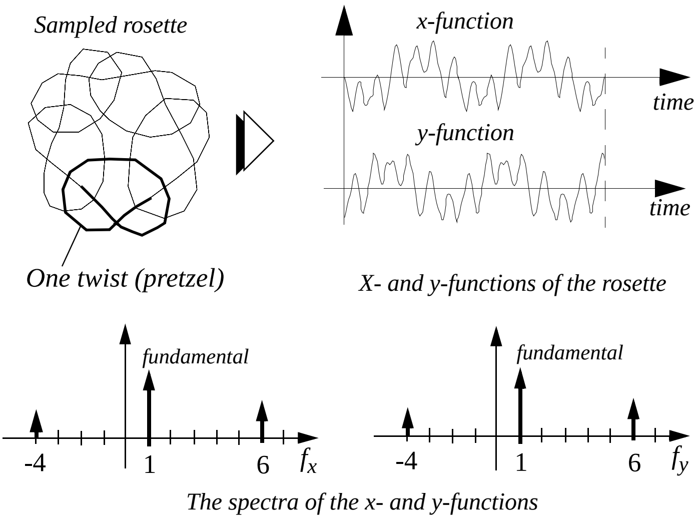



Figure 15 *The spectrum of the twisted rosette.*

Normally spectra consist of positive frequency components only. Studying the original phase output from the Fourier analysis, we can conclude that the components in the *x*- and *y*-spectra represent vectors rotating in both directions. This is illustrated graphically by both positive and negative spectral components, and show very clearly the constant gap between the individual components. This is evident in the next examples.

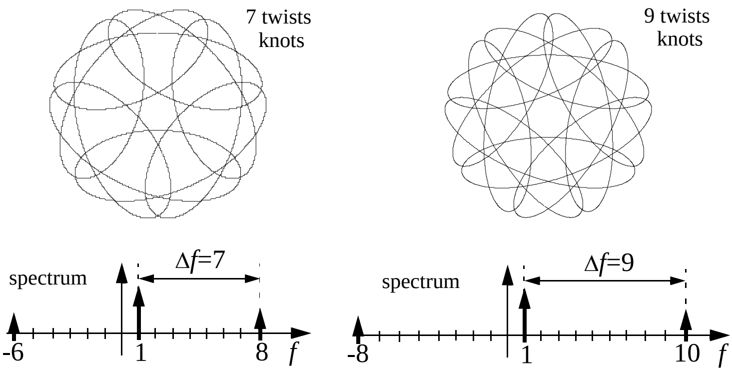



Figure 16 *The number of twists is equal to the distance between the frequency components in the spectrum.*

It is now easy to experiment with the parameters. By changing the amplitudes, the distance between the spectral components or by moving the x- and y-components along the frequency axis, we may design new versions of the rosette. Figur 16 and 17 show four examples of the twisted rosette.

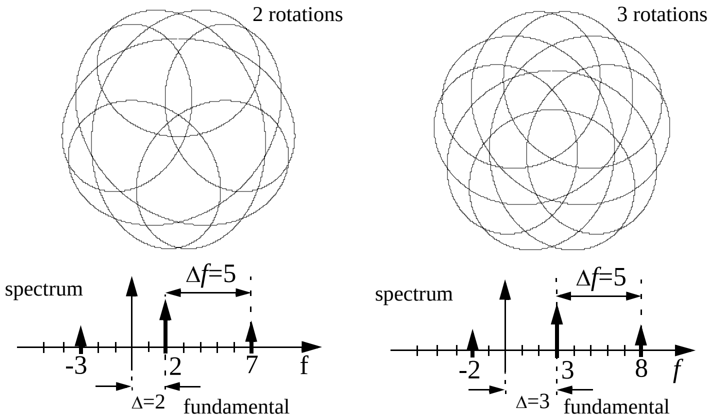


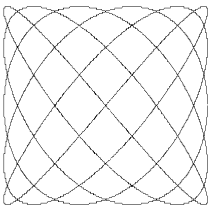

Figure 17 *The value of the fundamental frequency*
*is equal to the number of laps the curve is running around the centre of the rosette.*

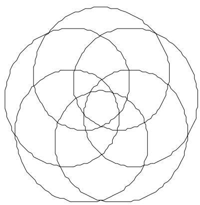
Figur 18 shows the rosettes from figure 17 realised in 5 mm hemp rope.

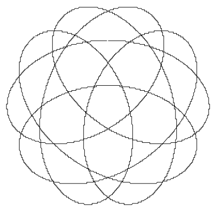
Figure 18 *The patterns from figure 17 realised in 5 mm hemp rope (photo Nils Kr. Rossing).*

## 2.8 Rosettes of different order

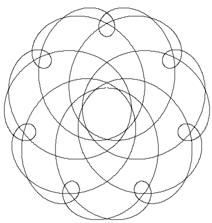
Since the Rectangular mat can be described by one frequency component in each direction *x* and *y*, this class of mats can be called *mats of first "order" (authors notion)*. In the same manner we can say that a Turk's Head rosette is a mat of *second order* and a Twist rosette a mat of *third order*. Using the Fourier technique it is quite easy to analyse mats of higher orders. The figure below presents some examples.

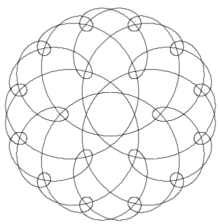


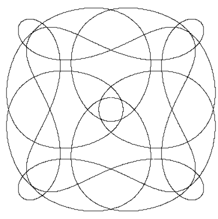




*1st order Rectangular mat* *2nd order Turk's Head rosette* *3rd order Twist rosette*







*5th order Simple eye rosette* *8th order Jens Kusk Jensens rosette* *9th order alternating eye rosette*

Figure 19 *Traditional and new rosettes of orders 1, 2, 3, 5, 8, and 9, mathematically generated.*

However, to characterize rosettes by its *order*, is not an exact branch of knowledge. The order is a kind of minimum number of Fourier components that can describe the rosette or mat up to a certain level of visual quality. Some rosettes may apparently be described in more detail by adding components, and some can be described by fewer components but with reduced quality. So order may, up to certain extent, characterise a mat or rosette.

Based on the knowledge of the Fourier components of some traditional mats and rosettes, we can synthesize their patterns with a two-dimensional curve drawing software like Matlab or Winplot. By changing the Fourier components in frequency and amplitude, it is possible to make new variants of the mats and rosettes within each individual family (or order).

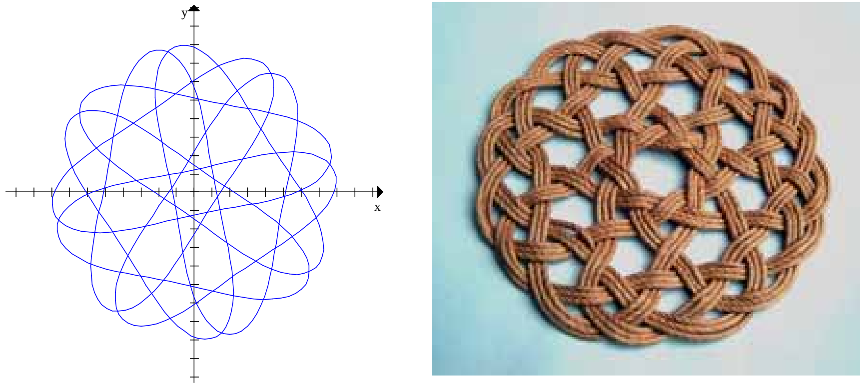



Figure 20 *Fourth order Rattan rosette*.

The tradition of making fancy rope works originates mainly from sailors. In their spare time they had little to do, and a lot of suitable material, and in addition few of them could read. Making mats and rosettes was therefore mainly for pleasure and decoration. In some cases mats were used to protect the deck, and to reduce slipperiness. P.P.O. Harrison writes:

> *The art of Fancy Rope work reached the peak of its excellence about the middle of last century (19$^{th}$ century) when a slump in trade brought about a reduction in the number of crew in a ship with the consequent reduction of the spare time that had permitted sailors to occupy themselves with their handicraft. An observer at that time considered that in the way of knots, especially for use at sea, there could be nothing more to invent. He was wrong [12].*

Harrison concludes that much more has been invented since then. Probably more than existed at that time in 19th century, but probably mostly by hobby sailors and rope work enthusiasts. Similar patterns can be found in Celtic, Tamilian (Indian) [1] and *Sona* (Angolan) tradition [13].

The main purpose of this work has been to establish a method suitable to explore and understand the inner properties of traditional mats and rosettes, and to develop a tool for expanding and redesigning traditional models into new versions of rope works.

I have approached the subject more as an engineer rather than as a mathematician. Consequently I would like to focus on developing ICT-tools rather then developing the mathematics. For me a further improvement would be to make an integrated ICT-tool for analysing and synthesising mat and rosette patterns. The analysing part of the software should be able to scan and sample the patterns, subsequently decompose them into their Fourier components. With the synthesising part, based on the results from the analyser, it should be possible to develop new patterns and prepare them for handicraft production.

## 3 Mathematical Synthesis and Making of Rope Mats and Rosettes

The workshop lets the participants synthesize and make a simple rope mat or rosette with thin hemp rope. The workshop is an extension of the presentation: *The Old Art of Rope Work and Fourier decomposition.*

### 3.1 Introduction

The nice thing about mathematical modelling of rope mats and rosettes is that we can use these models for synthesising old and new patterns. The workshop gives a short introduction using WinPlot for experimenting with a few patterns. On the basis of a suitable pattern the participants learn how to make a template showing where the strings are passing over and below each other. This template is later used for making the mat or rosette using thin hemp rope and pins.

### 3.2 Simulation

Install the program WinPlot[2] and start experimenting with patterns like square mats and

2. A free version of WinPlot can be downloaded from http://math.exeter.edu/rparris/winplot.html

Turk's Head rosettes (figure 21).

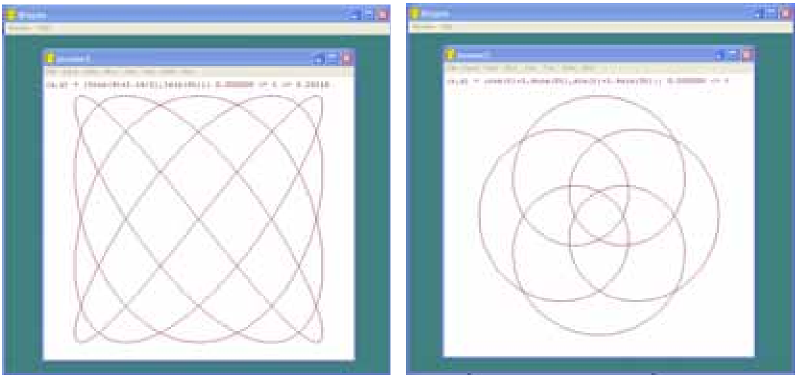
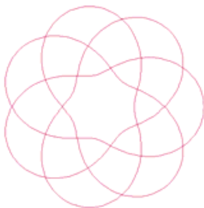


Figure 21 *Left - Square mat with 5·4 bights* ($x = 3\cos(4t+\pi/2)$, $y = 3\sin(5t)$), *Right - Turk's Head rosette with 4 bights* ($x = \cos(t) + \cos(5t)$, $y = \sin(t) + \sin(5t)$).

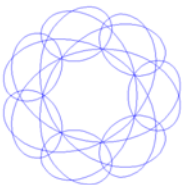
## 3.3 Criterion for high quality templates

There are some hand rules for making good templates for rope mats or rosettes:

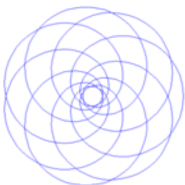
1. Avoid multicrossings
2. Avoid tight crossings
3. Avoid long bights along the outer rim
4. Avoid loops without internal crossings
5. Make right angel crossings



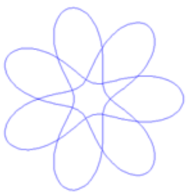
Make right angel crossings



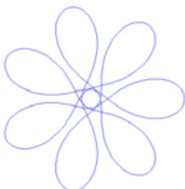
Avoid multi-crossings



Avoid tight crossings



Avoid long bights



Avoid loops without crossings

## 3.4 Preparations

When the pattern design is ready, it is time to decide where the string should pass above or below the crossing string. This is mostly much easier than expected. The starting point is arbitrary, and you can usually follow the string and mark every second crossing as above and below as shown in figure 22.

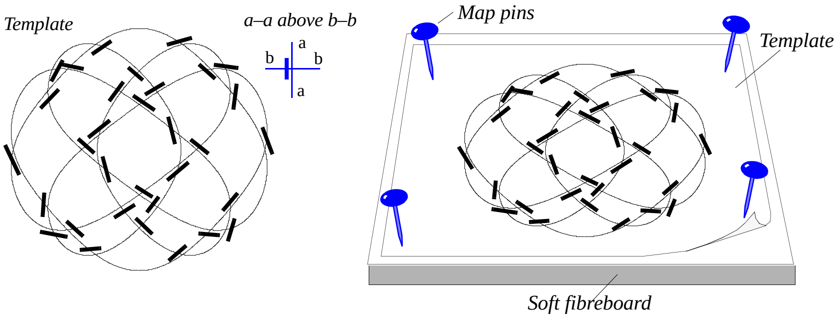



Figure 22 *Left – Template for a heart mat marked with lines indicating which string passes over: a–a passing over b–b. Right – The template is attached to a soft fibreboard with map pins.*

Secondly. Print out the template and attach it to a soft fibreboard with map pins (figure 22 – Right). Hemp is a pleasant material to work with. Choose a suitable rope dimension, which have to be fitted to the total dimension and complexity of the mat. Before starting, you must stipulate the length of the rope needed to complete one lap through the pattern. This may be done by allowing the rope briefly to follow the route without threading it.

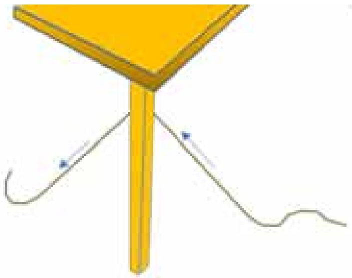


Figure 23 Soften the rope by pulling it back and forth around a sharp edge.

The last preparation is to soften the rope by pulling it back and forth around a sharp leg of a table or a chair (figure 23). Now you are ready to start.

### 3.5 Making the rope mat


Figure 24 Start placing the rope along the route. Use map pins to keep the rope on the track.

Choose a place to start along the periphery of the pattern and that let the rope follow the route on the template. Use map pins to keep the rope close to the route (figure 24). The marks on the template show if rope should pass above or creep below at a crossing point. At the beginning there will be many crossing points without crossing *ropes*. This will however, change during the process.

At the end you will be back at the starting point, finished with the first turn. If you still have some rope left, you can start at the next turn. When the end of the rope is reached, a new length is cut from the supply and you may continue where the previous rope ended. The joining of the two ends may be done quite simple by using needle and thread and fastening them to the rear side of the mat.

Figur 25 shows some examples of rope mats and rosettes synthesised by mathematical models.

*Square mat (5 · 4 bights)* *Double heart mat* *Alternating Eye Rosette* *Rattan rosette (16 bights)*

Figure 25 *Examples of rope mats and rosettes. From left - Rectangular mat (5 · 4 bights), Double Heart mat, Alternating Eye Rosette and Rattan rosette (16 bights) (Photo Kaj Lund and Nils Kr. Rossing).*

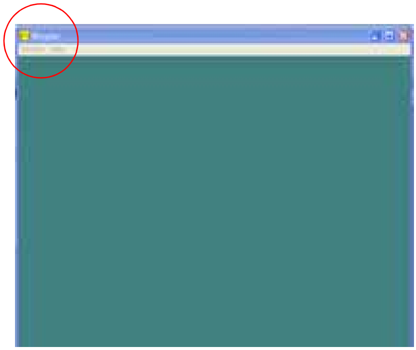
## 4 Synthesis rope mats and rosettes with Winplot (tutorial)

WinPlot is a free software for plotting mathematical functions. The software may be downloaded from http://math.exeter.edu/rparris/winplot.html. The file must be unzipped before used. The following is at tutorial guide explaining the main property of Winplot and how to use the program for experimenting with rope mat and rosette patterns.

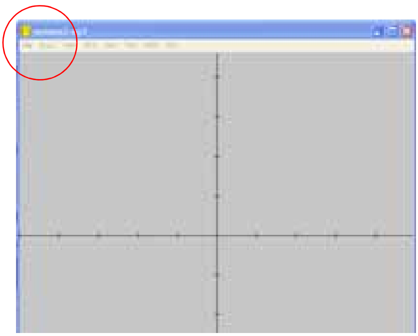
## 4.1 Synthesising simple rectangular mats

- Start the program by click the icon. 

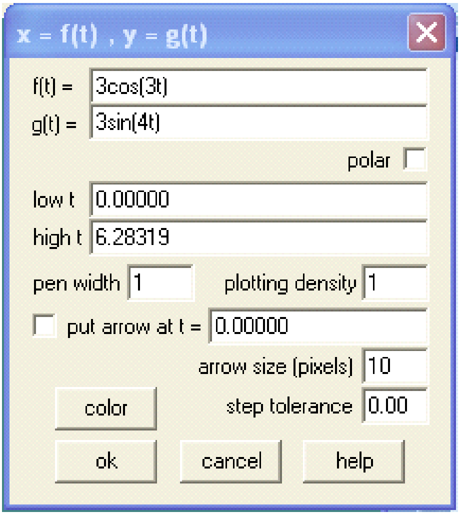
- **Choose:**
  - *Window*
  - *2-dim*

- **Choose:**
  - *Eque*
  - *Parametric*

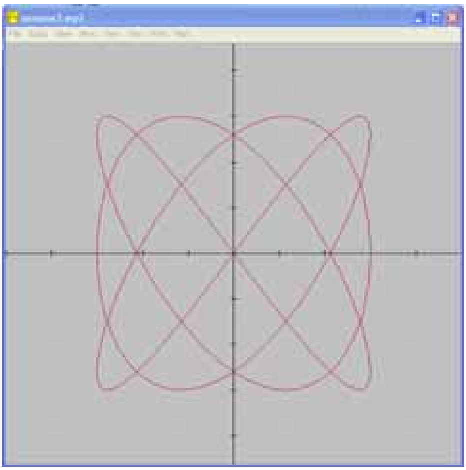
- **Equation editor**
  - You will now see the equation editor



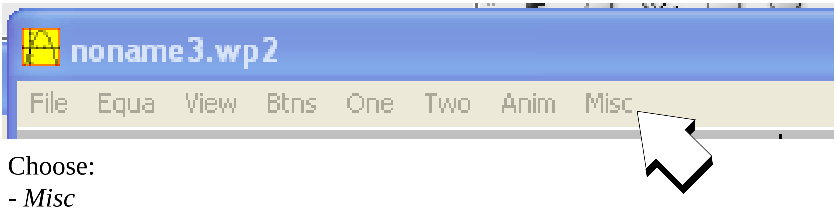

Write the *x*- and *y*-functions in the input boxes *f(t)* and *g(t)*. Use *t* as the variable (time).

Set the lower and upper limit of t. Default is normally OK.

- **Plotting**
  Press OK and the function will be plotted



- **Change background colour**



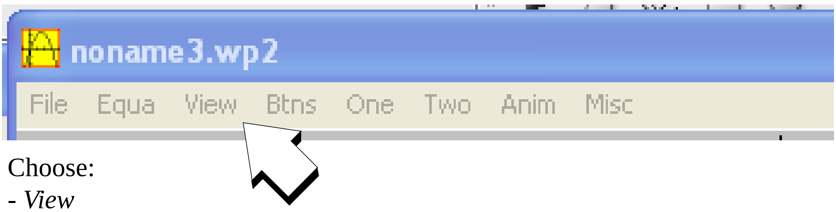

Choose:
- *Misc*
- *Colors*
- *Background*
- *White*
- *Close*

- **Adjust the size of the plot**

Increase:
- *Press PgUp*

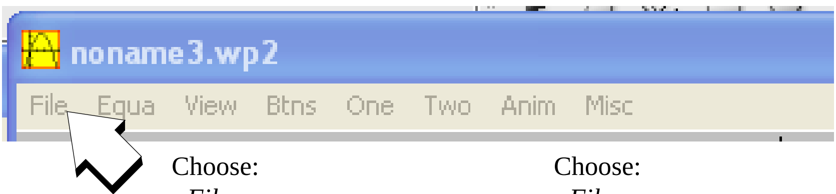
Decrease:
- *Press PgDn*

Fit to window:




Choose:
- *View*
- *Fit window* (Ctrl F)

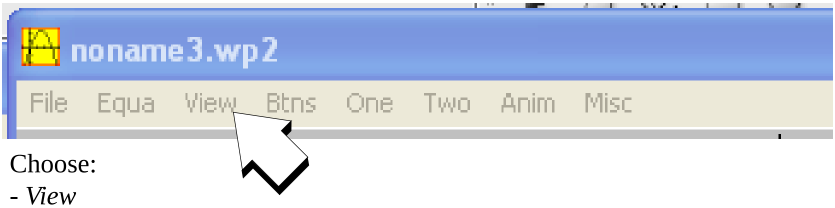
- **Print template (curve) (Ctrl P)**




Choose:
- *File*
- *Copy to clipboard* (Ctrl C)
- *Paste in Word or other program*

Choose:
- *File*
- *Print (Ctrl P)*

- **Remove axis' at plot (Ctrl A)**

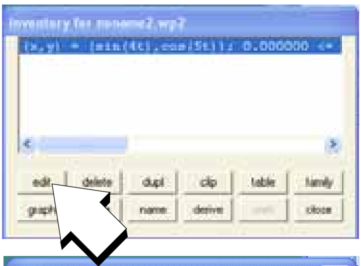



Choose:
- *View*
- *Axes*
- *Axes* (Ctrl A)

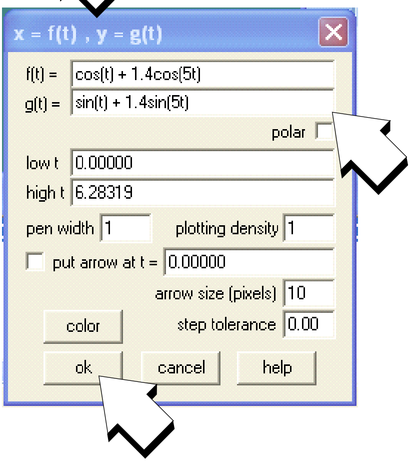
## 4.2 Defining new equations

We shall now redefine the parametric equations and designing a *Turk's Head rosette*.

- **Edit equation - equations with two terms**



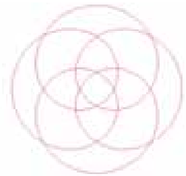

Choose:
- *Edit*



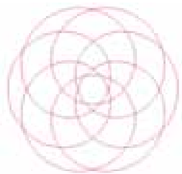

Write the equations into the input boxes *f(t)* and *g(t)*:
*cos(t) + 1.4cos(5t)*
*sin(t) + 1.4sin(5t)*

Press:
*OK*

- **Experimenting and observing**

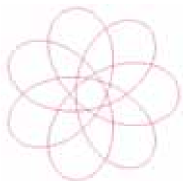


f(t) = cos(t) + 1.4cos(5t)
g(t) = sin(t) + 1.4sin(5t)



f(t) = cos(t) + 1.4cos(7t)
g(t) = sin(t) + 1.4sin(7t)

Changing frequency in second term



f(t) = cos(-2t) + 1.4cos(5t)
g(t) = sin(-2t) + 1.4sin(5t)

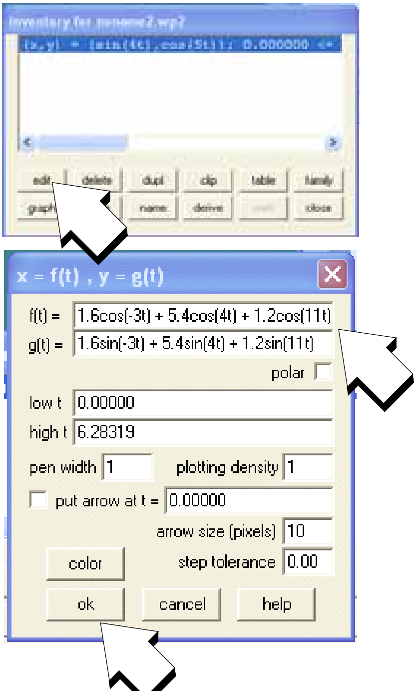
Changing frequency in first term



f(t) = cos(-2t) + 2.0cos(5t)
g(t) = sin(-2t) + 2.0sin(5t)

Changing amplitude in second term

- **Edit equation - equations with three terms**

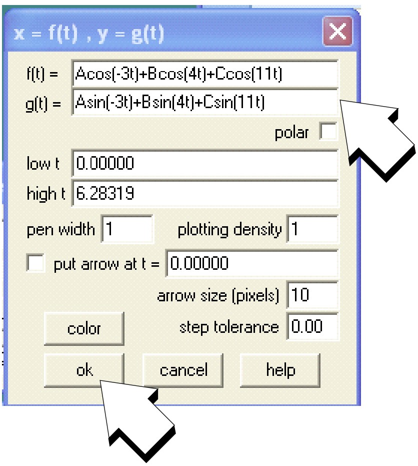



Choose:
- *Edit*

Write the equations into
the input boxes *f(t)* and *g(t)*:
*1.6cos(-3t) + 5.4cos(4t) + 1.2cos(11t)*
*1.6sin(-3t) + 5.4sin(4t) + 1.2sin(11t)*

Press:
*OK*

- **Prepare for tuning the template**




Choose:
- *Edit*

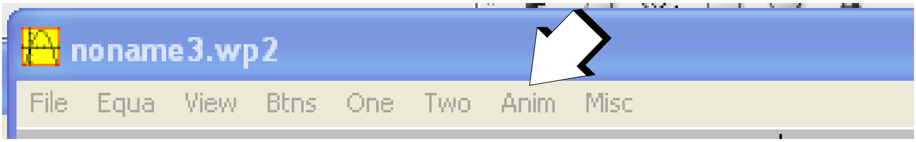
Write the equations into
the input boxes *f(t)* and *g(t)*:
*Acos(-3t) + Bcos(4t) + Ccos(11t)*
*Asin(-3t) + Bsin(4t) + Csin(11t)*

Press:
*OK*

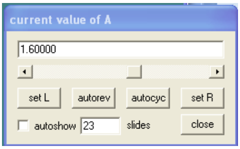
Remember:
A = 1.6, B = 5.4, C = 1.2

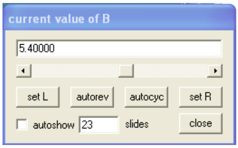
*A, B and C is variables that can be manipulated with sliders*

- **Tuning the template**



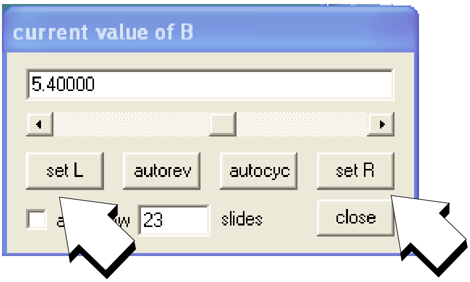

Choose from menu line:
- *Anim*
- *Individual*
- *A*
- *B*
- *C*




Each of the variables *A*, *B* and *C* got its own slider. The upper (*R*) and lower (*L*) values may be set by the button *set L* and *set R*.

- **Set limits for the sliders**




- Write values for the limits in the input box
- Press *set L* (lower limit)
- Press *set R* (upper limit)

## 5 Examples of synthesised patterns

In this chapter we will show some examples of rope work patterns and their equation as a basis for further experimenting. Let's start with the rectangular mat.

### 5.1 Rectangular mats

#### 5.1.1 Equations

The following parametric equation can be used to synthesis the patterns:

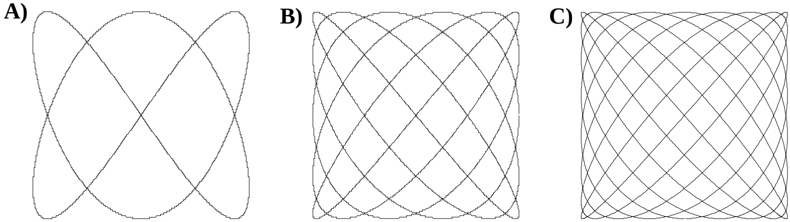
$$x = A_x cos(f_{x1}t) \tag{5.1}$$

$$y = A_y sin(f_{y1}t) \qquad (5.2)$$

Here we can change the amplitude in *x*- ($A_x$) and *y*-direction ($A_y$). This will make the mat more or less rectangular. We may also choose different values of $f_x$ and $f_y$. These two parameters will determine the number of bights along the to edges of the mat ($A_y$) provided that the $f_x$ and $f_y$ are mutually primes.

### 5.1.2 Examples of patterns

Figure 26 shows examples of synthesised patterns of rectangular mats.




$f_{x1} = 2$, $f_{y1} = 3$ — $A_x = 1$, $A_y = 1$

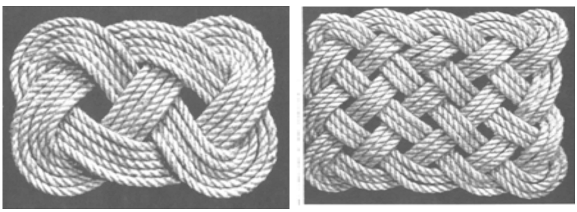
$f_{x1} = 5$, $f_{y1} = 6$ — $A_x = 1$, $A_y = 1$

$f_{x1} = 8$, $f_{y1} = 9$ — $A_x = 1$, $A_y = 1$

Figure 26 *Examples of synthesised patterns of rectangular mats.*

### 5.1.3 Examples of woven mats

Figure 27 shows examples of woven rectangular mats.



$f_{x1} = 3$, $f_{y1} = 2$ — $A_x = 1$, $A_y = 1.8$

$f_{x1} = 4$, $f_{y1} = 5$ — $A_x = 1$, $A_y = 1.6$

Figure 27 *Examples of woven rectangular mats (Photo Kaj Lund).*

## 5.2 Turk’s Head rosettes

### 5.2.1 Equations

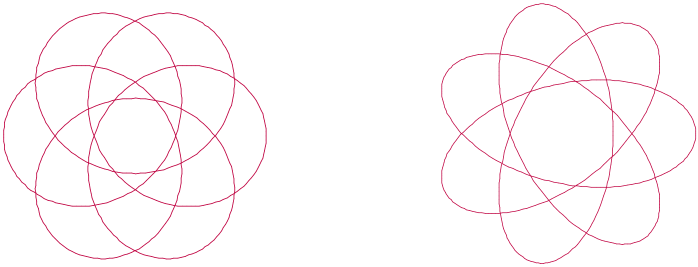
The following parametric equation can be used to synthesis the patterns of Turk’s Head rosettes:

$$x = A\cos(f_1 t) + B\cos(f_2 t) + C\cos(f_3 t) \tag{5.3}$$

$$y = A\sin(f_1 t) + B\sin(f_2 t) + C\sin(f_3 t) \tag{5.4}$$

### 5.2.2 Examples of patterns

Figure 28 and figure 29 show examples of synthesised patterns of Turk’s Head rosettes.



$f_1 = -1$, $f_2 = 5$

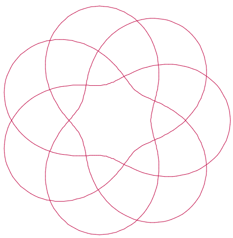
$A = 1$, $B = 1.8$, C = 0

$f_1 = -3$, $f_2 = 4$

$A = 1$, $B = 2.2$, C = 0

Figure 28 *Examples of synthesised patterns of Turk’s Head rosettes*.

A better pattern is obtained by adding one more term to the equations:



$f_1 = -3$, $f_2 = 4$, $f_3 = 11$

$A = 1.6$, $B = 5.4$, $C = 1.2$

Figure 29 *Example of synthesised Turk’s Head rosettes with three terms*.

### 5.2.3 Examples of woven mats

Figure 30 shows examples of woven Turk's Head rosettes.

$f_1 = -1$, $f_2 = 5$

$A = 1$, $B = 1.8$

$f_1 = -3$, $f_2 = 4$, $f_3 = 11$

$A = 1.6$, $B = 5.4$, $C = 1.2$

Figure 30 *Examples of woven Turk's Head rosettes (Photo Nils Kr. Rossing).*

## 5.3 Twisted rosettes

### 5.3.1 Equations

The following parametric equation can be used to synthesis the patterns of Twisted rosettes:

$$x = A\cos(f_1 t) + B\cos(f_2 t) + C\cos(f_3 t) \tag{5.5}$$

$$y = A\sin(f_1 t) + B\sin(f_2 t) + C\sin(f_3 t) \tag{5.6}$$

A fourth term may sometimes give better crossings (see figure 31 B):

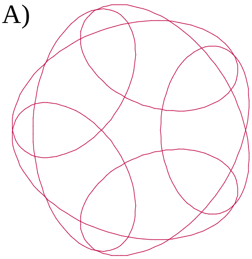
$$x = A\cos(f_1 t) + B\cos(f_2 t) + C\cos(f_3 t) + D\cos(f_4 t) \tag{5.7}$$

$$y = A\sin(f_1 t) + B\sin(f_2 t) + C\sin(f_3 t) + D\sin(f_4 t) \tag{5.8}$$

### 5.3.2 Examples of patterns

Figure 31 shows examples of synthesised patterns of Twisted rosettes.

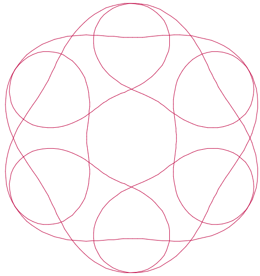
A)



$f_1 = -4, f_2 = 1, f_3 = 6$

$A = 1.0, B = 2.0, C = -2.2$

B)



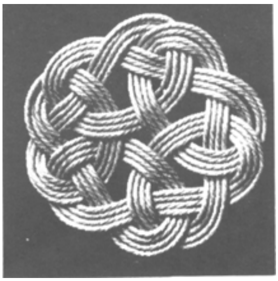
$f_1 = -5, f_2 = 1, f_3 = 7, f_4 = 19$

$A = 1.0, B = 2.6, C = -2.4$, D = 0,4

Figure 31 *Examples of synthesised patterns of Twisted rosettes*.

### 5.3.3 Examples of woven mats

Figure 32 shows examples of woven Twisted rosettes.



$f_1 = -4, f_2 = 1, f_3 = 6$

$A = 1.0, B = 2.0, C = -2.2$

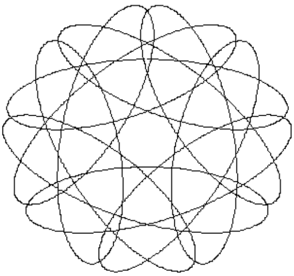
$f_1 = -5, f_2 = 1, f_3 = 7, f_4 = 19$

$A = 1.0, B = 2.6, C = -2.4, D = 0{,}4$

Figure 32 *Examples of woven Twisted rosettes (Photo Kaj Lund, Nils Kr. Rossing).*

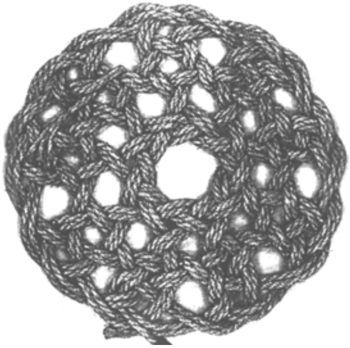
### 5.3.4 Advanced Twisted rosettes

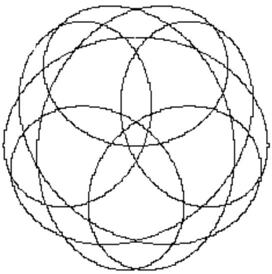
Figure 33 shows some advanced Twisted rosettes



$f_1 = -8, f_2 = 1, f_3 = 10$

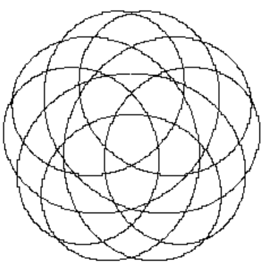
$A = 1.8, B = 2.0, C = -2.8$





$f_1 = -3, f_2 = 2, f_3 = 7$

$A = 1.0, B = 1.2, C = -2.5$



$f_1 = -2, f_2 = 3, f_3 = 8$

$A = 1.2, B = 0.6, C = -2.5$

Figure 33 *Examples of advanced Twisted rosettes (Photo Nils Kr. Rossing)*.

## 5.4 Rattan rosettes

### 5.4.1 Equations

The following parametric equation can be used to synthesis Rattan rosettes:

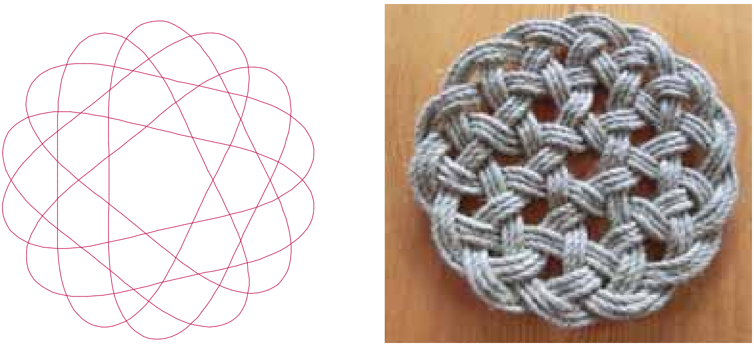
$$x = A\cos(f_1 t) - \mathbf{B\sin(f_2 t)} + C\cos(f_3 t) + D\cos(f_4 t) \qquad (5.9)$$

$$y = A\sin(f_1 t) + \mathbf{B\cos(f_2 t)} + C\sin(f_3 t) + D\sin(f_4 t) \qquad (5.10)$$

Be especially aware of the second term.

### 5.4.2 Examples of patterns and woven rosettes

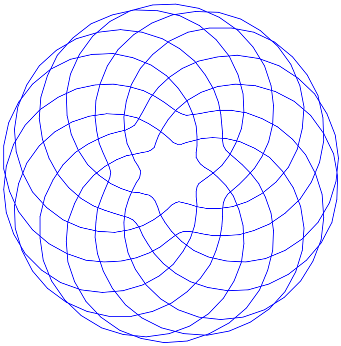
Figure 34 and figure 35 show examples of synthesised patterns and woven examples of Rattan rosettes.



$f_1 = -8$, $f_2 = -1$, $f_3 = 6$, $f_4 = 20$

$A = 1.6$, $B = 1.2$, $C = 5.4$, $D = 0{,}3$

Figure 35 *Examples of synthesised and woven patterns of Rattan rosette with 6 cross-over and 14 bights (Photo Nils Kr. Rossing).*

$f_1 = -4$, $f_2 = 3$, $f_3 = 10$, $f_4 = 24$

$A = 1.6$, $B = 0.9$, $C = 6.4$, $D = 2.1$

Figure 36 *Examples of synthesised and woven Rattan rosette with 8 crossovers and 14 bights (Photo Nils Kr. Rossing).*

## 5.5 Eye rosette

### 5.5.1 Equations

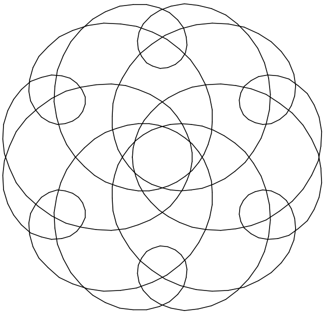
The following parametric equation can be used to synthesis the Eye rosette:

$$x = A\cos(f_1 t) + B\cos(f_2 t) + C\cos(f_3 t) + D\cos(f_4 t) + E\cos(f_5 t) \quad (5.11)$$

$$y = A\sin(f_1 t) + B\sin(f_2 t) + C\sin(f_3 t) + D\sin(f_4 t) + E\sin(f_5 t) \quad (5.12)$$

### 5.5.2 Example of Eye rosette, synthesised pattern and woven

Figure 37 shows examples of synthesised patterns and woven Eye rosettes.



$f_1 = -5, f_2 = 1, f_3 = 7, f_4 = 13, f_5 = 19$

$A = -0.8, B = 4.7, C = -3.6, D = -2.9, E = 0.9$

Figure 37 *Examples of synthesised pattern and woven Eye rosette with 7 crossover and 6 eye loops (Photo Nils Kr. Rossing).*

## 5.6 Double Eye rosette (Jens Kusk Jensen rosette)

### 5.6.1 Equations

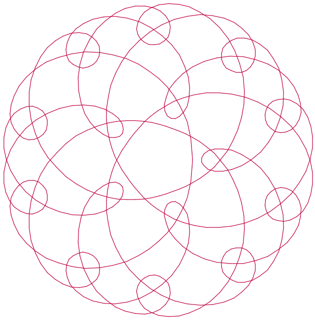
The following parametric equation can be used to synthesis the Double Eye rosette:

$$x = A\cos(f_1t) + B\cos(f_2t) + C\cos(f_3t) + D\cos(f_4t) + E\cos(f_5t) + F\cos(f_6t) + G\cos(f_7t) + H\cos(f_8t) \quad (5.13)$$

$$y = A\sin(f_1t) + B\sin(f_2t) + C\sin(f_3t) + D\sin(f_4t) + E\sin(f_5t) + F\sin(f_6t) + G\sin(f_7t) + H\sin(f_8t) \quad (5.14)$$

### 5.6.2 Examples of patterns of and woven double eye rosettes

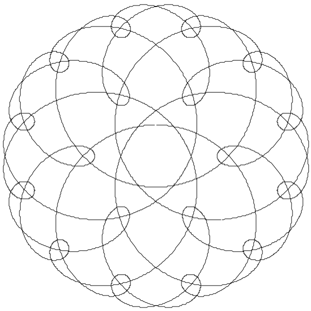
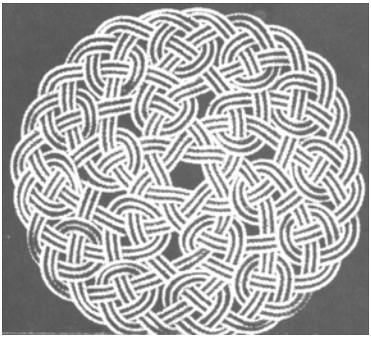
Figure 39 and figure 39 shows examples of synthesised patterns and woven Double Eye rosettes.

$f_1 = -9, f_2 = -4, f_3 = 1, f_4 = 6, f_5 = 11, f_6 = 16, f_7 = 21, f_8 = 26$

$A = 0.5, B = -0.8, C = -2.45, D = 2.4, E = 1.2, F = 0.5, G = 0.6, H = -0.8$

Figure 38 *Examples of synthesised and woven patterns of Double Eye rosette with 10 eye loops along the rim (Photo Nils Kr. Rossing).*

$f_1 = -11, f_2 = -5, f_3 = 1, f_4 = 7, f_5 = 13, f_6 = 19, f_7 = 25, f_8 = 31$

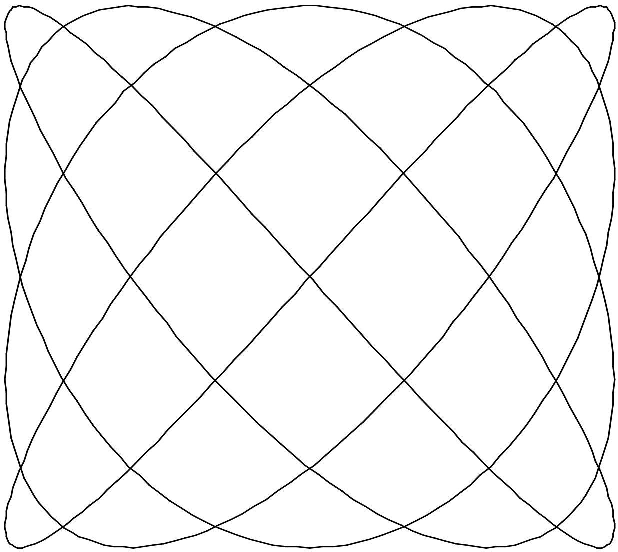
$A = 0.45, B = -0.65, C = -2.9, D = 2.4, E = 1.3, F = 0.45, G = 0.65, H = -0.6$

Figure 39 *Examples of synthesised and woven patterns of Double Eye rosette with 12 eye loops along the rim (Photo Kaj Lund).*

# 6 Templates

## 6.1 Rectangular mat

Rectangular mat



$x = 3\cos(5t)$

$y = 3\sin(4t)$

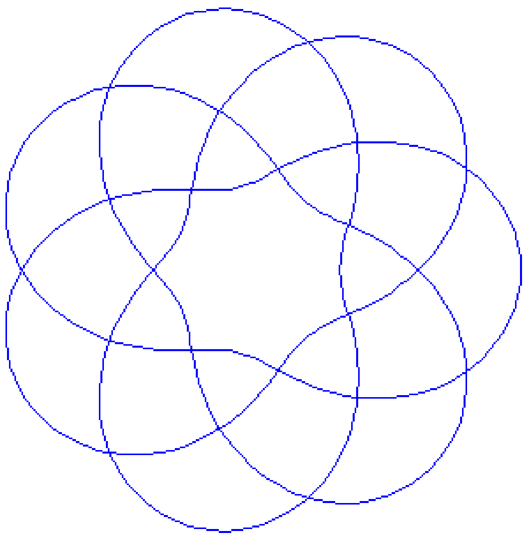
425 cm.

Ø 2 - 3 mm.

3 strands

## 6.2 Turk's Head rosette

# Turk’s head rosette



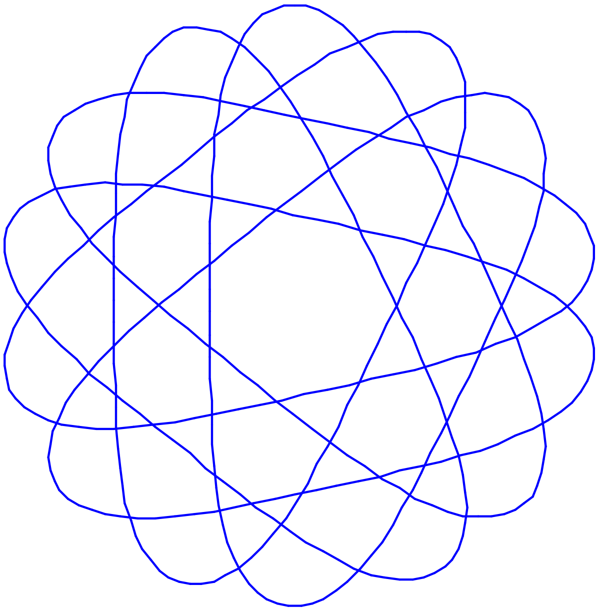
$$x = 1.6\cos(-3t) + 5.4\cos(+4t) + 1.2\cos(11t)$$
$$y = 1.6\sin(-3t) + 5.4\sin(+4t) + 1.2\sin(11t)$$

225 cm.
Ø 2 - 3 mm.
3 strands

## 6.3 Rattan rosette

# Rattan rosette

6 crossover an 14 bights



$$x = 1.8\cos(-8t) - 1.2\sin(-1t) + 5.4\cos(6t) + 0.2\cos(20t)$$

$$y = 1.8\sin(-8t) + 1.2\cos(-1t) + 5.4\sin(6t) + 0.2\cos(20t)$$

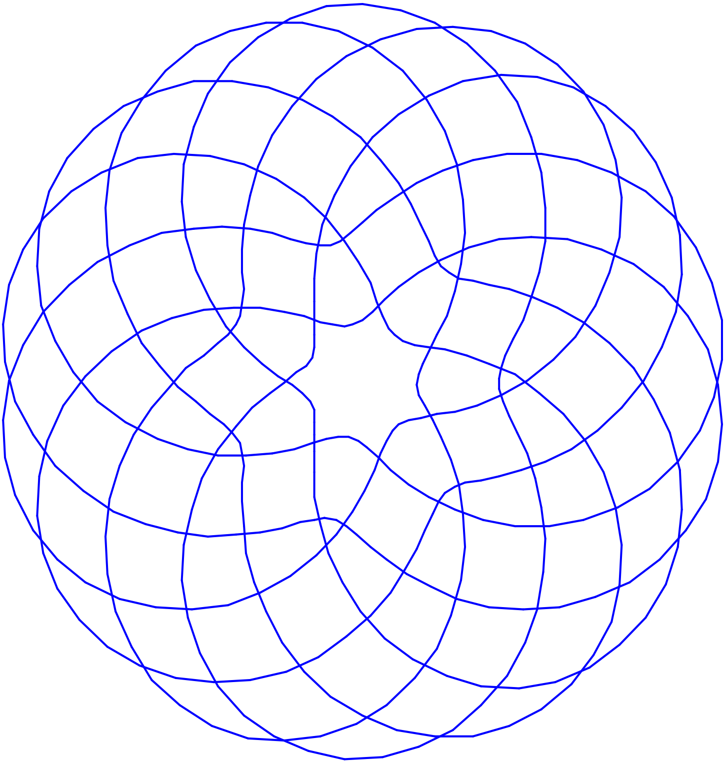
390 cm.

Ø 2 mm.

3 strands

# Rattan rosette

8 crossover and 14 bights



$$x = 1.6\cos(-4t) - 0.9\sin(3t) + 6.4\cos(10t) + 2.1\cos(24t)$$

$$y = 1.6\sin(-4t) + 0.9\cos(3t) + 6.4\sin(10t) + 2.1\cos(24t)$$

600 cm.

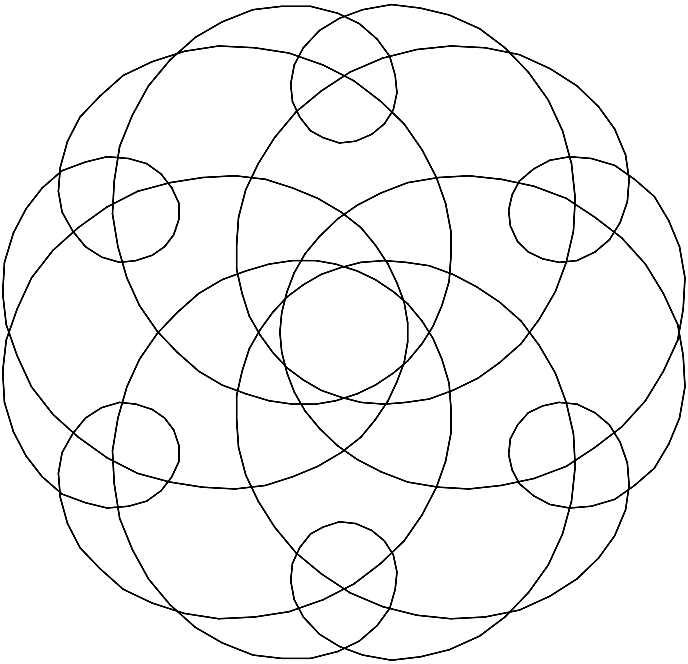
Ø 2 mm.

3 strands

## 6.4 Eye rosettes

# Eye rosette

7 crossover and 6 eye loops

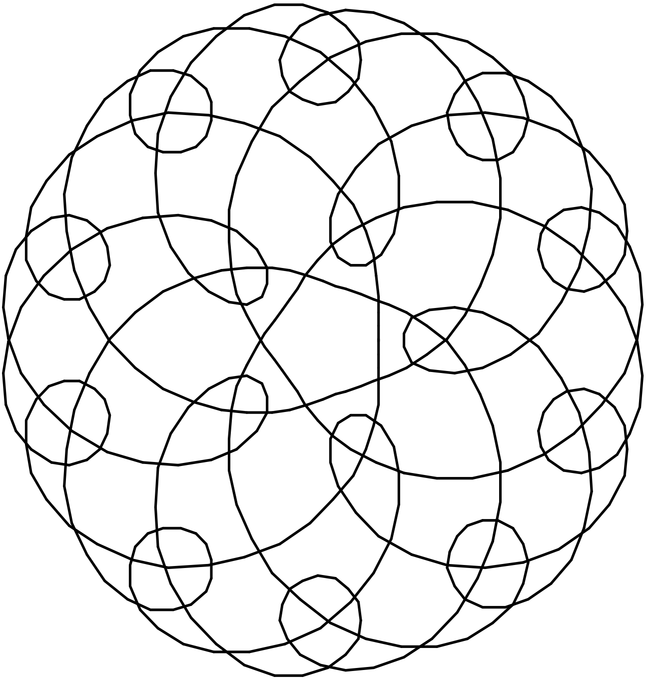


$$x = -0.8\cos(-5t) + 4.7\cos(1t) - 3.6\cos(7t) - 2.9\cos(13t) + 0.9\cos(19t)$$
$$y = -0.8\sin(-5t) + 4.7\sin(1t) - 3.6\sin(7t) - 2.9\sin(13t) + 0.9\sin(19t)$$

480 cm.
Ø 2 mm.
3 strands

# Double eye rosette

6 crossover and 10 eye loops



360 cm.

Ø 2 mm.

2 strands

# 7 Literature

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